1) (a)

$f(x) = x^4 + 3x^2 \quad \Rightarrow \quad f'(x) = 4x^3 + 6x$, so $f(-1) = 4$ and $f'(-1) = -10$.

Thus, $L(x) = f(-1) + f'(-1)(x - (-1)) = 4 + (-10)(x + 1) = -10x - 6$.

1) (b)

$f(x) = \cos x \quad \Rightarrow \quad f'(x) = -\sin x$, so $f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = -1$.

Thus, $L(x) = f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) = 0 - 1\left(x - \frac{\pi}{2}\right) = -x + \frac{\pi}{2}$.

2)

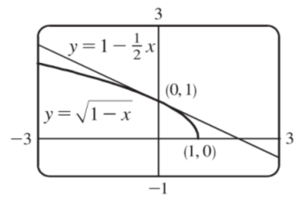
$f(x) = \sqrt{1-x} \quad \Rightarrow \quad f'(x) = \dfrac{-1}{2\sqrt{1-x}}$, so $f(0) = 1$ and $f'(0) = -\frac{1}{2}$.

Therefore,

$\sqrt{1-x} = f(x) \approx f(0) + f'(0)(x - 0) = 1 + \left(-\frac{1}{2}\right)(x - 0) = 1 - \frac{1}{2}x$.

So $\sqrt{0.9} = \sqrt{1 - 0.1} \approx 1 - \frac{1}{2}(0.1) = 0.95$

and $\sqrt{0.99} = \sqrt{1 - 0.01} \approx 1 - \frac{1}{2}(0.01) = 0.995$.



3)

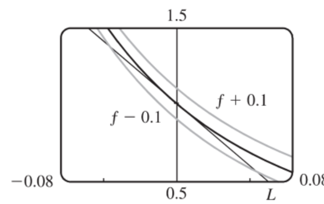
$f(x) = \dfrac{1}{(1+2x)^4} = (1+2x)^{-4} \quad \Rightarrow$

$f'(x) = -4(1+2x)^{-5}(2) = \dfrac{-8}{(1+2x)^5}$, so $f(0) = 1$ and $f'(0) = -8$.

Thus, $f(x) \approx f(0) + f'(0)(x - 0) = 1 + (-8)(x - 0) = 1 - 8x$.

We need $\dfrac{1}{(1+2x)^4} - 0.1 < 1 - 8x < \dfrac{1}{(1+2x)^4} + 0.1$, which is true

when $-0.045 < x < 0.055$.



4. (a)

To estimate $(2.001)^5$, we'll find the linearization of $f(x) = x^5$ at $a = 2$. Since $f'(x) = 5x^4$, $f(2) = 32$, and $f'(2) = 80$,

we have $L(x) = 32 + 80(x - 2) = 80x - 128$. Thus, $x^5 \approx 80x - 128$ when x is near 2 , so

$(2.001)^5 \approx 80(2.001) - 128 = 160.08 - 128 = 32.08$.

4. (b)

To estimate $(8.06)^{2/3}$, we'll find the linearization of $f(x) = x^{2/3}$ at $a = 8$. Since $f'(x) = \frac{2}{3}x^{-1/3} = 2/\left(3\sqrt[3]{x}\right)$,

$f(8) = 4$, and $f'(8) = \frac{1}{3}$, we have $L(x) = 4 + \frac{1}{3}(x - 8) = \frac{1}{3}x + \frac{4}{3}$. Thus, $x^{2/3} \approx \frac{1}{3}x + \frac{4}{3}$ when x is near 8, so

$(8.06)^{2/3} \approx \frac{1}{3}(8.06) + \frac{4}{3} = \frac{12.06}{3} = 4.02$.

4. (c)

$y = f(x) = \tan x \quad \Rightarrow \quad dy = \sec^2 x\, dx$. When $x = 45°$ and $dx = -1°$,

$dy = \sec^2 45°(-\pi/180) = \left(\sqrt{2}\right)^2(-\pi/180) = -\pi/90$, so $\tan 44° = f(44°) \approx f(45°) + dy = 1 - \pi/90 \approx 0.965$.

4(d)

Let $f(x) = 1/\sqrt{x}$, $a = 121$. Then $f(a) = 1/11$ and $f'(a) = -1/(2a^{3/2}) = -1/2662$, so the linear approximation to f near $a = 121$ is $L(x) = f(a) + f'(a)(x-a) = \frac{1}{11} - \frac{1}{2662}(x-121)$. Therefore $\frac{1}{\sqrt{119}} = f(119) \approx L(119) \approx 0.0917$.

4(e)

Let $f(x) = \ln x$, $a = 1$. Then $f(a) = 0$ and $f'(a) = 1/a = 1$, so the linear approximation to f near $a = 1$ is $L(x) = f(a) + f'(a)(x-a) = x - 1$. Therefore $\ln(1.05) = f(1.05) \approx L(1.05) = .05$.

4(f)

Let $f(x) = e^x$, $a = 0$. Then $f(a) = 1$ and $f'(a) = e^a = 1$, so the linear approximation to f near $a = 0$ is $L(x) = f(a) + f'(a)(x-a) = 1 + x$. Therefore $e^{0.06} = f(0.06) \approx L(0.06) \approx 1.06$.

5.

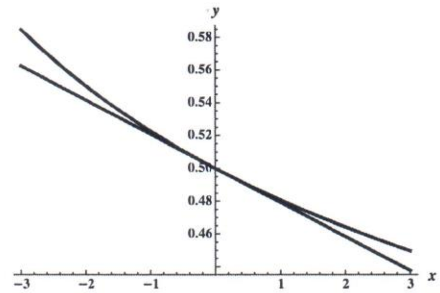
a. Note that $f(a) = 8^{-1/3} = 1/2$ and $f'(a) = (-1/3)(8)^{-4/3} = -1/48$, so the linear approximation has equation

$$y = L(x) = f(a) + f'(a)(x-a) = \frac{1}{2} + \frac{-1}{48}x.$$

c. We have $f(-0.1) \approx L(-0.1) \approx .502$

d. The percentage error is $100 \cdot \frac{|7.9^{-1/3} - .502|}{(7.9)^{-1/3}} \approx 0.02\%$.

b.



6.

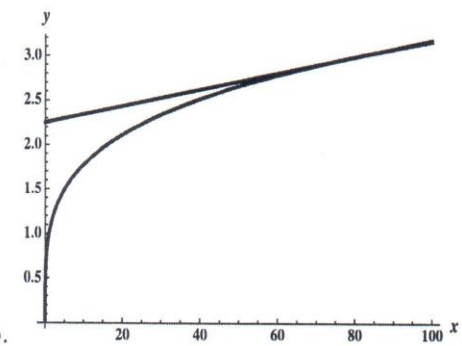
a. Note that $f(a) = \sqrt[4]{81} = 3$ and $f'(a) = \frac{1}{4}(81)^{-3/4} = \frac{1}{27}$, so the linear approximation has equation

$$y = L(x) = f(a) + f'(a)(x-a) = 3 + \frac{1}{108}(x-81).$$

c. We have $f(85) \approx L(85) = 3 + \frac{4}{108} = 3 + \frac{1}{27} \approx 3.04$.

d. The percentage error is $100 \cdot \frac{|3.04 - \sqrt[4]{85}|}{\sqrt[4]{85}} \approx 0.12\%$.

b.



7)

(a) $V = \pi r^2 h \quad \Rightarrow \quad \Delta V \approx dV = 2\pi rh\,dr = 2\pi rh\,\Delta r$

(b) The error is

$$\Delta V - dV = [\pi(r+\Delta r)^2 h - \pi r^2 h] - 2\pi rh\,\Delta r = \pi r^2 h + 2\pi rh\,\Delta r + \pi(\Delta r)^2 h - \pi r^2 h - 2\pi rh\,\Delta r = \pi(\Delta r)^2 h.$$

8)

$V = RI \quad \Rightarrow \quad I = \frac{V}{R} \quad \Rightarrow \quad dI = -\frac{V}{R^2}\,dR$. The relative error in calculating I is $\frac{\Delta I}{I} \approx \frac{dI}{I} = \frac{-(V/R^2)\,dR}{V/R} = -\frac{dR}{R}$.

Hence, the relative error in calculating I is approximately the same (in magnitude) as the relative error in R.