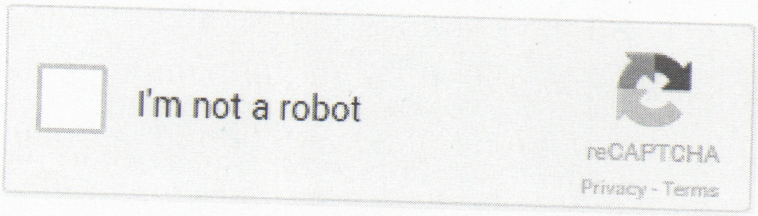
Based on browser history, Google believes that **there is a 0.2 probability that a particular visitor to a website is a robot.** They decide to give the visitor a recaptcha:



Google presents the visitor with a box, 10 pixels wide by 10 pixels tall. The visitor must click inside the box to show that they are not a robot.

Google has observed that robots click very close to the center of a recaptcha. The distance D of a robot click from the center of the box, in pixels, is normally distributed with mean 0 and variance 2. Humans, on the other hand, click uniformly in the box (all locations are equally likely).

a. (6 points) What is the probability that a robot clicks on a pixel that has a **distance from the center** of the box which is greater than or equal to 1.2 pixels?

$$P(X \geq 1.2) = 1 - P(X < 1.2) = 1 - \Phi\left(\frac{1.2-\mu}{\sigma}\right) = 1 - \Phi\left(\frac{1.2-0}{\sqrt{2}}\right)$$

$$= 1 - \Phi(0.85)$$

$$= 1 - 0.8023$$

$$= 0.1977$$

b. (6 points) What the the Probability Density Function (PDF) of a human clicking X pixels from the left of the box and Y pixels from the top of the box?

$$f_{XY}(x,y) = C$$

$$\int_{-5}^{5}\int_{-5}^{5} c \, dx dy = 1$$

$$\int_{-5}^{5} 10c \, dy = 1$$

$$100c = 1$$

$$c = \frac{1}{100}$$

$$f_{XY}(x,y) = \frac{1}{100}$$