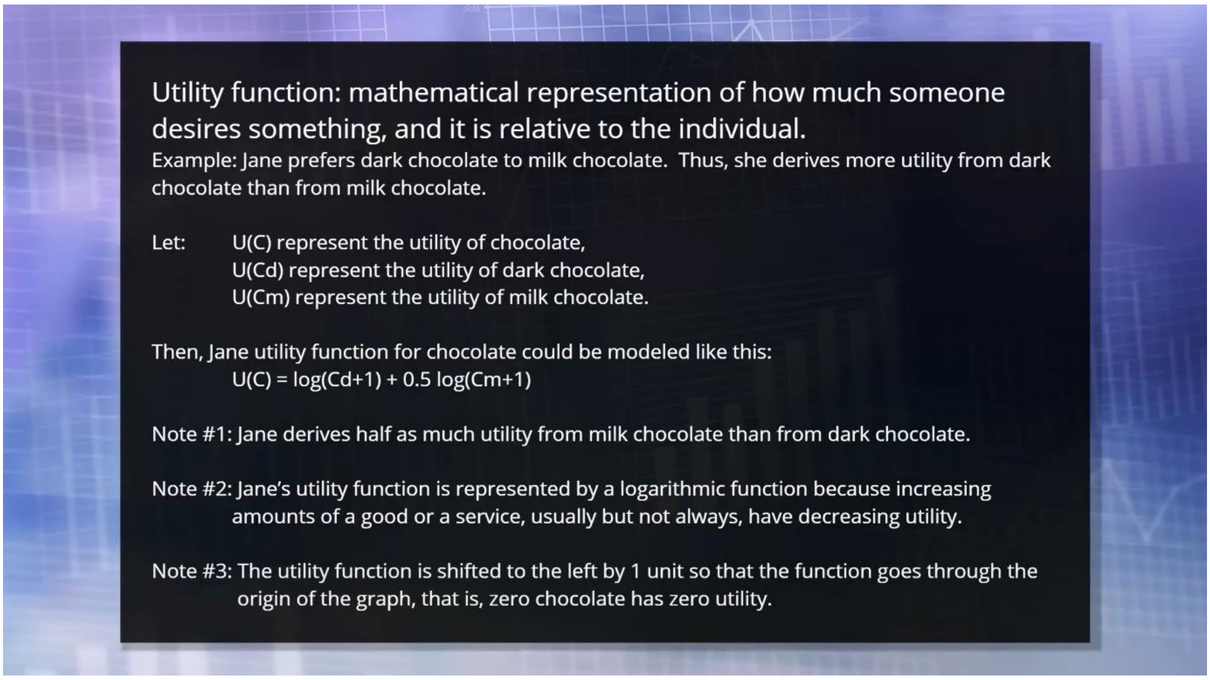
M4L17. Utility Function

Slide #1

In this topic, we will discuss how to use the utility function to model decisions under uncertainty.

Slide #2



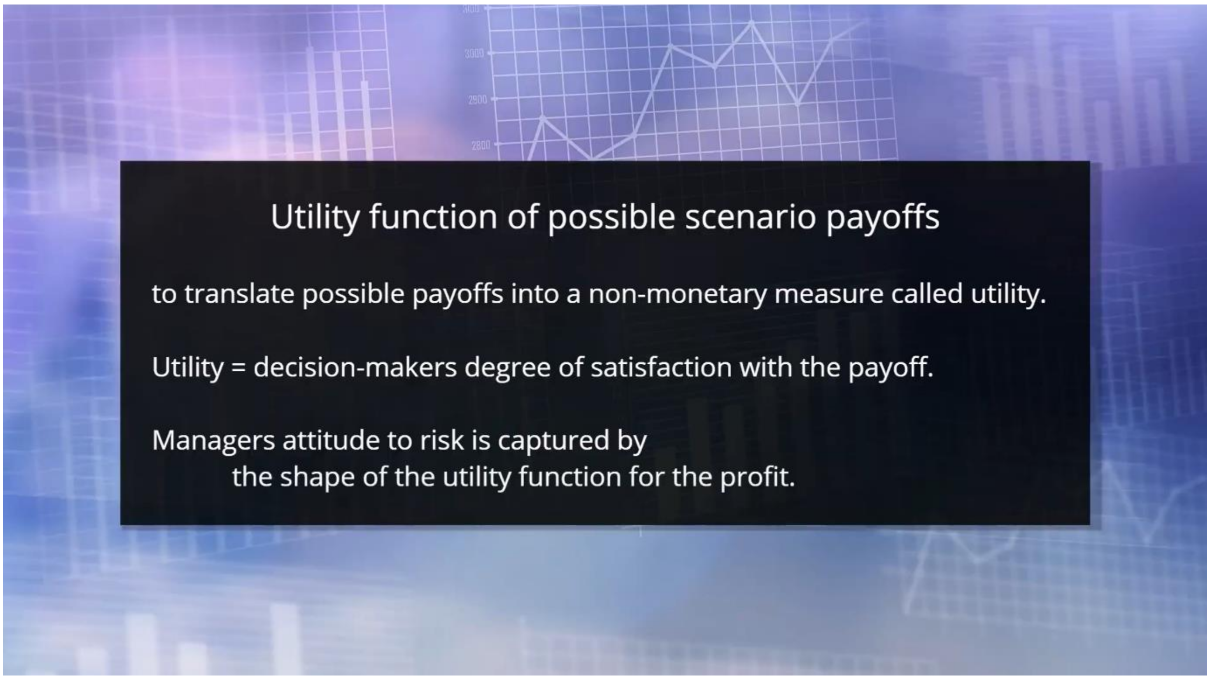
The utility function is a mathematical representation of how much someone desires something, and it is relative to the individual. For example, Jane prefers dark chocolate to milk chocolate. Thus, she derives more utility from dark chocolate than from milk chocolate. Let U of C represent the utility of chocolate, U of CD represent the utility of dark chocolate, and U of CM represent the utility of milk chocolate. Then, Jane's utility function for chocolate could be modeled like this: U of C equals the log of CD plus 1 plus 0. 5 times the log of CM plus 1.

Note number one, Jane derives half as much utility from milk chocolate than from dark chocolate.

Note number two, Jane's utility function is represented by a logarithmic function because increasing amounts of a good or a service, usually but not always, have decreasing utility.

Note number three, the utility function is shifted to the left by one unit so that the function goes through the origin of the graph. That is, zero chocolate has zero utility.

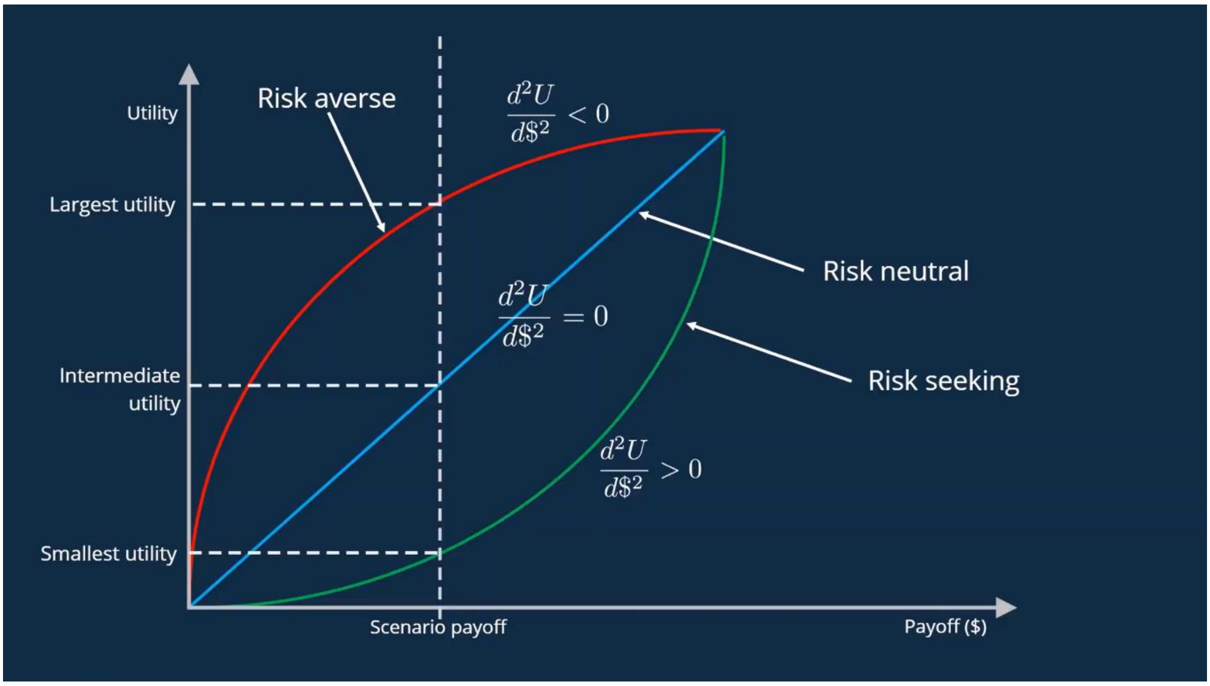
Slide #3



We can use a utility function u of x to translate the possible payoffs into a non monetary measure called utility, which can also be viewed as a decision maker's degree of satisfaction with the payoff.

The manager's attitude to risk is captured by the shape of the utility function for profit.

Slide #4



A risk averse decision maker assigns the largest relative utility to any payoff, but experiences diminishing marginal utility for profit.

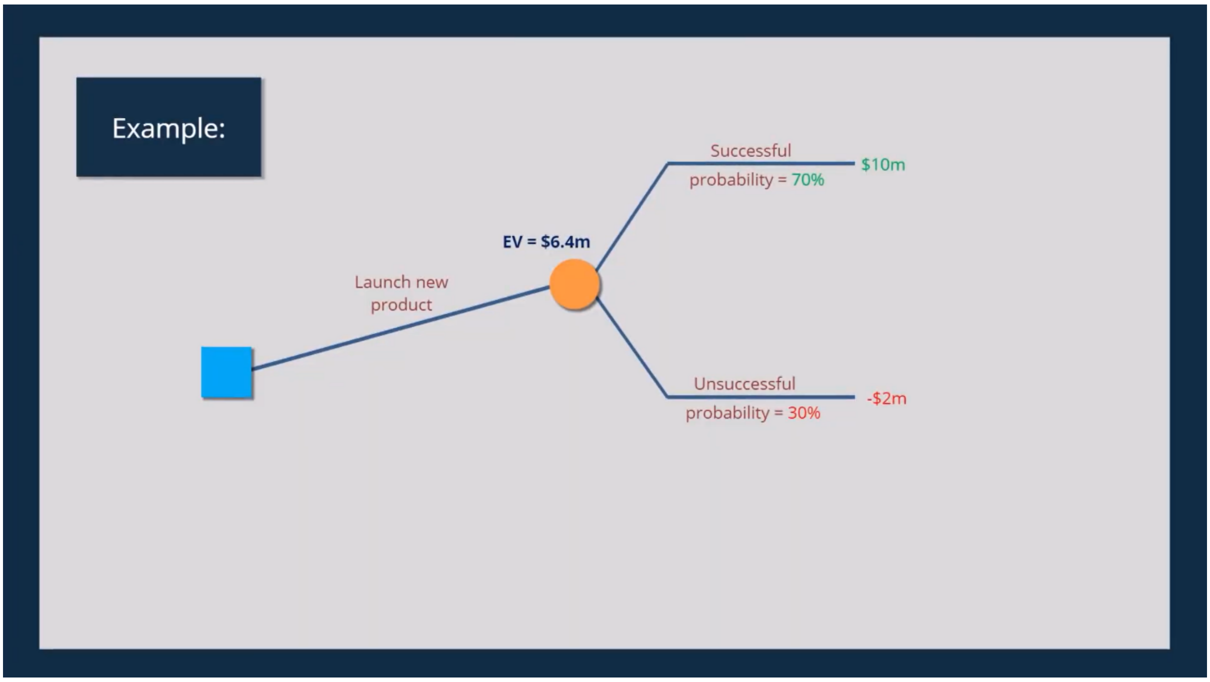
That is, every additional dollar in payoff result in smaller increases in utility.

The risk seeking decision maker assigns the smallest utility to any payoff, but has an increased marginal utility for increased payoff.

That is, every additional dollar in payoff results in larger increases in utility.

A risk neutral decision maker who follows the expected value decision rule falls in between these two extremes and has a consistent marginal utility for increased payoffs.

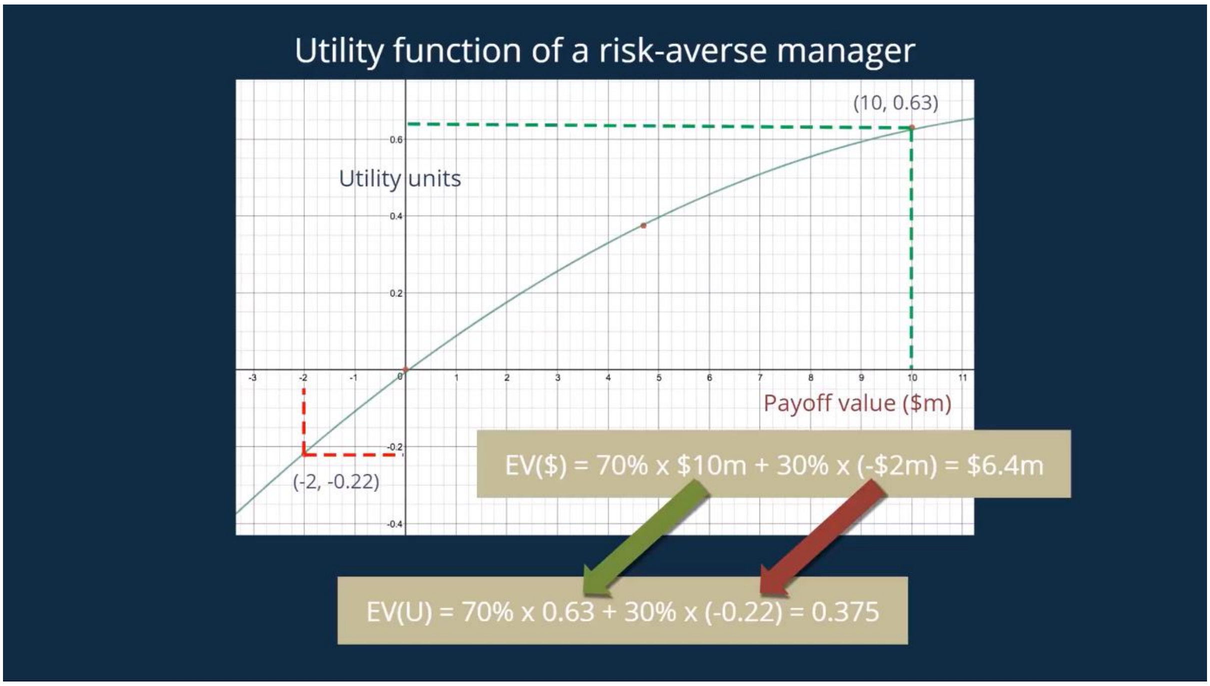
Slide #5



Recall the previous example.

About the launch of a new product, the probability of success is 70% with a payout of 10 million dollars and the probability of failure is 30% with a negative payout, that is, a loss of 2 million dollars.

Slide #6



This utility function models risk tolerance of a risk averse manager.

The utility of 10 million dollar profit is 0.63 utility for this risk averse manager, while the utility of 2 million dollar loss is negative 0.22.

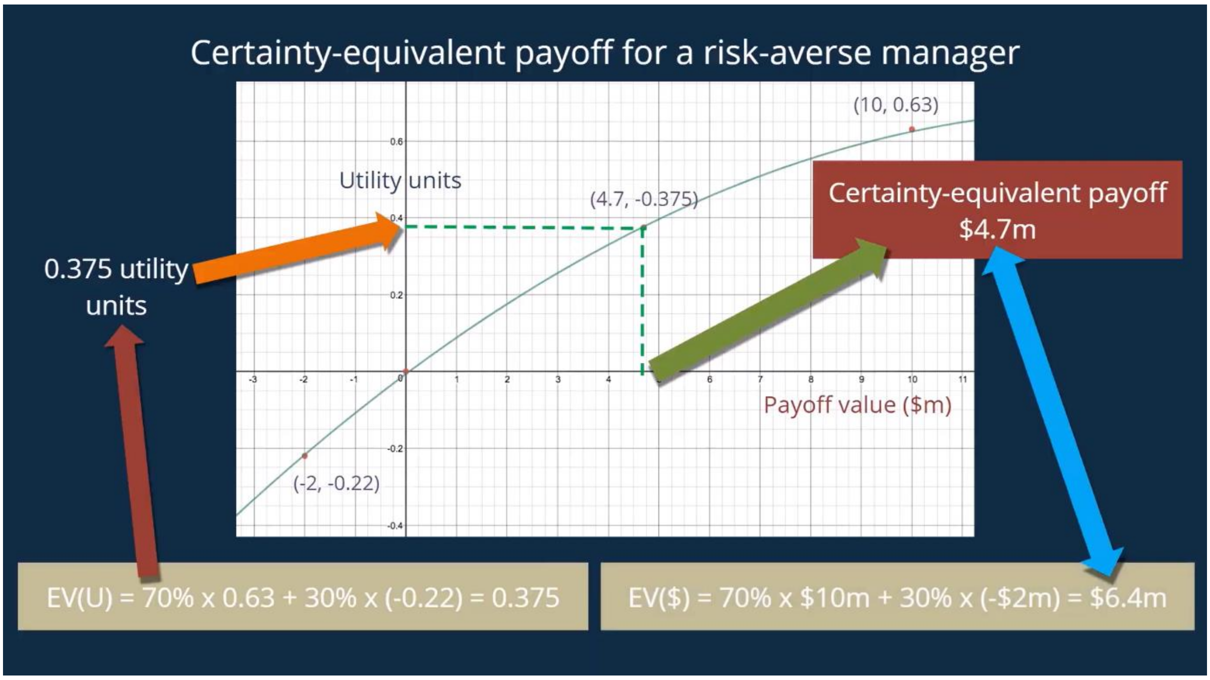
We can replace the actual payoff with utility number from each outcome of the decision tree, and then we can calculate the expected utility the same way as we calculate the expected value, but using the utility number rather than the monetary payoff.

The expected utility of this scenario is 70% times 0.63 plus 30% times negative 0.22.

So the expected utility is 0.375.

Equation of curve, negative 0.0035x squared plus 0.0981x minus 0.0063.

Slide #7

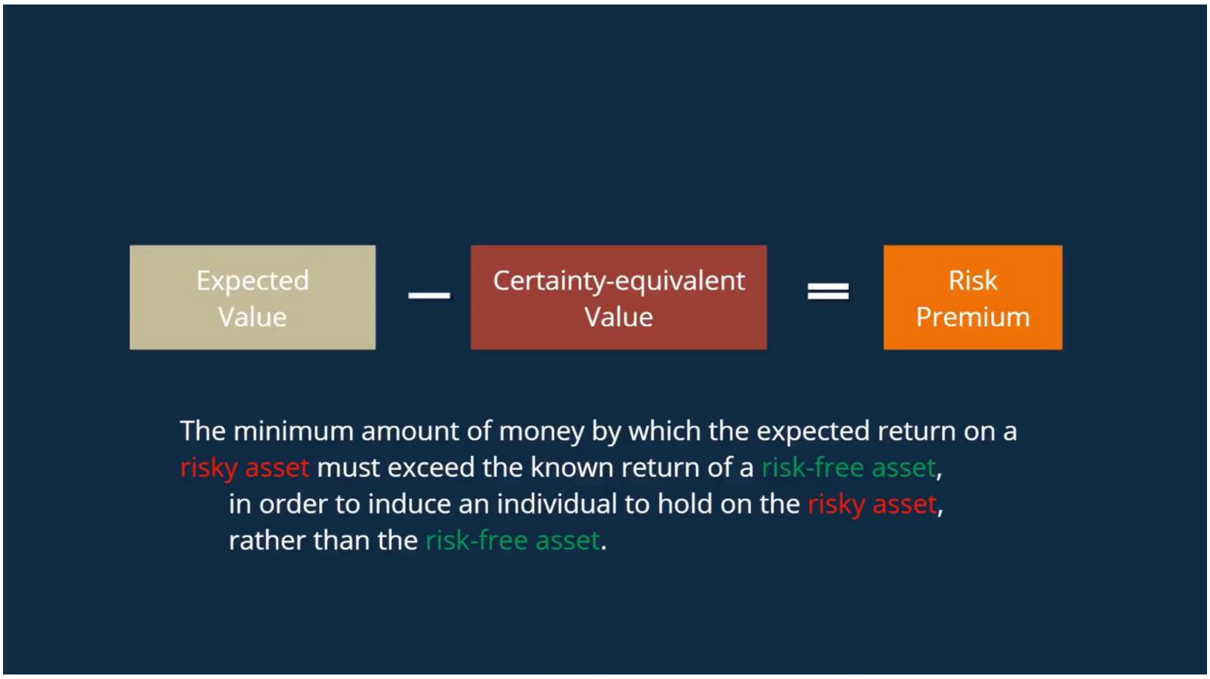


Then we can use the utility function again to translate the utility number back into management payoffs or certainty equivalent payoffs.

The certainty equivalent payoffs of a product launch with uncertainty is 4.7 million dollars.

The risk averse decision maker views the value of the uncertain product launch as much as a certainty equivalent value of 4.7 million dollars. So this is lower than the expected value that we discussed before.

Slide #8



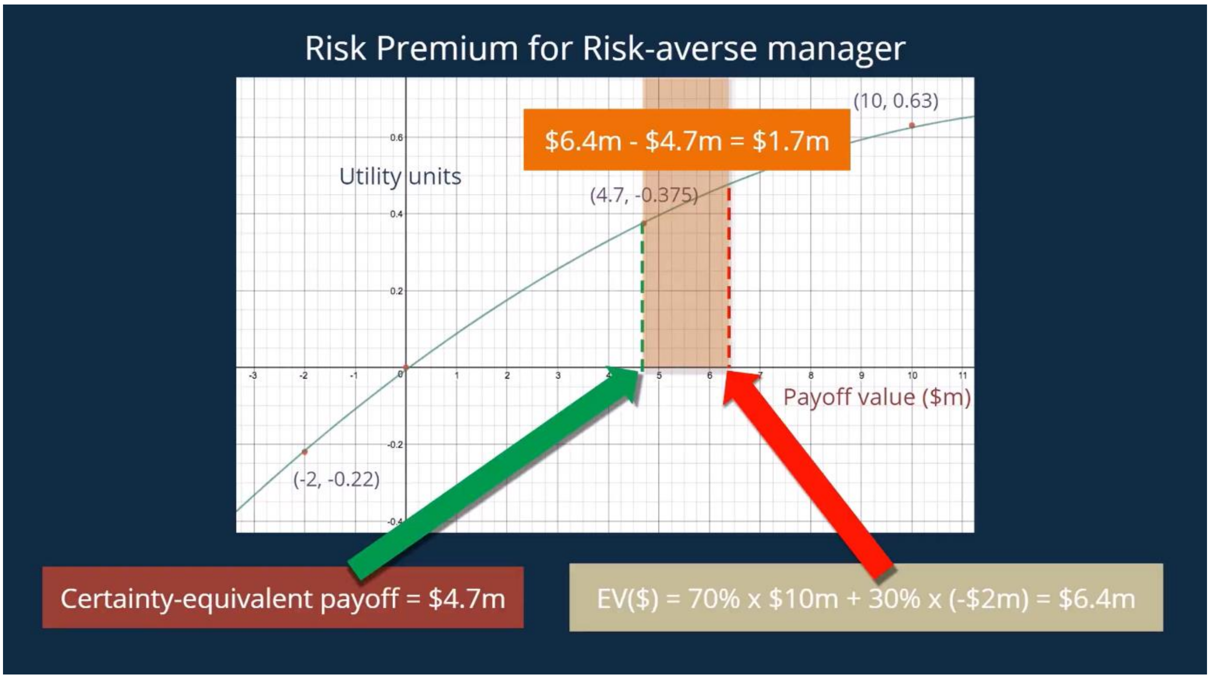
What is the difference between the certainty equivalent value and the expected value?

Actually, the difference is called risk premium.

The risk premium is the minimum amount of money by which the expected return on a risky asset must exceed the known return on a risk free asset in order to induce an individual to hold on the risky asset rather than the risk free asset.

This is a long way to say the risk premium is the difference between the certainty equivalent value and the expected value of a decision.

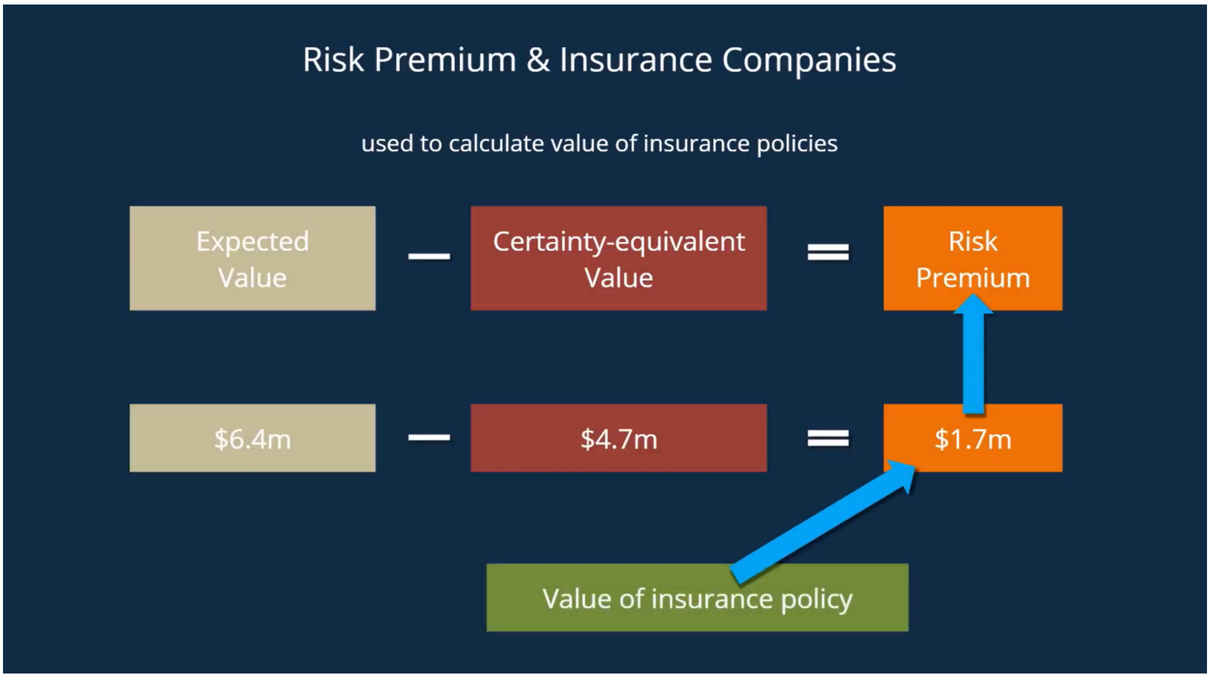
Slide #9



From the decision-making perspective, the difference can be calculated by the expected value of an uncertain decision minus the certainty equivalent method.

In this case it is 6.4 million dollars minus 4.7 million dollars and the risk premium is 1.7 million dollars.

Slide #10



Insurance companies use the risk of premium concept a lot.

They use the concept to calculate the value of their insurance policies.

For example, in this case, the value of the insurance policy is equal to 1.7 million dollars, which is the risk premium of the decision.