MAT 201 Final Exam

May 8, 2020

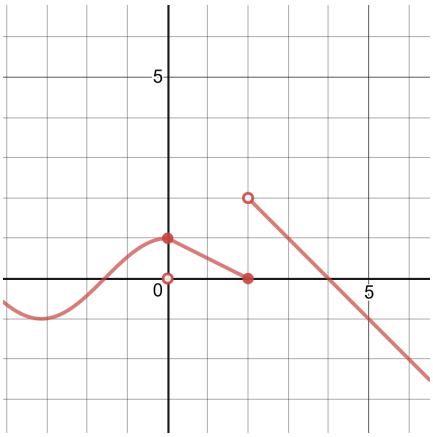
1. [1] If $f(t) = t^2$ represents the motion of an object at time t (in seconds), find the most simplified version for the average velocity of the ball on the interval $[2, 2 + h]$. You must show all work.

2. [2] Use your result in the last question to find the instantaneous velocity at the time $t = 2$.

3. [3] Evaluate the following limit algebraically.

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

4. [4] The graph of a function $f(x)$ is given below. Use the graph, find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 2} f(x)$. If the limit does not exist, write DNE. Explain your work.

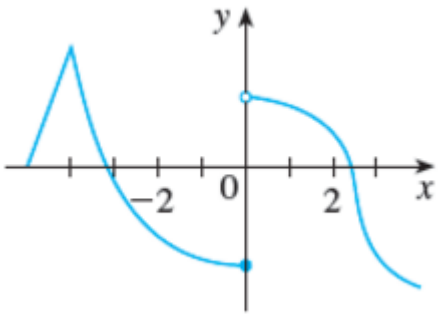


5. [5] Find the derivative with respect to x evaluated at $x = 1$ of the function $f(x) = 2x^2 + 1$ **using the definition**. You must show all work.

6. [6] Given a function $f(x) = 3x^3 + 2x^2 - 5x + 3$, use the first derivative of the function, determine whether the function is increasing or decreasing at $x = 0$.

7. [7] Based on the function in last question, use the second derivative of the function, determine whether the function is concave up or down at $x = 0$.

8. [8] Find all the point(s) (if there are any) of the function that don't have a limit.



9. [9] Use the graph above, find all the point(s) of the function that are not continuous.

10. [10] Use the graph above, find all the point(s) of the function that are continuous but not differentiable.

11. [11] Given a function $f(x) = 2\sqrt{x} - 1$, find the tangent line at $x = 4$.

12. [12] Use the tangent line in the last quesetion, find the approximate value of $f(4.01)$.

13. [13] Find the derivative of $y = 2x^4 + 5x^2 - 5$.

14. [14] Find the derivative of $y = 3^{x^2}$.

15. [15] Find the derivative of $y = \ln(x^2 + 1)$.

16. [16] Find the derivative of $y = \cos(\ln(x))$.

17. [17] Find the derivative of $y = \arcsin(x^2)$.

18. [18] Find the derivative of $y = x^2 \sin(x)$.

19. [19] Find the derivative of $y = \frac{e^x}{\sin(x)}$.

20. [20] Find the derivative of $y = \sec(3x^2)$.

21. [21] Given a function $f(x) = x + \sin(x)$. (Note that it is not easy to find a formula for the inverse function of $f(x)$). Let $g(x)$ be the inverse function of $f(x)$. Note that $f(\frac{\pi}{2}) = \frac{\pi}{2} + 1$. Find $g'(\frac{\pi}{2} + 1)$.

22. [22] Find dy/dx using implicit differentiation $\frac{y}{x} = \frac{x}{y}$.

23. [23] Use second derivative test to find the local max and local min for the function $f(x) = 4x^3 - 6x^2$.

24. [24] Find the global max and global min for the function $k(x) = 5e^x - e^{2x}$ on the closed interval $[-1, 2]$.

25. [25] Air is being pumped into a spherical balloon at a rate of 4 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet. (Note: the volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

26. [25] A conical tank (with vertex down) has radius 5 across the top and height 12 feet deep. Water is flowing into the tank at a rate of 10 cubic feet per minute. Find the rate of change of the depth of the water when the water is 8 feet deep. (Note: the volume of a cone is $V = \frac{1}{3}\pi r^2 h$)

27. [26] Find the length and width of a rectangle that has 32 square feet with a minimum perimeter.

28. [26] A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 2 square miles in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing?