Low-Pass Filtering

CS 450: Introduction to Digital Signal and Image Processing

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Low-Pass Filtering

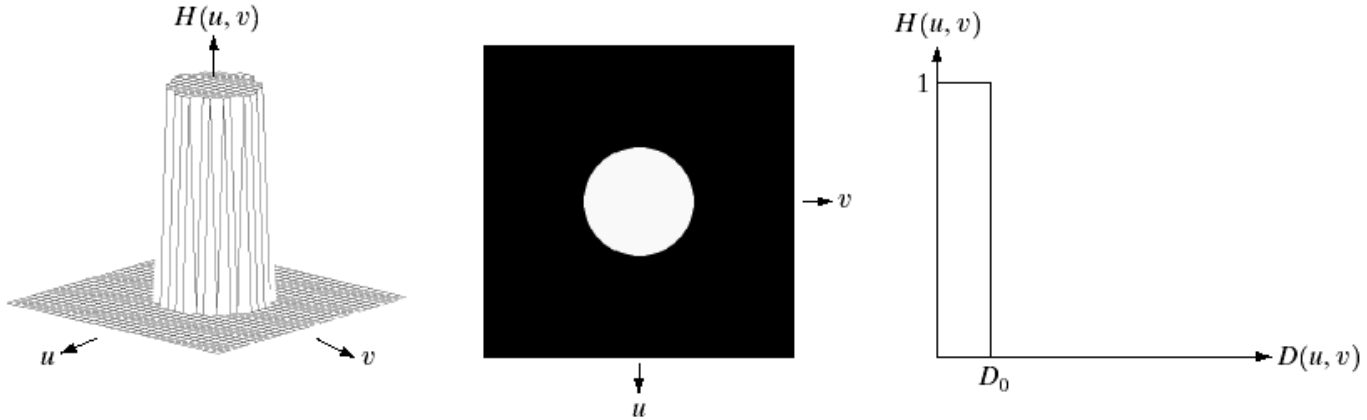
- Low-pass filtering smooths a signal or image
  - Low frequency components = gradual transitions
  - High frequency components = rapid transitions (edges, texture, etc.)
- Remember: smoothing helps remove noise
- Can also be used to target specific types of noise/interference (high-frequency hissing, etc.)

Ideal Low-Pass Filtering

For cutoff frequency u_c,

$$H(u) = \begin{cases} 1 & \text{if } u < u_c \\ 0 & \text{otherwise} \end{cases}$$

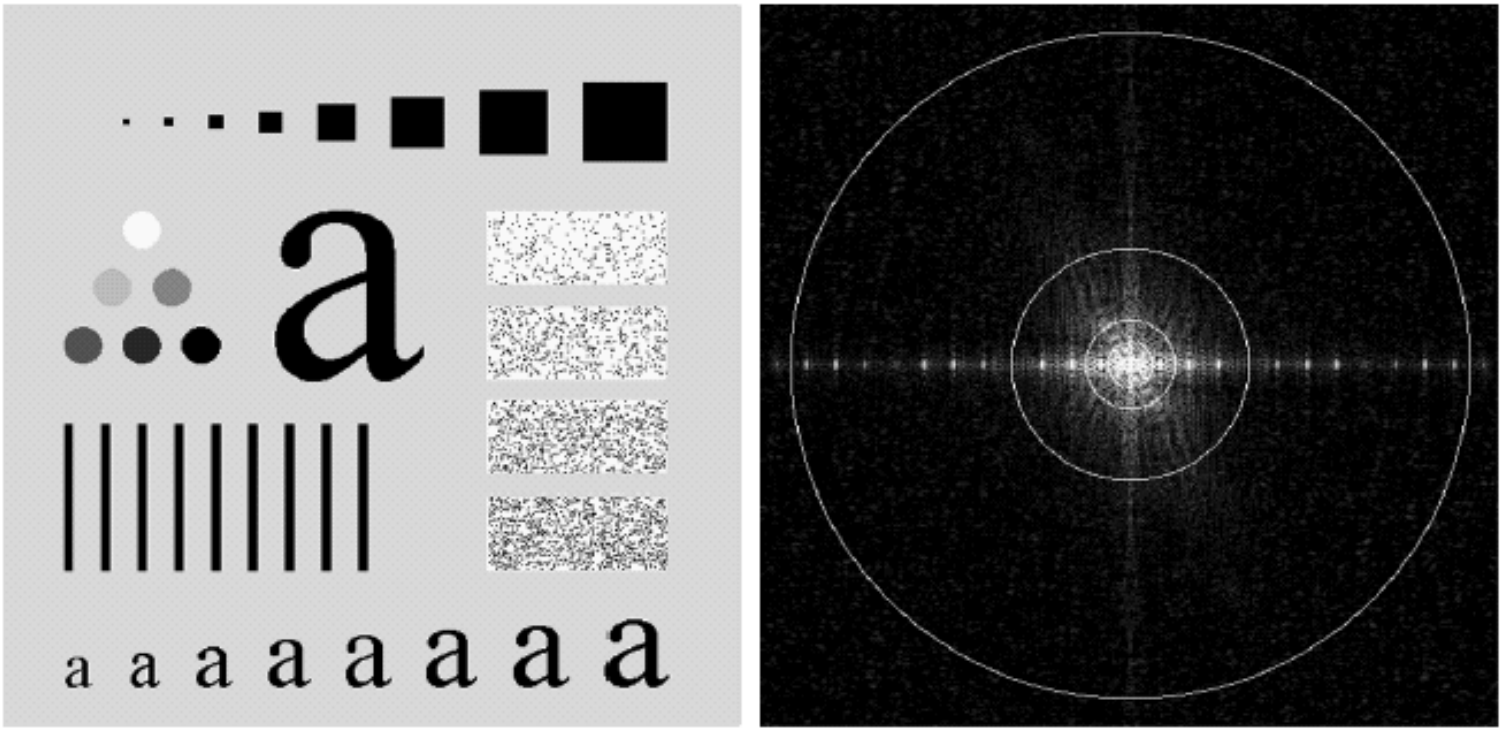
- What is the corresponding convolution kernel?
- What problem does this cause?
- What could you do differently?



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

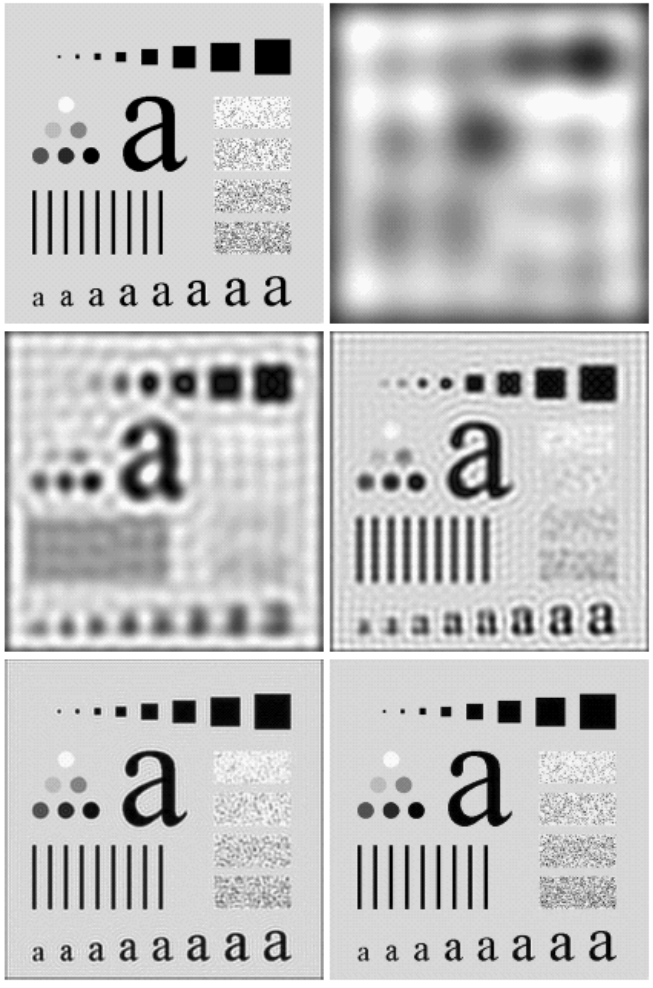
Ideal Filters and Ringing - Frequency Domain



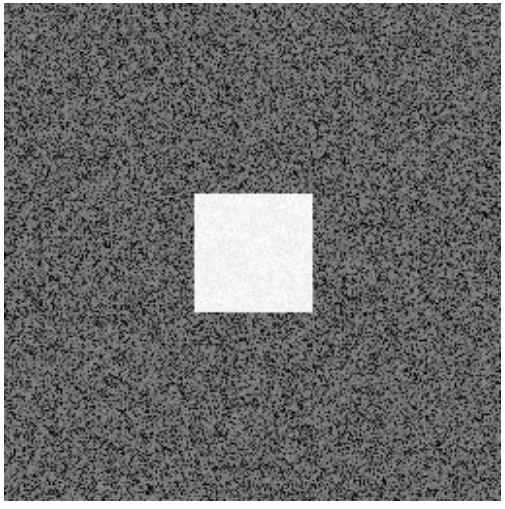
a b

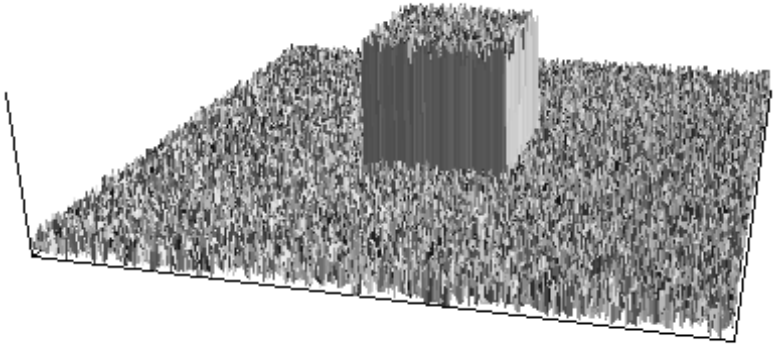
FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

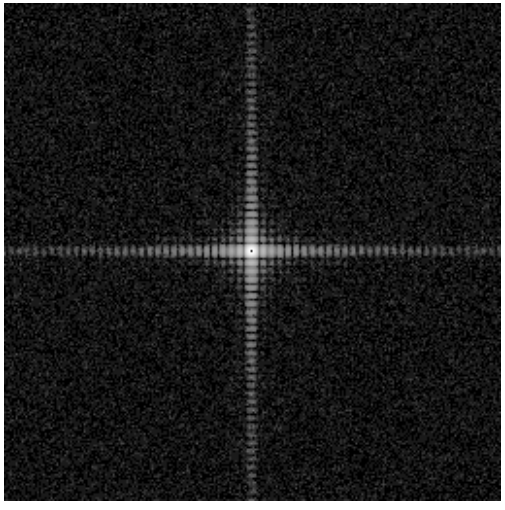
Ideal Filters and Ringing - Spatial Domain

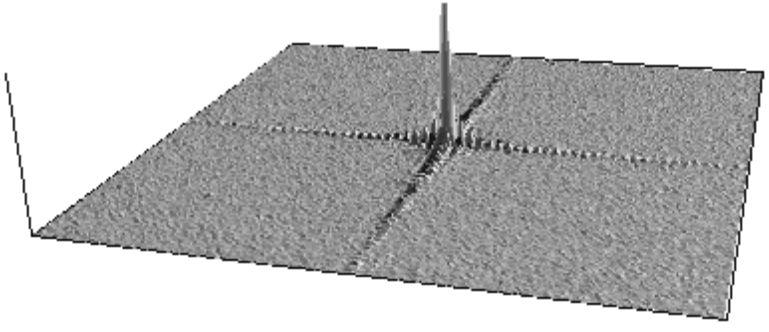


Ideal Filters and Ringing

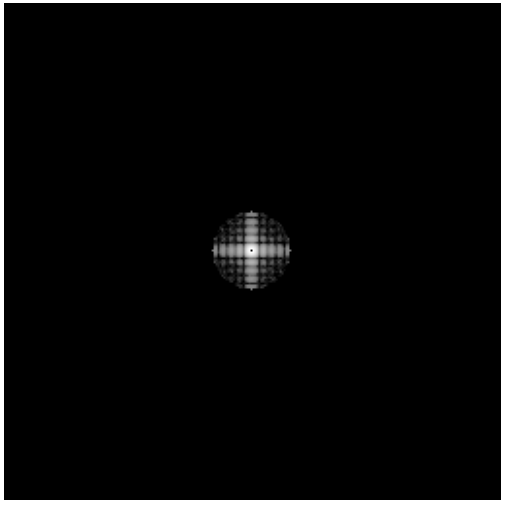


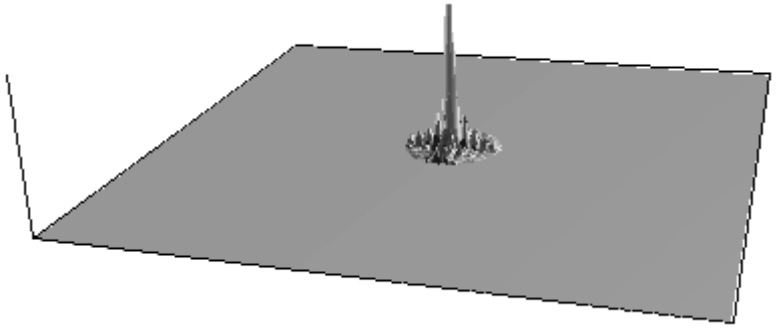


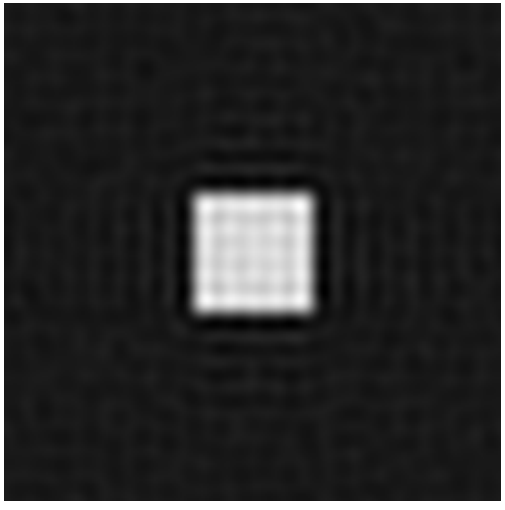


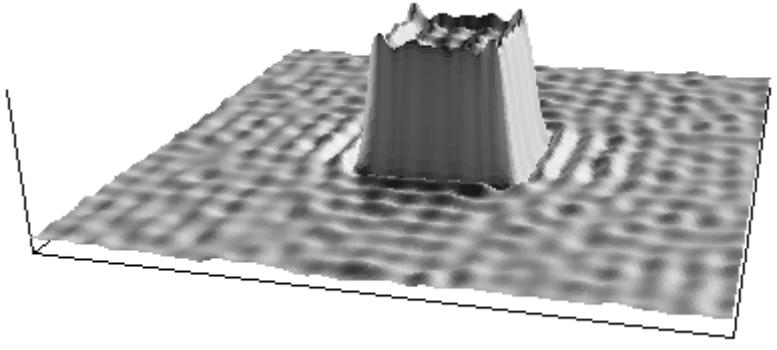


Ideal Filters and Ringing (cont'd)









Better Ways of Low-Pass Filtering

Gentler ways of cutting off high frequencies:

- Gaussian

$$H(u) = e^{-\frac{1}{2}u^2/{u_c}^2}$$

- Butterworth

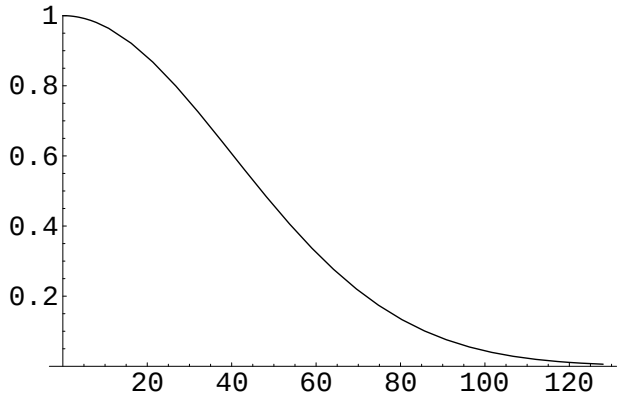
$$H(u) = \frac{1}{1 + (u^2/{u_c}^2)^n}$$

The goals of these and similar filters is to cut off the high frequencies *gradually*.

Gaussian Low-Pass Filter (GLPF)

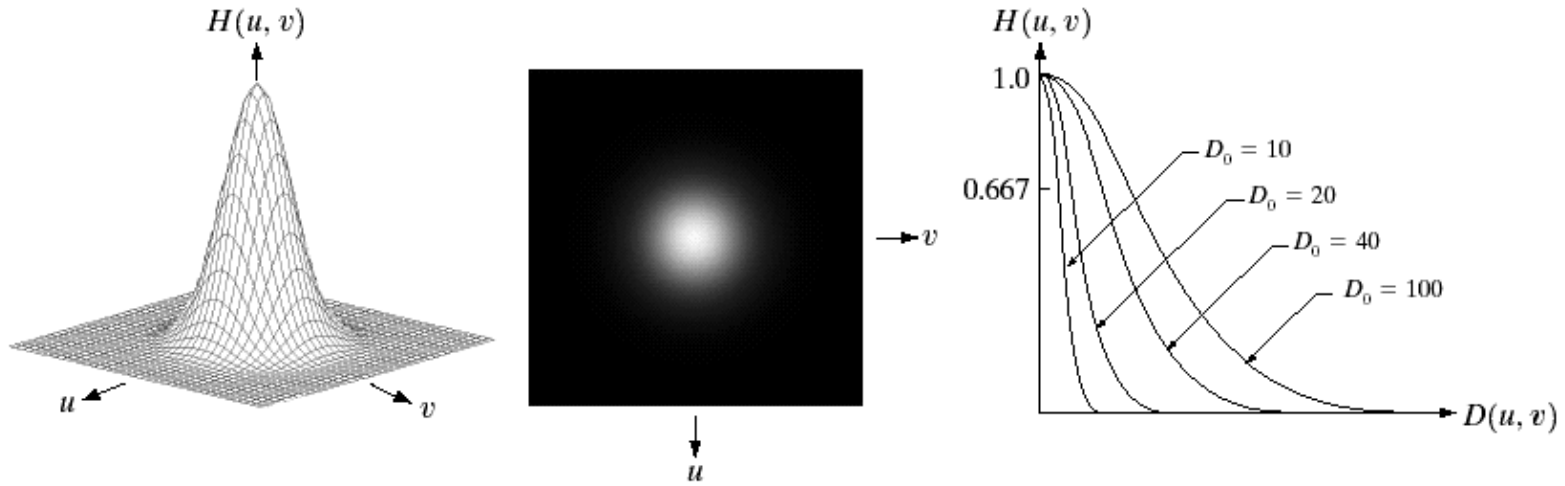
Gaussian fall-off in frequency response:

$$H(u) = e^{-\frac{1}{2}u^2/{u_c}^2}$$



- The frequency u_c is the std. dev. of the Gaussian filter.
- What is the spatial-domain equivalent of the GLPF?
- Does increasing u_c cause more or less blurring?

Gaussian Low-Pass Filter



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Gaussian Low-Pass Filter

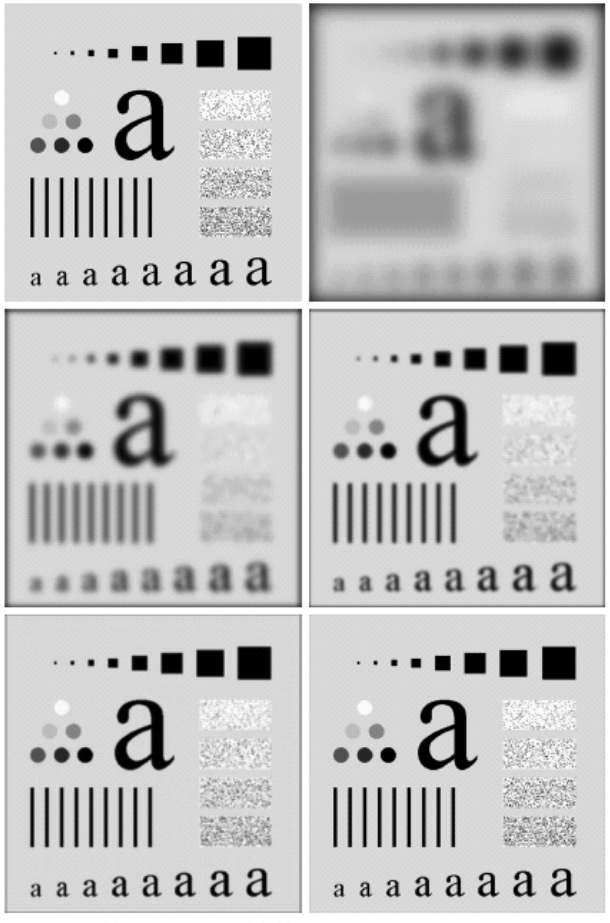


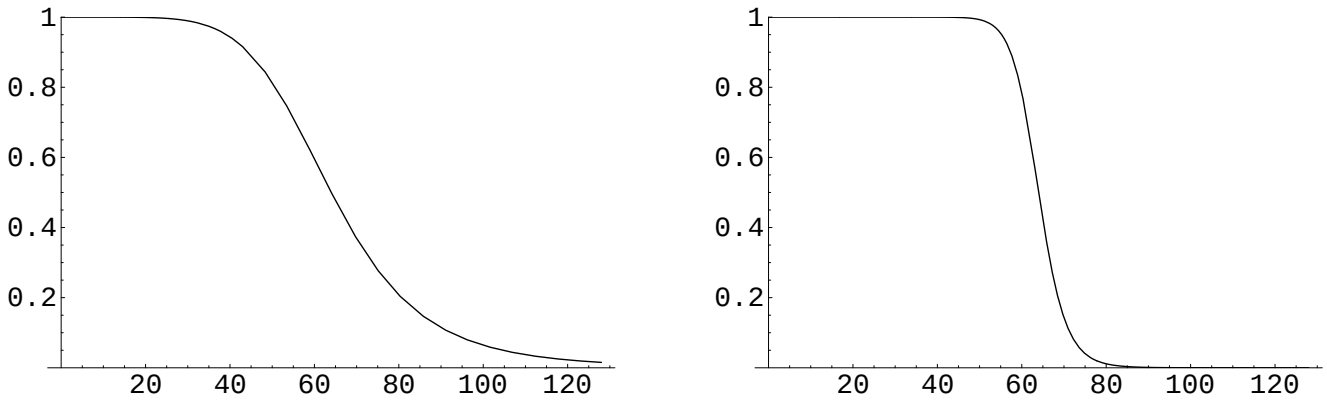
FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b
c d
e f

Butterworth Low-Pass Filter

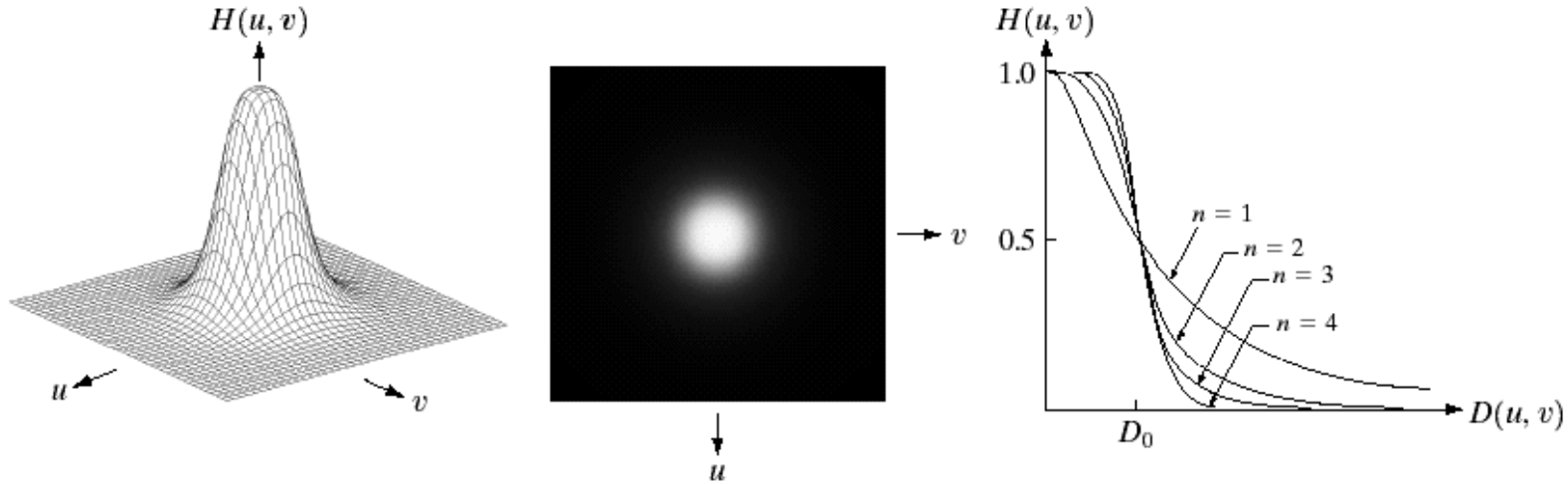
Controllable sharpness of the frequency-domain cutoff u_c:

$$H(u) = \frac{1}{1 + (u^2/u_c{}^2)^n}$$



- The "cutoff" frequency u_c controls where the cutoff occurs.
- The parameter n controls the sharpness of the cutoff.
- What is the tradeoff involved in setting n?

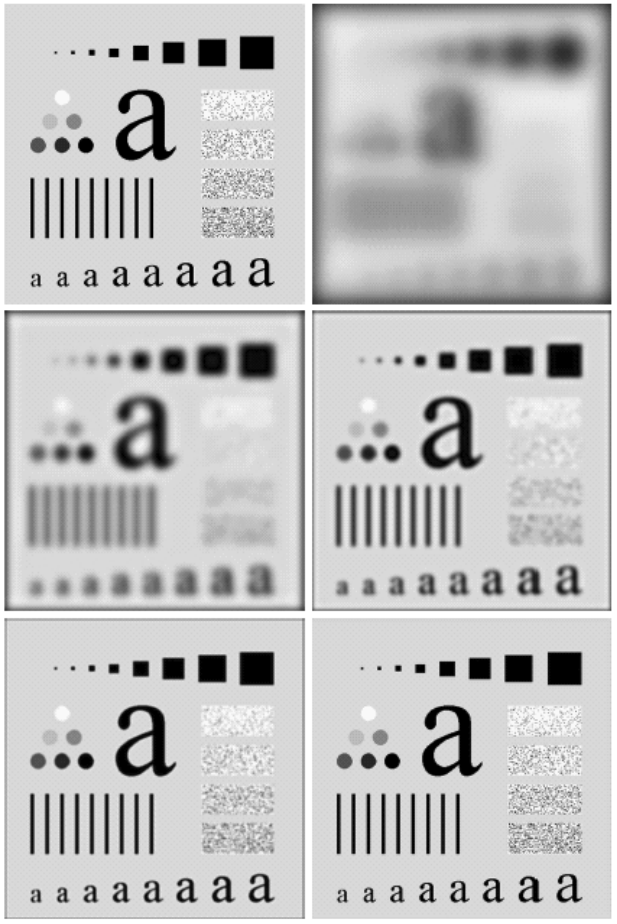
Butterworth Low-Pass Filter



a b c

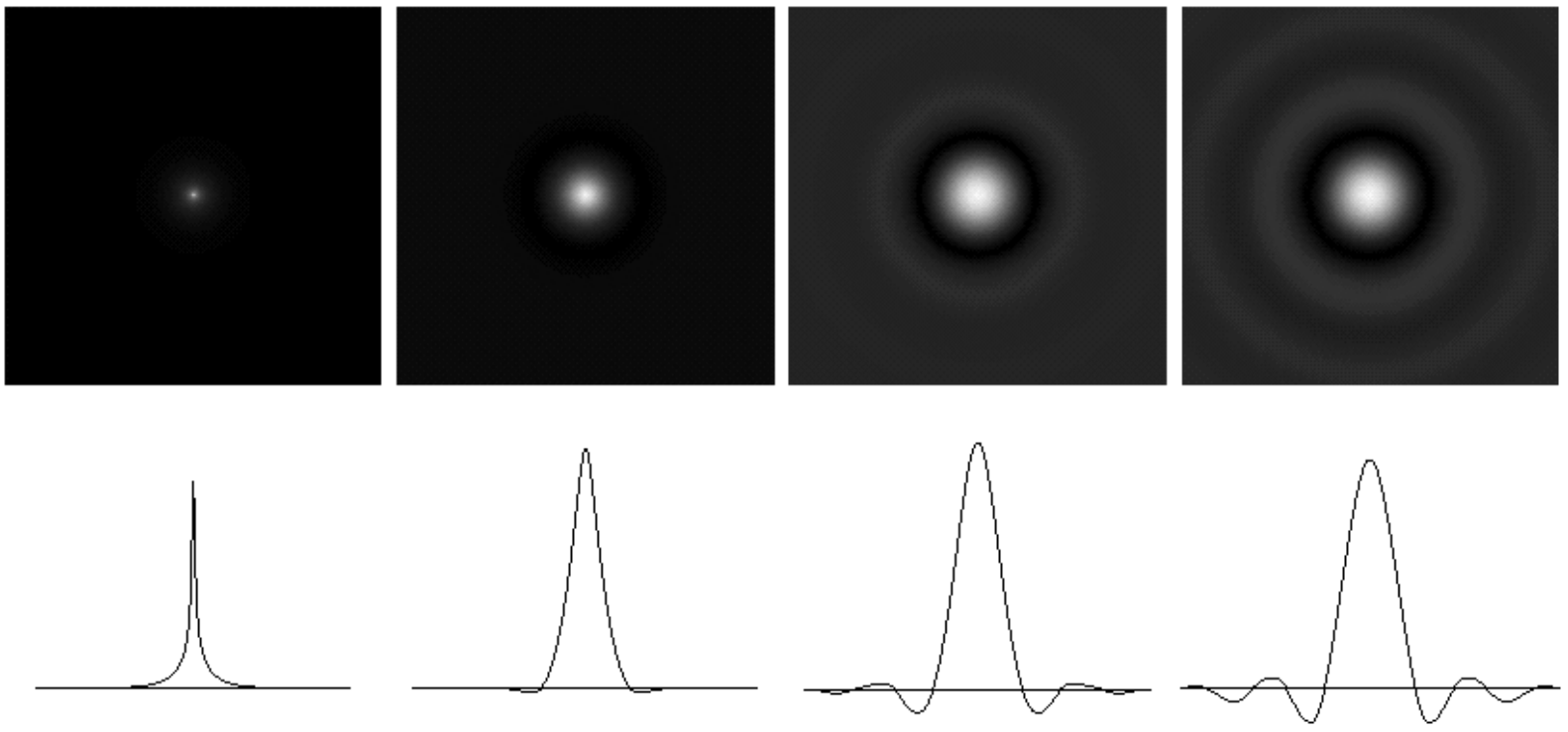
FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Low-Pass Filter



a b c d e f **FIGURE 4.15** (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Butterworth Low-Pass Filter



a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.