Logic-Based Learning: Tutorial 2

Observation Predicate Learning

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The aim of this tutorial is to practice with Observation Predicate Learning (OPL) (Unit 3).

Question 1

Consider the following clauses C1 and C2. State if θ-subsumption relation holds between these two clauses and if so which clause subsumes the other:

i) $C1 : has\_daughter(X) \leftarrow parent(X, Y), female(Y)$
   $C2 : has\_daughter(tom) \leftarrow parent(tom, anne), female(anne)$

ii) $C1 : p(c) \leftarrow l1(c, d), l2(d, d)$
   $C2 : p(X_0) \leftarrow l1(X_0, X_1), l2(X_1, X_2), l2(X_1, X_3)$

iii) $C1 : even([X|L]) \leftarrow L = [Y|L1], even(L1)$
   $C2 : even([X1, Y1|L1]) \leftarrow even(L1)$

Consider now the two sets of clauses $H1 = \{C_0, C_1\}$ and $H2 = \{C_0, C_2\}$, where C_0, C_1, C_2 are defined below. Does H1 θ-subsume $H2$? Explain your answer.

iv) $C0 : num(0)$
   $C1 : num(s(X)) \leftarrow num(X)$
   $C2 : num(s(s(X)) \leftarrow num(X).$

Question 2

Consider the following two clauses $C1$ and $C2$:

$C1 \quad f(t, a) \leftarrow p(t, a), m(t), s(a)$
$C2 \quad f(j, b) \leftarrow p(j, b), m(j), m(b)$

i) Using the definition of least general generalisation (lgg) given in slide 18 of Unit 3, define the $lgg(C1, C2)$. Explain your answer.

ii) Show that your $lgg(C1, C2)$ is more general than $C1$ and more general than $C2$.

Question 3

Consider the following set E of examples, and background knowledge B:

$$B = \left[\begin{array}{ll} parent(tom, john) & male(john) \\ parent(tom, maggie) & male(mark) \\ father(ned, mark) & male(paul) \\ father(ned, paul) & sibling(X, Y) \leftarrow parent(Z, X), parent(Z, Y), X \neq Y. \end{array} \right]$$

$$E = \left[\begin{array}{l} brother(john, maggie) \\ brother(mark, paul) \end{array} \right]$$

State whether the following hypotheses $H1$ and $H2$ are inductive solutions under the definition of learning from entailment.

i) $H1 = \left[\; brother(X, Y) \leftarrow sibling(X, Y), male(X) \; \right]$

ii) $H2 = \left[\begin{array}{l} brother(X, Y) \leftarrow sibling(X, Y), male(X) \\ parent(X, Y) \leftarrow father(X, Y) \end{array} \right]$

Question 4

Consider the following sets E^+ and E^- of positive and negative examples respectively and hypothesis $H1$, $H2$ and $H3$. Which of these three hypotheses is a complete and consistent hypothesis? For those that are not, state whether they need to be generalised further or specialised further or both. Assume the domain to be the infinite set $\{0, s(0), s^2(0), s^3(0), \ldots\}$, where $s^n(X)$ denotes the term $s(s(...(s(X))))$ constructed by nesting the function s, n times. Explain your answer.

$$E^+ = \left\{ \begin{array}{l} p(s(0)), p(s^3(0)), \\ p(s^5(0)), p(s^7(0)) \end{array} \right\} \quad E^- = \left\{ \begin{array}{l} p(0), p(s^2(0)), \\ p(s^4(0)) \end{array} \right\}$$

$$H1 = \left[\begin{array}{l} p(s^2(X)) \leftarrow p(X) \\ p(s(0)) \end{array} \right] \quad H2 = \left[\; p(s^2(X)) \; \right] \quad H3 = \left[\; p(s(X)) \; \right]$$