Math 1013 T11 (Tutorial 1)
(Appendix A Numbers, Inequalities, and Absolute Values)

1) Real Number System:

(a) Rational number $Q = \left\{ r = \frac{n}{m},\ \text{where } n \text{ and } m \text{ are integers and } m \neq 0 \right\}$

(b) Irrational number $= R \backslash Q$

2) Rational number can be express as finite decimals or recurrence decimals.

Eg. $x = 2.31456173561735617356173\ldots = 2.314\overline{56173}$

$(10^5)(x) - x = 231456.173 - 2.314$ can be expressed as a fraction form.

3) Intervals

(a) Open Interval : $(a, b) = \{x|\ a < x < b\}$

(b) Closed interval : $[a,\ b] = \{x|\ a \le x \le b\}$

(c) Half Open and Half Closed Interval :

$(a, b] = \{x|\ a < x \le b\}$

4) Inequalities

(a) Rules of Inequalities:

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$ and $c < d$, then $a + c < b + d$.
3. If $a < b$ and $c > 0$, then $ac < bc$.
4. If $a < b$ and $c < 0$, then $ac > bc$.
5. If $0 < a < b$, then $1/a > 1/b$.

(b) Solutions of inequalities :

(i) Solve $(x-\alpha)(x-\beta) > 0$ where $\alpha < \beta$.

Key : $x < \alpha$ or $x > \beta$.

(ii) Solve $(x-\alpha)(x-\beta) < 0$ where $\alpha < \beta$.

Key : $\alpha < x < \beta$.

(iii) Solve $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) > 0$ where $\alpha < \beta < \gamma < \delta$.

Key : $x < \alpha$, $\beta < x < \gamma$ or $x > \delta$.

(iv) Solve $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)^2 > 0$ where $\alpha < \beta < \gamma < \delta$.

Key : $\alpha < x < \beta$ or $x > \gamma$ and $x \neq \delta$).

5) Examples of inequalities:

Example 1

Solve the inequalities $4 \leqslant 3x - 2 < 13$.

SOLUTION Here the solution set consists of all values of x that satisfy both inequalities. Using the rules given in $\boxed{2}$, we see that the following inequalities are equivalent:

$$4 \leqslant 3x - 2 < 13$$

$$6 \leqslant 3x < 15 \qquad \text{(add 2)}$$

$$2 \leqslant x < 5 \qquad \text{(divide by 3)}$$

Therefore the solution set is $[2, 5)$.

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Example 2

Solve the inequality $x^2 - 5x + 6 \leqslant 0$.

SOLUTION First we factor the left side:

$$(x - 2)(x - 3) \leqslant 0$$

We know that the corresponding equation $(x - 2)(x - 3) = 0$ has the solutions 2 and 3. The numbers 2 and 3 divide the real line into three intervals:

$$(-\infty, 2) \qquad (2, 3) \qquad (3, \infty)$$

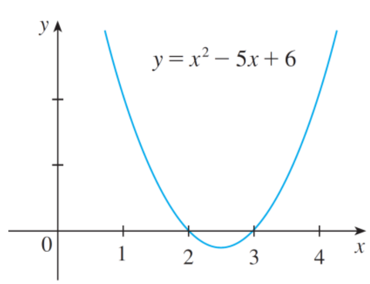
On each of these intervals we determine the signs of the factors. For instance,

$$x \in (-\infty, 2) \quad \Rightarrow \quad x < 2 \quad \Rightarrow \quad x - 2 < 0$$

Then we record these signs in the following chart:

Interval	$x - 2$	$x - 3$	$(x - 2)(x - 3)$
$x < 2$	$-$	$-$	$+$
$2 < x < 3$	$+$	$-$	$-$
$x > 3$	$+$	$+$	$+$

A visual method for solving Example 3 is to use a graphing device to graph the parabola $y = x^2 - 5x + 6$ (as in Figure 4) and observe that the curve lies on or below the x-axis when $2 \leqslant x \leqslant 3$.



Example 3

Solve $x^3 + 3x^2 > 4x$.

SOLUTION First we take all nonzero terms to one side of the inequality sign and factor the resulting expression:

$$x^3 + 3x^2 - 4x > 0 \qquad \text{or} \qquad x(x - 1)(x + 4) > 0$$

As in Example 3 we solve the corresponding equation $x(x - 1)(x + 4) = 0$ and use the solutions $x = -4$, $x = 0$, and $x = 1$ to divide the real line into four intervals $(-\infty, -4)$, $(-4, 0)$, $(0, 1)$, and $(1, \infty)$. On each interval the product keeps a constant sign as shown in the following chart:

Interval	x	$x - 1$	$x + 4$	$x(x - 1)(x + 4)$
$x < -4$	$-$	$-$	$-$	$-$
$-4 < x < 0$	$-$	$-$	$+$	$+$
$0 < x < 1$	$+$	$-$	$+$	$-$
$x > 1$	$+$	$+$	$+$	$+$

Then we read from the chart that the solution set is

$$\{x \mid -4 < x < 0 \text{ or } x > 1\} = (-4, 0) \cup (1, \infty)$$

5) Absolute Value

The **absolute value** of a number a, denoted by $|a|$, is the distance from a to 0 on the real number line. Distances are always positive or 0, so we have

$$|a| \geqslant 0 \qquad \text{for every number } a$$

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In General,

$$|a| = a \qquad \text{if } a \geqslant 0$$
$$|a| = -a \qquad \text{if } a < 0$$

(a) Properties of absolute values:

Suppose that *a* and *b* are eal numbers, n is an integer.
Then

1. $|ab| = |a||b|$ **2.** $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|} \quad (b \neq 0)$

3. $|a^n| = |a|^n$

Suppose $a > 0$. Then

4. $|x| = a$ if and only if $x = \pm a$

5. $|x| < a$ if and only if $-a < x < a$

6. $|x| > a$ if and only if $x > a$ or $x < -a$

(b) Examples

(i) Example 1

Solve $|x - 5| < 2$.

SOLUTION 1 By Property 5 of $\boxed{6}$, $|x - 5| < 2$ is equivalent to

$$-2 < x - 5 < 2$$

Therefore, adding 5 to each side, we have

$$3 < x < 7$$

and the solution set is the open interval (3, 7).

(ii) Example 2

Solve $|3x + 2| \geqslant 4$.

SOLUTION By Properties 4 and 6 of $\boxed{6}$, $|3x + 2| \geqslant 4$ is equivalent to

$$3x + 2 \geqslant 4 \qquad \text{or} \qquad 3x + 2 \leqslant -4$$

In the first case $3x \geqslant 2$, which gives $x \geqslant \frac{2}{3}$. In the second case $3x \leqslant -6$, which gi $x \leqslant -2$. So the solution set is

$$\{x \mid x \leqslant -2 \text{ or } x \geqslant \tfrac{2}{3}\} = (-\infty, -2] \cup [\tfrac{2}{3}, \infty)$$

(c) Triangular Inequality

If a and b are any real numbers, then

$$|a + b| \leqslant |a| + |b|$$

Example 1

If $|x - 4| < 0.1$ and $|y - 7| < 0.2$, use the Triangular Inequality to estimate $|(x + y) - 11|$.

Solution:

In order to use the given information, we use the Triangle Inequality \ $a = x - 4$ and $b = y - 7$:

$$\begin{aligned} |(x + y) - 11| &= |(x - 4) + (y - 7)| \\ &\leqslant |x - 4| + |y - 7| \\ &< 0.1 + 0.2 = 0.3 \end{aligned}$$

Thus $|(x + y) - 11| < 0.3.$

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6) Exercise :

1) **Solve the inequalities in terms of intervals and illustrate the solution sets on the real number line.**

(a) $-1 < 2x - 5 < 7$

(b) $-5 \leq 3 - 2x \leq 9$

(c) $2x - 3 < x + 4 < 3x - 2$

(d) $x^2 + x > 1$

(e) $(x+1)(x+2)(x+3) \geq 0$

(f) $-3 < \frac{1}{x} \leq 1$

2) **Solve the equations for *x* :**

(a) $|3x + 5| = 1$

(b) $\left|\frac{2x-1}{x+1}\right| = 3$

3) **Solve the inequalities:**

(a) $|x + 5| \geq 2$

(b) $|5x - 2| < 6$

(c) $0 < |x - 5| < \frac{1}{2}$