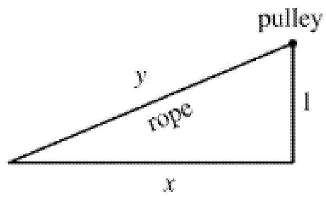


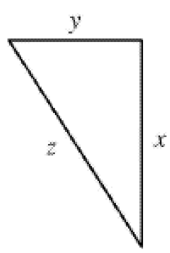
1)

Given $\frac{dy}{dt}=-1$ m / s, find $\frac{dx}{dt}$ when $x=8$ m.

$$y^2=x^2+1 \Rightarrow 2y\frac{dy}{dt}=2x\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=\frac{y}{x}\frac{dy}{dt}=-\frac{y}{x}.$$

When $x=8$, $y=\sqrt{65}$, so $\frac{dx}{dt}=-\frac{\sqrt{65}}{8}$.

Thus, the boat approaches the dock at $\frac{\sqrt{65}}{8}\approx 1.01$ m / s.



2)

We are given that $\frac{dx}{dt}=60$ mi / h and $\frac{dy}{dt}=25$ mi / h.

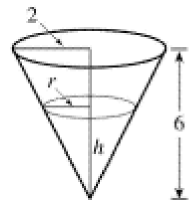
$$z^2=x^2+y^2 \Rightarrow 2z\frac{dz}{dt}=2x\frac{dx}{dt}+2y\frac{dy}{dt} \Rightarrow$$

$$z\frac{dz}{dt}=x\frac{dx}{dt}+y\frac{dy}{dt} \Rightarrow \frac{dz}{dt}=\frac{1}{z}\left(x\frac{dx}{dt}+y\frac{dy}{dt}\right).$$

After 2 hours, $x=2(60)=120$ and $y=2(25)=50 \Rightarrow z=\sqrt{120^2+50^2}=130$, so

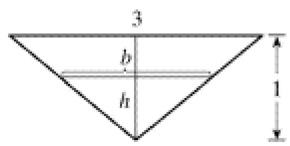
$$\frac{dz}{dt}=\frac{1}{z}\left(x\frac{dx}{dt}+y\frac{dy}{dt}\right)=\frac{120(60)+50(25)}{130}=65 \text{ mi / h}.$$

3)



If C= the rate at which water is pumped in, then $\frac{dV}{dt} = C - 10,000$, where $V = \frac{1}{3}\pi r^2 h$ is the volume at time t . By similar triangles, $\frac{r}{2} = \frac{h}{6} \Rightarrow r = \frac{1}{3}h \Rightarrow V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{\pi}{27}h^3 \Rightarrow \frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$.
When $h = 200\text{cm}$, $\frac{dh}{dt} = 20\text{cm/min}$, so $C - 10,000 = \frac{\pi}{9}(200)^2(20) \Rightarrow C = 10,000 + \frac{800,000}{9}\pi \approx 289$,

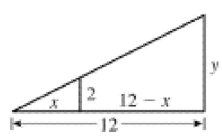
4)



By Similar Triangles,

$\frac{3}{1} = \frac{b}{h}$, so $b = 3h$. The trough has volume $V = \frac{1}{2}bh(10) = 5(3h)h = 15h^2 \Rightarrow 12 = \frac{dV}{dt} = 30h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{2}{5h}$. When $h = \frac{1}{2}$, $\frac{dh}{dt} = \frac{2}{5 \cdot \frac{1}{2}} = \frac{4}{5}$ ft / min.

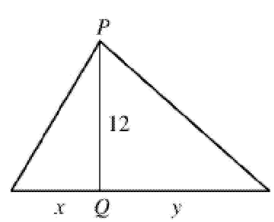
5)



We are given that $\frac{dx}{dt} = 1.6$ m / s. By similar triangles, $\frac{y}{12} = \frac{2}{x} \Rightarrow y = \frac{24}{x} \Rightarrow \frac{dy}{dt} = -\frac{24}{x^2}\frac{dx}{dt} = -\frac{24}{x^2}(1.6)$.

When $x = 8$, $\frac{dy}{dt} = -\frac{24(1.6)}{64} = -0.6$ m / s, so the shadow is decreasing at a rate of 0.6 m / s.

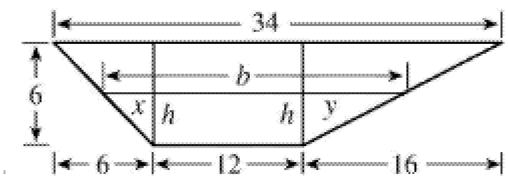
6)



Using Q for the origin, we are given $\frac{dx}{dt}=-2$ ft / s and need to find $\frac{dy}{dt}$ when $x=-5$. Using the Pythagorean Theorem twice, we have $\sqrt{x^2+12^2}+\sqrt{y^2+12^2}=39$, the total length of the rope. Differentiating with respect to t , we get $\frac{x}{\sqrt{x^2+12^2}}\frac{dx}{dt}+\frac{y}{\sqrt{y^2+12^2}}\frac{dy}{dt}=0$, so

$\frac{dy}{dt}=-\frac{x\sqrt{y^2+12^2}}{y\sqrt{x^2+12^2}}\frac{dx}{dt}$. Now when $x=-5$, $39=\sqrt{(-5)^2+12^2}+\sqrt{y^2+12^2}=13+\sqrt{y^2+12^2} \Leftrightarrow \sqrt{y^2+12^2}=26$, and $y=\sqrt{26^2-12^2}=\sqrt{532}$. So when $x=-5$, $\frac{dy}{dt}=-\frac{(-5)(26)}{\sqrt{532}(13)}(-2)=-\frac{10}{\sqrt{133}}\approx -0.87$ ft / s. So cart B is moving towards Q at about 0.87 ft / s.

7)



The figure is drawn without the top 3 feet. $V=\frac{1}{2}(b+12)h(20)=10(b+12)h$ and, from similar triangles, $\frac{x}{h}=\frac{6}{6}$ and $\frac{y}{h}=\frac{16}{6}=\frac{8}{3}$, so $b=x+12+y=h+12+\frac{8h}{3}=12+\frac{11h}{3}$. Thus,

$V=10\left(24+\frac{11h}{3}\right)h=240h+\frac{110h^2}{3}$ and so $0.8=\frac{dV}{dt}=\left(240+\frac{220}{3}h\right)\frac{dh}{dt}$. When $h=5$, $\frac{dh}{dt}=\frac{0.8}{240+5(220/3)}=\frac{3}{2275}\approx 0.00132$ ft / min.

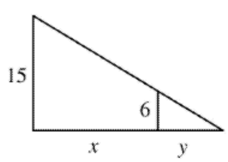
8)

(a) Given: a man 6 ft tall walks away from a street light mounted on a 15 –ft–tall pole at a rate of 5

ft / s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt=5$ ft / s.

(b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of

his shadow (in ft), then we want to find $\frac{d}{dt}(x+y)$ when $x=40$ ft.

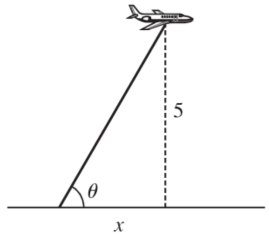


(c)

(d) By similar triangles, $\frac{15}{6}=\frac{x+y}{y}\Rightarrow 15y=6x+6y\Rightarrow 9y=6x\Rightarrow y=\frac{2}{3}x$.

(e) The tip of the shadow moves at a rate of $\frac{d}{dt}(x+y)=\frac{d}{dt}\left(x+\frac{2}{3}x\right)=\frac{5}{3}\frac{dx}{dt}=\frac{5}{3}(5)=\frac{25}{3}$ ft / s.

9)



$$\cot\theta = \frac{x}{5} \quad\Rightarrow\quad -\csc^2\theta\,\frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt} \quad\Rightarrow\quad -\left(\csc\frac{\pi}{3}\right)^2\left(-\frac{\pi}{6}\right) = \frac{1}{5}\frac{dx}{dt} \quad\Rightarrow$$

$$\frac{dx}{dt} = \frac{5\pi}{6}\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{10}{9}\pi \text{ km/min } [\approx 130 \text{ mi/h}]$$

10)

(a) $s(t) = t^3 - 4.5t^2 - 7t \quad\Rightarrow\quad v(t) = s'(t) = 3t^2 - 9t - 7 = 5 \quad\Leftrightarrow\quad 3t^2 - 9t - 12 = 0 \quad\Leftrightarrow$

$3(t-4)(t+1) = 0 \quad\Leftrightarrow\quad t = 4$ or -1. Since $t \geq 0$, the particle reaches a velocity of 5 m/s at $t = 4$ s.

(b) $a(t) = v'(t) = 6t - 9 = 0 \quad\Leftrightarrow\quad t = 1.5$. The acceleration changes from negative to positive, so the velocity changes from decreasing to increasing. Thus, at $t = 1.5$ s, the velocity has its minimum value.

11)

(a) Using $A(r) = \pi r^2$, we find that the average rate of change is:

(i) $\dfrac{A(3) - A(2)}{3 - 2} = \dfrac{9\pi - 4\pi}{1} = 5\pi$ (ii) $\dfrac{A(2.5) - A(2)}{2.5 - 2} = \dfrac{6.25\pi - 4\pi}{0.5} = 4.5\pi$

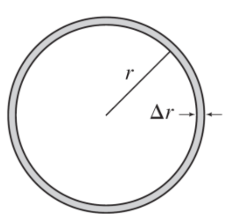
(iii) $\dfrac{A(2.1) - A(2)}{2.1 - 2} = \dfrac{4.41\pi - 4\pi}{0.1} = 4.1\pi$

(b) $A(r) = \pi r^2 \quad\Rightarrow\quad A'(r) = 2\pi r$, so $A'(2) = 4\pi$.

(c) The circumference is $C(r) = 2\pi r = A'(r)$. The figure suggests that if Δr is small, then the change in the area of the circle (a ring around the outside) is approximately equal to its circumference times Δr. Straightening out this ring gives us a shape that is approximately rectangular with length $2\pi r$ and width Δr, so $\Delta A \approx 2\pi r(\Delta r)$. Algebraically,

$\Delta A = A(r + \Delta r) - A(r) = \pi(r + \Delta r)^2 - \pi r^2 = 2\pi r(\Delta r) + \pi(\Delta r)^2$.

So we see that if Δr is small, then $\Delta A \approx 2\pi r(\Delta r)$ and therefore, $\Delta A/\Delta r \approx 2\pi r$.



12)

The quantity of charge is $Q(t) = t^3 - 2t^2 + 6t + 2$, so the current is $Q'(t) = 3t^2 - 4t + 6$.

(a) $Q'(0.5) = 3(0.5)^2 - 4(0.5) + 6 = 4.75$ A

(b) $Q'(1) = 3(1)^2 - 4(1) + 6 = 5$ A

The current is lowest when Q' has a minimum. $Q''(t) = 6t - 4 < 0$ when $t < \frac{2}{3}$. So the current decreases when $t < \frac{2}{3}$ and increases when $t > \frac{2}{3}$. Thus, the current is lowest at $t = \frac{2}{3}$ s.

13)

(a) $C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3 \quad \Rightarrow \quad C'(x) = 12 - 0.2x + 0.0015x^2$ $/yard, which is the marginal cost function.

(b) $C'(200) = 12 - 0.2(200) + 0.0015(200)^2 = \$32/\text{yard}$, and this is the rate at which costs are increasing with respect to the production level when $x = 200$. $C'(200)$ predicts the cost of producing the 201st yard.

(c) The cost of manufacturing the 201st yard of fabric is $C(201) - C(200) = 3632.2005 - 3600 \approx \$32.20$, which is approximately $C'(200)$.

14)

The relative growth rate is $\dfrac{1}{P}\dfrac{dP}{dt} = 0.7944$, so $\dfrac{dP}{dt} = 0.7944P$ and, by Theorem 2, $P(t) = P(0)e^{0.7944t} = 2e^{0.7944t}$.

Thus, $P(6) = 2e^{0.7944(6)} \approx 234.99$ or about 235 members.

15)

(a) By Theorem 2, $P(t) = P(0)e^{kt} = 100e^{kt}$. Now $P(1) = 100e^{k(1)} = 420 \quad \Rightarrow \quad e^k = \frac{420}{100} \quad \Rightarrow \quad k = \ln 4.2$.

So $P(t) = 100e^{(\ln 4.2)t} = 100(4.2)^t$.

(b) $P(3) = 100(4.2)^3 = 7408.8 \approx 7409$ bacteria

(c) $dP/dt = kP \quad \Rightarrow \quad P'(3) = k \cdot P(3) = (\ln 4.2)\left(100(4.2)^3\right)$ [from part (a)] $\approx 10{,}632$ bacteria/hour

(d) $P(t) = 100(4.2)^t = 10{,}000 \quad \Rightarrow \quad (4.2)^t = 100 \quad \Rightarrow \quad t = (\ln 100)/(\ln 4.2) \approx 3.2$ hours

16)

(a) If $y(t)$ is the mass (in mg) remaining after t years, then $y(t) = y(0)e^{kt} = 100e^{kt}$.

$y(30) = 100e^{30k} = \frac{1}{2}(100) \quad \Rightarrow \quad e^{30k} = \frac{1}{2} \quad \Rightarrow \quad k = -(\ln 2)/30 \quad \Rightarrow \quad y(t) = 100e^{-(\ln 2)t/30} = 100 \cdot 2^{-t/30}$

(b) $y(100) = 100 \cdot 2^{-100/30} \approx 9.92$ mg

(c) $100e^{-(\ln 2)t/30} = 1 \quad \Rightarrow \quad -(\ln 2)t/30 = \ln \frac{1}{100} \quad \Rightarrow \quad t = -30\,\frac{\ln 0.01}{\ln 2} \approx 199.3$ years

17)

$\frac{dT}{dt} = k(T - 20)$. Letting $y = T - 20$, we get $\frac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt}$. $y(0) = T(0) - 20 = 5 - 20 = -15$, so $y(25) = y(0)e^{25k} = -15e^{25k}$, and $y(25) = T(25) - 20 = 10 - 20 = -10$, so $-15e^{25k} = -10 \Rightarrow e^{25k} = \frac{2}{3}$. Thus, $25k = \ln\left(\frac{2}{3}\right)$ and $k = \frac{1}{25}\ln\left(\frac{2}{3}\right)$, so $y(t) = y(0)e^{kt} = -15e^{(1/25)\ln(2/3)t}$. More simply, $e^{25k} = \frac{2}{3} \Rightarrow e^k = \left(\frac{2}{3}\right)^{1/25} \Rightarrow e^{kt} = \left(\frac{2}{3}\right)^{t/25} \Rightarrow y(t) = -15 \cdot \left(\frac{2}{3}\right)^{t/25}$.

(a) $T(50) = 20 + y(50) = 20 - 15 \cdot \left(\frac{2}{3}\right)^{50/25} = 20 - 15 \cdot \left(\frac{2}{3}\right)^2 = 20 - \frac{20}{3} = 13.\bar{3}\,^\circ\mathrm{C}$

(b) $15 = T(t) = 20 + y(t) = 20 - 15 \cdot \left(\frac{2}{3}\right)^{t/25} \Rightarrow 15 \cdot \left(\frac{2}{3}\right)^{t/25} = 5 \Rightarrow \left(\frac{2}{3}\right)^{t/25} = \frac{1}{3} \Rightarrow$

$(t/25)\ \ln\left(\frac{2}{3}\right) = \ln\left(\frac{1}{3}\right) \Rightarrow t = 25\ln\left(\frac{1}{3}\right)/\ln\left(\frac{2}{3}\right) \approx 67.74$ min.

18) **Carbon Dating**

Let $y(t)$ be the level of radioactivity. Thus, $y(t) = y(0)e^{-kt}$ and k is determined by using the half-life:

$$y(5730) = \tfrac{1}{2}y(0) \Rightarrow y(0)e^{-k(5730)} = \tfrac{1}{2}y(0) \Rightarrow e^{-5730k} = \tfrac{1}{2} \Rightarrow -5730k = \ln\tfrac{1}{2} \Rightarrow k = -\frac{\ln\frac{1}{2}}{5730} = \frac{\ln 2}{5730}.$$

If 74% of the ^{14}C remains, then we know that $y(t) = 0.74y(0) \Rightarrow 0.74 = e^{-t(\ln 2)/5730} \Rightarrow \ln 0.74 = -\frac{t\ln 2}{5730} \Rightarrow$

$$t = -\frac{5730(\ln 0.74)}{\ln 2} \approx 2489 \approx 2500 \text{ years.}$$

19)

(a) Using $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$ with $A_0 = 3000$, $r = 0.05$, and $t = 5$, we have:

(i) Annually: $n = 1$; $A = 3000\left(1 + \frac{0.05}{1}\right)^{1\cdot 5} = \$3828.84$

(ii) Semiannually: $n = 2$; $A = 3000\left(1 + \frac{0.05}{2}\right)^{2\cdot 5} = \$3840.25$

(iii) Monthly: $n = 12$; $A = 3000\left(1 + \frac{0.05}{12}\right)^{12\cdot 5} = \$3850.08$

(iv) Weekly: $n = 52$; $A = 3000\left(1 + \frac{0.05}{52}\right)^{52\cdot 5} = \$3851.61$

(v) Daily: $n = 365$; $A = 3000\left(1 + \frac{0.05}{365}\right)^{365\cdot 5} = \$3852.01$

(vi) Continuously: $A = 3000e^{(0.05)5} = \$3852.08$

(b) $dA/dt = 0.05A$ and $A(0) = 3000$.