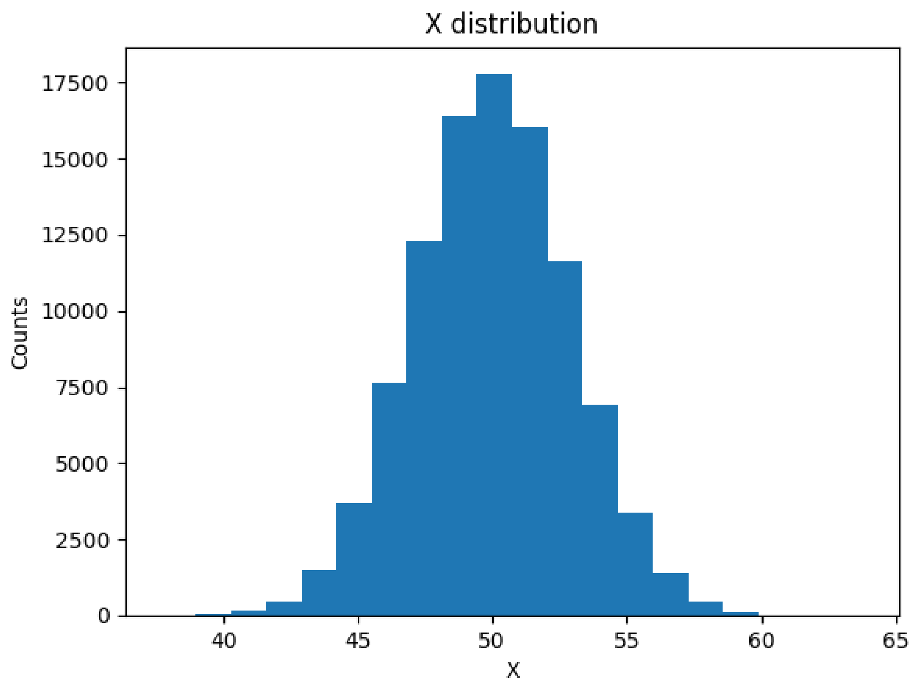
Xiaoqi Zhou (xqzhou@stanford.edu)
06237147

1. (a) **Answer:**
The after trial distribution is:
$f(x) = Beta(2+7, 2+2) = Beta(9, 4)$

(b) **Answer:**
$F_{Beta}(0.5) = 0.073$
$P(\text{drug having effect} \geq 0.5) = 1 - F_{Beta}(0.5) = 0.927$

2. (a) **Answer:**
$P(35 \leq X \leq 40) = 0.00022$
$P(40 \leq X \leq 45) = 0.04192$
$P(60 \leq X \leq 65) = 0.00022$



(b) **Solution:**
From the simulation results, we can find
$E[X] = 50.00$
$Var(X) = 8.336$
Then we can use normal distribution to represent the distribution.
Answer:
$X \sim N(50.00, 8.336)$
$f(x) = \frac{1}{\sqrt{2\pi 8.336}} e^{-(x-50)^2/(2\times 8.336)}$

(c) **Solution:**
$P\{40 \leq X \leq 45\} = F(45) - F(40) = \Phi(\frac{(45-50)}{\sqrt{8.336}}) - \Phi(\frac{(40-50)}{\sqrt{8.336}})$
$\text{text} P\{40 \leq X \leq 45\} = \Phi(\frac{10}{2.89}) - \Phi(\frac{5}{2.89}) = 0.9997 - 0.9582 = 0.0415$

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3. (a) **Answer:**
   The expected amount of money that each person gives is
   $E[X] = 5.95$

   (b) **Answer:**
   $Var(X) = E[X^2] - (E[X])^2 = 23.19$

   (c) **Solution:**
   $E[n\overline{X}] = nE[X] = 5.95 \times 50 = 279.5$
   $Var(n\overline{X}) = nVar(X) = 1159.5$
   Answer:
   $\sum_{i=1}^{50} X_i \sim N(279.5, 1159.5)$

   (d) **Answer:**
   $Y = \sum_{i=1}^{50} X_i$
   $P\{Y \geq 350\} = 1 - P\{Y < 350\} = 1 - \Phi(\frac{350-279.5}{\sqrt{1159.5}}) = 1 - 0.9808 = 0.02$

4. **Solution:**
   $Cov(X, Y) = Cov(X, X^2) = E[X^3] - E[X]E[X^2]$
   $E[X^3] = \frac{1}{6}\sum_{i=1}^{6} i^3 = 73.5$
   $E[X^2] = \frac{1}{6}\sum_{i=1}^{6} i^2 = 15.17$
   $E[X] = \frac{1}{6}\sum_{i=1}^{6} i = 3.5$
   Answer:
   $Cov(X, Y) = 20.65$

5. **Answer:**
   $2X + Y \sim N(2 + 1, 2 \times 2 + 2)$
   $2X + Y \sim N(3, 6)$

6. (a) **Answer:**
   $f(x) = \frac{1}{4\sqrt{2\pi}} e^{-(x-98)^2/32}$

   (b) **Answer:**
   Let Y be the measured distance
   $f(y = 100|x = t) = \frac{1}{2\sqrt{2\pi}} e^{-(100-t)^2/8}$

   (c) **Solution:**
   $f(x, y = 100) = f(y = 100|x)f(x) = \frac{1}{2\sqrt{2\pi}} e^{-(100-x)^2/8} \times \frac{1}{4\sqrt{2\pi}} e^{-(x-98)^2/32}$
   $$f(x|y = 100) = \frac{f(y = 100, x)}{f(y = 100)} = \frac{f(y = 100, x)}{\int_{-\infty}^{+\infty} f(y = 100)dx}$$