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1. (a) **Solution:**
   Define event
   $J = \{\text{engineer program in Java}\}$
   $C = \{\text{engineer program in C++}\}$
   Then $P(C|J) = \frac{P(CJ)}{P(J)}$
   $0.24 = \frac{P(CJ)}{0.36}$
   Answer:
   The probability that a randomly selected engineer programs in Java and C++ is
   $P(CJ) = 0.0864$

   (b) **Solution:**
   $P(J|C) = \frac{P(CJ)}{P(C)}$
   Answer:
   The probability that a randomly selected engineer programs in Java given that he/she programs in C++ is
   $P(J|C) = \frac{0.0864}{0.33} = 0.2618$

2. (a) **Solution:**
   $P(E) = \frac{4\times3}{52\times51} = 0.0181$
   The Ace of Spades can be either the first card or the second card
   $P(F) = \frac{1\times51+51\times1}{52\times51} = 0.0385$
   $P(EF) = \frac{1\times3+3\times1}{52\times51} = 0.00226$
   Answer:
   $P(E|F) = \frac{P(EF)}{P(F)} = 0.0588$

   (b) **Solution:**
   Since event G must happen when event E happens
   $P(G|E) = 1$
   $1 = \frac{P(GE)}{P(E)}$
   $P(GE) = P(E)$
   We can calculate the complement of event G
   $P(G^c) = \frac{48\times47}{52\times51} = 0.851$
   $P(G) = 1 - P(G^c) = 0.149$
   Answer:
   $P(E|G) = \frac{P(EG)}{P(G)} = \frac{P(E)}{P(G)} = 0.121$

3. (a) **Solution:**
   Define event $E_i = \{\text{a user likes movie} M_i\}$,$T = \{\text{a user like the Tearjerker genre}\}$
   $P(E_i|T) = p_i$
   Answer:
   $P((E_1 \cap E_2 \cap E_3)|T) = P(E_1|T) \cap P(E_2|T) \cap P(E_3|T)$
   Since all the $E_i|T$ are conditionally independent, the probability that a user likes all three movies M_1, M_2 and M_3 given that they like the Tearjerker genre is
   $P((E_1 \cap E_2 \cap E_3)|T) = P(E_1|T)P(E_2|T)P(E_3|T) = p_1p_2p_3$

   (b) **Answer:**
   $P((E_1 \cup E_2 \cup E_3)|T) = P(E_1|T) \cup P(E_2|T) \cup P(E_3|T) = p_1 + p_2 + p_3 - (p_1p_2 + p_3p_2 + p_3p_1)$

   (c) **Solution:**
   Define event $E_{all} = \{\text{user likes all the 3 movie}\}$
   $P(E_{all}|T) = p_1p_1p_3$

$P(E_{all}|T^c) = q_1q_2q_3$
$P(E_{all}T) = P(E_{all}|T)P(T) = 0.6p_1p_2p_3$
$P(E_{all}T^c) = P(E_{all}|T^c)P(T) = (1-0.6)q_1q_2q_3 = 0.4q_1q_2q_3$
$P(E_{all}) = P(E_{all}T) + P(E_{all}T^c)$
Answer:
The probability that they like the Tearjerker genre that they like M_1, M_2 and M_3 is
$P(T|E_{all}) = \frac{P(TE_{all})}{P(E)} = \frac{0.6p_1p_2p_3}{0.6p_1p_2p_3+0.4q_1q_2q_3}$

4. (a) **Solution:**
We can calculate the probability of event $F = \{textallthe5serversfailedinoneyear\}$
$P(F) = (1-p)^5$
Answer:
The probability that at least 1 server is still working after on year is
$P(E_1) = 1 - P(F) = 1 - (1-p)^5$

(b) **Solution:**
We can consider each particular combination of the 3 servers that are still working.
$P(G_i) = p^3(1-p)^2$
Since all the events are mutually exclusive **Answer:**
The probability that exactly 3 server is still working after on year is
$P(E_3) = \binom{5}{3}P(G_i) = 6p^3(1-p)^2$

(c) **Solution:**
We can consider 3 situations: exactly 3, 4, 5 servers are still working after one year and combine them together.
Answer:
The probability that at least 3 server is still working after on year is
$P(E) = P(E_3) + P(E_4) + P(E_5) = \sum\limits_{i=3}^{5} \binom{5}{i}p^i(1-p)^{5-i}$

5. **Solution:**
The probability of all the bit in a n bit string is
$P(F) = (1-p)^n$
Then the probability that at least one 1 in the string is
$P(E) = 1 - P(F) = 1 - (1-p)^n$
Answer:
$P(E) > 0.7$
The n requirment for the probability that there is at least one 1 in the string is at least 0.7 is
$n > log_{1-p}(0.3)$

6. (a) **Solution:**
$F_i = \{$at least one string hashed into i-th bucket$\}$
$P(E) = 1 - P((F_1F_2F_3F_4)^c) = 1 - P(F_1^c \cup F_2^c \cup F_3^c \cup F_4^c)$
Since all the F_i^c are not mutually exclusive, the answer will be very complex before expansion. Because of the limited number of buckets and strings, we can try to use another way to get the answer.
$G = \{$All the buckets have at least 1 string$\}$
$H = \{$No string in bucket 5, all the first 4 buckets have one or 2 strings$\}$
$I = \{$No string in bucket 5, all the first 4 buckets have one or 3 strings$\}$
$P(E) = P(G) + P(H) + P(I)$
For each of event G, there is only one bucket can have 2 strings in it and we can add up all the 5

possible situation to get the total probability.
$P(G) = \frac{6!}{2!}\sum_{i=1}^{5}(p_i \prod_{j=1}^{5} p_j) = 360\sum_{i=1}^{5} p_i \prod_{j=1}^{5} p_j = 360\prod_{j=1}^{5} p_j$
For each of event H, there are 2 bucket have 2 strings in it.
$P(H) = \frac{6!}{2!2!}(p_1p_2p_3^2p_4^2 + p_1p_2^2p_3p_4^2 \ldots) = 180(p_1p_2 + p_1p_3 + p_1p_4 + p_2p_3 + p_2p_4 + p_3p_4)p_1p_2p_3p_4$
For each of event H, there are 1 bucket have 3 strings in it.
$P(I) = \frac{6!}{3!}(p_1^3p_2p_3p_4 + p_1p_2^3p_3p_4 + p_1p_2p_3^3p_4 + p_1p_2p_3p_4^3) = 120(p_1^2 + p_2^2 + p_3^2 + p_4^2)p_1p_2p_3p_4$
Answer:
$P(E) = 60(6p_5 + 3(p_1p_2 + p_1p_3 + p_1p_4 + p_2p_3 + p_2p_4 + p_3p_4) + 2(p_1^2 + p_2^2 + p_3^2 + p_4^2))p_1p_2p_3p_4$

(b) **Answer:**
After substitute all the p_i values, we can get
$P(E) = 60 \times (6 \times 0.1 + 3 \times 0.2925 + 2 \times 0.225) \times 0.001875 = 0.2168$

7. (a) **Solution:**
The probability that **`fairRandom`** returns 1 can be described as
$P(r2 = 1|r2 \neq r1) = \frac{P(\{r2=1, r1=0\})}{P(\{r2 \neq r1\})}$
Answer:
The probability that **`fairRandom`** returns 1 is
$P(E) = \frac{p(1-p)}{p(1-p)+(1-p)p} = 0.5$
So that **`fairRandom`** dose indeed return a 0 and a 1 with equal probability.

(b) **Solution:**
Based on the **`simpleRandom`**, the only chance that function can return is when $r2 \neq r1$. So we can find
$P(\{r2 = 1|r1 = 1\}) = 0$
$P(\{r2 = 1|r1 = 0\}) = 1$
The probability of $P($`simpleRandom` returns 1$)$ is
$P(\{r2 = 1\}) = P(\{r1 = 0\}) = 1 - p$
Answer:
We can not guarantee that **`simpleRandom`** generates 0's and 1's with equal probability unless **`unknownRandom`** returns 0's and 1's with equal probability $p = 0.5$.

(c) **Solution:**
After run the simulation, the probability that second player wins is
0.528

8. **Solution:**
We can define the event $E = \{\text{window is detected}\}$. Then
$P(EL_1) = 0.8 \times 0.2 = 0.16$
$P(EL_2) = 0.2 \times 0.9 = 0.18$
Since L_1 and L_2 are complement to each other,
$P(E) = P(EL_1) + P(EL_2) = 0.34$
We can update the probability estimation based on the new information that the window is detected.
$P(L_1|E) = \frac{P(EL_1)}{P(E)} = 0.47$
$P(L_2|E) = \frac{P(EL_2)}{P(E)} = 0.53$
Answer:
The rober new values for $P(L_1)$ and $P(L_2)$ is
$P(L_1') = P(L_1|E) = 0.47$
$P(L_2') = P(L_2|E) = 0.53$

9. **Solution:**
For each location we can define the probability of the cell at that location is $P(L_i)$; the probability of

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a cell records two bars is $P(B)$
$P(B|L_i) = \frac{P(BL_i)}{P(L_i)}$
$P(BL_i) = P(L_i)P(B|L_i)$
Since the summation of the probability that records two bars at all the locations is the total probability that the cell can have two bars.
$PLi|B = \frac{P(BL_i)}{P(B)} = \frac{P(L_i)P(B|L_i)}{\sum\limits_{i=1}^{n} P(L_i)P(B|L_i)}$
In the program, we can calculate $P(BL_i)$ in each location and get the summation of them.
Answer:
The probabilities of all 16 cells is

$$\begin{bmatrix} 0.0744 & 0.1885 & 0.0744 & 0.0050 \\ 0.0050 & 0.1488 & 0.0942 & 0.0744 \\ 0.0010 & 0.0050 & 0.1488 & 0.0942 \\ 0.0010 & 0.0010 & 0.0010 & 0.0744 \end{bmatrix}$$