1. (a) **Solution:**
We calculate after how many rounds, the game provide will use up all their money.
$m = \lfloor log_2 X \rfloor$
Then we can derive the expectation of payoff of the game by considering 2 parts, normal payoff and the limited payoff after provider run out of money, after you win more or equal to 2^m, no matter what happend, you will win X
$E[Y] = \sum_{n=0}^{m} (\frac{1}{2})^{n+1} 2^n + \frac{1}{2}^{n+1} X = (m+1)\frac{1}{2} + \frac{1}{2}^{m+1} X$
Answer:
$m = 2$
$E[Y] = 3 \times \frac{1}{2} + \frac{1}{2}^3 \times 5 = 2.125$

(b) **Answer:**
$m = 8$
$E[Y] = 9 \times \frac{1}{2} + \frac{1}{2}^8 \times 500 = 5.477$

Answer:
$m = 12$
$E[Y] = 13 \times \frac{1}{2} + \frac{1}{2}^{13} \times 4096 = 7$

2. **Solution:**
The expectation of new users in 5 minutes is $E(X) = 2$. The expectation of gain for the trip can be preseted as
$E[(X+1) \times \$6 - \$7] = E[X] \times \$6 - \$1$
Answer:
The expectation of Lyft to make in this trip is
$E[X] \times \$6 - \$1 = \$11$

3. (a) **Solution:**
The possibility that x bit are corrupted is
$P(X = x) = \binom{2n}{x} p^x (1-p)^{(2n-x)}$
Answer:
The probability that the message ss is received without any corruption is
$P(X = 0) = (1 - 0.05)^8 = 0.663$

(b) **Solution:**
For each first string s which has any corruption bit, if the second time the corruption bit(s) are exactly the same, we can not detect that problem
Answer:
The probability that we can not detect the corrupted bit(s)
$P(E) = \sum_{x=1}^{n} \binom{n}{x} (p^x (1-p)^{(n-x)})^2$
$P(E) = \sum_{x=1}^{4} \binom{4}{x} (0.05^x (1-0.05)^{(4-x)})^2 = 0.0074$

(c) **Solution:**
The probability that recipient can detect the corruption took place is
$P(F) = 1 - P(X = 0) - P(E)$
Answer:
$P(F) = 1 - (1-p)^{2n} - \sum_{x=1}^{n} \binom{n}{x} (p^x (1-p)^{(n-x)})^2 = 1 - 0.663 - 0.0074 = 0.329$

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4. There are 2 cases that the jury renders a correct decision, votes guilty when the defendants are actually guilty or votes innocent and the defendants are actally innocent.
The probability of the defendants are actually guilty is
$P(G) = 0.75$
For each juror, the probability that votes guilty $P(H)$ in different conditions is
$P(H|G^c) = 0.1$
$P(H^c|G) = 0.2$
The probability that the jury decides the defendants are guilty is $P(F)$
Then the probability make correct decision when the defenant is actually guilty (less than 4 juror votes innocent) is
$P(F|G) = \sum_{n=0}^{3} \binom{12}{i} P(H^c|G)^i P(H|G)^{12-i}$
$P(F|G) = \sum_{n=0}^{3} \binom{12}{i} 0.2^i 0.8^{12-i} = 0.7946$
$P(FG) = P(F|G)P(G) = 0.5959$
Then the probability make correct decision when the defenant is actually innocent (1 - P(less than 4 juror votes innocent)) is
$P(F^c|G^c) = 1 - \sum_{n=0}^{4} \binom{12}{i} P(H^c|G^c)^i P(H|G^c)^{12-i}$
$P(F|G) = 1 - \sum_{n=0}^{3} \binom{12}{i} 0.92^i 0.1^{12-i} = 1 - 3.4 \times 10^{-6} = 1$
$P(FG) = P(F|G)P(G) = 0.5959$