M7L2a. Linear Optimization

Slide #1

In this topic, we will discuss linear optimization.

Slide #2

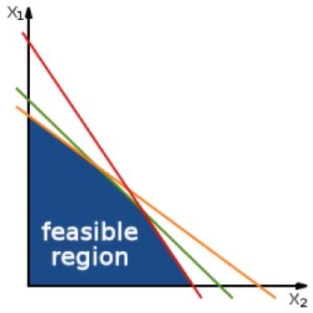
Linear optimization

linear optimization is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships." - Wikipedia

Decision variables: $X_1, X_2, \ldots X_n$

Objective: Maximize (or Minimize) $f(X_1, X_2, \ldots X_n)$
- Linear function: f(X)

Constraints:
$f_1(X_1, X_2, \ldots X_n) \leq b_1$
$f_2(X_1, X_2, \ldots X_n) \geq b_2$
$f_m(X_1, X_2, \ldots X_n) = b_m$
$X_n \geq 0$



Business optimization relies on economic models to describe the numeric relationship of your decisions, resources, and objectives.

We also call economic models optimization models.

A family of popular optimization is linear optimization, which is a method to achieve the best outcome in a mathematical model whose elements are represented by linear relationships.

A typical linear optimization model is as follows.

The decisions in an optimization model are often represented as variables such as x1 and x2 through xn.

The objective function, f, is a linear equation and describes the relationship between the decision variables and business objectives.

The constraints functions, f1, f2, and f3, are also linear equations.

In this case, the constraint is some function of the decision variables that must be less than or equal to, greater than or equal to, or equal to some specific value represented by the letter b in those equations.

In a business world, a decision variable is typically equal to or greater than zero.

You will notice this constraint in each business optimization case throughout the rest of this module, as illustrated in the two variable charts.

These linear constraint equations are the colored lines and they define the boundaries of your decision variables.

So sometimes we also call the constraint functions boundary equations.

Slide #3

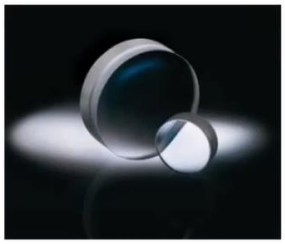
Business optimization example

Company XYZ makes laser optics units. The company produces two recent models LOU2.0A and LOU2.0B in its Houston plant.

The plant has a production capacity of making 23000 LOU units per year.

There are 220 skillful employees working at the plant to make both products.

The marketing & sale unit projects the market demands on both units are:



Sales forecast	LOU Model 2.0A	LOU Model 2.0B	
maximum	15000	14000	units
min	10000	9000	units

Let us use a laser optics unit business case to illustrate the business optimization.

Company XY Z makes laser optics units.

The company produced two recent models, LOU2.0A and LOU2.0B in its Houston plant.

The plant has a production capacity of making 23,000 laser optics units, LOU, Per year.

There are 220 skillful employees working at the plant to make both products.

The marketing and sales department projected the marketed demand on both units as follows.

For LOU Model 2.0A, the minimum demand is 10,000, and the maximum demand is 15,000 units.

For LOU Model 2.0B, the minimum demand is 9,000 units and the maximum is 14,000 units. The actual sales should fall between the minimum and the maximum.

Slide #4

Business optimization example

The cost structure and sale price of each unit are:

Cost	LOU Model 2.0A	LOU Model 2.0B	
Materials cost per unit	$1,000	$900	dollars
Direct labor man hour per unit	9.7	11.0	hrs
Direct labor cost per man hour	$60	$60	dollars
Direct labor cost per unit	$582	$660	dollars
Labor overhead cost per unit	$204	$231	dollars
Management and sales cost fixed	$150,000	$150,000	dollars
Sale price per unit	$4,300	$4,500	dollars

Business decision:
How many units of Model2.0A and 2.0B will be made in order to maximize the profit of the Houston plant?

The cost structure of the laser unit includes material cost, direct labor cost, labor overhead cost, and fixed cost for management and sales overhead.

The detailed cost numbers are in the summary table.

For LOU model 2.0A, the material cost per unit It's $1,000 and it will take 9.7 hours of a skillful technician to assemble the unit.

The direct labor cost per hour is $60, so totally it will cost $582 to assemble a unit.

Also, there is overhead cost for technicians.

The overhead includes vacation time, compensation, health care insurance, office supplies, and equipment.

The labor overhead cost is approximately 35% of the direct labor cost.

In my experience, junior analysts tend to miss the labor's overhead cost.

So please make sure you include your labor's overhead cost in your cost estimation.

The other line of fixed cost is management and sale cost.

That is the money you pay for the sales department to market your product and for the management team for their administration service.

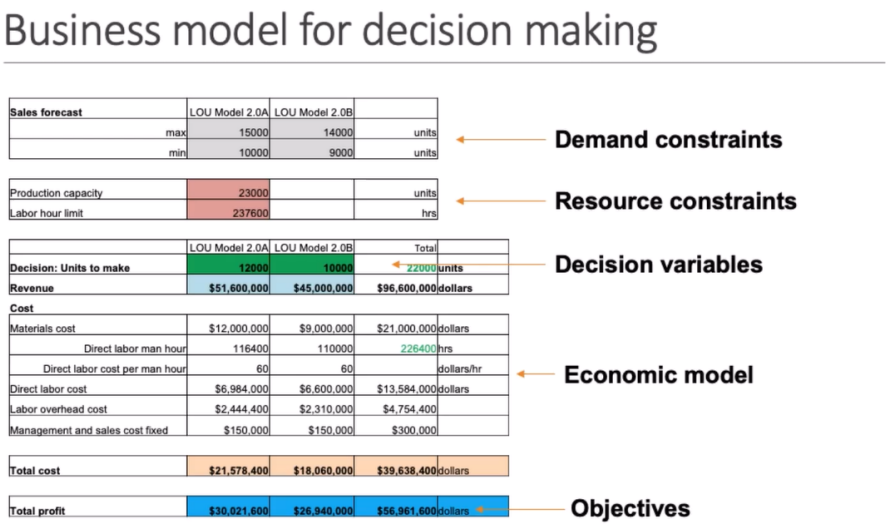
In this case, the fixed cost is 150,000.

That is not per unit.

For LOU model 2.0b, those cost items are in the second column.

The sale price of model 2.0b is slightly higher than that of model 2.0a.

Slide #5



This is the business model of this case. The sale forecast numbers are the demand constraints. The quantities of your two-laser optics unit should fall between the minimum and maximum forecast numbers.

The second table is about resource constraints, which include the production capacity limit and the human hour limit.

The total production volume of two laser optics units must be less than the production capacity of your Houston plant.

Since the plant hires limited number of employees, the corresponding available labor hours also set a limit on how many units can be produced in a year.

The decisions to make are the numbers of model A and B units to make in a year respectively.

The revenue is estimated based on the numbers of units and the sale price.

The economic model of this simple example is basically the material cost, labor cost, and management cost estimation. The bottom line is the business objective, which is to maximize the total profit of production.