Xiaoqi Zhou (xqzhou@stanford.edu)
06237147

1. (a) **Solution:**
   Define event
   $J = \{\text{engineer program in Java}\}$
   $C = \{\text{engineer program in C++}\}$
   Then $P(C|J) = \frac{P(CJ)}{P(J)}$
   $0.24 = \frac{P(CJ)}{0.36}$
   Answer:
   The probability that a randomly selected engineer programs in Java and C++ is
   $P(CJ) = 0.0864$

   (b) **Solution:**
   $P(J|C) = \frac{P(CJ)}{P(C)}$
   Answer:
   The probability that a randomly selected engineer programs in Java given that he/she programs in C++ is
   $P(J|C) = \frac{0.0864}{0.33} = 0.2618$

2. (a) **Solution:**
   $P(E) = \frac{4\times 3}{52\times 51} = 0.0181$
   The Ace of Spades can be either the first card or the second card
   $P(F) = \frac{1\times 51+51\times 1}{52\times 51} = 0.0385$
   $P(EF) = \frac{1\times 3+3\times 1}{52\times 51} = 0.00226$
   Answer:
   $P(E|F) = \frac{P(EF)}{P(F)} = 0.0588$

   (b) **Solution:**
   Since event G must happen when event E happens
   $P(G|E) = 1$
   $1 = \frac{P(GE)}{P(E)}$
   $P(GE) = P(E)$
   We can calculate the complement of event G
   $P(G^c) = \frac{48\times 47}{52\times 51} = 0.851$
   $P(G) = 1 - P(G^c) = 0.149$
   Answer:
   $P(E|G) = \frac{P(EG)}{P(G)} = \frac{P(E)}{P(G)} = 0.121$

3. (a) **Solution:**
   Define event $E_i = \{\text{a user likes movie} M_i\}$,$T = \{\text{a user like the Tearjerker genre}\}$
   $P(E_i|T) = p_i$
   Answer:
   $P((E_1 \cap E_2 \cap E_3)|T) = P(E_1|T) \cap P(E_2|T) \cap P(E_3|T)$
   Since all the $E_i|T$ are conditionally independent, the probability that a user likes all three movies M_1, M_2 and M_3 given that they like the Tearjerker genre is
   $P((E_1 \cap E_2 \cap E_3)|T) = P(E_1|T)P(E_2|T)P(E_3|T) = p_1p_2p_3$

   (b) **Answer:**
   $P((E_1 \cup E_2 \cup E_3)|T) = P(E_1|T) \cup P(E_2|T) \cup P(E_3|T) = p_1 + p_2 + p_3 - (p_1p_2 + p_3p_2 + p_3p_1)$

   (c) **Solution:**
   Define event $E_{all} = \{\text{user likes all the 3 movie}\}$
   $P(E_{all}|T) = p_1p_1p_3$

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$P(E_{all}|T^c) = q_1q_2q_3$
$P(E_{all}T) = P(E_{all}|T)P(T) = 0.6p_1p_2p_3$
$P(E_{all}T^c) = P(E_{all}|T^c)P(T) = (1-0.6)q_1q_2q_3 = 0.4q_1q_2q_3$
$P(E_{all}) = P(E_{all}T) + P(E_{all}T^c)$
Answer:
The probability that they like the Tearjerker genre that they like M_1, M_2 and M_3 is
$P(T|E_{all}) = \frac{P(TE_{all})}{P(E)} = \frac{0.6p_1p_2p_3}{0.6p_1p_2p_3+0.4q_1q_2q_3}$

4. (a) **Solution:**
We can calculate the probability of event $F = \{textallthe5serversfailedinoneyear\}$
$P(F) = (1-p)^5$
Answer:
The probability that at least 1 server is still working after on year is
$P(E_1) = 1 - P(F) = 1 - (1-p)^5$

(b) **Solution:**
We can consider each particular combination of the 3 servers that are still working.
$P(G_i) = p^3(1-p)^2$
Since all the events are mutually exclusive **Answer:**
The probability that exactly 3 server is still working after on year is
$P(E_3) = \binom{5}{3}P(G_i) = 6p^3(1-p)^2$

(c) **Solution:**
We can consider 3 situations: exactly 3, 4, 5 servers are still working after one year and combine them together.
Answer:
The probability that at least 3 server is still working after on year is
$P(E) = P(E_3) + P(E_4) + P(E_5) = \sum_{i=3}^{5} \binom{5}{i}p^i(1-p)^{5-i}$

5. **Solution:**
The probability of all the bit in a n bit string is
$P(F) = (1-p)^n$
Then the probability that at least one 1 in the string is
$P(E) = 1 - P(F) = 1 - (1-p)^n$
Answer:
$P(E) > 0.7$
The n requirment for the probability that there is at least one 1 in the string is at least 0.7 is
$n > log_{1-p}(0.3)$