1)

$$f(x) = \frac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}}.$$

SOLUTION The square root in the denominator forces us to revise the strategy used with rational functions. First, consider the limit as $x \to \infty$. The highest power of the polynomial in the denominator is 6. However, the polynomial is under a square root, so we divide the numerator and denominator by $\sqrt{x^6} = x^3$, for $x \geq 0$. The limit is evaluated as follows:

$$\lim_{x\to\infty} \frac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}} = \lim_{x\to\infty} \frac{\frac{10x^3}{x^3} - \frac{3x^2}{x^3} + \frac{8}{x^3}}{\sqrt{\frac{25x^6}{x^6} + \frac{x^4}{x^6} + \frac{2}{x^6}}} \quad \text{Divide by } \sqrt{x^6} = x^3.$$

$$= \lim_{x\to\infty} \frac{10 - \overbrace{\frac{3}{x}}^{\text{approaches } 0} + \overbrace{\frac{8}{x^3}}^{\text{approaches } 0}}{\sqrt{25 + \underbrace{\frac{1}{x^2}}_{\text{approaches } 0} + \underbrace{\frac{2}{x^6}}_{\text{approaches } 0}}} \quad \text{Simplify.}$$

$$= \frac{10}{\sqrt{25}} = 2. \quad \text{Evaluate limits.}$$

As $x \to -\infty$, x^3 is negative, so we divide numerator and denominator by $\sqrt{x^6} = -x^3$ (which is positive):

$$\lim_{x\to-\infty} \frac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}} = \lim_{x\to-\infty} \frac{\frac{10x^3}{-x^3} - \frac{3x^2}{-x^3} + \frac{8}{-x^3}}{\sqrt{\frac{25x^6}{x^6} + \frac{x^4}{x^6} + \frac{2}{x^6}}} \quad \text{Divide by } \sqrt{x^6} = -x^3 > 0.$$

$$= \lim_{x\to-\infty} \frac{-10 + \overbrace{\frac{3}{x}}^{\text{approaches } 0} - \overbrace{\frac{8}{x^3}}^{\text{approaches } 0}}{\sqrt{25 + \underbrace{\frac{1}{x^2}}_{\text{approaches } 0} + \underbrace{\frac{2}{x^6}}_{\text{approaches } 0}}} \quad \text{Simplify.}$$

$$= \frac{-10}{\sqrt{25}} = -2. \quad \text{Evaluate limits.}$$

The limits reveal two asymptotes, $y = 2$ and $y = -2$. Observe that the graph crosses both horizontal asymptotes (Figure 2.39).

➤ Recall that

$$\sqrt{x^2} = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Therefore,

$$\sqrt{x^6} = |x^3| = \begin{cases} x^3 & \text{if } x \geq 0 \\ -x^3 & \text{if } x < 0 \end{cases}$$

Because x is negative as $x \to -\infty$, we have $\sqrt{x^6} = -x^3$.

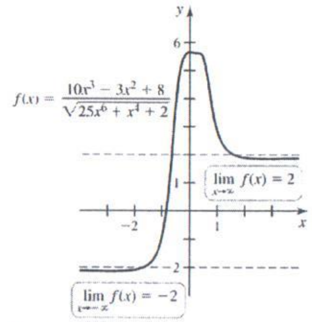


FIGURE 2.39

2(a) $f(x) = e^{\frac{1}{x}}$

when $x = 0$, $f(x)$ is undefinded.

$\lim_{x\to0^+} f(x) = +\infty$, $\lim_{x\to0^-} f(x) = \lim_{t\to0^+} f(-t) = \lim_{t\to0^+} e^{\frac{1}{-t}}$

$= \lim_{t\to0^+} \frac{1}{e^{\frac{1}{t}}} =$

$\therefore x=0$ is a vertical asymptote #

when $x \to \infty$, $f(x) \to 1$

when $x \to -\infty$, $\lim_{x\to-\infty} f(x) = \lim_{t\to\infty} e^{\frac{1}{-t}} = \lim_{t\to\infty} \frac{1}{e^{\frac{1}{t}}} = 1$

$\therefore$ there is only one horizontal Asymptote, $y = 1$ #

2(b) $f(x) = \frac{|1-x^2|}{x(x+1)} = \frac{|(1+x)(1-x)|}{x(x+1)} = \begin{cases} \frac{1-x}{x}, & x > 1 \\ \frac{x-1}{x}, & -1 < x < 1 \\ \frac{1-x}{x}; & x < -1 \end{cases}$

when $\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-} \frac{1-x}{x} = 0$

$\lim_{x\to-1^+} f(x) = \lim_{x\to-1^+} f(x) = \frac{x-1}{x} = 0$

$\Big\}$ $f(x)$ has a Removable discontinuity. at $x=-1$

$\therefore f(x)$ has a vertical asymptote $x = 0$. #

$f(x) = \frac{\left|\frac{1}{x^2} - 1\right|}{(1+\frac{1}{x})}$, $\lim_{x\to\infty} f(x) = 1/1 = 1$

$\lim_{x\to-\infty} f(x) = \lim_{t\to\infty} f(-t) = \lim_{t\to\infty} \frac{\left|\frac{1}{t^2} - 1\right|}{(1-\frac{1}{t})} = 1$

$\Big\}$ $y=1$ is a horizontal asymptote

2(c) $f(x)=\dfrac{\sqrt{16x^4+16x^2}+x^2}{2x^2-4}$

$= \dfrac{x\left(4\sqrt{x^2+1}+x\right)}{2(x^2-2)}$

$\therefore$ $f(x)$ has 2 vertical Asymptotes $x=\sqrt{2}$ and $x=-\sqrt{2}$ #

$f(x)=\dfrac{4\sqrt{1+\frac{1}{x^2}}+1}{2-\frac{4}{x^2}}$

$\lim\limits_{x\to\infty} f(x)=\dfrac{4\sqrt{1+0}+1}{2-0}=\dfrac{5}{2}$

$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{t\to\infty}\dfrac{4\sqrt{1+\frac{1}{t^2}}+1}{2-\frac{4}{t^2}}$

$=\dfrac{4\sqrt{1+0}+1}{2-0}=\dfrac{5}{2}$

$\therefore$ $f(x)$ has only one horizontal Asymptote $y=5/2$ #

3(a) $\lim\limits_{x\to\infty}\left(\sqrt{|x|}-\sqrt{|x-1|}\right)$

$=\lim\limits_{x\to\infty}\left(\sqrt{x}-\sqrt{x-1}\right)\left(\dfrac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}\right)$

$=\lim\limits_{x\to\infty}\dfrac{x-(x-1)}{\sqrt{x}+\sqrt{x-1}}$

$=\lim\limits_{x\to\infty}\dfrac{1}{\sqrt{x}+\sqrt{x-1}}=0$ #

3(b) $\lim\limits_{x\to-\infty}\left(\sqrt{|x|}-\sqrt{|x-1|}\right)$

$=\lim\limits_{t\to\infty}\left(\sqrt{|-t|}-\sqrt{|-t-1|}\right)$

$=\lim\limits_{t\to\infty}\left(\sqrt{t}-\sqrt{t+1}\right)$

$=\lim\limits_{t\to\infty}\dfrac{\left(\sqrt{t}-\sqrt{t+1}\right)\left(\sqrt{t}+\sqrt{t+1}\right)}{\left(\sqrt{t}+\sqrt{t+1}\right)}$

$=\lim\limits_{t\to\infty}\dfrac{t-(t+1)}{\sqrt{t}+\sqrt{t+1}}=\dfrac{-1}{\sqrt{t}+\sqrt{t+1}}$

$=0$ #

4(a) $f(x)=\dfrac{x^2-7x+10}{x-2}=\dfrac{(x-2)(x-5)}{x-2}$

$\lim\limits_{x\to2^-}f(x)=\lim\limits_{x\to2^-}\dfrac{(x-2)(x-5)}{(x-2)}$

$=\lim\limits_{x\to2^-}(x-5)=-3.$

$\lim\limits_{x\to2^+}f(x)=\lim\limits_{x\to2^+}\dfrac{(x-2)(x-5)}{x-2}=-3.$

$\therefore \lim\limits_{x\to2}f(x)\ \underline{\underline{\text{exist}}}\ -3$

Set $f(2)=3$. $\Rightarrow$ $f(x)$ is continuous at $x=2$.

$\therefore$ $f(x)$ has a removable discontinuity

4(b) $g(x)=x^3\sin\frac{1}{x^2}$

As $-1\le\sin\frac{1}{x^2}\le1$

$-x^3\le x^3\sin\frac{1}{x^2}\le x^3$

But $\lim\limits_{x\to0}(-x^3)=0=\lim\limits_{x\to0}x^3$

$\therefore \lim\limits_{x\to0}x^3\sin\frac{1}{x^3}=0$

(By sandwich thm)

$\therefore \lim\limits_{x\to0}g(x)=0$.

we set $g(0)=0$.

$\Rightarrow$ $g(x)$ is continuous at $x=0$.

i.e. $g(x)$ has a removable discontinity at $x=0$.

5) let $y=\frac{\pi}{2}-x$

$\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\frac{1}{\sqrt{\sin x}}-1}{x-\frac{\pi}{2}}\right)=\lim\limits_{y\to0}\dfrac{\frac{1}{\sqrt{\sin(\frac{\pi}{2}-y)}}-1}{-y}$

$=\lim\limits_{y\to0}\dfrac{\frac{1}{\sqrt{\cos y}}-1}{-y}=\lim\limits_{y\to0}\dfrac{\sqrt{\cos y}-1}{y\sqrt{\cos y}}$

$=\lim\limits_{y\to0}\dfrac{\cos y-1}{y\sqrt{\cos y}\left(\sqrt{\cos y}+1\right)}$

Consider $\dfrac{\cos y-1}{y}=\dfrac{\left(1-2\sin^2\frac{y}{2}\right)-1}{y}$

$=\dfrac{-2\sin^2\frac{y}{2}}{y}=-\left(\dfrac{\sin\frac{y}{2}}{\frac{y}{2}}\right)\left(\sin\frac{y}{2}\right)$

$\therefore \lim\limits_{y\to0}\left(\dfrac{\cos y-1}{y}\right)=\lim\limits_{y\to0}\left(\dfrac{-\sin\frac{y}{2}}{\frac{y}{2}}\right)\left(\sin\frac{y}{2}\right)$

$=(-1)(0)=0$

$\therefore$ the required limit $=\lim\limits_{y\to0}\left(\dfrac{\cos y-1}{y}\right)\left(\dfrac{1}{\sqrt{\cos y}}\right)\left(\dfrac{1}{\sqrt{\cos y}+1}\right)=0$ #

6) $1\le g(x)\le\sin^2x+1$

$\lim\limits_{x\to0}1=1$

$\lim\limits_{x\to0}\left(\sin^2x+1\right)=0+1=1$

By Squeeze thm

$\lim\limits_{x\to0}g(x)=1$ #

7)(a) $y=f(x)=x^2-4$

$\left.\dfrac{dy}{dx}\right|_{(2,0)}=2x=4$

$\therefore$ The slope of tangent $=\left.\dfrac{dy}{dx}\right|_{(2,0)}=4$

The required tangent is

$y-0=4(x-2)$

$\Rightarrow y=4x-8$ #

7.(b) $y=f(x)=\sqrt{x+3}$

$\dfrac{dy}{dx}=\dfrac{d(x+3)^{\frac{1}{2}}}{dx}$

$=\dfrac{d\,(x+3)^{\frac{1}{2}}}{d(x+3)}\times\dfrac{d(x+3)}{dx}$

$=\frac{1}{2}(x+3)^{\frac{1}{2}-1}(1)$

$=\dfrac{1}{2\sqrt{x+3}}$

$\left.\dfrac{dy}{dx}\right|_{(1,2)}=\dfrac{1}{2\sqrt{1+3}}=\dfrac{1}{4}$

= Slope of the required tangent.

$\therefore$ The required tangent is

$y-2=\frac{1}{4}(x-1)$

$\Rightarrow 4y-8=x-1$

$\Rightarrow 4y=x+7$ #.

8(a) $y=f(x)=x^2+1$

$\dfrac{dy}{dx}=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to0}\dfrac{\left((x+h)^2+1\right)-\left(x^2+1\right)}{h}$

$=\lim\limits_{h\to0}\dfrac{\left(x^2+2hx+h^2-x^2\right)}{h}$

$=\lim\limits_{h\to0}\dfrac{2hx+h^2}{h}$

$=\lim\limits_{h\to0}\dfrac{h(2x+h)}{h}$

$=\lim\limits_{h\to0}(2x+h)$

$=2x$ #

8(b) $y=f(x)=\sqrt{3x+1}$

$\dfrac{dy}{dx}=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to0}\dfrac{\sqrt{3(x+h)+1}-\sqrt{3x+1}}{h}$

let $A=\sqrt{3x+3h+1}$

$B=\sqrt{3x+1}$

(By $(A-B)\left(\dfrac{A+B}{A+B}\right)=A-B$
$\quad \| $
$\dfrac{A^2-B^2}{A+B}$)

We have

$\dfrac{dy}{dx}=\lim\limits_{h\to0}\dfrac{(3(x)+3h+1)-(3x+1)}{\left(\sqrt{3x+3h+1}+\sqrt{3x+1}\right)h}$

$=\lim\limits_{h\to0}\dfrac{3h}{h\left(\sqrt{3x+3h+1}+\sqrt{3x+1}\right)}$

$=\lim\limits_{h\to0}\dfrac{3}{\sqrt{3x+3h+1}+\sqrt{3x+1}}$

$=\dfrac{3}{\sqrt{3x+1}+\sqrt{3x+1}}$

$=\dfrac{3}{2\sqrt{3x+1}}$ #

9) $f(x)=\sqrt{x+2}$

$f'(2)=\lim\limits_{h\to0}\dfrac{f(2+h)-f(2)}{h}$

$=\lim\limits_{h\to0}\dfrac{\sqrt{2+h2}-\sqrt{2+2}}{h}$

$=\lim\limits_{h\to0}\dfrac{\sqrt{4+h}-\sqrt{4}}{h}$

$=\lim\limits_{h\to0}\dfrac{(4+h)-(4)}{h\left(\sqrt{4+h}+\sqrt{4}\right)}$

$=\lim\limits_{h\to0}\dfrac{\cancel{h}}{\cancel{h}\left(\sqrt{4+h}+\sqrt{4}\right)}$

$=\dfrac{1}{\sqrt{4}+\sqrt{4}}=\dfrac{1}{2\sqrt{4}}$ #

10(a) $\dfrac{d\,e^{2014}}{dx}=0$

(b) $\dfrac{d}{dx}\left(\dfrac{x^{\frac{24}{5}}}{1984}\right)=\dfrac{1}{1984}\left(\dfrac{24}{5}\right)x^{\frac{24}{5}-1}$

$=\dfrac{1}{1984}\left(\dfrac{24}{5}\right)x^{\frac{19}{5}}$

(c) $\dfrac{dw^7}{dw}=7w^6$

(d) $\dfrac{d(3x^5+5e^x)}{dx}$

$=3\dfrac{dx^5}{dx}+5\dfrac{de^x}{dx}$

$=3(5x^4)+5e^x$

$=15x^4+5e^x$

(e) $\dfrac{d}{dx}\left(\dfrac{4x^3+3x-2}{x^2+1}\right)$

$=\dfrac{(x^2+1)\frac{d}{dx}(4x^3+3x-2)-(4x^3+3x-2)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$

$=\dfrac{(x^2+1)(12x^2+3)-(4x^3+3x-2)(2x)}{(x^2+1)^2}$

(f) $h(x)=(5x^7+5x)(6x^3+3x^2+3)$

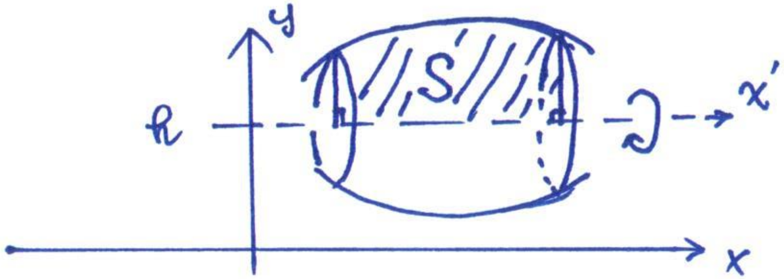
$\dfrac{dh(x)}{dx}=(5x^7+5x)\dfrac{d}{dx}(6x^3+3x^2+3)+(6x^3+3x^2+3)\dfrac{d}{dx}(5x^7+5x)$

$=(5x^7+5x)(18x^2+6x)+(6x^3+3x^2+3)(35x^6+5)$

1) Revolving about the line $y = h$.

Let $y = f(x)$ be a function continuous on [a, b] and let **S** be the region bounded by the curve $y = f(x)$ and the line $x = a$, $x = b$ and $y = h$. Then the volume of the solid generated by revolving the region **S** one complete revolution about the line $y = h$ is given by

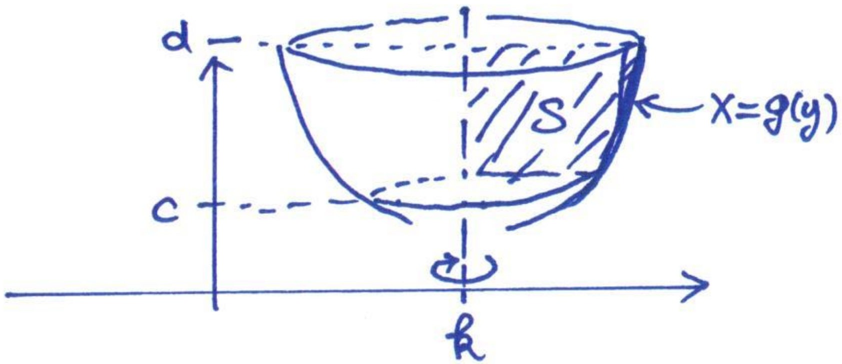
$$V = \int_a^b \pi(y-h)^2 dx = \int_a^b \pi(f(x)-h)^2 dx \ .$$



2) Revolving about the line $x = k$.

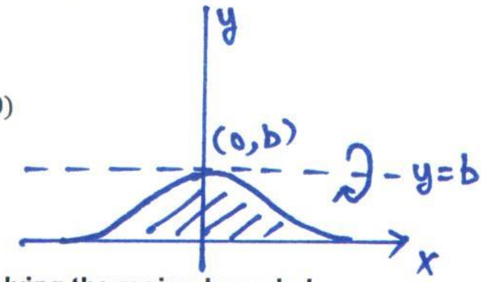
Let $x = g(y)$ be a function continuous on [c, d] and let **S** be the region bounded by the curve $x = g(y)$ and the line $y = c$, $y = d$ and $x = k$. Then the volume of the solid generated by revolving the region **S** one complete revolution about the line $x = k$ is given by

$$V = \int_c^d \pi(x-k)^2 dy = \int_a^b \pi(g(y)-k)^2 dy \ .$$

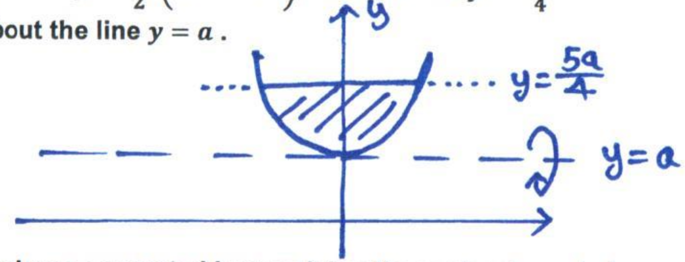


3) Exercises :

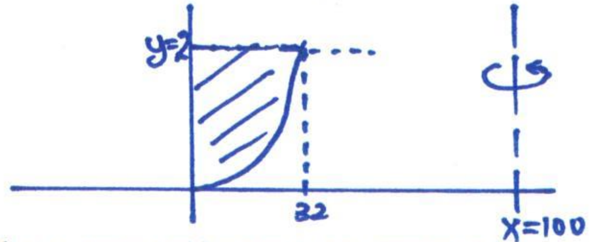
1. Find the volume of the solid generated by revolving one complete revolution of the upper half region of the close curve $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^{\frac{3}{2}} = 1$ about the line $y = b$. $(a > 0,\ b > 0)$



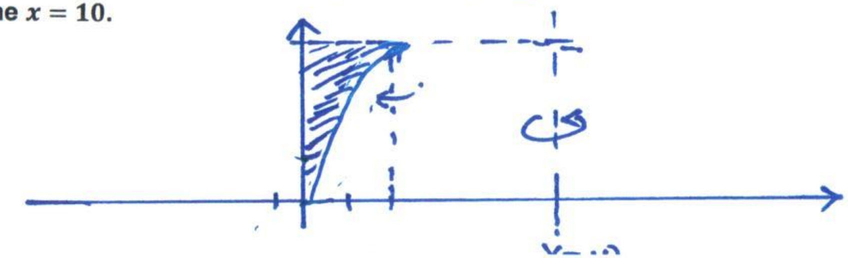
2. Find the volume generated by revolving the region bounded by the curve $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$ and the line $y = \frac{5a}{4}$ $(a > 0)$ about the line $y = a$.



3. Find the volume generated by revolving the region bounded by the curve $y = x^{\frac{1}{5}}$, the y-axis and $y = 2$ about the line $x = 100$.

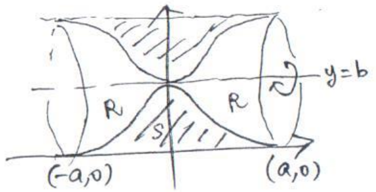


4. Find the volume generated by revolving the region bounded by the curve $x = y^4$, the y-axis and $y = 1$ about the line $x = 10$.



Solutions to Math 1014 (Volume of Revolution about other axis) (Extra Tutorial)

1)



$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$$

$$\Rightarrow y = \frac{(a^2-x^2)^{\frac{3}{2}} b}{a^3}$$

$$y-b = \frac{(a^2-x^2)^{\frac{3}{2}} b - a^3 b}{a^3}$$

The Volume V_1 of the Solid generated by revolving the Region R about the line $y=b$ is (The hollow volume)

$$V_1 = \pi \int_{-a}^{a} (y-b)^2 dx = 2\pi \int_0^a \left(\frac{(a^2-x^2)^{\frac{3}{2}} b - a^3 b}{a^3}\right)^2 dx$$

$$= \frac{2\pi b^2}{a^6} \int_0^a \left[(a^2-x^2)^3 - 2a^3(a^2-x^2)^{\frac{3}{2}} + a^6\right] dx$$

$$= \frac{2\pi b^2}{a^6} \left[\begin{array}{l} \left(a^6 x - 3a^4\left(\frac{x^3}{3}\right) + 3a^2\left(\frac{x^5}{5}\right) - \frac{x^7}{7}\right) \\ -2a^3\left\{\frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\sin^{-1}\frac{x}{a}\right\} \\ + a^6 x \end{array} \right]_0^a$$

$$= \frac{2\pi b^2}{a^2}\left[a^7 - a^7 + \frac{3a^7}{5} - \frac{a^7}{7} - 2a^3\left(0 + \frac{3a^4}{8}\cdot\frac{\pi}{2}\right) + a^7\right]$$

$$= 2\pi a b^2 \left(\frac{51}{35} - \frac{3\pi}{8}\right).$$

The Volume V_2 of the circular cylinder generated by revolving the region R+S. about the line $y=b$ is

$$V_2 = 2\pi a b^2$$

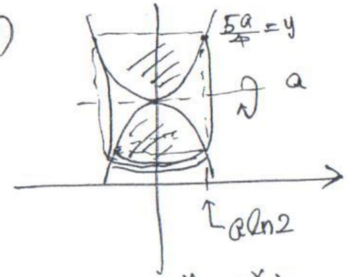
$\therefore$ The Required volume is

$$V_2 - V_1 = 2\left[\pi a b^2 - \pi a b^2\left(\frac{51}{35} - \frac{3\pi}{8}\right)\right]$$

$$= \left(\frac{3\pi}{4} - \frac{32}{35}\right)\pi a b^2 \ \#$$

Solutions to Math 1014 (Volume of Revolution about other axis) (Extra Tutorial)

2)



$$\begin{cases} y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right), \\ y = \frac{5a}{4} \end{cases}$$

$$\frac{5}{4} = \frac{1}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$$

$$2e^{\frac{2x}{a}} - 5e^{\frac{2x}{a}} + 2 = 0$$

$$\left(2e^{\frac{x}{a}} - 1\right)\left(e^{\frac{x}{a}} - 2\right) = 0$$

$$e^{\frac{x}{a}} = \frac{1}{2} \text{ or } 2$$

As the solid is symmetric about y, Volume of solid (about $y=a$) is.

$$V = 2\pi \int_0^{a\ln 2} \Big[\underbrace{\left(\frac{5a}{4} - a\right)^2}_{\text{(cylinder volume)}} - \underbrace{(y-a)^2}_{\text{(hollow volume)}}\Big] dx$$

$$= 2\pi \int_0^{a\ln 2} \left\{\left(\frac{a}{4}\right)^2 - \left[\frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right) - a\right]^2\right\} dx$$

$$= 2\pi \int_0^{a\ln 2} \left[\frac{a^2}{16} - \frac{a^2}{4}\left(e^{\frac{2x}{a}} + e^{\frac{-2x}{a}} + 2\right) - a^2 + a^2\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)\right] dx$$

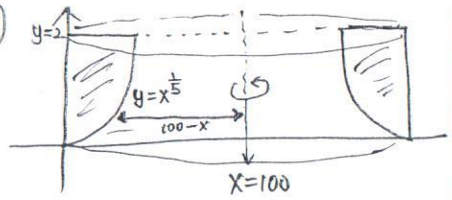
$$= 2\pi \int_0^{a\ln 2} \left[\frac{-23}{16}a^2 - \frac{a^2}{4}\left(e^{\frac{2x}{a}} + e^{\frac{-2x}{a}}\right) + a^2\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)\right] dx$$

$$= 2\pi \left[\frac{-23}{16}a^2 x - \frac{a^2}{4}\left(\frac{a}{2}\right)e^{\frac{2x}{a}} + \frac{a^2}{4}\left(\frac{a}{2}\right)e^{\frac{-2x}{a}} + a^3 e^{\frac{x}{a}} - a^3 e^{\frac{-x}{a}}\right]_0^{a\ln 2}$$

$$= 2\pi a^2\left[\frac{-23}{16}\ln 2 - \frac{4}{8} + \frac{1}{32} + 2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{8} - 1 + 1\right] = \left(\frac{33}{16} - \frac{23}{8}\ln 2\right)\pi a^3 \ \#$$

Solutions to Math 1014 (Volume of Revolution about other axis) (Extra Tutorial)

3)



$y=x^{\frac{1}{5}} \Rightarrow x=y^5$

$\therefore$ The hollow volume

$V_1 = \int_0^2 \pi(100-x)^2 dy$

$= \int_0^2 \pi(100-y^5)^2 dy$

$= \int_0^2 \pi[10^4 - 200y^5 + y^{10}]dy$

$= \pi\left[10^4 y - \frac{200}{6}y^6 + \frac{y^{11}}{11}\right]\Big|_0^2$

$= \frac{595744\pi}{33}$

Let V_2 be the volume of the cylinder contains both the hollow volume and the volume of the solid.

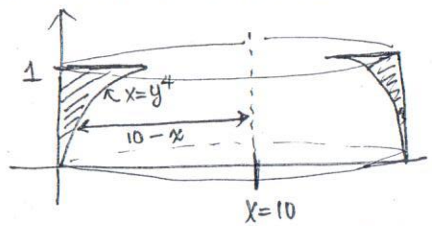
$V_2 = \pi(100)^2(2) = 20000\pi$

$\therefore$ The required volume is

$V_2 - V_1 = \left(20000\pi - \frac{595744}{33}\pi\right)$

$= \frac{64256}{33}\pi$ #

4)



The volume of the hollow part is

$V_1 = \pi \int_0^1 (10-x)^2 dy$

$= \pi \int_0^1 (10-y^4)^2 dy$

$= \pi \int_0^1 (100-20y^4+y^8)\, dy$

$= \pi\left[100y - 4y^5 + \frac{y^9}{9}\right]\Big|_0^1$

$= \pi\left[100 - 4 + \frac{1}{9}\right]$

$= \frac{865}{9}\pi$

V_2 = the volume of the cylinder contains both the required solide and the hollow part

$V_2 = \pi(10)^2(1) = 100\pi$

$\therefore$ The Required Volume is

$V_2 - V_1 = 100\pi - \frac{865}{9}\pi$

$= \frac{35}{9}\pi$ #