Solutions to Math 1013 (Tutorial 2)

1) $h(x) = \sqrt{4-x^2}$ **. Let** $y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$ **,** $x^2 + y^2 = 4$ **.**

So the graph is the top half of a circle of $x^2 + y^2 = 2^2$ **.**

For $4 - x^2 = -(x+2)(x-2) \geq 0$

Implies $(x+2)(x-2) \leq 0$**,** $-2 \leq x \leq 2$**.**

The Domain of h(x) = $[-2,\ 2]$ **, the range of h(x) =**$[0,\ 2]$**.**

2) $g(x) = sin^{-1}(3x+1)$**,** $sin(g(x)) = 3x+1$

Let $f(x) = sin\theta$**, The inverse function** $f^{-1}(\theta) = sin^{-1}(\theta)$ **exists iff the domain of** $f(x) = sin\theta$ **is defined in the region such that** $f(x) = sin\theta$ **is bijective (one to one and onto).**

As we know $-1 \leq sin\theta \leq 1$**,**

imply $-1 \leq sin(g(x)) = 3x+1 \leq 1$**.**

$$\Rightarrow -1 \leq 3x+1 \leq 1 \qquad \Rightarrow -2 \leq 3x \leq 0 \ \Rightarrow \frac{-2}{3} \leq x \leq 0$$

Therefore the domain of g(x) is $\left[\frac{-2}{3},\ 0\right]$**.**

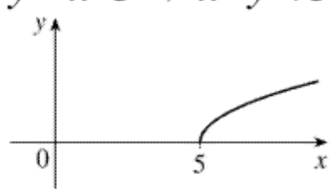
The range of g(x) is $[\sin^{-1}(-1),\ \sin^{-1} 1\,] = \left[\frac{-\pi}{2},\ \frac{\pi}{2}\right]$**.**

3(a)

$g(x) = \sqrt{x-5}$ is defined when $x-5 \geq 0$ or $x \geq 5$, so the domain is $[5, \infty)$.

Since $y = \sqrt{x-5} \Rightarrow$

$y^2 = x-5 \Rightarrow x = y^2 + 5$, we see that g is the top half of a parabola.



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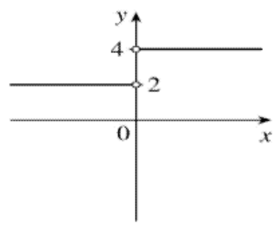
3(b)

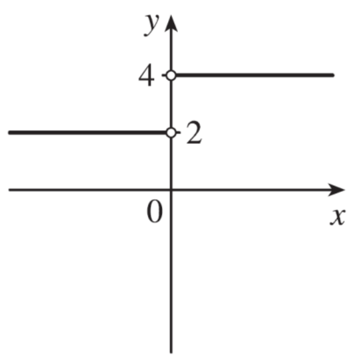
$G(x) = \dfrac{3x+|x|}{x}$. Since $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x<0 \end{cases}$, we have

$$G(x) = \begin{cases} \dfrac{3x+x}{x} & \text{if } x>0 \\ \dfrac{3x-x}{x} & \text{if } x<0 \end{cases} = \begin{cases} \dfrac{4x}{x} & \text{if } x>0 \\ \dfrac{2x}{x} & \text{if } x<0 \end{cases} = \begin{cases} 4 & \text{if } x>0 \\ 2 & \text{if } x<0 \end{cases}$$



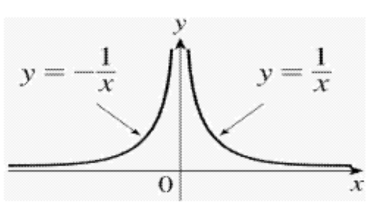
Note that G is not defined for $x=0$.

The domain is $(-\infty,0)\cup(0,\infty)$.



3(c) $g(x) = \frac{|x|}{x^2}$. **Since** $|x| = \begin{cases} x & if & x \geq 0 \\ -x & if & x < 0 \end{cases}$,

we have $g(x) = \begin{cases} \frac{x}{x^2} & if & x \geq 0 \\ \frac{-x}{x^2} & if & x < 0 \end{cases}$ **=** $\begin{cases} \frac{1}{x} & if & x \geq 0 \\ \frac{-1}{x} & if & x < 0 \end{cases}$



Note that g is not defined for $x=0$. The domain is $(-\infty,0)\cup(0,\infty)$.