南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目：<u>概率论与数理统计</u>　　　开课单位：<u>数学系</u>

考试时长：<u>2 小时</u>　　　命题教师：<u>概率论与数理统计教学组</u>

题号	Part 1	Part 2	Part 3					
			1	2	3	4	5	6
分值								

本试卷共三大部分，满分 100 分（考试结束后请将试卷、答题本、草稿纸一起交给监考老师）

第一部分　选择题（每题 4 分，总共 20 分）

Part One – Single Choice (4 marks each question, 20 marks in total)

1. 设A，B，C表示三个事件，则$\overline{A}\,\overline{B}\,\overline{C}$表示（　　）.

   (A) A，B，C中有一个发生;

   (B) A，B，C中恰有两个发生;

   (C) A，B，C中不多于一个发生;

   (D) A，B，C都不发生.

Assume A，B，C are three events, then $\overline{A}\,\overline{B}\,\overline{C}$ means that (　　) .

   (A) one of the events A, B, C happens;

   (B) two of the events A, B, C happen;

   (C) no more than one of the events A, B, C happen;

   (D) none of the events A, B, C happens.

2. 甲、乙、丙 3 人独立地译出一种密码，他们能译出的概率分别为1/5,1/3,1/4，则能译出这种密码的概率为(　　).

   (A) 1/5　　　(B) 2/5　　　(C) 3/5　　　(D) 4/5

There are three people who are independently guessing a password. The probability of individually getting the password is 1/5, 1/3, 1/4 respectively. The probability of getting the password is (　　).

   (A) 1/5　　　(B) 2/5　　　(C) 3/5　　　(D) 4/5

3. 设随机变量$X \sim N(0,4)$，$Y \sim N(1,4)$，且X与Y相互独立，则$X-Y$服从(　　)分布.

   (A) $N(-1,0)$　　　(B) $N(-1,32)$　　　(C) $N(-1,8)$　　　(D) $N(1,8)$

Assume random variables $X \sim N(0,4)$，$Y \sim N(1,4)$, and they are independent to each other. Then, $X-Y$ follows the distribution of (　　).

   (A) $N(-1,0)$　　　(B) $N(-1,32)$　　　(C) $N(-1,8)$　　　(D) $N(1,8)$

4. 设随机变量X和Y独立同分布，且X的分布函数为$F(x)$，则$Z = max\,(X,Y)$的分布函数为(　　).

   (A) $F^2(z)$　　　(B) $1-F^2(z)$　　　(C) $\left(1-F(z)\right)^2$　　　(D) $1-\left(1-F(z)\right)^2$

Assume that random variables X and Y independent and identically distributed. The distribution of r.v. X is $F(x)$. The distribution function of $Z = max\,(X, Y)$ is (　　).

(A) $F^2(z)$　　(B) $1 - F^2(z)$　　(C) $\left(1 - F(z)\right)^2$　　(D) $1 - \left(1 - F(z)\right)^2$

5. 已知随机变量X的分布函数为$F(x)$，且$F(a) = 0.5$，则(　　)

(A) 存在点$x_0 < a$，使得$F(x_0) > 0.5$ 成立;

(B) 存在点x_0，使得$F(x_0) > 1$ 成立;

(C) 对任意$x > a$有$F(x) = 1$;

(D) 对任意$x > a$有$F(x) \geq 0.5$.

Assume the distribution of r.v. X is $F(x)$, and $F(a) = 0.5$. Which statement is true? (　　)

(A) There is a point of $x_0 < a$, which makes $F(x_0) > 0.5$;

(B) There is a point x_0, which makes $F(x_0) > 1$;

(C) Any point of $x > a$ makes$F(x) = 1$;

(D) Any point of $x > a$ makes$F(x) \geq 0.5$.

第二部分 填空题（每空 2 分，总共 20 分）

Part Two – Blank Filling (2 marks each blank, 20 marks in total)

1. 已知事件A,B相互独立，事件C与A,B互不相容，且$P(A)=0.5, P(B)=0.4, P(C)=0.2$，设$D$为事件$A,B,C$中至少有一个发生，则$P(D)=$__________.

   Events A and B are independent to each other. Events C and A, Events C and B are disjoint events. Furthermore $P(A)=0.5, P(B)=0.4, P(C)=0.2$. Assume D is the event that at least one of the three Events A,B,C happens, thus $P(D)=$_________.

2. 已知A,B两个事件满足条件$P(AB)=P(\overline{A}\,\overline{B})$，且$P(A)=p$，则$P(B)=$__________.

   There are two events A,B, and they have $P(AB)=P(\overline{A}\,\overline{B})$. If $P(A)=p$, then $P(B)=$__________.

3. 设两个相互独立的事件A和B都不发生的概率为 $1/9$，A发生B不发生的概率与B发生A不发生的概率相等，则$P(A)=$__________.

   Assume there are two independent events A and B. The probability that both of them don't happen is $1/9$. The probability of A happens and B doesn't is the same as the probability of B happens and A doesn't. Then $P(A)=$__________.

4. 设$F_1(x), F_2(x)$都是一元分布函数，常数$a,b>0$, 若$a\cdot F_1(x)+b\cdot F_2(x)$也是分布函数，则常数$a,b$应满足的条件是________.

   Assume that $F_1(x), F_2(x)$ are one dimensional distributions with the constants $a,b>0$. If $a\cdot F_1(x)+b\cdot F_2(x)$ is a distribution, then the constants a,b satisfy the condition of ________.

5. 设随机变量$X\sim N(-3,25)$，令$Y=-2(X+3)$，则随机变量Y服从的分布及参数为__________.

   Assume the random variable $X\sim N(-3,25)$, and $Y=-2(X+3)$. Then, the distribution of r.v. Y is $Y\sim$ __________.

6. 设随机变量 Y 服从参数为 1 的指数分布，a 为常数且大于 0，则 $P\{Y \le a+1 | Y > a\} =$__________.

Assume the random variable Y follows exponential distribution with the parameter 1. If a is a constant being greater than 0, then $P\{Y \le a+1 | Y > a\} =$__________.

7. 设两个相互独立的随机变量 X 与 Y 分别服从正态分布 $N(0,1)$ 和 $N(1,1)$，则 $P\{X+Y \le 1\} ==$__________.

Assume there are two independent random variables X and Y, they follow normal distribution $N(0,1)$ and $N(1,1)$ respectively. Then $P\{X+Y \le 1\} ==$__________.

8. 设随机变量(X,Y)的联合密度函数为$f(x,y) = \begin{cases} \frac{1}{2}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{其他} \end{cases}$，则$X,Y$中至少有一个小于 1/2 的概率为__________.

Assume that the joint density function of random variables (X,Y) is $f(x,y) = \begin{cases} \frac{1}{2}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{others} \end{cases}$. Then the probability that at least one of the events $\{X \le 0.5\}$ and $\{Y \le 0.5\}$ happens is__________.

9. 设随机变量 X 与 Y 相互独立，且 X 与 Y 均服从区间 [0,3] 上的均匀分布，则 $P\{\max\{X,Y\} \le 1\} =$__________.

Assume there are two independent random variables X and Y, they all follow uniform distribution U[0,3]. Then $P\{\max\{X,Y\} \le 1\} =$__________.

10. 设 X 与 Y 是两个随机变量，且 $P\{X \ge 0, Y \ge 0\} = \frac{3}{7}$，$P\{X \ge 0\} = P\{Y \ge 0\} = \frac{4}{7}$. 则 $P\{\max\{X,Y\} \ge 0\} =$__________.

Assume that X and Y are two random variables, and $P\{X \ge 0, Y \ge 0\} = \frac{3}{7}$, $P\{X \ge 0\} = P\{Y \ge 0\} = \frac{4}{7}$. Then $P\{\max\{X,Y\} \ge 0\} =$__________.

第三部分 大题（每题 10 分，总共 60 分）

Part Three – Question Answering (10 marks each question, 60 marks in total)

1. 学生在做一道有 4 个选项的单项选择题时，如果学生不知道正确答案，就作随机猜测. 现从卷面上看题是答对了，试在以下情况下求学生确实知道正确答案的概率.
   a) 学生知道正确答案和胡乱猜测的概率都是 0.5.
   b) 学生知道正确答案的概率都是 0.2.

   A student needs to select 1 choice from 4 optional choices for answering a question. The student will randomly select a choice if having not got the answer, and of course will select it if having obtained the answer. Now the student's selected choice is the answer, what is the probability that the student has obtained the answer before selecting a choice based on the following scenarios?
   a) The probability of having obtained the answer before selecting a choice is 0.5.
   b) The probability of having obtained the answer before selecting a choice is 0.2.

2. 设随机变量X的概率分布$P\{X=1\}=P\{X=2\}=\frac{1}{2}$. 在给定$X=i$的条件下，随机变量$Y$服从均匀分布$U(0,i)$（$i=1,2$）. 求$Y$的分布函数$F_Y(y)$和密度函数$f_Y(y)$.

   Assume the random variable Xhas $P\{X=1\}=P\{X=2\}=\frac{1}{2}$. Under the condition $X=i$, the random variable Y follows uniform distribution $U(0,i)(i=1,2)$. What are the distribution function $F_Y(y)$ and the density function $f_Y(y)$?

3. 若每只母鸡产蛋的个数服从参数为λ的泊松分布，而每个蛋能孵化成小鸡的概率为p. 试证：每只母鸡有k只小鸡的概率服从参数为λp的泊松分布.

   If a hen lays eggs X which follows Poisson distribution with the parameter λ. The probability that each egg transforms into a chick is p. Determine the probability of each hen has k chicks ($Y=k$) such that Yfollows Poisson distribution with the parameter λp.

4．设$Y = X^2$，其中随机变量X的密度函数为

$$f_X(x) = \begin{cases} cx, & 0 < x < 2, \\ 0, & \text{其他}. \end{cases}$$

a)　求常数c.

b)　求Y的密度函数$f_Y(y)$.

Assume $Y = X^2$，and the density function of r.v. X is:

$$f_X(x) = \begin{cases} cx, & 0 < x < 2, \\ 0, & \text{others}. \end{cases}$$

a)　What is the constant c?

b)　What is the density function $f_Y(y)$?

5.　已知随机变量X和Y的分布函数分别为

$$F_X(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{3}, & 0 \le x < 1, \\ 1, & x \ge 1, \end{cases} \qquad F_Y(y) = \begin{cases} 0, & y < 1, \\ \frac{1}{2}, & 1 \le y < 2, \\ 1, & y \ge 2. \end{cases}$$

且已知$P(X = 1, Y = 1) = \frac{1}{3}$，求：

a)　X和Y的联合频率函数；

b)　X和Y是否独立？

c)　$Y = 1$ 时，X的条件频率函数$P(X = k|Y = 1)$.

Assume the distribution functions of r.v. X and r.v. Y are as follows.

$$F_X(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{3}, & 0 \le x < 1, \\ 1, & x \ge 1, \end{cases} \qquad F_Y(y) = \begin{cases} 0, & y < 1, \\ \frac{1}{2}, & 1 \le y < 2, \\ 1, & y \ge 2. \end{cases}$$

Furthermore, $P(X = 1, Y = 1) = \frac{1}{3}$.

a)　What is the joint frequency function of r.v. X and r.v. Y?

b)　Are X and Y independent?

c)　When $Y = 1$，what is the conditional probability$P(X = k|Y = 1)$?

6.　设二维随机变量(X, Y)的联合密度函数为

$$f(x,y)=\begin{cases} ke^{-(x+y)}, & 0<x<1,0<y<\infty, \\ 0, & \text{其他}. \end{cases}$$

a) 确定常数k;

b) 求边际密度函数$f_X(x)$，$f_Y(y)$;

c) 求函数$Z=\max\{X,Y\}$的分布函数.

Assume the joint density function of the two-dimensional random variable (X,Y)

$$f(x,y)=\begin{cases} ke^{-(x+y)}, & 0<x<1,0<y<\infty, \\ 0, & \text{others} \end{cases}$$

a) Compute the constant k;

b) Find the marginal density function of $f_X(x)$，$f_Y(y)$;

c) Find the distribution function of $Z=\max\{X,Y\}$.

答案

第一部分　选择题（每题 4 分，总共 20 分）

D C C A D

第二部分 填空题（每空 2 分，总共 20 分）

1. 0.9
2. $1-p$
3. 2/3
4. $a+b=1$
5. $N(0,100)$
6. $1-e^{-1}$
7. 0.5
8. 5/8
9. 1/9
10. 5/7