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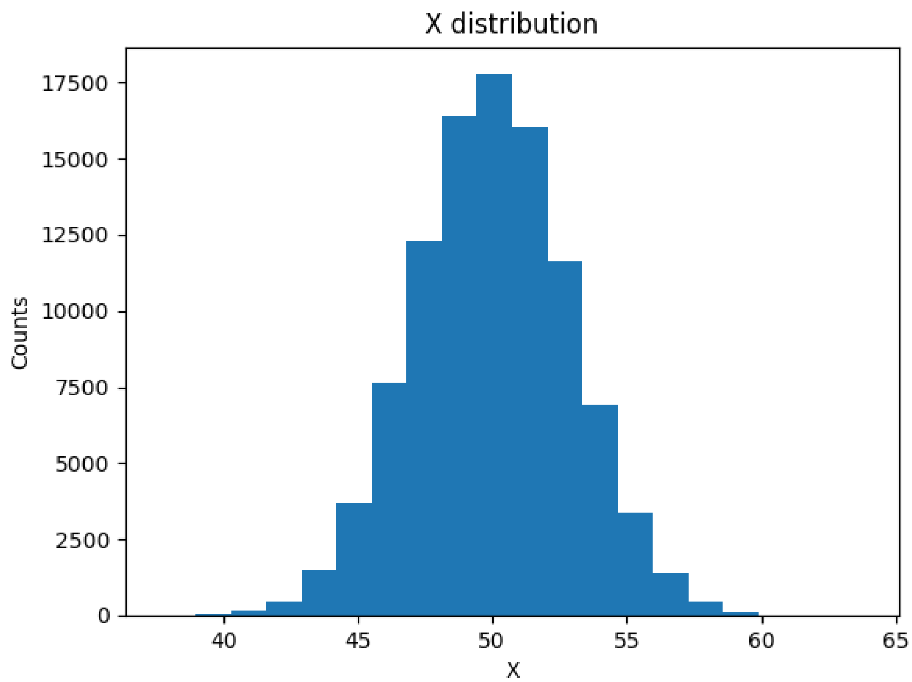
1. (a) **Answer:**
   The after trial distribution is:
   $f(x) = Beta(2+7, 2+2) = Beta(9,4)$

   (b) **Answer:**
   $F_{Beta}(0.5) = 0.073$
   $P(\text{drug having effect} \geq 0.5) = 1 - F_{Beta}(0.5) = 0.927$

2. (a) **Answer:**
   $P(35 \leq X \leq 40) = 0.00022$
   $P(40 \leq X \leq 45) = 0.04192$
   $P(60 \leq X \leq 65) = 0.00022$



   (b) **Solution:**
   From the simulation results, we can find
   $E[X] = 50.00$
   $Var(X) = 8.336$
   Then we can use normal distribution to represent the distribution.
   Answer:
   $X \sim N(50.00, 8.336)$
   $f(x) = \frac{1}{\sqrt{2\pi 8.336}} e^{-(x-50)^2/(2\times 8.336)}$

   (c) **Solution:**
   $P\{40 \leq X \leq 45\} = F(45) - F(40) = \Phi(\frac{(45-50)}{\sqrt{8.336}}) - \Phi(\frac{(40-50)}{\sqrt{8.336}})$
   $\text{text}P\{40 \leq X \leq 45\} = \Phi(\frac{10}{2.89}) - \Phi(\frac{5}{2.89}) = 0.9997 - 0.9582 = 0.0415$

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3. (a) **Answer:**
The expected amount of money that each person gives is
$E[X] = 5.95$

(b) **Answer:**
$Var(X) = E[X^2] - (E[X])^2 = 23.19$

(c) **Solution:**
$E[n\overline{X}] = nE[X] = 5.95 \times 50 = 279.5$
$Var(n\overline{X}) = nVar(X) = 1159.5$
Answer:
$\sum_{i=1}^{50} X_i \sim N(279.5, 1159.5)$

(d) **Answer:**
$Y = \sum_{i=1}^{50} X_i$
$P\{Y \geq 350\} = 1 - P\{Y < 350\} = 1 - \Phi(\frac{350-279.5}{\sqrt{1159.5}}) = 1 - 0.9808 = 0.02$

4. **Solution:**
$Cov(X,Y) = Cov(X, X^2) = E[X^3] - E[X]E[X^2]$
$E[X^3] = \frac{1}{6}\sum_{i=1}^{6} i^3 = 73.5$
$E[X^2] = \frac{1}{6}\sum_{i=1}^{6} i^2 = 15.17$
$E[X] = \frac{1}{6}\sum_{i=1}^{6} i = 3.5$
Answer:
$Cov(X,Y) = 20.65$

5. **Answer:**
$2X + Y \sim N(2 + 1, 2 \times 2 + 2)$
$2X + Y \sim N(3, 6)$

6. (a) **Answer:**
Let X be the true distance of the satellite
$f_X(x) = \frac{1}{4\sqrt{2\pi}} e^{-(x-98)^2/32}$

(b) **Answer:**
Let Y be the measured distance
$f_{Y|X}(y = 100|x = t) = \frac{1}{2\sqrt{2\pi}} e^{-(100-t)^2/8}$

(c) **Solution:**
Let X' be the posterior belief of the true distance of satellite
$f_{XY}(x, y = 100) = f_{Y|X}(y = 100|x) f_X(x) = \frac{1}{2\sqrt{2\pi}} e^{-(100-x)^2/8} \times \frac{1}{4\sqrt{2\pi}} e^{-(x-98)^2/32}$

$$f_{X'}(x') = f_{X|Y}(x = x'|y = 100) = \frac{f_{XY}(y = 100, x = x')}{f_Y(y = 100)} = \frac{f_{XY}(x = x', y = 100)}{\int_{-\infty}^{+\infty} f_{XY}(y = 100)dx}$$

Let

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$$C = \frac{1}{\int_{-\infty}^{+\infty} f_{XY}(x, y = 100)dx}$$

Answer:

$$f_{X'}(x') = C\frac{1}{16\pi^2}e^{\frac{-(4(100-x')^2+(x'-98)^2)}{32}}$$

7. (a) **Solution:**
   Let X_n be the total value of n rolling before any value ≥ 3 is rolled
   $E[X] = \sum_{i=0}^{\infty} E[X_i] + 4.5$
   Let Y be the value when for each rolling when value < 3
   $E[Z] = 1.5$
   $E[X_i] = (\frac{1}{3})^i(iE[Z]) = 1.5i(\frac{1}{3})^i$
   Answer:
   $$E[X] = \sum_{i=0}^{\infty} 1.5i(\frac{1}{3})^i + 4.5 = 1.5\frac{\frac{1}{3}}{(1-\frac{1}{3})^2} + 4.5 = 5.625$$

   (b) **Answer:**
   $$E[Y] = \sum_{i=0}^{\infty} i(\frac{1}{3})^i + 1 = \frac{\frac{1}{3}}{(1-\frac{1}{3})^2} + 1 = 1.75$$

8. **Solution:**
   $E[Y^2] = E[Y^2|X=1]P(X=1) + E[Y^2|X=2]P(X=2) + E[Y^2|X=2]P(X=2)$
   $E[Y^2] = E[3^2]\frac{1}{3} + E[(Y+5)^2]\frac{1}{3} + E[(Y+7)^2]\frac{1}{3}$
   $3E[Y^2] = 9 + E[Y^2] + 10E[Y] + 25 + E[Y^2] + 14E[Y] + 49$
   $E[Y^2] = 443$
   Answer:
   $$Var(Y) = E[Y^2] - E[Y]^2 = 443 - 225 = 218$$

9. (a) **Solution:**
   $E[n\overline{A}] = n\mu_A = 20 \times 50 = 1000$
   $Var(n\overline{A}) = n^2\frac{\sigma_A^2}{n} = n\sigma_A^2 = 2000$
   $P(n\overline{A} < 950) = \Phi(\frac{950-1000}{\sqrt{2000}}) = 1 - \Phi(\frac{50}{44.72}) = 1 - 0.8686$
   $$P(n\overline{A} < 950) = 0.1314$$

   (b) $E[n\overline{B}] = n\mu_B = 20 \times 52 = 1040$
   $Var(n\overline{B}) = n^2\frac{\sigma_B^2}{n} = n\sigma_B^2 = 4000$
   $P(n\overline{B} < 950) = \Phi(\frac{950-1040}{\sqrt{4000}}) = 1 - \Phi(\frac{90}{44.72}) = 1 - 0.9207$
   $$P(n\overline{B} < 950) = 0.0793$$

   (c) Let $C = A - B$
   $E[n\overline{C}] = n\mu_C = \mu_A - \mu_B = -40$
   $Var(n\overline{C}) = n\sigma_C^2 = n(\sigma_A^2 + \sigma_B^2) = 6000$
   Answer:
   $$P(C < 0) = F_C(0) = \Phi(\frac{0-(-40)}{\sqrt{6000}}) = 0.6985$$

10. **Solution:**
    Let $Y = \sum_{i=1}^{79} X_i$

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$E[Y] = 79E[X] = 79 \times 3.5 = 276.5$
$Var(Y) = 79Var(X)$
$Var(X) = E[X^2] - E[X]^2 = 2.92$
$Var(Y) = 230.68$
$Y \sim N(276.5, 230.68)$
When the first 79 rolls have sum lower than 300, at least 80 rolls are necessary to reach a sum that exceeds 300.
$P(Y \leq 299.5) = F_Y(299.5) = \Phi(\frac{299.5-276.5}{sqrt230.68})$
Answer:
$P(Y \leq 299.5) = 0.9332$

11. (a) **Answer:**
$P(X \geq 85) \leq \frac{E[X]}{85} = 0.882$

(b) **Solution:**
$P(65 \leq X \leq 85) > P(|X - 75| < 10) = 1 - P(|X - 75| \geq 10)$
$P(|X - 75| \geq 10) \leq \frac{Var(X)}{10^2} = 0.25$
$0.75 \leq 1 - P(|X - 75| \geq 10) < P(65 \leq X \leq 85)$
Answer:
$P(65 \leq X \leq 85) > 0.75$

(c) **Solution:**
Let $Y = \overline{X}$
$E[Y] = E[X] = 75$
$Var(Y) = \frac{Var(X)}{n} = \frac{25}{n}$ $P(|Y - 75| \geq 5) \leq \frac{Var(Y)}{5^2} = \frac{\frac{25}{n}}{25} \leq 0.1$
Answer:
$\frac{1}{n} \leq 0.1$
The number of students would have to take the midterm in order to ensure 90% probability that the class average would be within 5 of 75 is
$n \geq 10$

(d) **Solution:**
$P(-5 \leq \overline{X} - E[X] \leq 5) = \Phi(\frac{5}{\frac{\sigma}{\sqrt{n}}}) - \Phi(\frac{-5}{\frac{\sigma}{\sqrt{n}}}) = 2\Phi(\frac{5}{\frac{5}{\sqrt{n}}}) - 1 \geq 0.9$
$\Phi(\sqrt{n}) > 0.95$ $\sqrt{n} > 1.65$
Answer:
The number of students would have to take the midterm in order to ensure 90% probability that the class average would be within 5 of 75 is
$n = 3 > 2.72$

12. (a) **Answer:**
$P(S = true|G = female, C = 1) = 0.968$
$P(S = true|G = female, C = 1) = 0.921$
$P(S = true|G = female, C = 1) = 0.5$
$P(S = true|G = female, C = 1) = 0.369$
$P(S = true|G = female, C = 1) = 0.157$
$P(S = true|G = female, C = 1) = 0.137$

(b) **Solution:**
Total third class children is 53, and 22 of them survived.

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We can use the conditional of all the third class passengers as a prior distribution
Total third class passenger number is 487, 119 of them survived
Let
$a = 487 + 1 = 488$
$b = 119 + 1 = 120$
$m = 22$
$n = 31$
Answer:
$S \sim Beta(a + m, b + n)$
$S \sim Beta(510, 151)$

(c) **Answer:**
$E[X|C = 1] = 84.15$
$E[X|C = 1] = 20.66$
$E[X|C = 1] = 13.71$

13. (a) **Answer:**
$\overline{X} = 69.22$

(b) **Solution:**
$Y = \overline{X}$
$S^2 = \sum_{i=1}^{n} \frac{(Y_i - \overline{Y})}{n-1}$

Code:

```
n = 100000 #repeat 100000 loops
grade_mean_array = numpy.zeros(n)
index = [0]*5
for i in range(0,n):
    for j in range(0,5):
        #Generate 5 random index for each grades
        index[j] = int(numpy.floor(numpy.random.rand()*10000))
    grade_mean_array[i] = numpy.mean(grade_array[index])#Get mean of the 5 grades
#Calculate the expectation of all the mean values
grade_mean_expectation = numpy.mean(grade_mean_array)
grade_mean_s = float(0)
for i in range(0, len(grade_mean_array)):
    grade_mean_s = grade_mean_s \
                    + ((grade_mean_array[i] - grade_mean_expectation)**2)\
                /(len(grade_mean_array)-1) #Calculate S^2


print("grade mean expectation: %f"%(grade_mean_expectation))
print("grade mean variance: %f"%(grade_mean_s))
```

Answer:
$E[Y] = 69.22$
$S^2 = 101.18$

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(c) **Solution:**

$Z = median(X)$

$S^2 = \sum_{i=1}^{n} \frac{(Z_i - \overline{Z})}{n-1}$

Code:

```
n = 100000 #repeat 100000 loops
grade_median_array = numpy.zeros(n)
index = [0]*5
for i in range(0,n):
    for j in range(0,5):
        # Generate 5 random index for each grades
        index[j] = int(numpy.floor(numpy.random.rand()*10000))
    # Get median of the 5 grades
grade_median_array[i] = numpy.median(grade_array[index])
#Calculate the expectation of all the median values
grade_median_expectation = numpy.mean(grade_median_array)

grade_median_s = float(0)
for i in range(0, len(grade_median_array)):
    grade_median_s = grade_median_s \
                    + ((grade_median_array[i] - grade_median_expectation)**2)\
                    /(len(grade_median_array)-1) #Calculate S^2


print("grade mean expectation: %f"%(grade_median_expectation))
print("grade mean variance: %f"%(grade_median_s))
```

Answer:

$E[Z] = 73.60$

$S^2 = 56.04$

(d) **Answer:**

$E[Y] = 69.22$

$E[Z] = 73.60$

The expected median of 5 grades is different than the expected mean of 5 grades.

(e) **Solution:**

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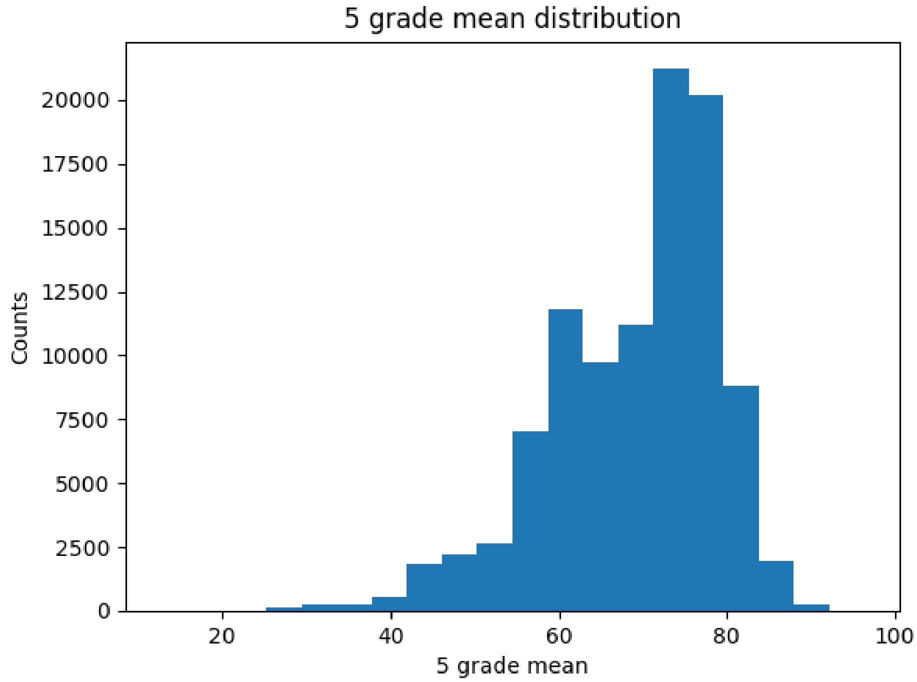

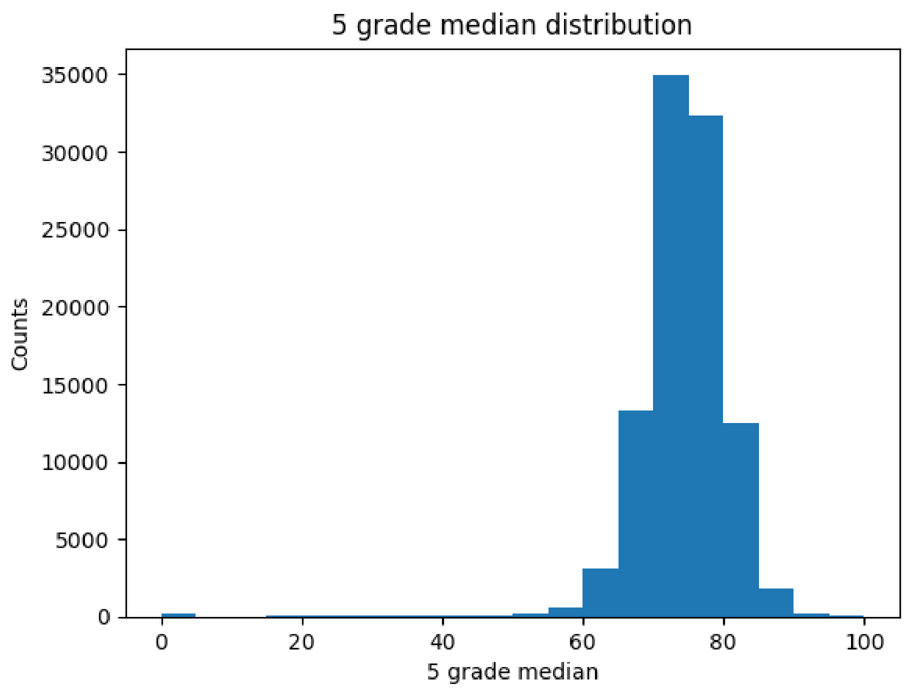

Answer:
Using median of 5 grades is a better way to assign scores, because the distribution is more like a normal distribution

14. (a) **Answer:**

The outcome of activity2 is not independent to activity1. All the student have already learned activity1 when learned activity2. It is difficult to justify outcome of activity2 separately.

(b) **Answer:**
The outcome may not only depend on different activities but also the geographic distribution of the students. The result will be biased by different between the students from eastern vs. western hemisphere.

15. (a) Let X be the outcome of activity1; Y be the outcome of activity2
$E[X] = 144.93$
$E[Y] = 153.13$
Answer:
$|E[X] - E[Y]| = 8.2$

(b) **Code:**

```
n = 100000 #repeat 100000 times bootstraping
new_diff = numpy.zeros(n)
p_num = float(0)
for i in range(0,n):
    #select same size of activity1 from uniform dataset
    new_selection_1 = numpy.array(random.sample(outcome, int(num_1)))
    #select same size of activity2 from uniform dataset
    new_selection_2 = numpy.array(random.sample(outcome, int(num_2)))
    #calculate mean of new activity1
    new_mean_1 = numpy.mean(new_selection_1)
    #calculate mean of new activity2
    new_mean_2 = numpy.mean(new_selection_2)
    new_diff[i] = abs(new_mean_1 - new_mean_2)
    #Compare the new observed difference with the pre-calculated difference
    if new_diff[i] >= diff:
        p_num = p_num + 1
#Calculate P-Value
p_value = p_num/n
print("P-Value = %f"%(p_value))
```

Answer:
$P - Value = 0.00478$

(c) **Solution:**
We can just separate all the students data into 3 groups first based on the background category (link two .csv file by using first column number). Then calculate the mean difference and p-values.
Answer:
For "more" experience group
$|E[X] - E[Y]| = 28.42$
$P - Value = 0.0000$
For "average" experience group
$|E[X] - E[Y]| = 24.98$
$P - Value = 0.00005$
For "less" experience group
$|E[X] - E[Y]| = 26.02$

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$P - Value = 0.0000$

From the Result we can find that the difference of 2 activities are more significant if we separate the students in different catagory. That means the activities have different outcome distribution in different catagory groups. Mixing the data from all 3 group will lead us to a biased result.