Lab7: Histogram

Mingqin Dai

I.Motivation

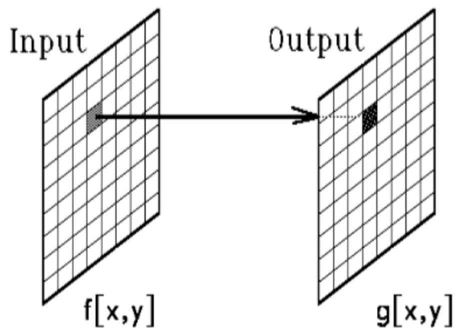
We are doing this lab to look at image enhancement methods which are histogram equalization and gamma correction. The goal of image enhancement is to process images so that the result is more suitable than the original for a specific application, i.e. make its features easier to see. This technique is used to process images with poor contrast. And it is often used to improve the visibility of certain features in the image, or to compensate for various imperfections of the image capture and display processes.

II.Method

Part1. Histogram Modification

The Modification P modifies the gray level of each pixel $x(n_1, n_2)$ of the original image independently of other pixel values. Each pixel $\tilde{x}(n_1, n_2)$ of the enhanced image is given by $\tilde{x}(n_1, n_2) = P[x(n_1, n_2)]$. Under P, each gray level g is mapped into gray level $\tilde{g}$ according to $\tilde{g} = P(g)$. The point operation we used here is $P(x) = 255(\frac{x}{255})^{\gamma}, \gamma = 2.5$, which is inverse gamma correction point operation. And we apply the point operation to each individual pixel of image lena independently.

This point operation satisfies the properties. 1. The gray value of the output image $\tilde{g}$ at a particular pixel $\tilde{x}(n_1, n_2)$ depends only on the gray value of the same pixel in the original image.



This is because we apply the point operation to each separate pixel once in order, till all pixels in the image has been applied, which means all output pixels are calculated independently. We can also verify this by seeing *P(x)*, according to *P(x)*, the output pixel's intensity does not depend on pixel's location *(u, v)* or intensities of neighboring pixels, but only depends on the previous gray value at same location. This is a homogeneous operation. 2. P is monotone increasing. The point operation $P(x) = 255(\frac{x}{255})^{\gamma}$, has the derivation $P(x) = \gamma(\frac{x}{255})^{\gamma-1}$. With $\gamma = 2.5$, $P'(x) = 2.5\times(\frac{x}{255})^{1.5}$. For $x \in [0, 255]$, $P'^{(x)} \geq 0$, which means this point function $P(x)$ is monotone increasing. Thus the operation performed is indeed a valid point operation. (Q1)

The printouts of the original and modified images are on the next page.

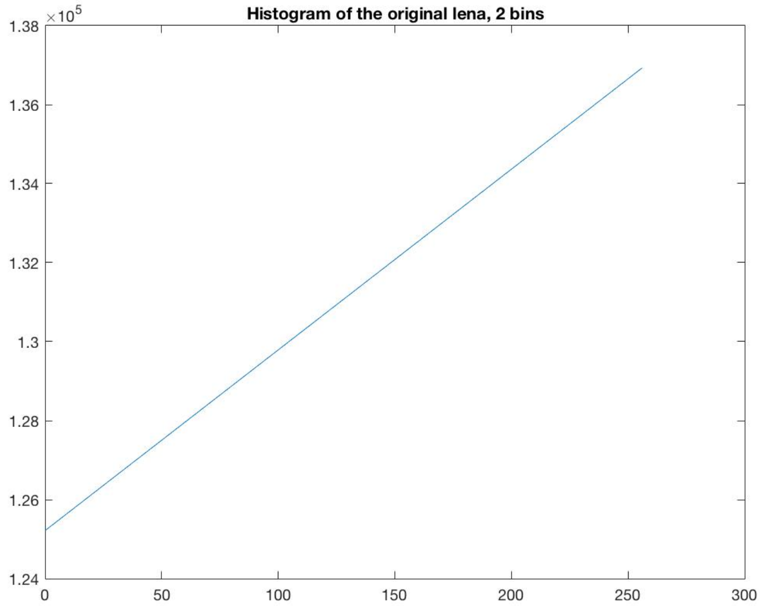
1. Original Image

2. modified image.

Part2. Image Histogram

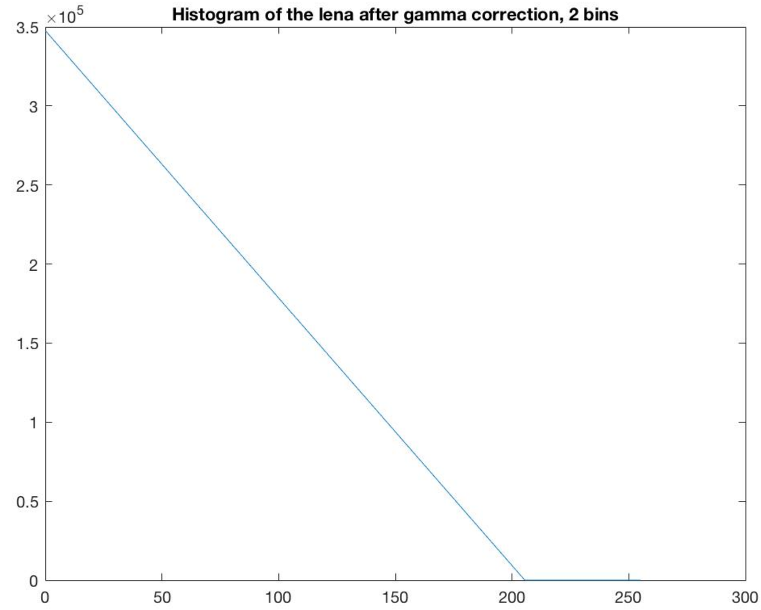
The histogram of the 2-D image $H(g)$ plots the population of pixels with each gray level g. The histogram generally is represented as a 1-D function where the independent variable is the gray value g and the dependent variable is the number of pixels *H* with that level. In this lab, we calculate the histograms of the original and the pre-gamma corrected image for bins = 2, 128, and 256. In each bin, the histogram value is calculated by $\sum_{g_{min}}^{g_{max}} n_g$, where g_{min} refers to the minimum gray value in a bin and g_{max} refers to the maximum gray value in the same bin. And the sum of populations of the histogram bins must equal the total number of image pixels N, $\sum_0^{255} H(g) = N$.

a. Histograms with 2 bins



(1) original image

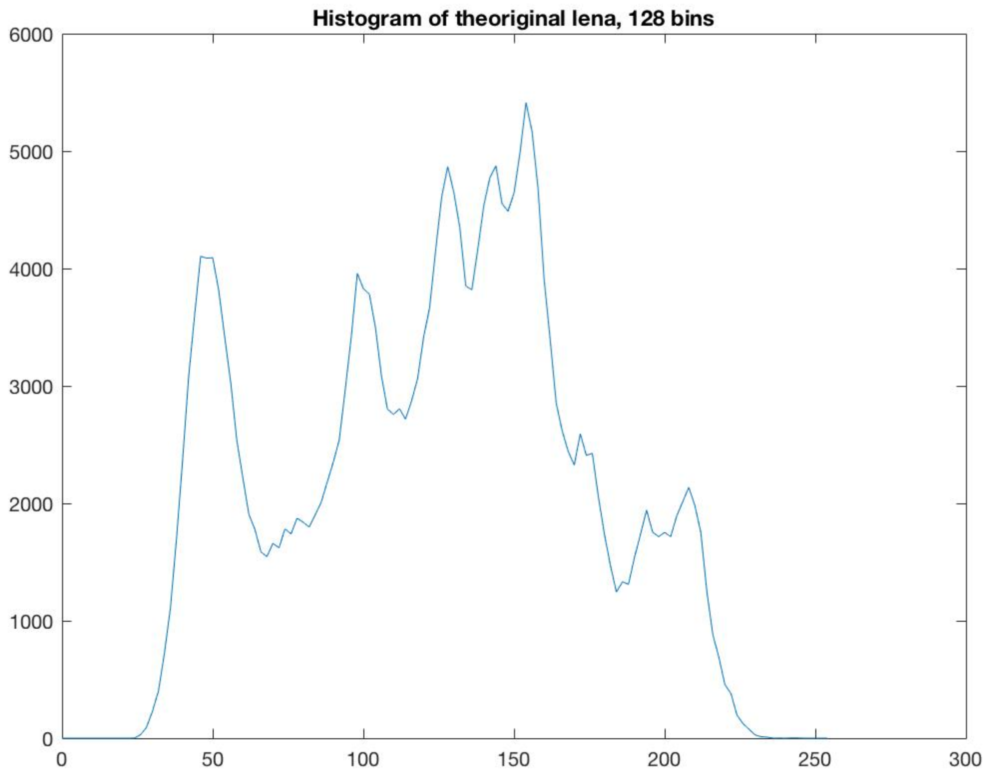
Values for the histogram bins: 128144, 134000.



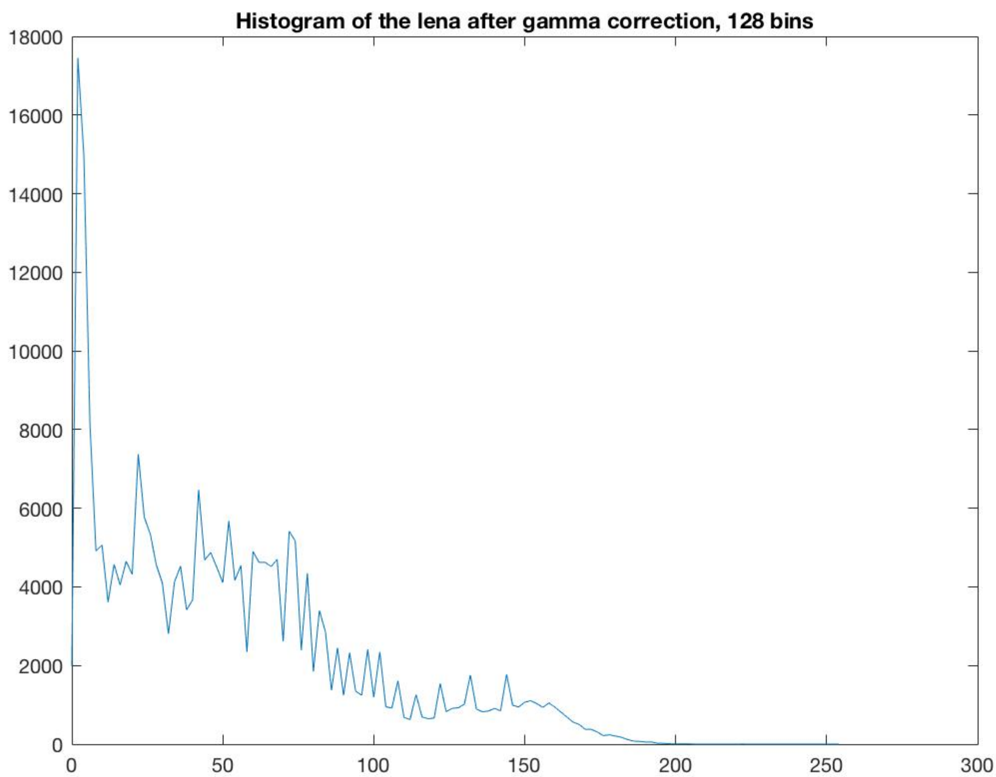
(2) After gamma correction

Values for the histogram bins: 239372, 22772.

b. Histograms with 128 bins



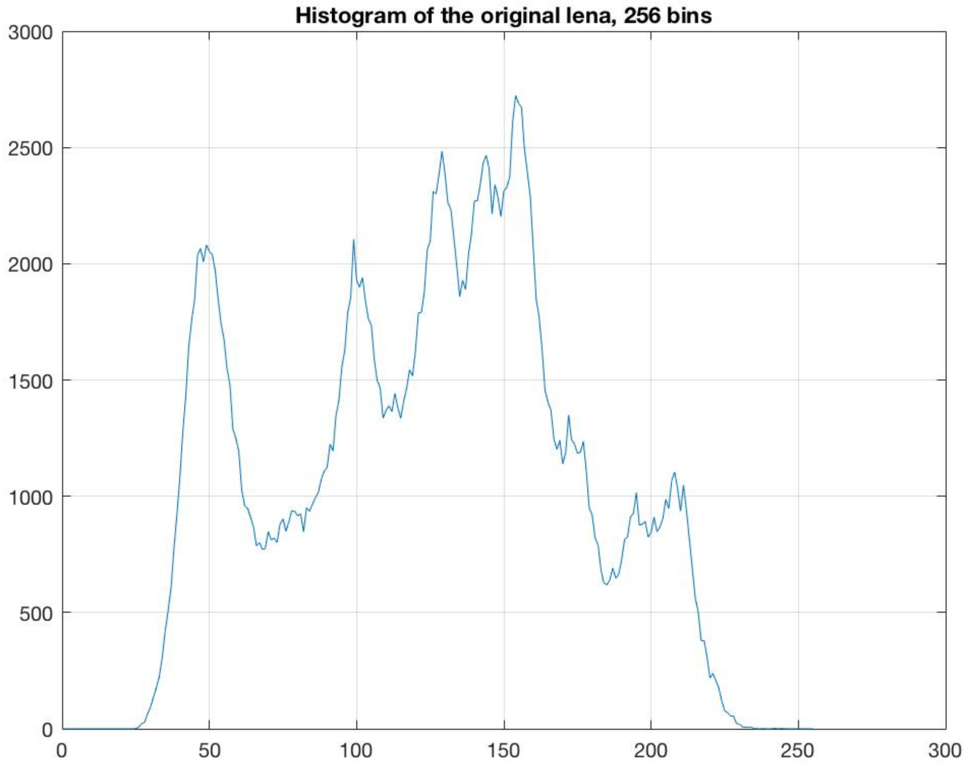
(1) Original image



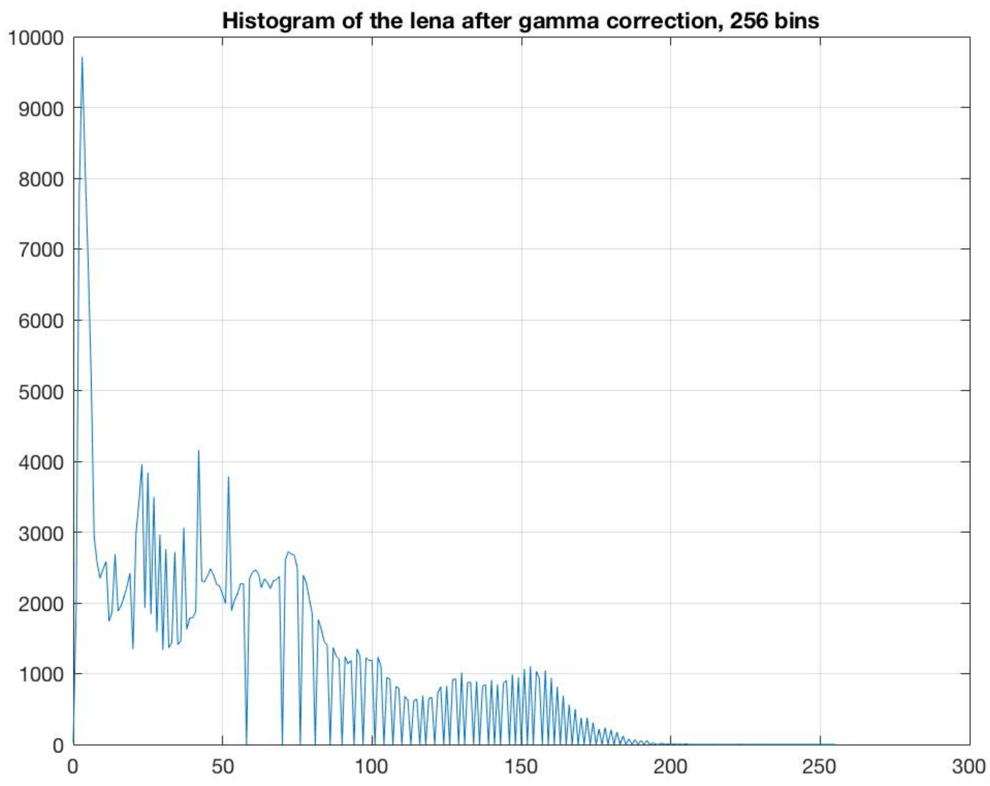
(2)After Gamma Correction

c. Histograms with 256 bins

(1) Original Image



(2)After Gamma Correction



Compared with the histograms of the original image, the histograms after gamma correction with bin = 2, 128 and 256 have right-skewed distribution. Lower gray levels have higher population, while higher gray levels tend to have lower population, which means the image after gamma correction is darker than the original image. And the peaks of the histograms after gamma correction moved to left (within 0 to 10) compared with the histograms of the original image. Overall, the histograms are shifting to left after gamma correction, and they are not stretching. For bin=2, the difference between number of pixels in 2 bins become much larger after gamma correction. For bin=128, 256, the histograms become flatter at the range (20, 200) after gamma correction. (Q2)

The psudo code of the histogram computation is as follows.

For M rows

For N columns

histogram(gray level) = histogram(gray level)+1

end

end

(where gray level in terms of the gray value of pixel)

For every pixel, we first get its gray value, then for the corresponding horizontal bin g of histogram, we add 1 to $H(g)$. So for each pixel, The complexity is *O(1).* There are overall $M\times N$ pixels, so the overall complexity of this algorithm is *O(MN).* (Q3)

Part3. Histogram Equalization

The goal of histogram equalization is to increase image contrast by making the new histogram uniform, i.e $\widetilde{H}(\tilde{g}) \equiv \frac{N}{255} = C.$ In this lab, we first calculate the cumulative density function of histogram. Then using the cumulative density function, we calculate the gray value of equalized image for each pixel.

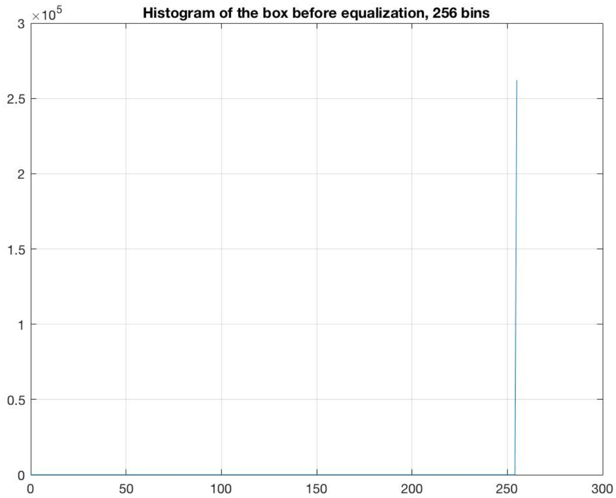
(Images and Histograms are on next page)

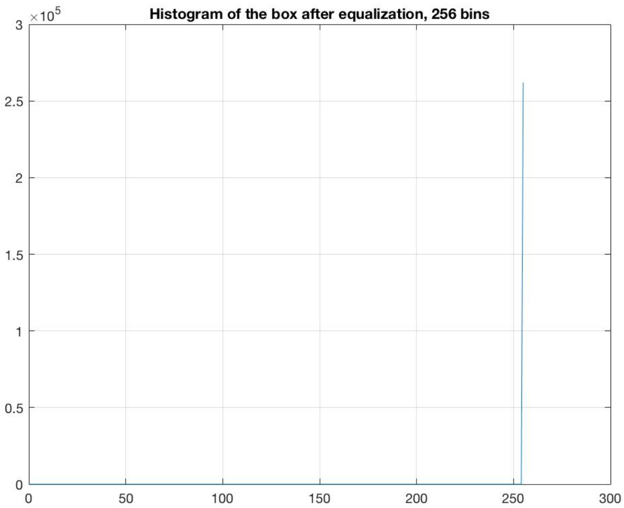
a. Image D

(1)Images

Original image (vanish) Equalized Image

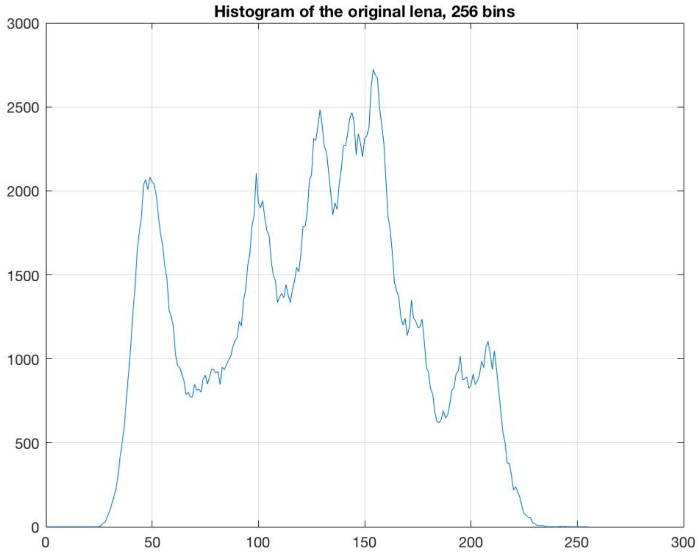
(2)Histograms





Original Image(H(254)=64;H(255=262080)) Equalized Image(H(0)=64;H(255)=262080) *b. Lena*

(1)Images *(2)Histograms*



Original Image and its Histogram

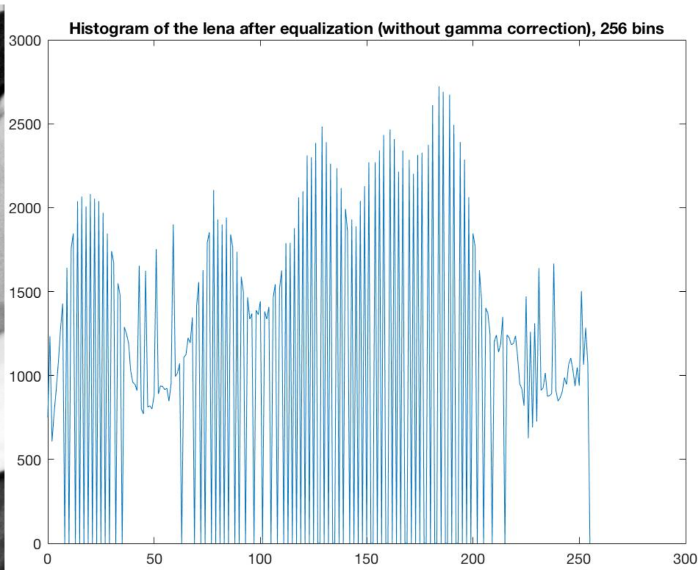


Image and its Histogram after equalization without Gamma Correction

Image and Histogram before equalization with Gamma correction

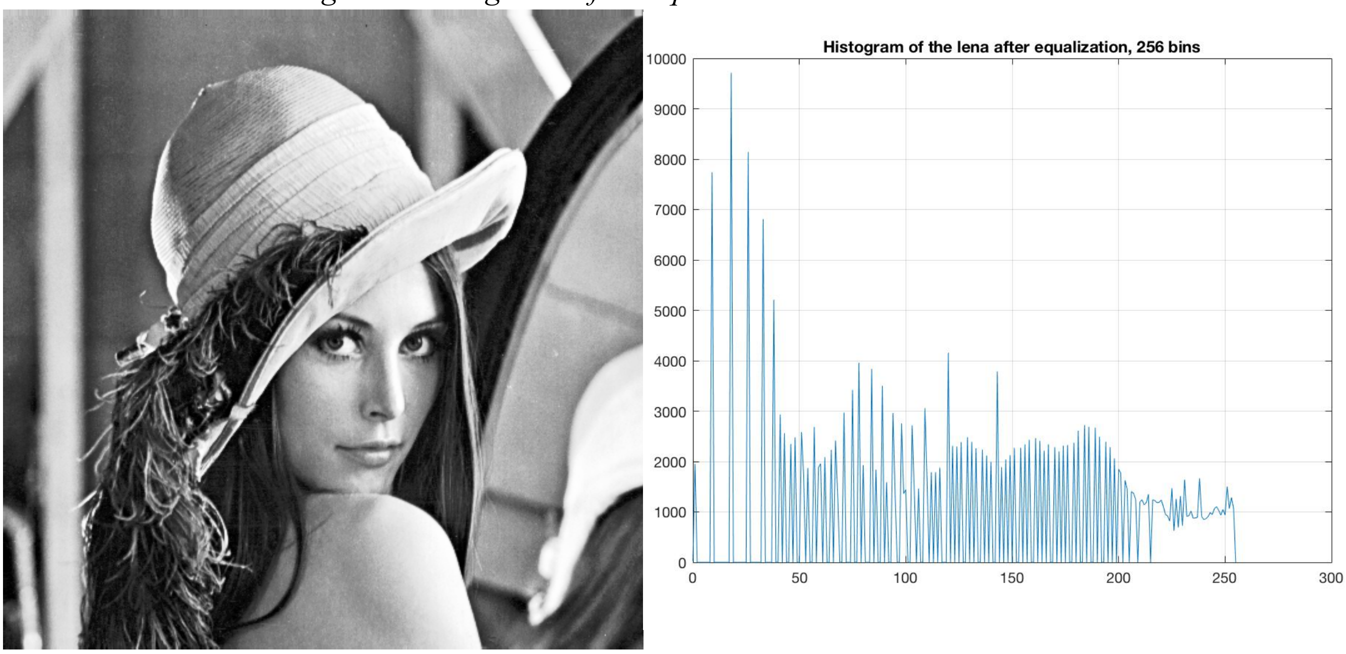


Image and Histogram after equalization with Gamma Correction

Comparing the images before and after equalization (both with or without gamma correction), the contrast has been clearly enhanced. Comparing the histograms before and after equalization (both with or without gamma correction), the histogram has been stretched, which make its range from 0 to 255. Overall, the histogram become flatter and there are clearly more zero points in histogram after equalization. (Q4)

If we only consider the histogram, histogram equalization is done by making the new histogram uniform. The histogram is not equalized because the limited validity of the assumption, which the gray levels g are real-valued, used for deriving $\widetilde{H}(\tilde{g}) = \frac{H(g)}{P\prime(g)}|_{g=P^{-1}(g)}$. The grayscale values in the range [0, 255] are not always continues. (Q5)

The original histograms (grayscale values) need to be completely continuous among the range [0, 255] to result in exactly flat histograms, while the discrete version won't result in exactly flat histograms because of quantization, it will flatten them and spread the histogram to span a larger range. (Q6)

The original image has both bright background and foreground, which can't be recognized by human eye. The equalized image has completely black (Intensity=0) foreground and completely white (Intensity =255) background, which make a clear contrast. Only the proximate gray values of original histogram have value, with other gray value vanish. The equalized histogram only has values at two critical points, which are 0 and 255. We can conclude that the method is useful in images with backgrounds and foregrounds that are both bright or both dark. It spread the range of histogram from [254, 255] to [0, 255]. (Q7)

After equalization, the edges of the image are clearer. And it is apparent that the dark pixels appear darker while the light pixels appear lighter in the output image, which means areas of lower local contrast gain a higher contrast. Comparing the histograms, through this adjustment, the intensities are better distributed on the histogram. We can conclude that histogram equalization causes a histogram with closely grouped values to spread out into a histogram with gray level range [0, 255]. Histogram equalization accomplishes the goal of enhancing contrast by effectively spreading out the most frequent intensity values. (Q8)

Histogram Equalization does improve contrast in some cases, but it may introduce noise and other undesired effect. Image regions that are dark and not clearly visible become clearer but this may happen at the expense of other regions. These undesired effect is a consequence of digitization. When digitize the continuous operations rounding leads to approximations. And the histogram equalization does not change the image resolution, and dose not decrease the depth of the image. For example, applying histogram equalization on RGB image introduce unexpected results. And for image with wide range histogram, it may increase the contrast of background noise, while decreases the contrast of foreground object. Thus it will not always improve the contrast, though it improve the contrast in most cases. (Q9)

III.Results

The inverse gamma correction point operation and histogram equalization both improve the contrast of the image in this lab. Histogram equalization is a point operation. Image contrast can often be enhanced using simple points operation.

For point operation on single images, the gray value of the output image g at a particular pixel (n_1, n_2) depends only on the gray value of the same pixel in the image. Histogram equalization does not operate on the histogram itself but uses the results of one histogram to transform the original image into an image that will have equalized histogram. It effectively spreading out the most frequent intensity values to improve the contrast. While the discrete version won't result in exactly flat histograms, it will flatten them and in doing so enhance the contrast in the image.