1) Use the result $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1$, or otherwise, to evaluate the following limits:

(a) $\lim_{x\to 0}\left(\frac{\tan 2x}{\sin x}\right)$ , (b) $\lim_{x\to 0}\left(\frac{\tan\ nx}{\sin x}\right)$

(c) $\lim_{x\to 0}\left(\frac{\sin 4x}{x}\right)$ (d) $\lim_{x\to 0}\left(\frac{\sin px}{\sin qx}\right)$

(e) $\lim_{x\to 0}\left(\frac{\cos^2 x-1}{x}\right)$ (f) $\lim_{x\to -3}\left(\frac{\sin(x+3)}{x^2+8x+15}\right)$

(g) $\lim_{x\to 2}\left(\frac{\sin(x-2)}{x^2-4}\right)$ (h) $\lim_{x\to 0}\left(\frac{x^2\sin\left(\frac{1}{x}\right)}{\sin x}\right)$

(i) $\lim_{x\to \frac{\pi}{2}}\left(\frac{\cos x}{x-\frac{\pi}{2}}\right)$ (j) $\lim_{x\to 0}\left(\frac{6x-\sin 2x}{2x-3\sin 4x}\right)$

(k) $\lim_{x\to 1}\left(\frac{3\sin \pi x-\sin 3\pi x}{x^3}\right)$

2) Evaluate $\lim_{x\to \infty}\left(\frac{\cos x}{x}\right)$. (Hints: By Squeeze Theorem)

3) Investigate the continuity of the function

$$f(x) = \begin{cases} \sin \pi x, & 0 < x < 1 \\ \ln x, & 1 < x < 2 \end{cases}$$ at the point $x = 1$ .

4) Prove by the definition of derivative that $\frac{d}{dx}(x^4) = 4x^3$ .

5) Evaluate $\frac{d}{dv}\left(v^2(2\sqrt{v}+1)\right)$

6) The following limits equal the derivative of a function $f$ at a point . Find one possible $f$ and $a$ for each case and evaluate the limits.

(a) $\lim_{h\to 0}\left(\frac{\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}}{h}\right)$, (b) $\lim_{h\to 0}\left(\frac{\cos\left(\frac{\pi}{6}+h\right)-\frac{\sqrt{3}}{2}}{h}\right)$,

(c) $\lim_{x\to \frac{\pi}{4}}\left(\frac{\cot x-1}{x-\frac{\pi}{4}}\right)$, (d) $\lim_{h\to 0}\left(\frac{\tan\left(\frac{5\pi}{6}+h\right)+\frac{1}{\sqrt{3}}}{h}\right)$.

7) **Continuity & Differentiability**

Let $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$.

(a) Is $f(x)$ continuous at $x = 0$ ?
(b) Does $f(x)$ have a derivative at $x = 0$ ?

8) Let $f(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$.

(a) Is $f(x)$ differentiable at $x = 0$ ?
(b) Is $f'(x)$ continuous at $x = 0$ ?

9) Differentiate the functions :

(a) $y = x^{5/3} - x^{2/3}$

(b) $y = \frac{x^2+4x+3}{\sqrt{x}}$

(c) $F(x) = (4x - x^2)^{100}$

(d) $v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2$

(e) $f(x) = x^4 e^x$

(f) $f(t) = \tan(e^t) + e^{\tan t}$

(g) $y = \sqrt{1 + xe^{-2x}}$

(h) $f(t) = \sin^2(e^{\sin^2 t})$

(i) $y = \cos\sqrt{\sin(\tan \pi x)}$

**10)** **Find the equation of tangent line and normal line to the given curve at the specified point.**

(a) $y = 2xe^x$ **at (0, 0) ,**

(b) $y = \frac{2x}{x^2+1}$ **at (1, 1).**

(c) $y = \sin(\sin x)$ **,** $(\pi,\ 0)$

**11)** **Find** $R'(0)$ **, where**

$$R(x) = \frac{x - 3x^3 + 5x^5}{1 + 3x^3 + 6x^6 + 9x^9}$$

*Hint:* Instead of finding $R'(x)$ first, let $f(x)$ be the numerator and $g(x)$ the denominator of $R(x)$ and compute $R'(0)$ from $f(0)$, $f'(0)$, $g(0)$, and $g'(0)$.

**12)** **Logarithmic Differentiation**

Differentiate $y = \dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$.

**(Hints : Taking the Logarithms to both sides)**

**13)** **Given that the velocity** $v = \frac{ds}{dt}$ **,**

The displacement of a particle on a vibrating string is given by the equation $s(t) = 10 + \frac{1}{4}\sin(10\pi t)$ where $s$ is measured in centimeters and $t$ in seconds. Find the velocity of the particle after $t$ seconds.

**14)** **Spread of Rumor**

Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}}$$

where $p(t)$ is the proportion of the population that knows the rumor at time $t$ and $a$ and $k$ are positive constants. [In Section 9.4 we will see that this is a reasonable equation for $p(t)$.]

(a) Find $\lim_{t\to\infty} p(t)$.

(b) Find the rate of spread of the rumor.

**15)** **Equation of tangent to a hyperbola**

Find an equation of the tangent line to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point $(x_0, y_0)$.