

1. **Solution:**

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} LL(\theta)$$

$$LL(\theta) = \sum_{i=1}^n \log \lambda e^{-\lambda X_i} = \sum_{i=1}^n \log \lambda - \lambda X_i$$

$$\frac{\delta LL(\lambda)}{\delta \lambda} = \frac{n}{\lambda} - \sum_{i=1}^n X_i = 0$$

**Answer:**

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^n X_i}$$

2. **Solution:**

$$Y_i \sim N(\theta_1 X_i + \theta_2, \sigma^2)$$

$$LL(\theta_1, \theta_2) = \sum_{i=1}^n \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \frac{(Y_i - \theta_1 X_i - \theta_2)^2}{2\sigma^2}$$

**Answer:**

$$LL(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi) - n \log(\sigma) - \frac{1}{2\sigma^2} \sum_{i=1}^n (Y_i - \theta_1 X_i - \theta_2)^2$$

3. **Answer:**

One important assumption of the Naive Bayesian Classifier is all the inputs  $X_i$  from the training set are independent. If the first  $k$  inputs of the training are always identical copies.

$$P(X_1, X_2, \dots, X_n | Y) \neq \prod_{i=1}^n P(X_i | Y)$$

And we have no data can be used as reference to estimate the  $P(X_1, X_2, \dots, X_n | Y)$ , when  $X_1, X_2, \dots, X_k$  are NOT copies of each other.

$$\begin{aligned} 4. \quad (a) \quad \frac{\delta LL(\theta)}{\delta \theta_1} &= \frac{\delta LL(\hat{y})}{\hat{y}} \frac{\delta \hat{y}}{\delta(\theta_3 g + \theta_4 h)} \frac{\delta(\theta_3 g + \theta_4 h)}{\delta g} \frac{\delta g}{\delta \theta_1 x} \frac{\delta \theta_1 x}{\delta \theta_1} \\ \frac{\delta LL(\theta)}{\delta \theta_1} &= \left[ \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right] \hat{y}(1-\hat{y})\theta_3 g(1-g)x \\ \frac{\delta LL(\theta)}{\delta \theta_2} &= \left[ \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right] \hat{y}(1-\hat{y})\theta_4 h(1-h)x \\ \frac{\delta LL(\theta)}{\delta \theta_3} &= \left[ \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right] \hat{y}(1-\hat{y})g \\ \frac{\delta LL(\theta)}{\delta \theta_4} &= \left[ \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right] \hat{y}(1-\hat{y})h \end{aligned}$$