1) **Evaluation of Composite Functions**

If $f(x) = \sqrt{x}$ and $g(x) = x^3 - 2$, find the compositions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$.

2) **Domain of a Function**

Find the Domain and Range of the following Functions :

(a) $g(x) = (x^2 - 4)\sqrt{x + 5}$;

(b) $f(x) = (9 - x^2)^{3/2}$;

(c) $g(y) = \dfrac{y + 1}{(y + 2)(y - 3)}$;

(d) $F(w) = \sqrt[4]{2 - w}$;

(e) $f(x) = \dfrac{x^2 - 1}{x^3\sqrt{\ln x + 1}}$

3) **State the condition (criteria) that a function $f(x)$ has an inverse function $f^{-1}(x)$.**

4) **Suppose that**

$$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$$

(a) Find $\lim_{x \to 2} f(x)$.

(b) Is $f(x)$ continuous at $x = 2$?

(c) Is $f(x)$ differentiable at $x = 2$?

5) $$g(x) = \frac{x - 100}{\sqrt{x} - 10}$$

(a) **Find $\lim_{x \to 100^-} g(x)$.**

(b) **Find $\lim_{x \to 100^+} g(x)$.**

(c) **What is the value of $g(100)$ so that $g(x)$ is continuous at $x = 100$. Why?**

6) **Evaluate the Limits :**

(a) $$\lim_{h \to 0} \frac{3}{\sqrt{16 + 3h} + 4}$$

(b) $$\lim_{h \to 0} \frac{\frac{1}{5 + h} - \frac{1}{5}}{h}$$

(c) $$\lim_{x \to 4} \frac{3(x - 4)\sqrt{x + 5}}{3 - \sqrt{x + 5}}$$

(d) $$\lim_{x \to \infty} \left(5 + \frac{\sin x}{\sqrt{x}}\right)$$

(e) $$\lim_{x \to -\infty} \frac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}}$$

(f) $$\lim_{x \to 0^+} \left(\frac{1}{x}\right)^x$$

7) **If**

$$f(x) = \begin{cases} 0 & \text{if } x \leq -5 \\ \sqrt{25 - x^2} & \text{if } -5 < x < 5 \\ 3x & \text{if } x \geq 5. \end{cases}$$

Compute the following limits or state that they do not exist.

a. $\lim_{x \to -5^-} f(x)$ **b.** $\lim_{x \to -5^+} f(x)$ **c.** $\lim_{x \to -5} f(x)$

d. $\lim_{x \to 5^-} f(x)$ **e.** $\lim_{x \to 5^+} f(x)$ **f.** $\lim_{x \to 5} f(x)$

8) *Evaluate* $\lim_{x \to \infty} f(x)$ *and* $\lim_{x \to -\infty} f(x)$ *for*

(a) $$f(x) = \frac{\sqrt[3]{x^6 + 8}}{4x^2 + \sqrt{3x^4 + 1}}$$

(b) $$f(x) = 4x\left(3x - \sqrt{9x^2 + 1}\right)$$

9) $g(x)$ is definded by

$$g(x) = \begin{cases} 5x - 2 & \text{if } x < 1 \\ a & \text{if } x = 1 \\ ax^2 + bx & \text{if } x \geq 1. \end{cases}$$

Determine values of the constants a and b for which g is continuous at $x = 1$.

10) **Determining the unknown constant**

$$f(x) = \begin{cases} 2x^2 & \text{if } x \leq 1 \\ ax - 2 & \text{if } x > 1. \end{cases}$$

For what value of a for which $f'(1)$ exist ?

11) Find $\frac{dy}{dx}$, if :

(a) $y = e^{x^2+1} \sin x^3$

(b) $y = (1 + 2\tan x)^{15}$

(c) $y = sin^5(sin(e^x))$

(d) $y = tan\left(e^{\sqrt{3x}}\right)$

(f) $y = sin^2(e^{3x+1})$

(g) $y = ln(e^{sin3x} + x^2 + 7)$

(h) $f(x) = x^{\ln x}$

(i) $$f(x) = \frac{(x + 1)^{3/2}(x - 4)^{5/2}}{(5x + 3)^{2/3}}$$

12) Prove that $\lim_{x\to 0} sin\frac{1}{x}$ and $\lim_{x\to 0} cos\frac{1}{x}$ do not exist.

13) Given that $f(1) = 3$, $f'(1) = 1$, $g(1) = 2$, $g'(1) = 0$, $h(1) = 2$, $h'(1) = 5$.

(a) If $w(x) = \frac{f(x)}{g(x)}$, find $w'(1)$.

(b) If $S(x) = \frac{g(x)}{(f(x))^2}$, find $s'(1)$.

(c) If $m(x) = f(x)g(x)h(x)$, find $m'(1)$.

(d) If $n(x) = \frac{\left(f(x)^{\frac{5}{3}}\right)\left(g(x)^{30}\right)}{\sqrt{h(x)}}$, find $n'(1)$.

14) Find the equations of tangents to the following curves at the given points :

(a) $xy + x^{3/2}y^{-1/2} = 2;\ (1, 1)$

(b) $xy^{5/2} + x^{3/2}y = 12;\ (4, 1)$

(c) $(x^2 + y^2 - 2x)^2 = 2(x^2 + y^2);$ (2, 2)

(d) $\sqrt{3x^7 + y^2} = \sin^2 y + 100xy.$ (1, 0)

15) $f(x) = \sqrt{3x^2 + 4}$, Find $f'(2)$ and $f'(0)$.

16) **Find**

(a) $$\frac{d}{dx}(x^{\pi} + \pi^{x})$$

(b) $$\frac{d}{dx}\left(1 + \frac{1}{x}\right)^{x}$$

(c) $$\frac{d}{dx}(1 + x^{2})^{\sin x}$$

(d) $$\frac{d}{dx}\left(x^{(x^{10})}\right)$$

(e) $$\frac{d}{dx}(\ln x)^{x^{2}}$$

17) **Find the derivates of the following functions :**

(a) $f(x) = \tan^{-1}(e^{4x})$

(b) $f(u) = \csc^{-1}(2u + 1)$

(c) $f(x) = \sec^{-1}\sqrt{x}$

(d) $f(t) = \ln(\tan^{-1} t)$

18) (a)

Suppose the slope of the curve $y = f^{-1}(x)$ at $(4, 7)$ is $\frac{4}{5}$. Find $f'(7)$.

(b) Find $(f^{-1})'(3)$ if $f(x) = x^3 + x + 1$.

19) **Inverse functions**

Find the inverse functions and the erivatives of their inverse functions for the following functions :

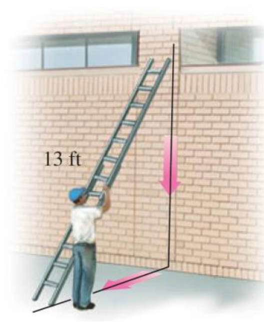
(a) $f(x) = \dfrac{x}{x+5}$

(b) $f(x) = x^3 + 3$

(c) $f(x) = |x + 2|$ **for** $x \geq -2$ **.**

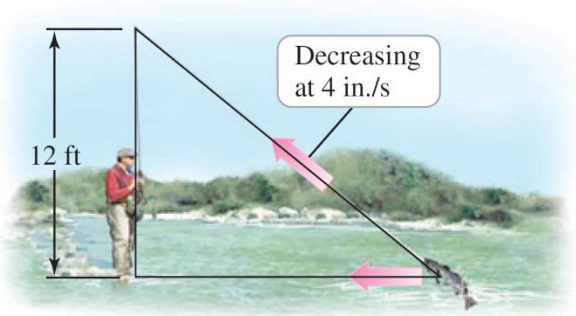
20) **Rate of Change**

Ladder against the wall A 13-foot ladder is leaning against a vertical wall (see figure) when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.5 ft/s. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?



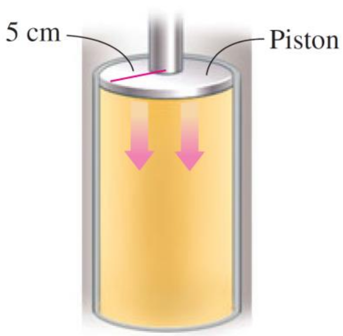
21)

Another fishing story An angler hooks a trout and reels in his line at 4 in./s. Assume the tip of the fishing rod is 12 ft above the water and directly above the angler, and the fish is pulled horizontally directly toward the angler (see figure). Find the horizontal speed of the fish when it is 20 ft from the angler.



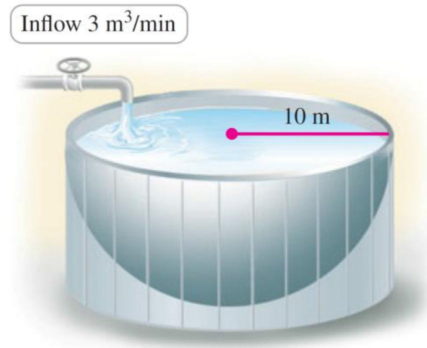
22)

Piston compression A piston is seated at the top of a cylindrical chamber with radius 5 cm when it starts moving into the chamber at a constant speed of 3 cm/s (see figure). What is the rate of change of the volume of the cylinder when the piston is 2 cm from the base of the chamber?



23)

Filling a hemispherical tank A hemispherical tank with a radius of 10 m is filled from an inflow pipe at a rate of 3 m^3/min (see figure). How fast is the water level rising when the water level is 5 m from the bottom of the tank? (*Hint:* The volume of a cap of thickness h sliced from a sphere of radius r is $\pi h^2(3r - h)/3$.)



24)

A lighthouse problem A lighthouse stands 500 m off of a straight shore, the focused beam of its light revolving four times each minute. As shown in the figure, P is the point on shore closest to the lighthouse and Q is a point on the shore 200 m from P. What is the speed of the beam along the shore when it strikes the point Q? Describe how the speed of the beam along the shore varies with the distance between P and Q. Neglect the height of the lighthouse.