6. Eleven-eleven (25 points)

You are running Alibaba, an online retailer. On the busiest hour of the busiest day, 6pm, November 11th (光棍节), you expect to receive 15 million requests per minute.

Each server you own can handle 10,000 requests in a minute. If the number of requests to your site during any minute is greater than 10,000 times the number of servers, you will "drop" requests. You may assume that the number of requests in any minute is **independent** of the number of requests in any other minute.

a. (5 points) Let p be the probability that you drop a request in any given minute of the busiest hour. What is the maximum value of p such that the probability that you don't drop a single request in the busiest hour is > 0.99?

$(1-p)^{60} > 0.99$

$1-p > 0.99^{\frac{1}{60}}$

$p < 1-0.99^{\frac{1}{60}}$

b. (7 points) Write an expression for the **exact** number of machines that you need (K) such that the probability that you don't drop a single request in the busiest hour is > 0.99. Use p from part a. The expression does *not* have to be closed form.

The distribution of request in each minute is

$X \sim poi(\lambda)$ where $\lambda = 15 \times 10^6$

To guarantee that the chance of request over the capablity of server. lower than p

$P(X > 1 \times 10^5 K) < p$

$\sum_{i=10^5 K}^{\infty} e^{-\lambda} \frac{\lambda^i}{i!} < p$