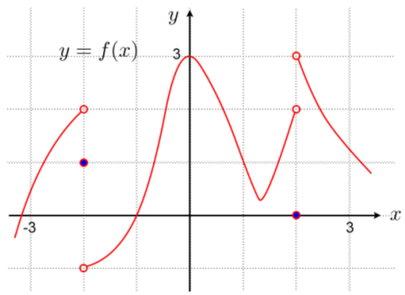
1) Find $\lim\limits_{x\to -2^-} \frac{|f(|x|)-2|}{f(x)+1}$ according to the given graph of $f(x)$ below.



2) Find the horizontal asymptote of the function

$$y = \left(cos\frac{1}{2x} - \frac{3}{x^2}\right)\left(1 + xsin\frac{1}{x}\right).$$

3) If $\frac{d}{dx}\left(f\left(\frac{x^3}{3}\right)\right) = 2x^5$, what is $'(x)$?

4) Find the Limit : $\lim\limits_{x\to 0}(e^x + 2x)^{\frac{1}{2x}}$.

5) Evaluate $$\lim_{x\to 0}\frac{\int_0^{x^2}(t+1)f(t)\,dt}{3x^2}.$$

Curve Sketching (6-10)

6) Sketch the graph of $y = f(x) = 2x^6 - 3x^4$.

7) Sketch the graph of $y = f(x) = \frac{x^2+x}{4-x^2}$.

8) Sketch the graph of $y = f(x) = \frac{cos\pi x}{1+x^2}$ on [-2, 2].

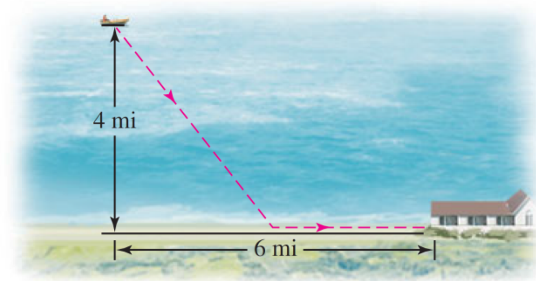
9) Sketch the graph of $y = f(x) = x^{\frac{2}{3}} + (x+2)^{\frac{1}{3}}$.

10) Sketch the graph of $y = f(x) = \frac{-x\sqrt{x^2-4}}{x-2}$.

Optimization Problems (11-22)

11) (4.4 #21)

Walking and rowing A boat on the ocean is 4 mi from the nearest point on a straight shoreline; that point is 6 mi from a restaurant on the shore. A woman plans to row the boat straight to a point on the shore and then walk along the shore to the restaurant.



a. If she walks at 3 mi/hr and rows at 2 mi/hr, at which point on the shore should she land to minimize the total travel time?

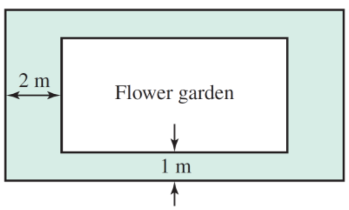
b. If she walks at 3 mi/hr, what is the minimum speed at which she must row so that the quickest way to the restaurant is to row directly (with no walking)?

12) (4.4.23)

Shortest ladder—more realistic An 8-ft-tall fence runs parallel to the wall of a house at a distance of 5 ft. Find the length of the shortest ladder that extends from the ground, over the fence, to the house. Assume that the vertical wall of the house is 20 ft high and the horizontal ground extends 20 ft from the fence.

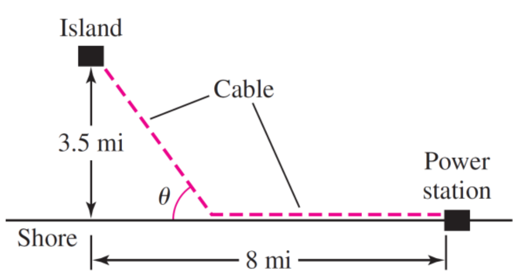
13) (4.4. 29)

Optimal garden A rectangular flower garden with an area of 30 m^2 is surrounded by a grass border 1 m wide on two sides and 2 m wide on the other two sides (see figure). What dimensions of the garden minimize the combined area of the garden and borders?



14) (4.4.37)

Laying cable An island is 3.5 mi from the nearest point on a straight shoreline; that point is 8 mi from a power station (see figure). A utility company plans to lay electrical cable underwater from the island to the shore and then underground along the shore to the power station. Assume that it costs \$2400/mi to lay underwater cable and \$1200/mi to lay underground cable. At what point should the underwater cable meet the shore in order to minimize the cost of the project?



15) (4.4.32)

Folded boxes

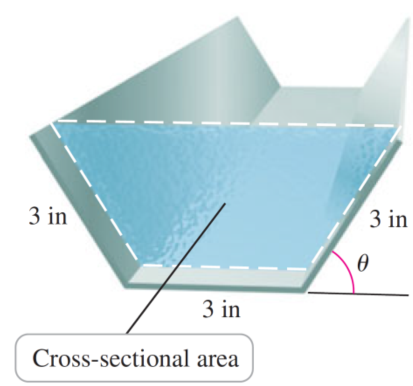
a. Squares with sides of length x are cut out of each corner of a rectangular piece of cardboard measuring 3 ft by 4 ft. The resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way.

b. Suppose that in part (a) the original piece of cardboard is a square with sides of length ℓ. Find the volume of the largest box that can be formed in this way.

c. Suppose that in part (a) the original piece of cardboard is a rectangle with sides of length ℓ and L. Holding ℓ fixed, find the size of the corner squares x that maximizes the volume of the box as $L \to \infty$. (*Source: Mathematics Teacher*, November 2002)

16) (4.4.44)

Metal rain gutters A rain gutter is made from sheets of metal 9 in wide. The gutters have a 3-in base and two 3-in sides, folded up at an angle θ (see figure). What angle θ maximizes the cross-sectional area of the gutter?

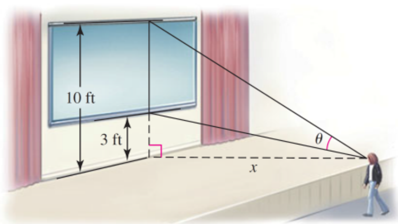


17) (4.4.46)

Cylinder and cones (Putnam Exam 1938) Right circular cones of height h and radius r are attached to each end of a right circular cylinder of height h and radius r, forming a double-pointed object. For a given surface area A, what are the dimensions r and h that maximize the volume of the object?

18) (4.4.47)

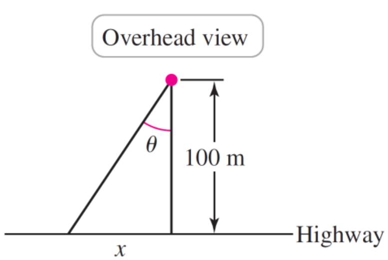
Viewing angles An auditorium with a flat floor has a large screen on one wall. The lower edge of the screen is 3 ft above eye level and the upper edge of the screen is 10 ft above eye level (see figure). How far from the screen should you stand to maximize your viewing angle?



19) (4.4.48)

Searchlight problem—narrow beam A searchlight is 100 m from the nearest point on a straight highway (see figure). As it rotates, the searchlight casts a horizontal beam that intersects the highway in a point. If the light revolves at a rate of $\pi/6$ rad/s,

find the rate at which the beam sweeps along the highway as a function of θ. For what value of θ is this rate maximized?



20) (4.4.53)

Cylinder in a cone A right circular cylinder is placed inside a cone of radius R and height H so that the base of the cylinder lies on the base of the cone.

a. Find the dimensions of the cylinder with maximum volume. Specifically, show that the volume of the maximum-volume cylinder is $\frac{4}{9}$ the volume of the cone.

b. Find the dimensions of the cylinder with maximum lateral surface area (area of the curved surface).

21) (4.4.57)

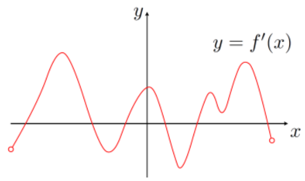
Minimum-length roads A house is located at each corner of a square with side lengths of 1 mi. What is the length of the shortest road system with straight roads that connects all of the houses by roads (that is, a road system that allows one to drive from any house to any other house)? (*Hint:* Place two points inside the square at which roads meet.) (*Source:* Halmos, *Problems for Mathematicians Young and Old*.)

22) (4.4.59)

Slowest shortcut Suppose you are standing in a field near a straight section of railroad tracks just as the locomotive of a train passes the point nearest to you, which is $\frac{1}{4}$ mi away. The train, with length $\frac{1}{3}$ mi, is traveling at 20 mi/hr. If you start running in a straight line across the field, how slowly can you run and still catch the train? In which direction should you run?

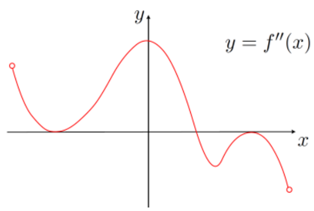
23)

Given the graph of the derivative function f' as shown below, exactly how many local maxima does f have?



24)

Given the graph of the second derivative function f'' as shown below, exactly how many inflection points does f have?



25) **When applying the linear approximation (tangent line approximation) at $x = 0$ to approximate $f(0.02)$, where $f(x) = \sqrt{1+x} + sinx$, what is the resulting approximate value?**

26) **Solve the inequalities :**

(a) $(x+1)(x+2)(x-3)(x+4)(x-5) \leq 0$

(b) $(x+1)^2(x+2)(x+3)^3 < 0$

(c) $\frac{(x+1)^2(x-2)(x+3)^3}{(x-5)^3} > 0$

(d) $\frac{(x+1)^{20}(x+2)^{2015}(x-3)^{\frac{300000}{4}}}{(x-5)^{27}(x-30)} \geq 0.$

(e) $\frac{(x+1)^{\frac{1}{3}}(x+2)^{\frac{1}{2}}(x+3)^{\frac{3}{4}}}{(x-5)^{27}(x-2)} \geq 0.$