5. Exponentials (16 points)

a. (8 points) What is the probability that an exponential random variable X ~ Exp(λ) takes on a value that is within one standard deviation of its mean?

$\sigma = \frac{1}{\lambda}$

$P(X \le \frac{1}{\lambda}) = \int_0^{\frac{1}{\lambda}} \lambda e^{-\lambda x} dx$

$= -e^{\lambda x} \Big|_0^{\frac{1}{\lambda}}$

$= -e + 1$

$= 1 - e$

b. (8 points) Let X_1 ~ Exp(λ_1) and X_2 ~ Exp(λ_2) be independent exponential random variables. Let M = max(X_1 , X_2) where max is a function which returns the larger of the two values. Give an expression for the cumulative density function of M.

$P_m(X_m \le a) = P_1(X_1 \le a) \cap P_2(X_2 \le a)$

since X_1 and X_2 are independent

$P_m(X_m \le a) = P_1(X_1 \le a) \cdot P_2(X \le a)$

$F_M(a) = (1 - e^{-\lambda_1 a})(1 - e^{-\lambda_2 a})$