c. (7 points) A Poisson random variable with a parameter of $\lambda \geq 1000$, a can be approximated by a normal. The approximating normal should match the mean and the variance of the Poisson. Let $X \sim Poi(1000)$. What is an approximation for $P(990 < X < 1000)$. Give your answer to two decimal places.

$\mu = E[X] = \lambda$

$\sigma^2 = Var(X) = \lambda$

$X \sim N(\lambda, \lambda)$

$$P(990 < X < 1000) = P(X < 1000) - P(X < 990)$$
$$= \Phi\left(\frac{1000-\lambda}{\sqrt{\lambda}}\right) - \Phi\left(\frac{990-\lambda}{\sqrt{\lambda}}\right)$$
$$= 0.5 - \left(1 - \Phi\left(\frac{10}{\sqrt{\lambda}}\right)\right)$$
$$= 0.5 - \left(1 - \Phi\left(\frac{10}{\sqrt{1000}}\right)\right)$$
$$= 0.5 - (1 - \Phi(0.32))$$
$$= 0.5 + 0.6217 - 1 \approx 0.12$$

d. (6 points) The probit function, $\Phi^{-1}(x)$ is the inverse of the CDF of a standard normal. It maps from probabilities to the standard normal CDF input that would produce said probability. For example, you can confirm using the Standard Normal Table that $\Phi(0.1) = 0.5398$, so $\Phi^{-1}(0.5398) = 0.1$. Give a closed form expression, using the probit function, for an approximation of K.

$X \sim N(\lambda, \lambda)$ $\lambda = 15 \times 10^6$

$$P(X > 10^4 k) = 1 - P(X < 10^4 k) =$$
$$\cancel{= 1 - \Phi\left(\frac{\lambda - 10^4 k}{\sqrt{\lambda}}\right)}$$
$$\cancel{= \Phi\left(\frac{10^4 k - \lambda}{\sqrt{\lambda}}\right)}$$
$$= 1 - \Phi\left(\frac{10^4 k - \lambda}{\sqrt{\lambda}}\right)$$

$$1 - \Phi\left(\frac{10^4 k - \lambda}{\sqrt{\lambda}}\right) < P$$
$$\Phi\left(\frac{10^4 k - \lambda}{\sqrt{\lambda}}\right) > 1 - P$$
$$\frac{10^4 k - \lambda}{\sqrt{\lambda}} > \Phi^{-1}(1-P)$$

$$10^4 k - \lambda > \sqrt{\lambda}\,\Phi^{-1}(1-P)$$
$$k > \frac{\sqrt{\lambda}\,\Phi^{-1}(1-P) + \lambda}{10^4}$$