e. (8 points) You are at the Tree House and hear music playing. You believe that there is a:

80% chance the song is: Hold Up by Beyonce (event X_1)
20% chance the song is: Can't Get Used by Andy Williams (event X_2)

You run "Shazam" and the app returns that it predicts the Andy Williams song is playing. Let q be the probability that the correct song is returned by Shazam. Let $(1 - q)$ be the probability that an incorrect song is returned. What is your new probability for X_1 and X_2? Express your answer in terms of q.

$A = \{$ Andy williams song is playing $\}$

$P(A|X_1) = (1-q)$

$P(A|X_2) = q$

$P(AX_1) = P(A|X_1)P(X_1) = (1-q)0.8$

$P(AX_2) = P(A|X_2)P(X_2) = q \times 0.2$

$P(A) = P(AX_1) + P(AX_2) = 0.8 - 0.6q$

$P(X_1|A) = \frac{P(X_1A)}{P(A)} = \frac{(1-q)0.8}{0.8-0.6q}$ $P(X_2|A) = \frac{P(X_2A)}{P(A)} = \frac{0.2q}{0.8-0.6q}$

The new probability is

~~$P(X_1) = P(A)$~~

$P(X_1') = P(X_1|A) = \frac{0.8-0.8q}{0.8-0.6q}$

$P(X_2') = P(X_2|A) = \frac{0.2q}{0.8-0.6q}$

This is the basic version of Shazam. Wisdom of the crowds is a phenomenon where a crowd (in this case a crowd of note-pairs) is correct consistently and wrong randomly.

4 Recaptcha (20 points)