Logic-Based Learning: Tutorial 1

Deductive and Abductive Inference

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The aim of this tutorial is to enable you to practice with logic-based concepts introduced in Unit 2, and with the two key inference methods of deduction, through resolution and SLD resolution, and abduction:

Question 1

Using the methods described in slides 8 and 10, convert the following first-order sentences into clausal representation:

i) $\forall X(literate(X) \rightarrow reads(X) \vee write(X))$

ii) $\forall X(clear(X) \rightarrow (block(X) \wedge \neg\exists Y\, on(Y, X)))$

iii) $\exists Y(g(Y) \wedge \forall Z(r(Z) \rightarrow f(Y, Z)))$

Question 2

Consider the following first-order logic sentence S whose language $\mathcal{L}$ includes only the constants b, c, l:

$$S = \forall X, \forall Y(\neg p(b, Y) \wedge p(c, l) \wedge (p(b, X) \vee \neg p(X, l)))$$

1. Rewrite the sentence S into a set C of clauses.
2. What is the Herbrand domain of C?
3. Write in full $ground(C)$ (i.e. the grounding of C).
4. Using the Theorem given in slide 14 state whether C is satisfiable or not and explain why.

Question 3

Consider the following set S of first-order formulae:

$$S = \{ \quad daughter(rebecca) \\ \forall X(daughter(X) \rightarrow (\exists Y(mum(Y,X)) \wedge \exists Z(dad(Z,X))) \quad \}$$

1. Rewrite S into a set C of clauses.
2. Assume an Herbrand domain $D = \{rebecca\}$, construct an Herbrand model for the set C of clauses. Remember, an Herbrand model of a set C of clauses is an Herbrand interpretation that satisfies all the clauses in C.
3. Repeat point 2 above but with Herbrand domain $D = \{rebecca, natascia\}$.
4. State what the minimal Herbrand model is in points 2 and 3 and explain your answer.

Question 4

Consider the following set S of sentences. Find the minimal Herbrand model of S, assuming in the second case an Herbrand domain $\{a1, b1\}$:

1. $S = \{a \leftarrow \neg b \wedge c, d \leftarrow \neg c, \neg d\}$
2. $S = \{\forall X, \forall Y(q(X) \leftarrow \neg q(Y))\}$

Question 5

For the following definite programs P calculate the Least Herbrand Model $M(P)$:

$$\text{i)}\ P = \begin{cases} t \leftarrow q, r. \\ p \leftarrow q, r. \\ p \leftarrow s. \\ s \leftarrow q. \\ q. \end{cases}$$

$$\text{ii)}\ P = \begin{cases} bird(penguin). \\ bird(X) \leftarrow flies(X), animal(X). \\ flies(plane). \\ flies(hawk). \\ flies(sparrow). \\ animal(hawk). \\ animal(sparrow). \end{cases}$$

Question 6

If possible unify the following pairs and give the substitution ϕ, otherwise explain why they do not unify:

1) $p(X)$ and $r(Y)$
2) $p(X,Y)$ and $p(a,Z)$
3) $p(X,X)$ and $p(a,b)$
4) $p(f(X))$ and $p(g(Y))$
5) $r(f(X),Y,g(Y))$ and $r(f(X),Z,g(X))$
6) $ancestor(X,Y)$ and $ancestor(bill, father(bill))$

Question 7

Consider the following set of a single first-order sentence:

$$S = \{\forall X, \forall Y(subset(X, Y) \leftrightarrow (\forall U(member(U, Y) \leftarrow member(U, X)))\}$$

1. Express in English what the sentence in S represents.
2. Consider a constant e to denote the empty set and define a clause $C4$ that expresses the notion of empty set using the signature of S. Note that the signature of a set of clauses is the set of predicate, functions and constants that appear in the clauses.
3. Use resolution to show that $S \cup \{C4\} \models \forall X(subset(e, X)$

Question 8

Consider the following set C of definite clauses. Draw the SLD-tree of the goal $\leftarrow p(X)$ using the top to bottom left-to-right ordering and state what the computed answer substitutions are.

$p(X) \leftarrow q(X, Y), r(Y)$
$p(X) \leftarrow q(X, X)$
$q(X, X) \leftarrow s(X)$
$r(b)$
$s(a)$
$s(b)$

Question 9

Consider the following set C of normal clauses. Draw the SLDNF derivation of the goal $\leftarrow subset(s1, s2)$ using the top to bottom left-to-right ordering.

$subset(X, Y) \leftarrow not\ \ exception(X, Y)$
$exception(X, Y) \leftarrow member(U, X), not\ \ member(U, Y)$
$member(a, s1)$
$member(a, s2)$
$member(b, s2)$

Question 10

Consider the following abductive model $\langle KB, Ab, IC \rangle$, where KB, Ab and IC are defined as follows:

$$KB = \begin{bmatrix} carDoesNotStart(X) \leftarrow batteryFlat(X) \\ carDoesNotStart(X) \leftarrow hasNoFuel(X) \\ lightsGoOn(mycar) \\ fuelIndicatorEmpty(mycar) \end{bmatrix} \quad Ab = \begin{bmatrix} batteryFlat(mycar) \\ batteryFlat(yourcar) \\ hasNoFuel(mycar) \\ hasNoFuel(yourcar) \\ brokenIndicator(mycar) \\ brokenIndicator(yourcar) \end{bmatrix}$$

$$IC = \begin{bmatrix} \leftarrow batteryFlat(X), \\ \quad lightsGoOn(X) \\ \leftarrow hasNoFuel(X), \\ \quad not\ \ fuelIndicatorEmpty(X), \\ \quad not\ \ brokenIndicator(X) \end{bmatrix}$$

State all the possible abductive solutions (if any) for each of the following observations and draw the abductive proof derivation for each of them.

1. $O = carDoesNotStart(mycar)$
2. $O = carDoesNotStart(yourcar)$

Question 11

Consider the following abductive model $\langle KB, Ab, IC\rangle$, where KB, Ab and IC are defined as follows. Remember that Ab includes the complement of the atoms defined below as well as the complement of ground instances of defined predicates.

$$KB = \begin{bmatrix} headache(X) \leftarrow jaundice(X) \\ headache(X) \leftarrow migraine(X) \\ sickness(X) \leftarrow stomachBug(X) \end{bmatrix} \qquad Ab = \begin{bmatrix} jaundice(bob) \\ migraine(bob) \\ yellowEyes(bob) \end{bmatrix}$$

$$IC = \begin{bmatrix} \leftarrow migraine(X), jaundice(X) \\ \leftarrow jaundice(X), not\ \ yellowEyes(X) \\ \leftarrow jaundice(X), not\ \ sickness(X) \end{bmatrix}$$

Show whether there is an abductive solution for the observation $headache(bob)$ by drawing its abductive proof derivation.