

1. (a) **Solution:**

Define event

$J = \{\text{engineer program in Java}\}$

$C = \{\text{engineer program in C++}\}$

Then $P(C|J) = \frac{P(CJ)}{P(J)}$

$$0.24 = \frac{P(CJ)}{0.36}$$

Answer:

The probability that a randomly selected engineer programs in Java and C++ is

$$P(CJ) = 0.0864$$

(b) **Solution:**

$$P(J|C) = \frac{P(CJ)}{P(C)}$$

Answer:

The probability that a randomly selected engineer programs in Java given that he/she programs in C++ is

$$P(J|C) = \frac{0.0864}{0.33} = 0.2618$$

2. (a) **Solution:**

$$P(E) = \frac{4 \times 3}{52 \times 51} = 0.0181$$

The Ace of Spades can be either the first card or the second card

$$P(F) = \frac{1 \times 51 + 51 \times 1}{52 \times 51} = 0.0385$$

$$P(EF) = \frac{1 \times 3 + 3 \times 1}{52 \times 51} = 0.00226$$

Answer:

$$P(E|F) = \frac{P(EF)}{P(F)} = 0.0588$$

(b) **Solution:**

Since event G must happen when event E happens

$$P(G|E) = 1$$

$$1 = \frac{P(GE)}{P(E)}$$

$$P(GE) = P(E)$$

We can calculate the complement of event G

$$P(G^c) = \frac{48 \times 47}{52 \times 51} = 0.851$$

$$P(G) = 1 - P(G^c) = 0.149$$

Answer:

$$P(E|G) = \frac{P(EG)}{P(G)} = \frac{P(E)}{P(G)} = 0.121$$

(a) **Solution:**

Define event $E_i = \{\text{a user likes movie } M_i\}, T = \{\text{a user like the Tearjerker genre}\}$

$$P(E_i|T) = p_i$$

Answer:

$$P((E_1 \cap E_2 \cap E_3)|T) = P(E_1|T) \cap P(E_2|T) \cap P(E_3|T)$$

Since all the $E_i|T$ are conditionally independent, the probability that a user likes all three movies M_1, M_2 and M_3 given that they like the Tearjerker genre is

$$P((E_1 \cap E_2 \cap E_3)|T) = P(E_1|T)P(E_2|T)P(E_3|T) = p_1p_2p_3$$

(b) **Answer:**

$$P((E_1 \cup E_2 \cup E_3)|T) = P(E_1|T) \cup P(E_2|T) \cup P(E_3|T) = p_1 + p_2 + p_3 - (p_1p_2 + p_3p_2 + p_3p_1)$$

(c) **Solution:**

Define event $E_{all} = \{\text{user likes all the 3 movie}\}$

$$P(E_{all}|T) = p_1p_2p_3$$

$$P(E_{all}|T^c) = q_1 q_2 q_3$$

$$P(E_{all}T) = P(E_{all}|T)P(T) = 0.6p_1p_2p_3$$

$$P(E_{all}T^c) = P(E_{all}|T^c)P(T) = (1 - 0.6)q_1q_2q_3 = 0.4q_1q_2q_3$$

$$P(E_{all}) = P(E_{all}T) + P(E_{all}T^c)$$

Answer:

The probability that they like the Tearjerker genre that they like M_1 , M_2 and M_3 is

$$P(T|E_{all}) = \frac{P(T E_{all})}{P(E)} = \frac{0.6p_1p_2p_3}{0.6p_1p_2p_3 + 0.4q_1q_2q_3}$$

(a) **Solution:**

We can calculate the probability of event $F = \{textallthe5serversfailedinoneyear\}$

$$P(F) = (1 - p)^5$$

Answer:

The probability that at least 1 server is still working after on year is

$$P(E_1) = 1 - P(F) = 1 - (1 - p)^5$$