Lab3: 2D Interpolation/Decimatioin

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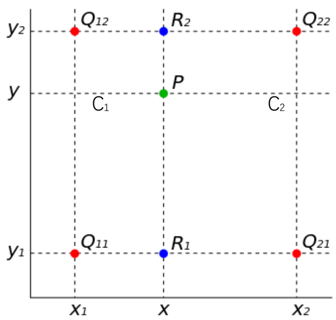
I.Introduction

In this lab, we did 2D image interpolation and decimation via different methods. An image $f(x, y)$ tells us the intensity values at the integral lattice locations, i.e., when x and y are both integers. Image interpolation refers to the prediction of intensity values at missing locations, i.e., x and y can be arbitrary. The engineering motivations of doing this lab is that we want big images when we see a video clip on a PC. Also, we want complete images when some block of an image gets damaged during the transmission, we could use image interpolation to repair it. Image interpolation can be applied in image resizing, image compression and image restoration. In this lab, we use two methods, pixel replication and Bilinear interpolation. Image decimation is the process of reducing the sampling rate of a signal. Low sampling rate reduces storage and computation requirements. We doing this lab because it could solve several image processing problems including image compression, limited bandwidth image transmission, protection of copyrighted material and limited resolution printing. In this lab, we try decimation with an ideal low pass filter and without an Anti-aliasing filter with different parameters.

II.Discussion

1.Image Interpolation

Zero order hold image interpolation is implemented by replicate each pixel value in the original image to fill its corresponding subblock in the magnified output image. This method is usefulness for its easiness and efficiency in implementation. Bilinear interpolation is to perform linear interpolation first in one direction and then again in the other direction on a rectilinear 2D grid, which is an extension of linear interpolation. The visual effect is better than zero order hold interpolation, it produce a clearer and sharper image than the above method, though the image is still not sharp. And the operation speed could slightly slower than it. However, it is still an simple algorithm with high efficiency.



Bilinear interpolation gives the same result for interpolating either along row or column first. The proof is as follows. We already know the pixel values at point $Q_{11} = (x_1, y_1)$, $Q_{12} = (x_1, y_2)$, $Q_{21} = (x_2, y_1)$, $Q_{22} = (x_2, y_2)$, which denoted by $f(Q_{11})$, $f(Q_{12})$, $f(Q_{21})$, $f(Q_{22})$. We want to find the interpolation pixel value $f(P)$ at point (x, y).

If we first do linear interpolation in the x-direction, $f(x, y_1) = f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} \times f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} \times f(Q_{21})$; $f(x, y_2) = f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} \times f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} \times f(Q_{22})$.

Then we proceed by interpolating in the y-direction to obtain the estimate interpolation value, $f(x, y) = f(P) \approx \frac{y_2 - y}{y_2 - y_1} \times f(R_1) + \frac{y - y_1}{y_2 - y_1} \times f(R_2) = \frac{y_2 - y}{y_2 - y_1} \times f(x, y_1) + \frac{y - y_1}{y_2 - y_1} \times f(x, y_2) = \frac{1}{(x_2 - x_1)(y_2 - y_1)} (f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1))$.

If we first do linear interpolation in the y-derection, $f(x_1, y) = f(C_1) \approx \frac{y_2 - y}{y_2 - y_1} \times f(Q_{11}) + \frac{y - y_1}{y_2 - y_1} \times f(Q_{12})$; $f(x_2, y) = f(C_2) \approx \frac{y_2 - y}{y_2 - y_1} \times f(Q_{21}) + \frac{y - y_1}{y_2 - y_1} \times f(Q_{22})$. Then we proceed by interpolating in the y-direction to obtain the estimate interpolation value, $f(x, y) = f(P) \approx \frac{x_2 - x}{x_2 - x_1} \times f(C_1) + \frac{x - x_1}{x_2 - x_1} \times f(C_2) = \frac{1}{(x_2 - x_1)(y_2 - y_1)} (f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1))$.

Comparing the two derived functions above, we can found they are equal, which means it does not matter if we interpolate along the rows first instead of along the columns first.

2.Image Decimation

The mean squared error(MSE) between two images $f(x, y)$ and $g(x. y)$ is $e_{MSE} = \frac{1}{MN} \sum_{n=1}^{M} \sum_{m=1}^{N} [f(n, m) - g(n. m)]^2$. The MSEs are:

No Anti-Alias Filter, 2x subsampling, $e_{MSE} = 582.685$;

No Anti-Alias Filter, 8x subsampling, $e_{MSE} = 1875.75$;

Ideal Low-Pass Anti-Alias Filter, 2x subsampling, $e_{MSE} = 519.457$;

Ideal Low-Pass Anti-Alias Filter, 8x subsampling, $e_{MSE} = 1382.81$.

Based on my subjective judgement, according to the visual quality of the interpolated output images, the image with best visual quality is restored image that decimated with an Ideal Low Pass Filter with M=2, follows by restored image that decimated without an Anti-aliasing filter with M=2, restored image that decimated with an Ideal Low Pass Filter with M=8. The image with worst visual quality is restored image that decimated without an Anti-aliasing filter with M=8, it is not smooth and the letters are distorted.

For the relationship between subsampling rate and MSE, a large subsampling rate will cause a higher MSE. The reason is that the larger rate results in more loss of image information; therefore, the MSE will be larger since the less original information can be restored.

An anti-aliasing filter makes the restored image become smoother, which make it more similar to the original image. The reason is that the anti-aliasing filter minimizes the distortion artifacts known as aliasing when representing a high-resolution image at a lower resolution.

The MSE is correlating with my subjected evaluation. The restored image with lower mean square error has better visual quality. The restored image with the highest MSE, which is the restored image that decimated with an Ideal Low Pass Filter with M=8, has the worst visual quality. Because MSE is a measure of image quality that compare restoration results between original image and restored image.

The method that convolving input image with "ideal interpolation" sinc function is done in spatial domain, while the method we use in this lab apply the ideal low pass filter in Fourier domain. For the Fourier transform that decomposes a function of time into the frequencies that make it up, in mathematical terms, the desired frequency response of ideal sinc function is $H(f) = rect(\frac{f}{2B})$, which is a rectangular function, i.e. the low pass filter we applied in the frequency domain. Also, Fourier transform has the property that convolution in the time domain corresponds to ordinary multiplication in the frequency domain, $f_1(t) * f_2(t) = F_1(\omega)F_2(\omega)$. Thus methods B and D that apply low pass filter in frequency domain are equivalent to convolving the input image with an "ideal interpolation" sinc function, then sub-sampling, which done in the spatial domain.

3. Overall Questions

Since the letters have much more sharp edges, which are usually considered as low-frequency components. In method A, without a low-pass filter, the edges from the letters are easily distorted by the some high-frequency components. The cat is an object that with similar color (a black object) which means it has similar frequencies in Fourier transform. Therefore, even though without a filter, a cat is less likely distorted and will be easier recovered.

The subsampled images obtained by using Methods B (i.e., with an anti-aliasing filter) have clearer and smoother edges compared with the subsampled images obtained by using Methods A(i.e., without an anti-aliasing filter), which can be observed apparently at the edges of the dialog boxes and letters. This is because in Fourier domain, the low pass filter passes signals with the frequencies lower than a certain cutoff frequency, compared with Method A, it preserves more useful frequencies. Thus after inverse Fourier transform, more information of the original image (especially in the edges and peaks) can be recovered.

III.Conclusion

The result of 2D image interpolation is that bilinear interpolation gives a smoother and clearer magnified image compared with the zero order hold interpolation. And the result of image decimation is that decimation with an ideal low pass anti-aliasing filter gives a smoother and more recognizable subsampled image and restored image after bilinear interpolation. Also, with the same decimation factor M, the MSE is lower than those without using low pass filtered for decimation. Through this lab, I learned different image interpolation and decimation algorithms.