c. (5 points) Every distinct pair of sounds from the 10 second sample casts a vote as to what song the pair thinks is playing. If both sounds in a sound-pair are from the music, the pair always casts a vote for the correct song. Otherwise, since at least one sound is from noise, the pair casts a vote uniformly at random from a set of 5 songs (always the same five songs, including the correct song).

We want **more than 1/5 of the total number of votes** to go to the correct song. How many of the pairs containing background noise must vote for the correct song in order for the correct song to get 1/5 of the votes?

total number = 1025×2049

sound-pair with both songs = 25×49

required votes = $(1025 \times 2049 - 25 \times 49) \times \frac{1}{5}$

$= (2000 \times 1000 + 1000 \times 49 + 2000 \times 25) \times \frac{1}{5}$

$= (2099000) \times \frac{1}{5}$

$= 419800$

d. (6 points) Let d be the number of sound-pairs that you calculated in part (c). What is the probability that the correct song receives more than 1/5 of all the votes?

Since the volume of sound pairs are very high, we can use poisson distribution to represent the probability

$X \sim Poi(\lambda)$

$\lambda = pn$ where $n = 2049 \times 1050$ $p = \frac{1}{5}$

$P(X \geq d) = \sum_{i=d}^{\infty} e^{-\lambda} \frac{\lambda^i}{i!}$