

1. (a) **Solution:**

We calculate after how many rounds, the game provide will use up all their money.

$$m = \log_2 X$$

Then we can derive the expectation of payoff of the game by considering 2 parts, normal payoff and the limited payoff after provider run out of money

$$E(Y) = \sum_{n=1}^m \left(\frac{1}{2}\right)^{n+1} 2^n + \frac{1}{2}^{n+1} X = m \frac{1}{2} + \frac{1}{2}^{n+1} X$$

Answer:

$$E(Y) = m \frac{1}{2} + \frac{1}{2}^{n+1} X$$