Artificial Intelligence (CS303)

Lecture 10: Logical Agents

Hints for this lecture

- Human not only act based on instinct (gene? Program?), but also act based on knowledge.
- Represent, store, and exploit knowledge should also be important (or at least useful) for AI.

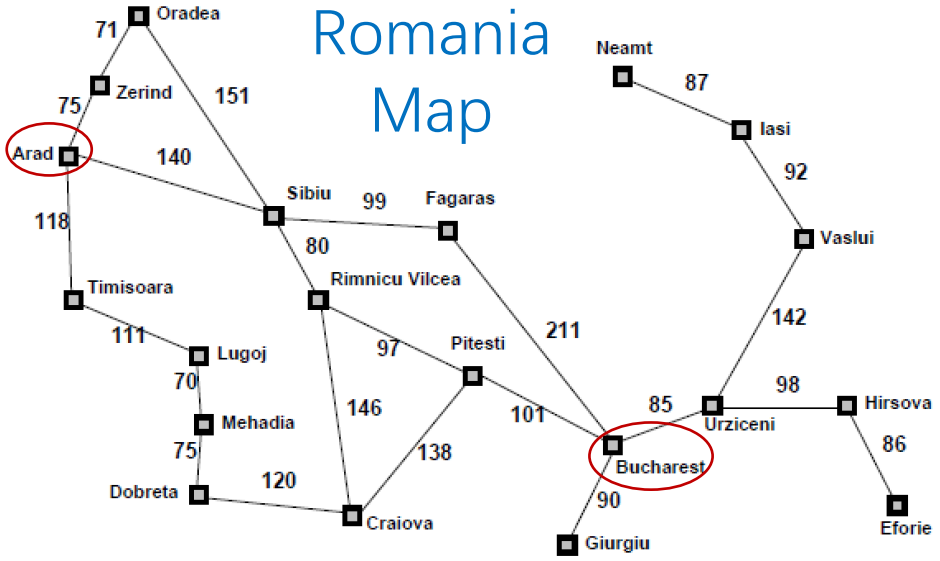
Outline of this lecture

- **Knowledge-based Agents**
- **Represent Knowledge with Logic**
- **(Propositional) Logic**
- **Inference with Propositional Logic**

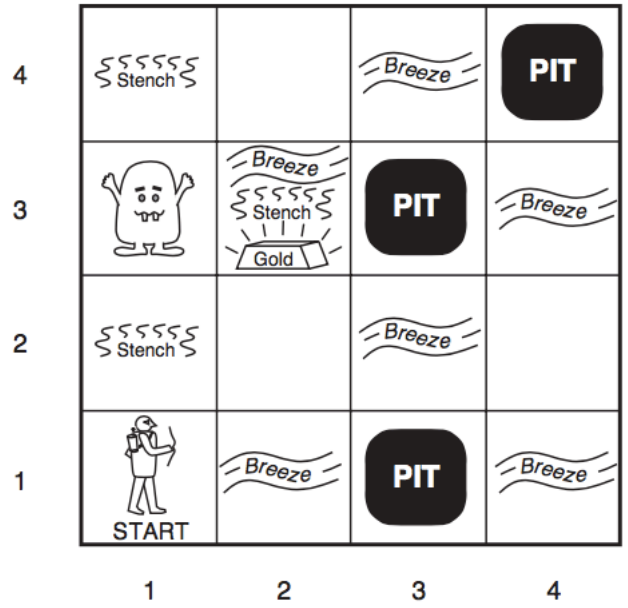
I. Knowledge-based Agents

Knowledge is important

- We (human) perceive the world, accumulate our knowledge, and act based on our perception and knowledge.
- In some cases, knowledge is not merely useful, but crucial.



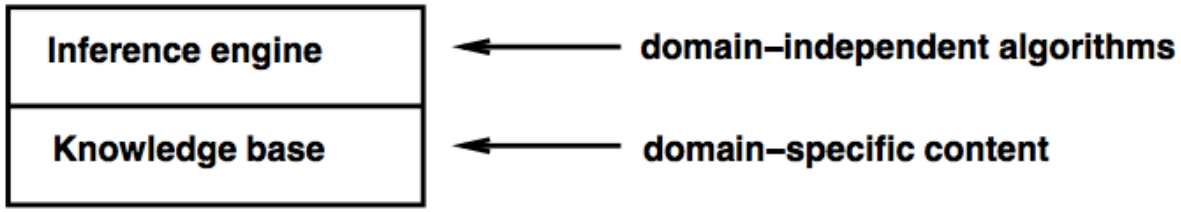
Difference?



But what is knowledge?

- Human gain knowledge from experience.
- Knowledge is something abstract.
- A neural network trained with data fulfills the above two conditions, but does not fit our intuition about knowledge.
  - Knowledge is represented/stored/transferred/...through language.
- A “language” to represent knowledge should be defined first.
  - The language should be understandable by human.

Knowledge-based Agents



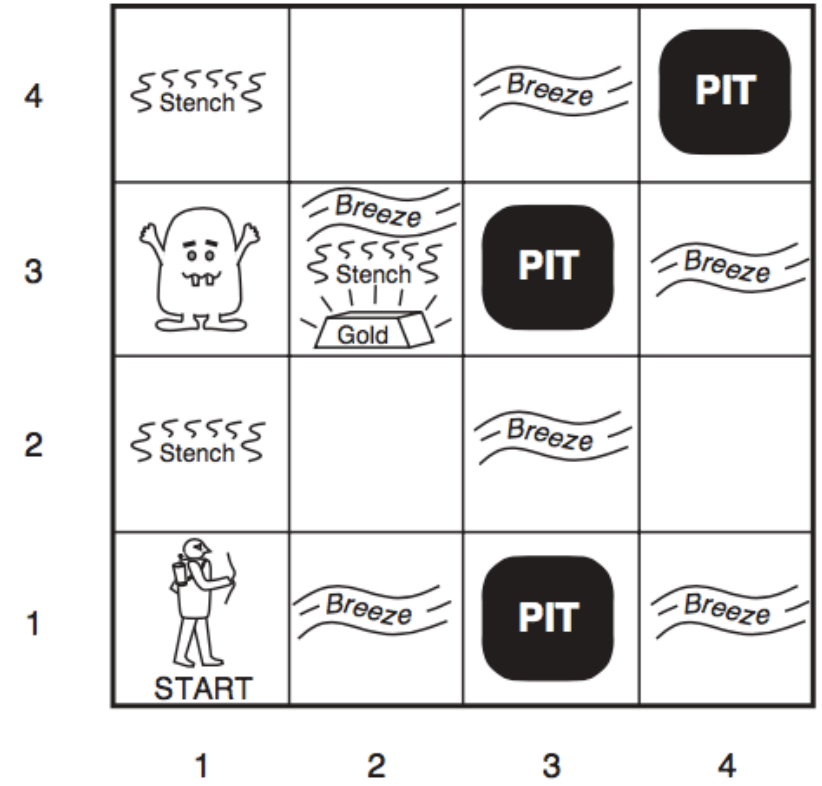
Knowledge base = set of sentences in a **formal** language

Declarative approach to building an agent (or other system):
TELL it what it needs to know

Then it can ASK itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level
i.e., **what they know**, regardless of how implemented

Or at the implementation level
i.e., data structures in KB and algorithms that manipulate them



II. Represent Knowledge with Logic

Sentences

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

Logic is a **formal** language for representing knowledge

Relationship Between Sentences

Entailment means that one thing **follows from** another:

$$KB \models \alpha$$

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

“Worlds”

Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

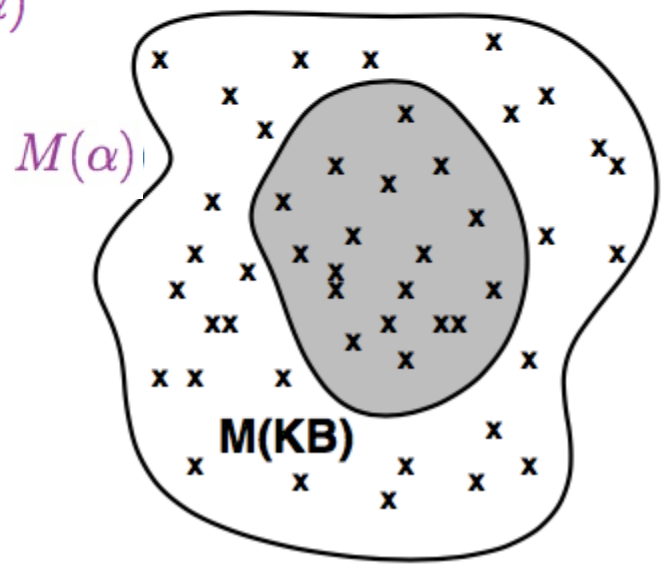
We say m is a model of a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. $KB =$ Giants won and Reds won

$\alpha =$ Giants won



Inference

Inference: the **procedure** of deriving a sentence from another sentence

Model Checking: A basic (and general) idea to inference

$KB \vdash_i \alpha =$ sentence α can be derived from KB by procedure i

Consequences of KB are a haystack; α is a needle.
Entailment $=$ needle in haystack; inference $=$ finding it

Soundness: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

III. (Propositional) Logic

Propositional Logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional Logic: Semantics

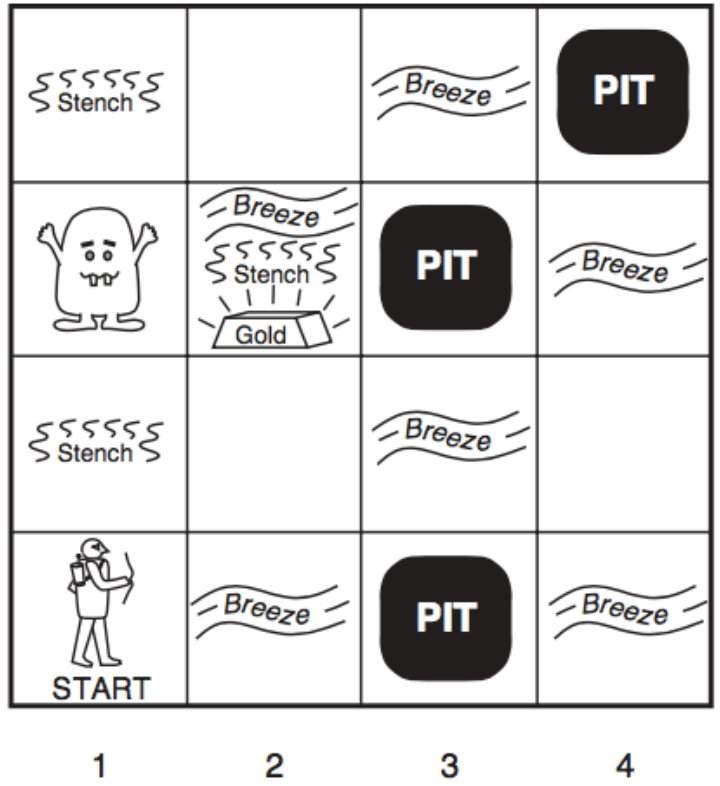
Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
$true$ $true$ $false$

(With these symbols, 8 possible models, can be enumerated automat

Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false			
$S_1 \wedge S_2$	is true iff	S_1	is true **and**	S_2	is true	
$S_1 \vee S_2$	is true iff	S_1	is true **or**	S_2	is true	
$S_1 \Rightarrow S_2$	is true iff	S_1	is false **or**	S_2	is true	
i.e.,	is false iff	S_1	is true **and**	S_2	is false	
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true **and**	$S_2 \Rightarrow S_1$	is true	

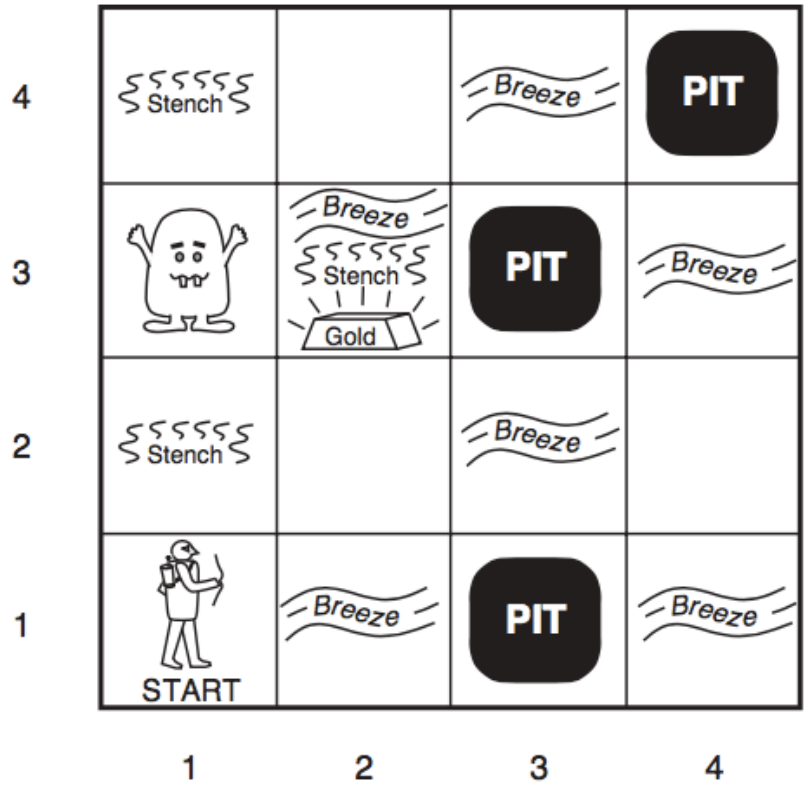


IV. Inference with Propositional Logic

Example for the Wumpus World

$P_{x,y}$ is true if there is a pit in $[x, y]$.
$W_{x,y}$ is true if there is a wumpus in $[x, y]$, dead or alive.
$B_{x,y}$ is true if the agent perceives a breeze in $[x, y]$.
$S_{x,y}$ is true if the agent perceives a stench in $[x, y]$.

$$B_{1,1} \quad \Leftrightarrow \quad (P_{1,2} \vee P_{2,1}) \; .$$
$$B_{2,1} \quad \Leftrightarrow \quad (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$



- Can we infer that "**position [3, 1] is safe (α)**" in the Wumpus world given a **KB**?

Inference by Enumeration

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

Inference by Enumeration

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
fals												
fals												
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

$O(2^n)$ for n symbols; problem is **co-NP-complete**

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

Conjunctive Normal Form (CNF)

- To make inference more efficient/effective, we need more
- Any sentence of propositional logic is equivalent to a conjunction of clauses.

$$
\begin{aligned}
CNFSentence &\rightarrow Clause_1 \wedge \cdots \wedge Clause_n \\
Clause &\rightarrow Literal_1 \vee \cdots \vee Literal_m \\
Literal &\rightarrow Symbol \mid \neg Symbol \\
Symbol &\rightarrow P \mid Q \mid R \mid \ldots
\end{aligned}
$$

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

What is the relationship between CSP and SAT?

Satisfiability Problem

- Propositional model checking could be done by solving a satisfiability problem.
  - Put the KB and ASK into CNF, and solve the SAT problem.

A sentence is valid if it is true in **all** models,
e.g., $True$, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:
$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in **some** model
e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in **no** models
e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:
$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

SAT: (Boolean) Satisfiability Problem

Backtracking Algorithm for SAT (DPLL)

- A search algorithm for SAT.
- Similar to Backtracking for CSP, but using different problem-dependent information/heuristics, such as
  - Early Termination
  - Pure symbol heuristic
  - Unit clause heuristic

Inference by Theorem Proving

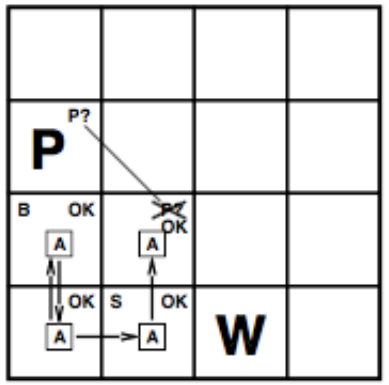
Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

Resolution is sound and complete for propositional logic



Inference by Theorem Proving

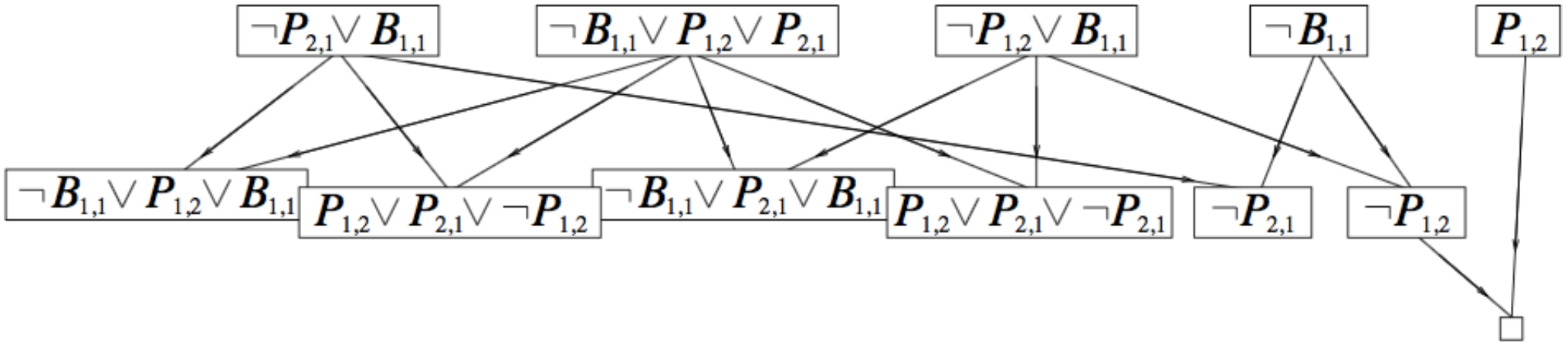
- Apply inference rules to generate new sentences based on old ones (i.e., without searching in the model space).
- Inference rules works like search operators.
- Example: Resolution algorithm
  - Convert the sentence to “ASK” (or to proof) into CNF.
  - Use Resolution rules as the inference rule.

- there are no new clauses that can be added, in which case KB does not entail α; or,
- two clauses resolve to yield the *empty* clause, in which case KB entails α.

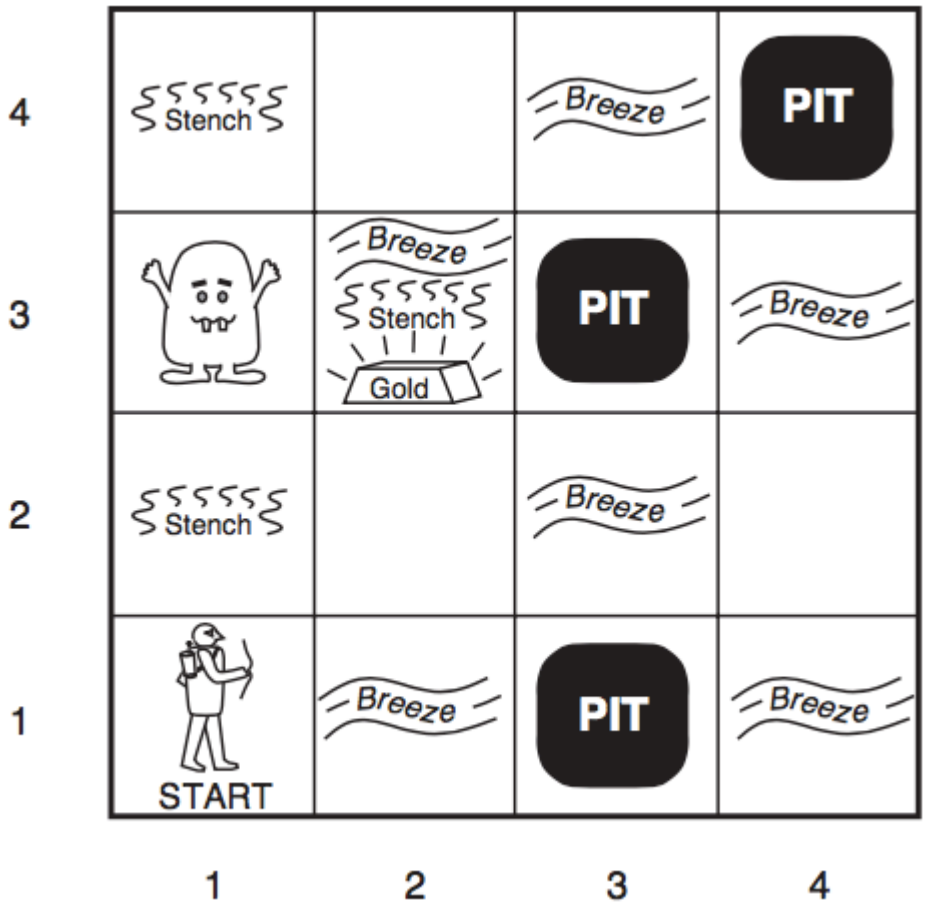
Inference by Theorem Proving

An example of resolution algorithm

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \; \alpha = \neg P_{1,2}$$



Wumpus Again



function HYBRID-WUMPUS-AGENT(*percept*) **returns** an *action*
  inputs: *percept*, a list, [*stench*,*breeze*,*glitter*,*bump*,*scream*]
  persistent: KB, a knowledge base, initially the atemporal "wumpus physics"
    t, a counter, initially 0, indicating time
    plan, an action sequence, initially empty

  TELL(KB, MAKE-PERCEPT-SENTENCE(*percept*, t))
  TELL the KB the temporal "physics" sentences for time t
  $safe \leftarrow \{[x, y] : \text{ASK}(KB, OK^t_{x,y}) = true\}$
  if ASK(KB, $Glitter^t$) $= true$ **then**
    $plan \leftarrow [Grab] + \text{PLAN-ROUTE}(current, \{[1,1]\}, safe) + [Climb]$
  if *plan* is empty **then**
    $unvisited \leftarrow \{[x, y] : \text{ASK}(KB, L^{t'}_{x,y}) = false \text{ for all } t' \leq t\}$
    $plan \leftarrow \text{PLAN-ROUTE}(current, unvisited \cap safe, safe)$
  if *plan* is empty and ASK(KB, $HaveArrow^t$) $= true$ **then**
    $possible\_wumpus \leftarrow \{[x, y] : \text{ASK}(KB, \neg\, W_{x,y}) = false\}$
    $plan \leftarrow \text{PLAN-SHOT}(current, possible\_wumpus, safe)$
  if *plan* is empty **then** // no choice but to take a risk
    $not\_unsafe \leftarrow \{[x, y] : \text{ASK}(KB, \neg\, OK^t_{x,y}) = false\}$
    $plan \leftarrow \text{PLAN-ROUTE}(current, unvisited \cap not\_unsafe, safe)$
  if *plan* is empty **then**
    $plan \leftarrow \text{PLAN-ROUTE}(current, \{[1, 1]\}, safe) + [Climb]$
  $action \leftarrow \text{POP}(plan)$
  TELL(KB, MAKE-ACTION-SENTENCE(*action*, t))
  $t \leftarrow t + 1$
  return *action*

---

function PLAN-ROUTE(*current*,*goals*,*allowed*) **returns** an action sequence
  inputs: *current*, the agent's current position
    goals, a set of squares; try to plan a route to one of them
    allowed, a set of squares that can form part of the route

  problem $\leftarrow$ ROUTE-PROBLEM(*current*, *goals*,*allowed*)
  return A*-GRAPH-SEARCH(*problem*)

To be continued