Solutions to Math 1013 Tutorial 6

1(a) $\lim_{x\to 0}\left(\frac{\tan 2x}{\sin x}\right)$

$= \lim_{x\to 0}\left(\frac{\sin 2x}{\cos 2x}\right)\left(\frac{1}{\sin x}\right)$

$= \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)\left(\frac{1}{\cos 2x}\right)\left(\frac{2x}{\sin x}\right)$

$= \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)\lim_{x\to 0}\left(\frac{1}{\cos 2x}\right)\lim_{x\to 0} 2\left(\frac{x}{\sin x}\right)$

$= (1)(1)(2)$

$= 2$ #

(b) $\lim_{x\to 0}\left(\frac{\tan nx}{\sin x}\right) = \lim_{x\to 0}\left(\frac{\sin nx}{\cos nx}\right)\left(\frac{1}{\sin x}\right)$

$= \lim_{x\to 0}\left(\frac{1}{\cos nx}\right)\left(\frac{\sin nx}{nx}\right)\left(\frac{nx}{\sin x}\right)$

$= \lim_{x\to 0}\left(\frac{1}{\cos nx}\right)\lim_{x\to 0}\left(\frac{\sin nx}{nx}\right)\lim_{x\to 0}(n)\left(\frac{x}{\sin x}\right)$

$= (1)(1)(n) = n$ #

(c) $\lim_{x\to 0}\left(\frac{\sin 4x}{x}\right) = \lim_{x\to 0}\left(\frac{\sin 4x}{4x}\right)(4)$

$= \lim_{4x\to 0}\left(\frac{\sin 4x}{4x}\right)(4) = 1\times 4$

$= 4$ #

(d) $\lim_{x\to 0}\left(\frac{\sin px}{\sin qx}\right) = \lim_{x\to 0}\left(\frac{\sin px}{px}\right)\left(\frac{px}{qx}\right)\left(\frac{qx}{\sin qx}\right)$

$= \lim_{x\to 0}\left(\frac{p}{q}\right)\left(\frac{\sin px}{px}\right)\left(\frac{qx}{\sin qx}\right)$

$= \frac{p}{q}\left(\lim_{px\to 0}\frac{\sin px}{px}\right)\left(\lim_{qx\to 0}\frac{qx}{\sin qx}\right)$

$= \left(\frac{p}{q}\right)(1)(1) = \frac{p}{q}$ #

(e) $\cos^2 x - 1 = -\sin^2 x$

$\therefore \lim_{x\to 0}\left(\frac{\cos^2 x - 1}{x}\right) = \lim_{x\to 0}\left(\frac{-\sin^2 x}{x}\right)$

$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)(-\sin x) = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\lim_{x\to 0}(\sin x)$

$= 1\cdot 0 = 0$ #

(f) Let $y = x+3$

$\lim_{x\to -3}\frac{\sin(x+3)}{x^2+8x+15} = \lim_{x\to -3}\frac{\sin(x+3)}{(x+3)(x+5)}$

$= \lim_{y\to 0}\frac{\sin y}{y(y+2)} = \lim_{y\to 0}\left(\frac{\sin y}{y}\right)\lim_{y\to 0}\left(\frac{1}{y+2}\right)$

$= (1)\left(\frac{1}{2}\right) = \frac{1}{2}$ #

(g) Let $y = x-2$

$\lim_{x\to 2}\frac{\sin(x-2)}{x^2-4} = \lim_{x\to 2}\frac{\sin(x-2)}{(x-2)(x+2)}$

$= \lim_{y\to 0}\frac{\sin y}{y(y+4)} = \lim_{y\to 0}\left(\frac{\sin y}{y}\right)\left(\frac{1}{y+4}\right)$

$= (1)\left(\frac{1}{4}\right) = \frac{1}{4}$ #

(h) $\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{\sin x} = \lim_{x\to 0}\left(x\sin\frac{1}{x}\right)\left(\frac{x}{\sin x}\right)$

$$\left\{\begin{array}{l} \text{As } -1 \le \sin\frac{1}{x} \le 1 \\ -x \le x\sin\frac{1}{x} \le x \\ \lim_{x\to 0}(-x) = 0 = \lim_{x\to 0} x \\ \therefore \lim_{x\to 0}\left(x\sin\frac{1}{x}\right) = 0 \end{array}\right\}$$

$= \lim_{x\to 0}\left(x\sin\frac{1}{x}\right)\lim_{x\to 0}\left(\frac{x}{\sin x}\right)$

$= (0)(1) = 0$ #

(i) Let $y = \frac{\pi}{2} - x$

$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{x-\frac{\pi}{2}} = \lim_{y\to 0}\frac{\cos\left(\frac{\pi}{2}-y\right)}{(-y)}$

$= \lim_{y\to 0}\frac{\sin y}{-y} = -1$ #

(j) $\lim_{x\to 0}\left(\frac{6x-\sin 2x}{2x-3\sin 4x}\right)$

$= \lim_{x\to 0}\left(\frac{6-\frac{\sin 2x}{x}}{2-3\,\frac{\sin 4x}{x}}\right)$

$= \lim_{x\to 0}\left(\frac{6-\left(\frac{\sin 2x}{2x}\right)(2)}{2-3\left(\frac{\sin 4x}{4x}\right)(4)}\right)$

$= \frac{6-2}{2-3\times 4} = \frac{4}{-10} = \frac{-2}{5}$ #

Solutions to Math 1013 Tutorial 6

(1)(k) $\lim_{x\to 1}\frac{3\sin\pi x - \sin 3\pi x}{x^3}$

$= \frac{3\sin\pi - \sin 3\pi}{1}$

$= 0$ #

2) As $-1 \le \cos x \le 1$

$\frac{-1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$

$(\forall x > 0)$

But $\lim_{x\to\infty}\frac{-1}{x} = 0 = \lim_{x\to\infty}\frac{1}{x}$

$\therefore \lim_{x\to\infty}\frac{\cos x}{x} = 0$

(By Squeeze Thm).

3) $f(x) = \begin{cases}\sin\pi x, & 0<x<1 \\ \ln x, & 1<x<2\end{cases}$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\sin\pi x$

$= 0$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\ln x$

$= 0$

$\therefore$ If $f(0) = 0$,

$f(x)$ is continuous at

$x = 0$. #

4) $\frac{d}{dx}x^4 = \lim_{h\to 0}\frac{(x+h)^4 - x^4}{h}$

$= \lim_{h\to 0}\frac{(x^4+4hx^3+6h^2x^2+4h^3x+h^4)-x^4}{h}$

$= \lim_{h\to 0}\left(\frac{h}{h}\right)(4x^3+6hx^2+4h^2x+h^3)$

$= 4x^3$ #

5) $\frac{d}{dV}\left(V^2(2\sqrt{V}+1)\right)$

$= V^2\frac{d}{dV}(2\sqrt{V}+1) + (2\sqrt{V}+1)\frac{dV^2}{dV}$

$= V^2\left(2\left(\frac{1}{2}\right)V^{-\frac{1}{2}}\right) + (2\sqrt{V}+1)(2V)$

$= V^{\frac{3}{2}} + 4V^{\frac{3}{2}} + 2V$

$= 5V^{\frac{3}{2}} + 2V$ #

6(a) Let $f(x) = \sin x$

$f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

$\therefore \lim_{h\to 0}\frac{f\left(\frac{\pi}{6}+h\right) - f\left(\frac{\pi}{6}\right)}{h}$

$= f'\left(\frac{\pi}{6}\right) = \lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}}{h}$

$= \lim_{h\to 0}\frac{\sin\frac{\pi}{6}\cos h + \cos\frac{\pi}{6}\sin h - \frac{1}{2}}{h}$

$= \lim_{h\to 0}\frac{\frac{1}{2}\cos h + \frac{\sqrt{3}}{2}\sin h - \frac{1}{2}}{h}$

$= \lim_{h\to 0}\frac{1}{2}\left[\frac{\cos h - 1}{h}\right] + \frac{\sqrt{3}}{2}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)$

$= \lim_{h\to 0}\frac{-2\sin^2\frac{h}{2}}{2h} + \frac{\sqrt{3}}{2}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)$

$= \lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)\left(-\frac{\sin\frac{h}{2}}{2}\right) + \frac{\sqrt{3}}{2}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)$

$= (1)(0) + \frac{\sqrt{3}}{2}(1)$

$= +\frac{\sqrt{3}}{2}$ #

6(b) Let $f(x) = \cos x$

$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\therefore f'\left(\frac{\pi}{6}\right) = \lim_{h\to 0}\frac{f\left(\frac{\pi}{6}+h\right) - f\left(\frac{\pi}{6}\right)}{h}$

$= \lim_{h\to 0}\frac{\cos\left(\frac{\pi}{6}+h\right) - \frac{\sqrt{3}}{2}}{h}$

$= \lim_{h\to 0}\frac{\cos\left(\frac{\pi}{6}\right)\cos h - \sin\frac{\pi}{6}\sin h - \frac{\sqrt{3}}{2}}{h}$

$= \lim_{h\to 0}\frac{\frac{\sqrt{3}}{2}\left[1-\cos h\right] - \frac{1}{2}\sin h}{h}$

$= \lim_{h\to 0}\frac{\frac{\sqrt{3}}{2}\left[2\sin^2\frac{h}{2}\right] - \frac{1}{2}\sin h}{h}$

$= \lim_{h\to 0}\left(\frac{\sqrt{3}\sin\frac{h}{2}}{\frac{h}{2}}\right)\left(\frac{\sin\frac{h}{2}}{2}\right) - \frac{1}{2}\lim_{h\to 0}\frac{\sin h}{h}$

$= (\sqrt{3})(0) - \frac{1}{2}(1)$

$= -\frac{1}{2}$ #

6(c) Let $f(x) = \cancel{\tan x}$

$f\left(\frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right) = 1$

$\therefore f'\left(\frac{\pi}{4}\right) = \lim_{h\to 0}\frac{f\left(\frac{\pi}{4}+h\right) - f\left(\frac{\pi}{4}\right)}{h}$

$= \lim_{h\to 0}\frac{\cot\left(\frac{\pi}{4}+h\right) - 1}{h}$

Now set $x = \frac{\pi}{4} + h$

when $x \to \frac{\pi}{4}$, $h \to 0$

$\therefore \lim_{x\to\frac{\pi}{4}}\left(\frac{\cot x - 1}{x - \frac{\pi}{4}}\right)$

$= \lim_{h\to 0}\frac{\cot\left(\frac{\pi}{4}+h\right) - 1}{h}$

$= f'\left(\frac{\pi}{4}\right)$

$= \lim_{h\to 0}\frac{\frac{\cot\left(\frac{\pi}{4}\right)\cot h - 1}{\cot\frac{\pi}{4} + \cot h} - 1}{h}$

$= \lim_{h\to 0}\frac{\frac{\cot h - 1}{1+\cot h} - 1}{h}$

$= \lim_{h\to 0}\frac{-2}{(h)(1+\cot h)}$

$= \lim_{h\to 0}\frac{-2}{h}\left(\frac{1}{1+\frac{\cos h}{\sin h}}\right)$

$= \lim_{h\to 0}\left(\frac{-2\sin h}{h}\right)\left(\frac{1}{\sin h + \cos h}\right)$

$= -2$ #

6(d) Let $f(x)=\tan x$

$f\left(\frac{5\pi}{6}\right)=\tan\left(\frac{5\pi}{6}\right)$

$=\tan\left(\pi-\frac{\pi}{6}\right)=-\tan\frac{\pi}{6}$

$=-\frac{1}{\sqrt{3}}$

$\therefore f'\left(\frac{5\pi}{6}\right)=\lim_{h\to 0}\frac{\tan\left(\frac{5\pi}{6}+h\right)-\left(\frac{-1}{\sqrt{3}}\right)}{h}$

$=\lim_{h\to 0}\frac{\frac{\tan\left(\frac{5\pi}{6}\right)+\tan h}{1-\tan\left(\frac{5\pi}{6}\right)\tan h}+\frac{1}{\sqrt{3}}}{h}$

$=\lim_{h\to 0}\frac{\frac{\frac{-1}{\sqrt{3}}+\tan h}{1+\frac{\tan h}{\sqrt{3}}}+\frac{1}{\sqrt{3}}}{h}$

$=\lim_{h\to 0}\frac{\frac{-1+\sqrt{3}\tan h}{\sqrt{3}+\tan h}+\frac{1}{\sqrt{3}}}{h}$

$=\lim_{h\to 0}\frac{\tan h\,(3+1)}{\sqrt{3}\,h\,(\sqrt{3}+\tan h)}$

$=\lim_{h\to 0}\left(\frac{4}{\sqrt{3}}\right)\left(\frac{\sin h}{h}\right)\left(\frac{1}{\cos h}\right)\left(\frac{1}{\sqrt{3}+\tan h}\right)$

$=\frac{4}{\sqrt{3}}(1)(1)\left(\frac{1}{\sqrt{3}}\right)=\frac{4}{3}$ #

Checking:

$\frac{d\tan x}{dx}=\sec^2 x$

$\sec^2\left(\frac{5\pi}{6}\right)=\frac{1}{\cos^2\left(\frac{5\pi}{6}\right)}$

$=\frac{1}{\left(-\cos\frac{\pi}{6}\right)^2}=\frac{1}{\left(-\frac{\sqrt{3}}{2}\right)^2}$

$=\frac{4}{3}$ #

7) $f(x)=x\sin\left(\frac{1}{x}\right), x\neq 0$

$f(0)=0.$

(a) As $-1\leq \sin\frac{1}{x}\leq 1$

$-x\leq x\sin\frac{1}{x}\leq x$

$\lim_{x\to 0}(-x)=0=\lim_{x\to 0}x$

$\Rightarrow \lim_{x\to 0}x\sin\left(\frac{1}{x}\right)=0$

$=f(0)$

$\therefore f(x)$ is continuous at $x=0$.

(b) As

$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{(h+0)\sin\left(\frac{1}{0+h}\right)-0}{h}$

$=\lim_{h\to 0}\frac{h\sin\frac{1}{h}}{h}$

$=\lim_{h\to 0}\left(\sin\frac{1}{h}\right)$ does not exist.

$\therefore f(x)$ does not have a derivative at $x=0$.

8) $f(x)=x^2\sin\left(\frac{1}{x}\right)$

$f(0)=0$

(a)

$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{(0+h)^2\sin\left(\frac{1}{0+h}\right)-0}{h}$

$=\lim_{h\to 0}\frac{h^2\sin\frac{1}{h}}{h}$

$=\lim_{h\to 0}h\left(\sin\frac{1}{h}\right)$

$=0$

$\therefore f(x)$ is differentiable at $x=0$. #

8(b)

$f(x)=x^2\sin\left(\frac{1}{x}\right) \quad x\neq 0$

$f'(x)=2x\sin\frac{1}{x}-x^2\left(\cos\left(\frac{1}{x}\right)\right)\left(\frac{-1}{x^2}\right)$

$=2x\sin\frac{1}{x}+\cos\frac{1}{x}, \ x\neq 0$

$\lim_{x\to 0}\left(\left(2x\sin\frac{1}{x}\right)+\cos\frac{1}{x}\right)=?$

As $\lim_{x\to 0}\left(2x\sin\frac{1}{x}\right)=2\cdot 0=0$

But $\lim_{x\to 0}\left(\cos\frac{1}{x}\right)$ does not exist.

$\therefore \lim_{x\to 0}f'(x)$ does not exist.

$\therefore f'(x)$ is not continuous at $x=0$ #

9(a) $y=x^{\frac{5}{3}}-x^{\frac{2}{3}}$

$\frac{dy}{dx}=\frac{dx^{\frac{5}{3}}}{dx}-\frac{dx^{\frac{2}{3}}}{dx}$

$=\frac{5}{3}x^{\frac{2}{3}}-\frac{2}{3}x^{-\frac{1}{3}}$

$=\frac{5}{3}x^{\frac{2}{3}}-\frac{2}{3x^{\frac{1}{3}}}$ #

(b) $y=\frac{x^2+4x+3}{\sqrt{x}}$

$=x^{\frac{3}{2}}+4x^{\frac{1}{2}}+x^{-\frac{1}{2}}$

$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+4\left(\frac{1}{2}\right)x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}$

$=\frac{3}{2}x^{\frac{1}{2}}+\frac{2}{x^{\frac{1}{2}}}-\frac{1}{2x^{3/2}}$ #

(c) $F(x)=(4x-x^2)^{100}$

Let $u=4x-x^2$

$F(x)=u^{100}$

$\frac{dF(x)}{dx}=\frac{dF(x)}{du}\times\frac{du}{dx}$

$=100u^{99}(4-2x)$

$=(\ )(4-2x)(4x-x^2)^{99}$ #

9(d) Let $u=x^{\frac{1}{2}}+x^{-\frac{3}{2}}$

$v=\left(\sqrt{x}+\frac{1}{\sqrt[3]{x}}\right)^2=u^2$

$\frac{dv}{dx}=\frac{dv}{du}\times\frac{du}{dx}$

$=(2u)\left(\frac{1}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{-\frac{5}{2}}\right)$

$=\left(x^{\frac{1}{2}}+\frac{1}{x^{\frac{3}{2}}}\right)\left(\frac{1}{x^{\frac{1}{2}}}-\frac{3}{x^{\frac{5}{2}}}\right)$

$=\left(x-\frac{3}{x^2}+\frac{1}{x}-\frac{3}{x^4}\right)$

$=\left(x+\frac{1}{x}\right)-3\left(\frac{1}{x^2}+\frac{1}{x^4}\right)$ #

(e) $f(x)=x^4e^x$

$f'(x)=4x^3e^x+x^4e^x$

$=e^x(4x^3+x^4)$ #

(f) $f(t)=\tan e^t+e^{\tan t}$

$f'(t)=\frac{df(t)}{dt}$

$=\frac{d\tan e^t}{dt}+\frac{de^{\tan t}}{dt}$

As $\frac{d\tan e^t}{dt}=\frac{d\tan e^t}{d(e^t)}\times\frac{de^t}{dt}$

$=(\sec^2 e^t)(e^t)$

and $\frac{de^{\tan t}}{d(\tan t)}\times\frac{d\tan t}{dt}$

$=(e^{\tan t})(\sec^2 t)$

$\therefore f'(t)=(\sec^2 e^t)(e^t)+e^{\tan t}\sec^2 t$ #

9(g) $y=\sqrt{1+xe^{-2x}}$

Let $u=1+xe^{-2x}$

$y=u^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$

$=\frac{1}{2}u^{-\frac{1}{2}}\left(0+e^{-2x}+xe^{-2x}(-2)\right)$

$=\frac{1}{2}\left(\frac{e^{-2x}-2xe^{-2x}}{u^{\frac{1}{2}}}\right)$

$=\frac{1}{2e^{2x}}\frac{(1-2x)}{\sqrt{1+xe^{-2x}}}$ #

(h) $f(t)=\sin^2(e^{\sin t})$

Let $u=e^{\sin t}$

$\frac{du}{dt}=\frac{d(e^{\sin t})}{d\sin t}\left(\frac{d\sin t}{dt}\right)$

$=(e^{\sin t})(\cos t)$

$f(t)=\sin^2 u$

$\therefore f'(t)=\frac{df(t)}{du}\times\frac{du}{dt}$

$=(2\sin u)(\cos u)\times\frac{du}{dt}$

$=\left(2\sin e^{\sin t}\cos e^{\sin t}\right)\left(e^{\sin t}\cos t\right)$ #

9(i) Let $u=\sin(\tan\pi x)$

$y=\cos\sqrt{\sin(\tan\pi x)}$

$=\cos(u^{\frac{1}{2}})$

$\frac{dy}{dx}=\frac{dy}{du^{\frac{1}{2}}}\times\frac{du^{\frac{1}{2}}}{du}\times\frac{du}{dt}$

$=\left(-\sin u^{\frac{1}{2}}\right)\left(\frac{1}{2}u^{-\frac{1}{2}}\right)\frac{du}{dt}$

Now $u=\sin(\tan\pi x)$

$\frac{du}{dx}=\frac{d\sin(\tan\pi x)}{d(\tan\pi x)}\times\frac{d\tan\pi x}{d\pi x}\times\frac{d\pi x}{dx}$

$=\cos(\tan\pi x)(\sec^2\pi x)(\pi)$

Hence

$\frac{dy}{dx}=\left(-\sin\sqrt{\sin(\tan\pi x)}\right)\cdot\left(\frac{1}{2\sqrt{\sin(\tan\pi x)}}\right)\cdot\left(\pi\cos(\tan\pi x)\right)(\sec^2\pi x)$ #

10(a)

$y=2xe^x$

$\frac{dy}{dx}=2e^x+2xe^x$

Slope of tangent at $(0,0)$

$=\left.\frac{dy}{dx}\right|_{(0,0)}=2(1)+0=2$

$\therefore$ Tangent at $(0,0)$ is:

$y-0=2(x-0)$

$y=2x$ #

Now slope of normal at $(0,0)=\frac{-1}{\left.\left(\frac{dy}{dx}\right)\right|_{(0,0)}}=\frac{-1}{2}$

$\therefore$ Equation of normal at $(0,0)$ is:

$y-0=\left(\frac{-1}{2}\right)(x-0)$

$\Rightarrow y=\frac{-1}{2}x$ #

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10(b) $y=\frac{2x}{x^2+1}$

$\frac{dy}{dx}=\frac{(x^2+1)(2)-2x(2x)}{(x^2+1)^2}$

Slope of tangent at (1,1)

$=\left.\frac{dy}{dx}\right|_{(1,1)}=\frac{(1+1)(2)-2(1)(2)}{(1+1)^2}$

$=\frac{4-4}{4}=0$

∴ Equation of tangent at (1, 1) is:

$(y-1)=0(x-1)$

$\Rightarrow y=1$ #

The Equation of normal at (1, 1) is given by:

$x=1$ #

(c) $y=\sin(\sin x)$

$\frac{dy}{dx}=\frac{d\sin(\sin x)}{d(\sin x)}\times\frac{d\sin x}{dx}$

$=\cos(\sin x)(\cos x)$

∴ Slope of tangent at $(\pi,0)$ is.

$\left.\frac{dy}{dx}\right|_{(\pi,0)}=(\cos(\sin\pi))(\cos\pi)$

$=(1)(-1)=-1$

∴ Equation of tangent is:

$(y-0)=(-1)(x-\pi)$

$\Rightarrow y=-x+\pi$ #

Slope of normal at $(\pi,0)$

$=1$

∴ Equation of normal at $(\pi,0)$:

$y-0=(1)(x-\pi)$

$\Rightarrow y=x-\pi$ #

11) $R(x)=\frac{x-3x^3+5x^5}{1+3x^3+6x^6+9x^9}$

Let $f(x)=x-3x^3+5x^5$

$g(x)=1+3x^3+6x^6+9x^9$

$\Rightarrow R(x)=\frac{f(x)}{g(x)}$

$R'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$

$\therefore R'(0)=\frac{g(0)f'(0)-f(0)g'(0)}{(g(0))^2}$

Now $f(0)=0$, $g(0)=1$

$f'(x)=1-9x^2+25x^4$

$f'(0)=1$

$g'(x)=9x^2+36x^5+81x^8$

$g'(0)=0$

$\therefore R'(0)=\frac{(1)(1)-(0)(0)}{1^2}$

$=1$ #

12) $y=\frac{(x^{\frac{3}{4}})(x^2+1)^{\frac{1}{2}}}{(3x+2)^5}$

$\ln y=\ln\left(\frac{x^{\frac{3}{4}}(x^2+1)^{\frac{1}{2}}}{(3x+2)^5}\right)$

$=\ln x^{\frac{3}{4}}+\ln(x^2+1)^{\frac{1}{2}}-\ln(3x+2)^5$

$=\frac{3}{4}\ln x+\frac{1}{2}\ln(x^2+1)-5\ln(3x+2)$

As $\frac{d\ln y}{dx}=\frac{d\ln y}{dy}\times\frac{dy}{dx}$

$=\frac{1}{y}\frac{dy}{dx}$

$\therefore \frac{1}{y}\frac{dy}{dx}=\frac{3}{4}\left(\frac{1}{x}\right)+\frac{1}{2}\left(\frac{2x}{x^2+1}\right)-\left(\frac{5}{3x+2}\right)(3)$

$\therefore \frac{dy}{dx}=y\left(\frac{3}{4x}+\frac{x}{x^2+1}-\frac{5}{9x+6}\right)$

$=\left(\frac{x^{\frac{3}{4}}(x^2+1)^{\frac{1}{2}}}{(3x+2)^5}\right)\left(\frac{3}{4x}+\frac{x}{x^2+1}-\frac{5}{9x+6}\right)$ #

13) $S=10+\frac{1}{4}\sin(10\pi t)$

$v=\frac{ds}{dt}=\frac{d}{dt}\left(10+\frac{1}{4}\sin(10\pi t)\right)$

$=0+\frac{1}{4}\frac{d\sin(10\pi t)}{d(10\pi t)}\times\frac{d\,10\pi t}{dt}$

$=\frac{1}{4}\cos(10\pi t)\,(10\pi)$

$=\frac{5}{2}\pi\cos(10\pi t)$ #

14) (a) $p(t)=\frac{1}{1+ae^{-kt}}$ $(a,k>0)$

$\lim_{t\to 0}p(t)=\frac{1}{1+0}=1$ #

(b) $\frac{dp(t)}{dt}=\frac{d(1+ae^{-kt})^{-1}}{dt}$

$=\frac{-1}{(1+ae^{-kt})^2}(-ake^{-kt})$

$=\frac{ake^{-kt}}{(1+ae^{-kt})^2}$ #

(15) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{d}{dx}\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$

$\Rightarrow \frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$

$\therefore \left.\frac{dy}{dx}\right|_{(x_0,y_0)}=\left.\left(\frac{x}{a^2}\right)\left(\frac{b^2}{y}\right)\right|_{(x_0,y_0)}$

$=\left(\frac{x_0}{a^2}\right)\left(\frac{b^2}{y_0}\right)$

As (x_0,y_0) on the hyperbola,

$\therefore \frac{x_0^2}{a^2}-\frac{y_0^2}{b^2}=1$

Equation of tangent is:

$y-y_0=\left(\frac{x_0}{a^2}\right)\left(\frac{b^2}{y_0}\right)(x-x_0)$

$\Rightarrow \frac{x_0x}{a^2}-\frac{y_0y}{b^2}=1$ #