c. (8 points) The visitor clicks in the box at pixel (3,3) which has a distance of 2 pixels from the center of the box. What is Google's **new** belief that the visitor is a robot?

Human $f_H(x,y) = \frac{1}{100}$

Robot $f_R(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$

$P_H(X=3, Y=3) = \frac{1}{100} \cdot \varepsilon = 0.01\varepsilon$

$P_R(d=2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(2-\mu)^2/2\sigma^2} \cdot \varepsilon$

$= \frac{1}{2\sqrt{\pi}} e^{-1} \cdot \varepsilon$

$= 0.1038\varepsilon$

Assume Probability is equally likely on the circle

$P_R(X=3, Y=3) = \frac{P_R(d=2)}{2\pi \cdot d} = \frac{0.1038\varepsilon}{4\pi} = 0.0083$

$P(A|H) = 0.01\varepsilon$

$P(A|R) = 0.0083\varepsilon$

$P(AH) = 0.01\varepsilon \cdot 0.8$

$P(AR) = 0.0083\varepsilon \cdot 0.2$

$P(A) = 0.0097\varepsilon$

$P(H|A) = \frac{P(AH)}{P(A)} = 0.825$

$P(R|A) = \frac{P(RA)}{P(A)} = 0.171$

$P(H') = 0.825$

$P(R') = 0.171$

Recaptcha uses more sophisticated statistics of natural human house gestures and clicks, but this problem covers the central idea behind the new click based recaptchas.