Lecture 7: Co-evolutionary Learning

CSE5012: Evolutionary Computation and Its Applications

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Review of the Last Lecture

- **Population diversity, Niching, Speciation**
- **Co-evolution**

Outline of This Lecture

Introduction

Strategy Games

Co-evolutionary Learning of Game-playing Strategies

Theoretical Framework of Generalisation in Co-evolutionary Learning

Examples of Generalisation Framework

Estimating Generalisation in Co-evolutionary Learning

Conclusions

Reading Lists

Evolutionary Learning

Very straightforward conceptually!

1. **Initialise population,** $X(t = 1)$**.**
2. **Evaluate fitness of each population member.**
3. **Select parents from** $X(t)$ **based on fitness.**
4. **Generate offspring from parents to obtain** $X(t + 1)$**.**
5. **Repeat steps (2-4) until some termination criteria are met.**

Two Approaches to Evolutionary Learning

Things might get a little trickier.

Michigan Approach **Holland-style learning classifier systems (LCS), where each individual is a rule. The whole population is a complete (learning) system.**

Pitt Approach **Each individual is a *complete* system. This lecture deals only with the Pitt-style evolutionary learning since it is more widely used.**

Current Practice in Evolutionary Learning

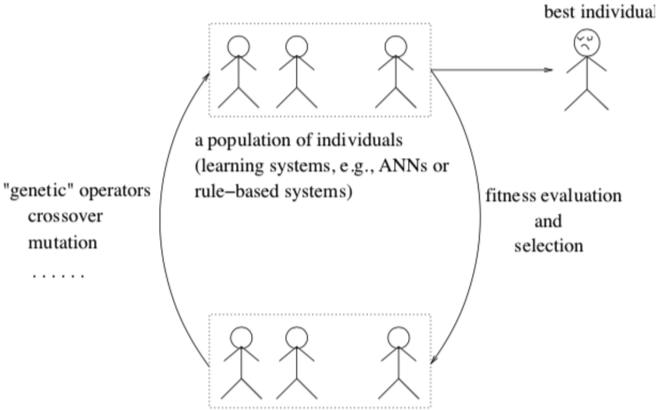


Figure 1: A general framework for Pitt style evolutionary learning.

Fitness Evaluation

1. **Based on the training error.**

2. **Based on the training error and complexity (regularisation), e.g.,**

$$\frac{1}{fitness} \propto error + \alpha \cdot complexity.$$

What If No Error Function Is Available

- **Or, we don't know how to obtain the fitness function required to evaluate the fitness of a population member, e.g., if we want to evolve game-playing strategies.**
- **In other words, the exact teacher/target information is unavailable.**

Well... We have Co-evolutionary Learning

1. **Initialise population, $X(t=1)$.**
2. **Evaluate fitness through interactions between population members.**
3. **Select parents from $X(t)$ based on fitness.**
4. **Generate offspring from parents to obtain $X(t+1)$.**
5. **Repeat steps (2-4) until some termination criteria are met.**

Co-evolutionary Learning In One Slide

1. **There has been a HUGE body of literature on co-evolution, especially since Hillis's seminal work in 1991. If we google "Co-evolutionary Learning", we would get more than 200k hits.**

2. **Many issues have been raised and discussed: robustness, cycles, mediocre stable states,** ...

3. **It is a great idea, but sometimes it just does not do what you hope it would do - frustrating!**

4. **There is only a small body of literature on theoretical aspects of co-evolution. We need more vigorous theories to move the research forward.**

Outline of This Lecture

Iterated Prisoner's Dilemma

	Cooperate	Defect
Cooperate	R, R	S, T
Defect	T, S	P, P

Figure 2: Figure 1 of [9]. The payoff matrix for the 2-player prisoner's dilemma (2PD) game. The values S, P, R, T must satisfy $T > R > P > S$ and $R > (S + T)/2$.

In the 2-player iterated prisoner's dilemma (2IPD) game,

- **the above interaction is repeated many times, and**
- **both players can remember previous outcomes.**

Sometimes Two Players Are Not Enough

- **Davis *et al.* [4] commented that**
  "`The` N`-player case (NPD) has greater generality and applicability to real-life situations. In addition to the problems of energy conservation, ecology, and overpopulation, many other real-life problems can be represented by the NPD paradigm.`"

- **Colman, and Glance and Huberman [2, 5, 6] have also indicated that the NIPD is "**`qualitatively different`**" from the 2IPD and that "**`...certain strategies that work well for individuals in the Prisoner's Dilemma fail in large groups`**".**

The NIPD Game

The NIPD game can be defined by the following three properties:

- **each player faces two choices between cooperation (C) and defection (D);**
- **the D option is dominant for each player, i.e., each is better off choosing D than C no matter how many of the other players choose C;**
- **the dominant D strategies intersect in a deficient equilibrium. In particular, the outcome if all players choose their non-dominant C strategies is preferable from every player's point of view to the one in which everyone chooses D, but no one is motivated to deviate unilaterally from D.**

The NIPD Game - Payoff Matrix

Number of cooperators among the remaining $n-1$ players

player A		0	1	2		$n-1$
	C	C_0	C_1	C_2	$\cdots$	C_{n-1}
	D	D_0	D_1	D_2	$\cdots$	D_{n-1}

The payoff matrix of the n-player Prisoner's Dilemma game, where the following conditions must be satisfied:

1. $D_i > C_i$ **for** $0 \leq i \leq n-1$**;**
2. $D_{i+1} > D_i$ **and** $C_{i+1} > C_i$ **for** $0 \leq i < n-1$**;**
3. $C_i > (D_i + C_{i-1})/2$ **for** $0 < i \leq n-1$**.**

The payoff matrix is symmetric for each player.

The NIPD Game - An Example

Number of cooperators among the remaining $n-1$ players

player A		0	1	2		$n-1$
	C	0	2	4	$\cdots$	$2(n-1)$
	D	1	3	5	$\cdots$	$2(n-1)+1$

Why NIPD Games?

“The NPD corresponds to a truly remarkable range of real-world social problems, but a few simple examples will suffice.”

- A. Colman, *Game Theory and Experimental Games*, Pergamon Press, 1982. (pp.157–159)

The Diner's Dilemma

Question

What would you do as a rational individual?

Conservation of Scarce Resources

E.g., the energy crisis, the drought in Britain in the summer of 1976, etc.

"It is evidently in each individual's rational self-interest to ignore the call for restraint irrespective of the choices of the others. But - and this is the rub - if everyone pursues individual rationality in this way, they are worse off than if everyone is motivated by collective rationality and exercises restraint. If everyone tries to be a ''free ride'' then no-one gets a ride at all."

The Tragedy of the Commons

First discussed by Hardin in 1968.

- **There are six farmers with one cow each weighted at 1000 lb.**
- **They share a common pasture which can only sustain six cows.**
- **Each additional cow will reduce the weight of every cow by 100 lb.**
- **Each farmer is always better off to have an additional cow regardless of the choices of other farmers.**
- **If they all have one additional cow, they end up with two 400 lb. cows.**

Such tragedies are not uncommon in our highly competitive world. It is suggested that "`the impoverishment of small farmers in England during the period of the enclosures in the eighteenth century may have been exacerbated by this phenomenon`**."**

Outline of This Lecture

Genotypical Representation of Strategies

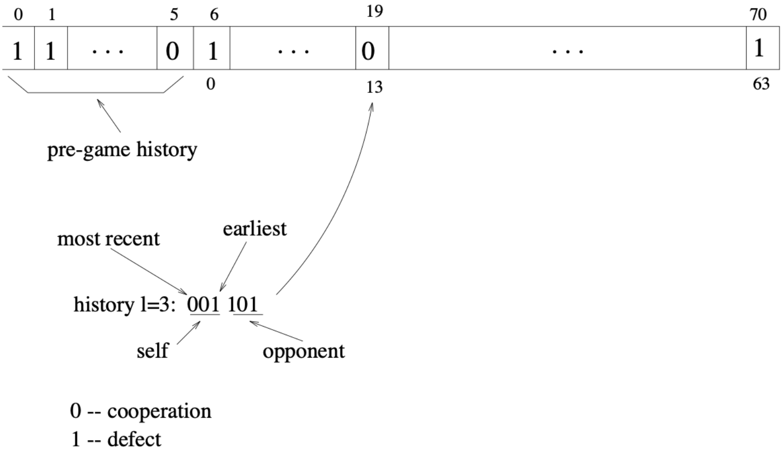


Figure 3: Encoding of strategies, assuming history (memory) length 3.

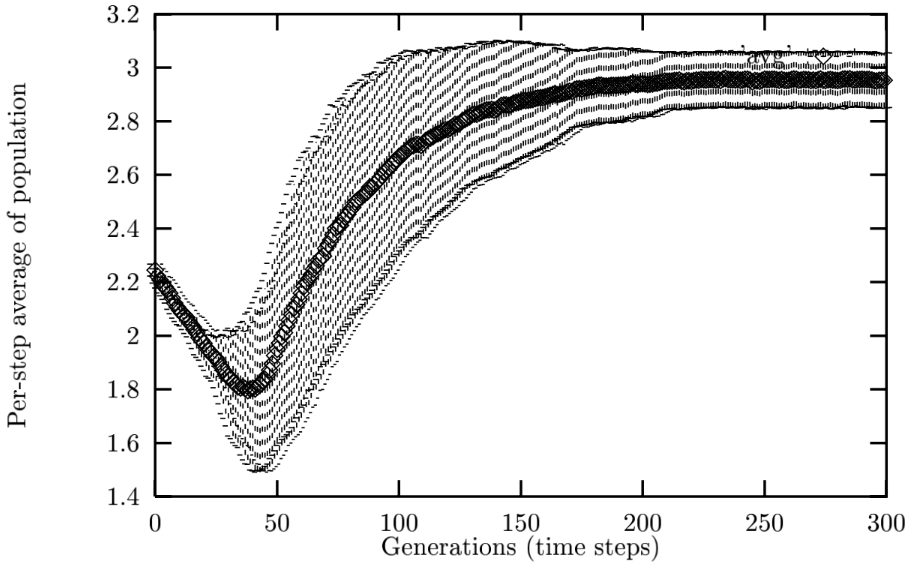


Figure 4: Results from 30 runs of the 2IPD with history (memory) length 3.

Why Co-evolution

1. **Co-evolution occurs frequently in Nature.**
2. **Co-evolution does not require any fixed fitness function.**
3. **Co-evolution provides a nice approach to learn without expert knowledge and a teacher.**

Discussion on the 2IPD Game

1. **Cooperative behaviours can be evolved from a population of random strategies without specifying an explicit and fixed fitness function.**

2. **A cooperative population can be vulnerable to defective intruders.**

3. **Co-evolutionary learning may not be able to produce strategies which generalise well against unseen opponents.**

A Team Approach to Evolving Strategies

1. **Evolving a strategy that is capable of dealing with all kinds of opponents can be hard.**

2. **Maybe we can use a team of individuals as a strategy so that each individual is only dealing a one or two types of opponents. Such individuals should be quite easy to evolve.**

3. **But how to evolve a team of individuals who are complement of each other?**

4. **Artificial speciation appears to be an effective technique to achieve that.**

Automatic Modularisation of Learning Systems

1. **A genetic algorithm was used to evolve strategies for playing the 2IPD.**
2. **Implicit fitness sharing was used to form different species (specialists).**
3. **A gating algorithm was used to decide which species should respond to an unknown opponent.**
4. **Experimental results showed that our method worked very well.**

A Combination Method - the Gating Algorithm

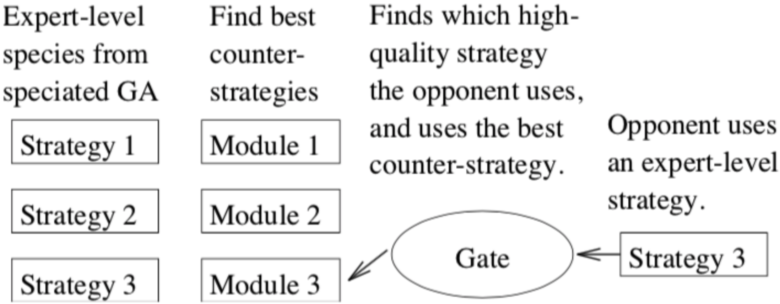


Figure 5: A gating algorithm for combining different expertise in a population together.

Experimental Results

$l = 4$

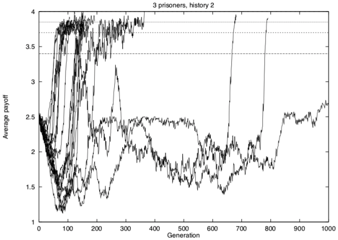
Strategy	Wins (%)	Ties (%)	Average Score Own	Other's
Best	0.360	0.059	1.322	1.513
Gate	0.643	0.059	1.520	1.234

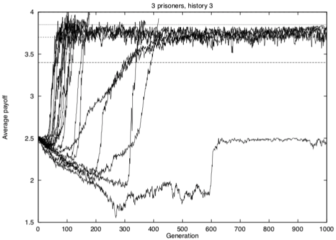
Table 1: Results against the best 25 strategies from the partial enumerative search, for 2IPD with remembered history $l = 4$. The results were averaged over 30 runs. Results from [9].

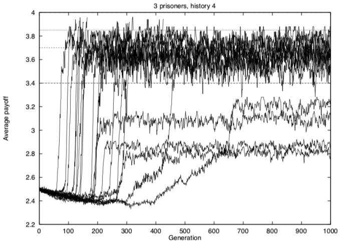
NIPD Game: Some Interesting Questions

1. **What is the impact of history length (memory) on the evolution?**
2. **What is the impact of group size on the evolution?**

3IPD with History Length 2, 3 and 4







Discussions on the NIPD Game

1. **Memory helps learning. Long history lengths encourage cooperation.**

2. **Large groups make the evolution of cooperative strategies more difficult. (Not shown in the lecture. Details are in the paper.)**

However,

1. **We rarely commit ourselves to one way or the other. We prefer to be somewhere in the middle.**

2. **We are more likely to have short games with short memory.**

3. **We may not be able to interact with everyone although we know them by names (reputation).**

Multiple Levels of Cooperation

The payoff to player A is given by:

$$p_A = 2.5 - 0.5c_A + 2c_B, (-1 \leq c_A, c_B \leq 1) \tag{1}$$

where c_A and c_B are the cooperation levels of the two players, which are discretised into four choices of cooperation.

	-1	$-\frac{1}{3}$	$+\frac{1}{3}$	$+1$
-1	1	$2\frac{1}{3}$	$3\frac{2}{3}$	5
$-\frac{1}{3}$	$\frac{2}{3}$	2	$3\frac{1}{3}$	$4\frac{2}{3}$
$+\frac{1}{3}$	$\frac{1}{3}$	$1\frac{2}{3}$	3	$4\frac{1}{3}$
$+1$	0	$1\frac{1}{3}$	$2\frac{2}{3}$	4

Figure 6: The payoff matrix for the 2-player prisoner's dilemma game, with four choices of cooperation.

Reputation

- **Information about a game (i.e., interaction is made available to all others. You can see someone's reputation without interacting with him/her directly.**
- **First considered by Nowak and Sigmund.**
- **But only on 2-player IPD games with two levels of cooperation.**
- **No evolutionary algorithm was used in their work. Players did not evolve in their case.**

The Key Question

Question

What is the impact of multiple levels of cooperation *and* reputation on the evolution of cooperation?

Experimental Setup

- **Each strategy is represented by a two-layer feed-forward neural network with 20 hidden nodes.**
- **There is one output indicating the current action taken by the player.**
- **There are five inputs:**
  1. **the player's own previous move;**
  2. **the opponent's previous move;**
  3. **whether the opponent exploited the player;**
  4. **whether the player exploited the opponent; and**
  5. **reputation of the opponent.**

Shorter Games Discourage Cooperation

10 and 5 Iterations

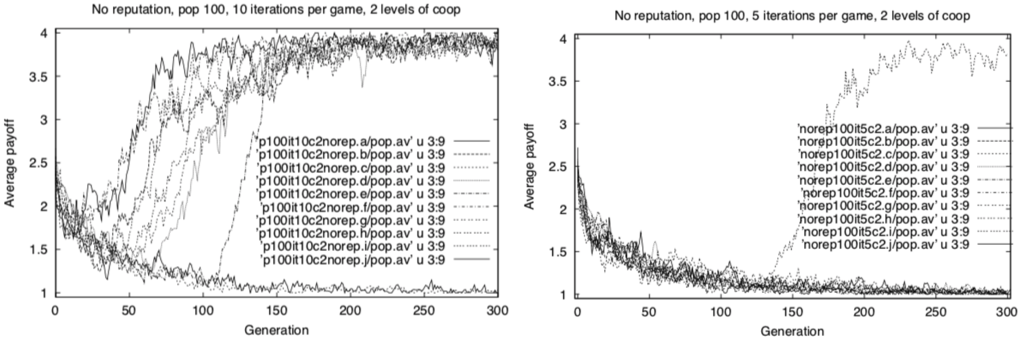
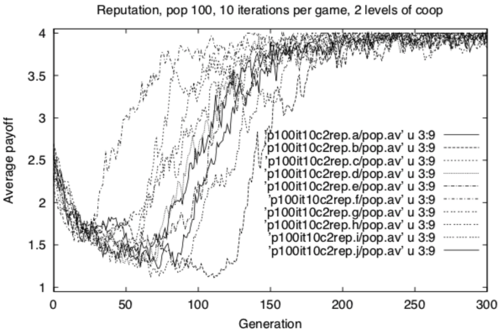


Figure 7: Game length is only 10 iterations (left) and 5 iterations (right), so there is less incentive to cooperate than for games of 150 iterations.

Reputation Helps Cooperation

10 and 1 Iteration



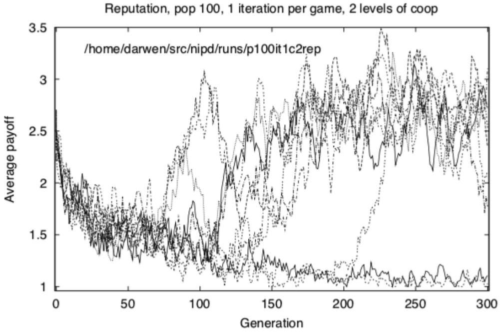


Figure 8: Game length is only 10 iterations (left), reputation makes cooperation more likely. Game length is only 1 iteration (right), but reputation keeps it a somewhat attractive option.

More Choices of Cooperation Discourages Cooperation

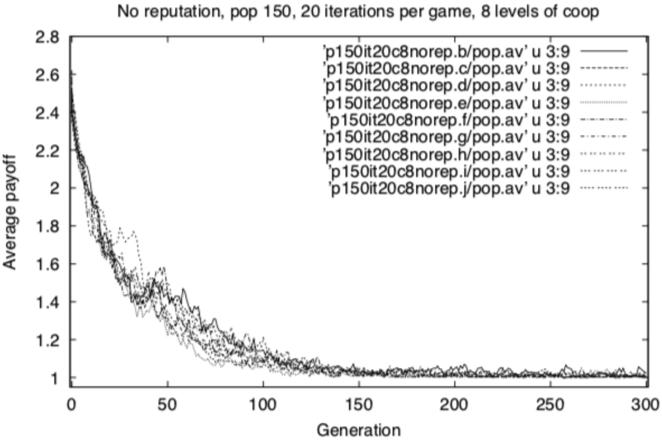


Figure 9: Game length is 20 iterations, with 8 choices of cooperation, and defection dominates.

Reputation Helps Cooperation Again

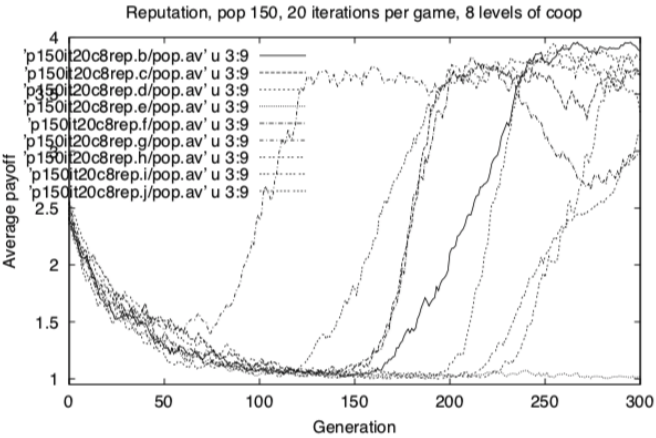


Figure 10: Game length is 20 iterations, with 8 choices of cooperation. With reputation, cooperation is much more likely.

Reputation Is Important

Even when the number of cooperation levels is high

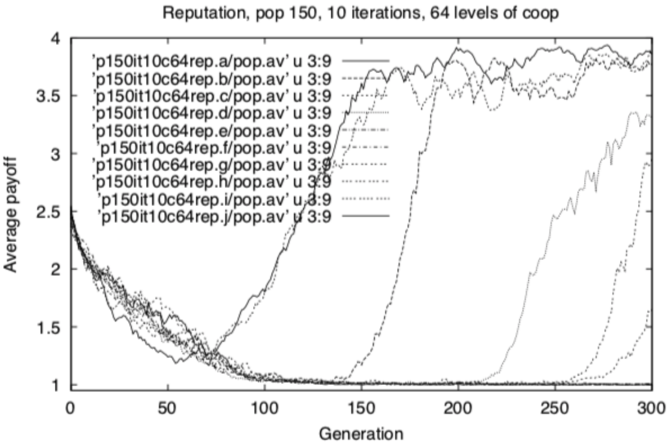


Figure 11: Game length is 10 iterations, with 64 choices of cooperation. With reputation, cooperation is still likely.

Remarks on Computational Studies

1. **IPD games are useful models of real-life situations.**
2. **NIPD games are more realistic and general than the 2IPD one.**
3. **Multiple levels of cooperation discourage cooperation.**
4. **Reputation can mitigate this and encourage cooperation.**
5. **Reputation can encourage cooperation even when the game and history lengths are extremely short.**
6. **IPD games with multiple levels of cooperation as well as reputation behave qualitatively differently from classical IPD games.**
7. **Co-evolutionary learning is an effective learning technique.**

Outline of This Lecture

A Simple Research Question

- **If I invent a wonderful co-evolutionary learning algorithm and use it to co-evolve a really intelligent game-playing strategy (e.g., for chess, car racing, iterated prisoner's dilemma, or others), how do I know it would perform well against a new opponent that it has never seen before?**
- **Can we say anything at all about the ability (performance) of our co-evolved solutions in a new and unseen environment?**
- **Sounds like generalisation to me.**

Generalisation? We Know That!

1. **There have been various discussions about robustness of co-evolved solutions.**

2. **However, we still do not have any quantitative analysis of generalisation performance, e.g., an absolute quality measure for co-evolved solutions.**

3. **It is still very hard to compare performance between different co-evolutionary learning algorithms (when applied to a problem).**

An Early Attempt

An empirical approach to estimate generalisation of co-evolved solutions:

1. **Sample a large number of random test strategies.**
2. **Co-evolved strategies compete against these test strategies.**
3. **Generalisation is taken to be the average performance against these test strategies.**
4. **Note that the number of test strategies should be significantly larger than what co-evolution can search (so that the vast majority of the test strategies are unseen by co-evolved strategies).**

Paul J Darwen and Xin Yao. "On evolving robust strategies for iterated prisoner's dilemma". In: *Progress in Evolutionary Computation*. Springer, 1993, pp. 276–292

The Need for a General Theoretical Framework

1. **Although the empirical estimation can give us some information, it is unknown how accurate the estimate is to the true value.**
2. **What is needed is a theoretical framework that would enable us to define/compute the true generalisation and the accuracy of an estimation.**
3. **But how? Learn from others!**

Theoretical Framework - Problem Formulation

1. **In co-evolutionary learning, we consider the performance (quality) of a solution relative to other solutions.**

2. **This is achieved through interactions or comparisons between solutions.**

3. **This can be framed in the context of “game-playing”.**

Outline of This Lecture

Theoretical Framework - $G_i(j)$

1. **Let us have two strategies i and j.**
2. **A game is played between these two strategies.**
3. **Let $G_i(j)$ be the game outcome of strategy i playing against strategy j.**
4. **Similarly, let $G_j(i)$ be the outcome of strategy j playing against strategy i.**
5. **Strategy i is said to solve the test provided by strategy j if $G_i(j) \geq G_j(i)$. Strategy i wins against j if $G_i(j) > G_j(i)$.**

True Generalisation Performance

1. **Given a co-evolved strategy i, let test strategies j be obtained from strategy space $\mathcal{S}$. The true generalisation performance of strategy i, G_i, is:**

$$G_i = E_{P_1(j)}[G_i(j)] = \int_{\mathcal{S}} G_i(j) P_1(j) dj, \tag{2}$$

where G_i is the expectation of strategy i's performance against j, $G_i(j)$, w.r.t. distribution $P_1(j)$ over strategy space $\mathcal{S}$.

2. **A simplified form:**

$$G_i = \frac{1}{M} \sum_{j}^{M} G_i(j), \tag{3}$$

which is simply its average performance against all strategies j.

Outline of This Lecture

Estimating Generalisation Performance

1. **In reality, it is very hard or even impossible to compute the true generalisation performance using either Equation (2) or Equation (3).**
2. **We have to reply on estimation using a random sample of N test strategies j, S_N.**
3. **The estimated generalisation performance of strategy i is given by:**

$$\hat{G}_i(S_N) = \frac{1}{N} \sum_{j \in S_N} G_i(j), \tag{4}$$

where S_N is the sample of N test strategies randomly drawn from $\mathcal{S}$ (we'll use notation $\hat{G}_i$ for $\hat{G}_i(S_N)$).

How Good Is the Estimation

1. **We want to know how accurate the estimate $\hat{G}_i$ is compared to G_i, i.e., how small $|\hat{G}_i - G_i|$ is.**

2. **We don't know G_i, so we can't compute $|\hat{G}_i - G_i|$. So frustrating!**

3. **Fortunately, we can make a statistical claim as to how confident we are that $|\hat{G}_i - G_i| \leq \epsilon$.**

Chebyshev's Theorem

Theorem

Chebyshev's Theorem Consider a random variable U distributed according to the probability density $p(u)$. Given a positive number $a > 0$, we can bound the probability that $U \leq -a$ or $U > a$, i.e., the probability that U falls outside $[-a, +a]$, by

$$P(|U| \geq a) \leq \frac{E[U^2]}{a^2},$$

where $E[U^2]$ is the mean of the new random variable $V = U^2$ with respect to p.

Boris V Gnedenko. *Theory of probability*. Routledge, 2017

Bound for Estimation Accuracy

Applying Chebyshev's Theorem, we derive the following:

$$P(|\hat{G}_i - G_i| \geq \epsilon) \leq \frac{\sigma_i^2}{N\epsilon^2} \tag{5}$$

More Usable Bound

1. **In general, for the random variable $G_i(j)$ distributed over the interval $[G_{MIN}, G_{MAX}]$, $\sigma_{MAX} = (G_{MIN} + G_{MAX})/2$.**
2. **With this, we obtain the following lemma:**

Lemma

For a strategy i, let $\hat{G}_i$ be the estimated generalisation performance with respect to N random test strategies and G_i be the true generalisation performance. Consider the absolute difference $|\hat{G}_i - G_i|$, which is a random variable with distribution P_N taken on a compact interval $[G_{MIN}, G_{MAX}]$ of length $R = G_{MAX} - G_{MIN}$. Then, for any positive number $\epsilon > 0$:

$$P_N(|\hat{G}_i - G_i| \geq \epsilon) \leq \frac{R^2}{4N\epsilon^2}. \quad (6)$$

Harold I Jacobson. "The maximum variance of restricted unimodal distributions". In: *The annals of mathematical statistics* 40.5 (1969), pp. 1746–1752

Generality of the Generalisation Framework

The framework is extremely general.

1. **The framework is independent of the complexity of the game, since it is independent of the size of the strategy space, and independent of the strategy distribution in the strategy space.**

2. **Both G_{MAX} and G_{MIN} are often known a priori as they are defined by the game, which means that we can always obtain an upper bound using (6).**

3. **The framework is independent of learning algorithms since the bound holds for any strategy in the strategy space.**

Estimation Accuracy: An Illustration

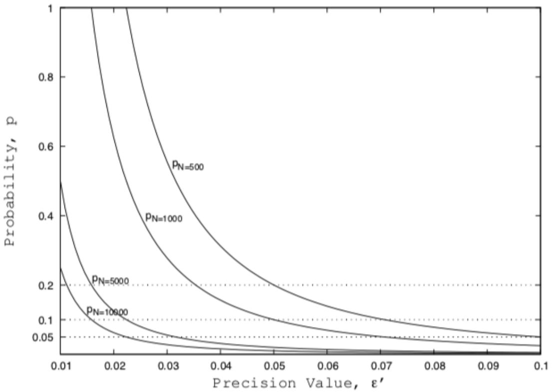


Figure 12: Equation (6) can be simplified to $P(|D_N|' \geq \epsilon') \leq \frac{1}{4N\epsilon'^2}$, where $\epsilon' = \epsilon/R$ and $|D_N|' = |\hat{G}_i - G_i|/R$. This figure shows the relationship between P_N for different N and precision ϵ' in $[0.01, 0.1]$.

Can We Obtain Tighter Bounds?

1. **Chebyshev's bound is not very tight.**
2. **Furthermore, in Equation (5), we bound σ_i^2 using R^2 to obtain Equation (6) (assuming the worst-case).**
3. **It is possible to find a tighter upper bound for σ_i^2.**

Recall What We Did

Consider a random variable X **with the underlying distribution** P_X **over a real interval** $[a, b]$**, i.e.,** $X \in [a, b]$**. For** N **realisations** $x_1, x_2, \ldots, x_N$**, the empirical mean is** $\hat{E}_{P_X}[X] = \hat{\mu}_N = \frac{1}{N}\sum_{j=1}^{N} x_j$ **while the true mean is** $E_{P_X}[X]$**. The true variance is** $\sigma^2 = E_{P_X}[(X - E_{P_X}[X])^2]$. **Applying Chebyshev's Theorem gives us:**

$$P(|\hat{\mu}_N - \mu| \geq \epsilon) \leq \frac{\sigma^2}{N\epsilon^2} \leq \frac{R_X^2}{4N\epsilon^2},$$

where the maximum variance for a random variable over $[a, b]$ **is** $\sigma^2_{MAX} = R_X^2/4$ **with** $R_X = b - a$**.**

If We Knew $E_{P_X}[X]$

we could obtain an unbiased estimate of σ^2:

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{j=1}^{N}(x_j - \mu)^2 = \frac{1}{N}\sum_{j=1}^{N} y_j,$$

where y_j, $j = 1, \ldots, N$ are realisations of a new random variable $Y = (X - E_{P_X}[X])^2$. $\hat{\sigma}_N^2$ is the empirical mean of Y based on a sample $\{y_j\}_{j=1}^N$ and $E_{P_Y}[Y] = E_{P_X}[(X - E_{P_X}[X])^2] = \sigma^2$ is the true mean of Y. We can apply Chebyshev's Theorem for Y to obtain:

$$P(|\hat{\sigma}_N^2 - \sigma^2| \geq \delta) \leq \frac{Var_{P_Y}[Y]}{N\delta^2} \leq \frac{R_X^4}{4N\delta^2},$$

where $Var_{MAX}[Y] \leq \frac{R_Y^2}{4} = \frac{(b-a)^4}{4} = \frac{R_X^4}{4}$ given that the range for Y is $R_Y = y_{max} - y_{min} = (b-a)^2 - 0 = R_X^2$.

If We Don't Know $E_{P_X}[X]$

1. **We need to examine the correction required for the fact that $\hat{\mu}$ rather than $\mu = E_{P_X}[X]$ is used to calculate the variance $\hat{\sigma}_N^2$. (Note that the worst-case is when $\hat{\sigma}_N^2$ is underestimated.)**

2. **We know that with probability at least $c_1 = 1 - \frac{R_X^2}{4N\epsilon^2}$, $|\hat{\mu} - \mu| \leq \epsilon$. With probability c_1, we have $(x_j - \hat{\mu}_N - \epsilon)^2 \geq (x_j - \mu)^2$. So, with probability c_1, we can be sure that:**
$$\hat{\sigma}_{N,U}^2 = \frac{1}{N}\sum_{j=1}^{N}(x_j - \hat{\mu}_N - \epsilon)^2 \geq \hat{\sigma}_N^2.$$

3. **With probability $c_2 = 1 - \frac{R_X^4}{4N\delta^2}$, we have the upper bound $\sigma^2 \leq \hat{\sigma}_N^2 + \delta$.**

Obtain a Tighter Bound

1. **In order to combine the probabilities in a simple way, we must make the Obtaining a Tighter Bound events independent, i.e., we need two sets of N-tuples of observations. The first set is used to estimate $\hat{\mu}_{1,N} = \frac{1}{N}\sum_{j=1}^{N} x_{1,j}$. The second set is used to estimate $\hat{\mu}_{2,N} = \frac{1}{N}\sum_{j=1}^{N} x_{2,j}$ and $\hat{\mu}^2_{N,U} = \frac{1}{N}\sum_{j=1}^{N}(x_{2,j} - \hat{\mu}_{2,N} - \epsilon)^2$. Then, with probability:**

$$c_1 \cdot c_2 = (1 - \frac{R_X^2}{4N\epsilon^2})(1 - \frac{R_X^4}{4N\delta^2}),$$

   we have $\sigma^2 \leq \hat{\sigma}^2_{N,U} + \delta$ since we know that $\sigma^2 \leq \hat{\sigma}^2_N + \delta$ and $\hat{\sigma}^2_N \leq \hat{\sigma}^2_{N,U}$.

2. **In other words, with probability at leat $c_1 c_2$ the following inequality holds:**

$$P(|\hat{\mu}_{1,N} - \mu| \geq \epsilon) \leq \frac{\hat{\sigma}^2_{N,U} + \delta}{N\epsilon^2}.$$

   However, for this inequality to be true, we require that $\sigma^2 \leq \hat{\sigma}^2_{N,U} + \delta < \sigma^2_{MAX} = \frac{R_X^2}{4}$.

A Tighter Bound on Estimation Accuracy

Lemma

For a strategy i, consider two independent non-overlapping sets of N test strategies: T_1 and T_2, where $T_1 \cap T_2 = \emptyset$ and $|T_1| = |T_2| = N$. The first set is used to estimate the generalisation performance $\hat{G}_i(T_1) = \frac{1}{N}\sum_{j \in T_1} G_i(j)$. The second set is used to estimate the variance $\hat{\sigma}^2_{N,U} = \frac{1}{N}\sum_{j \in T_2}(G_i(j) - \hat{G}_i(T_2) - \epsilon)^2$, for some positive number $\epsilon > 0$, where $\hat{G}_i(T_2) = \frac{1}{N}\sum_{j \in T_2} G_i(j)$. Then, for $\delta > 0$ with probability at least $c_1 c_2 = (1 - \frac{R^2}{4N\epsilon^2})(1 - \frac{R^4}{4N\delta^2})$, the following inequality holds:

$$P_N(|\hat{G}_i(T_1) - G_i| \geq \epsilon) \leq \frac{\hat{\sigma}^2_{N,U} + \delta}{N\epsilon^2}. \tag{7}$$

Measuring Generalisation Performance for IPD

1. **Let $g(i, j)$ be the average payoff per move to strategy i when it plays an IPD game with strategy j.**

2. **Generalisation performance in terms of the number of wins based on individual game outcomes:**

$$G_W(i, j) = \begin{cases} C_{WIN} & for\ g(i, j) > g(j, i), \\ C_{LOSE} & otherwise, \end{cases}$$

where $C_{WIN} > C_{LOSE}$.

3. **Generalisation performance in terms of average payoff:**

$$G_A(i, j) = g(i, j).$$

Computational Studies: Experimental Setup

1. **Consider three-choice IPD since we can compute the true generalisation performance, G_i (assuming uniform distribution of test strategies in the space for simplicity).**
2. **For all our experiments, we obtained $\hat{G}_i$ and G_i for 4000 strategies (which were randomly sampled).**
3. **The proportion of the 4000 strategies where $|D_N|' > \epsilon'$ was determined. This empirical value is then compared with Chebyshev's bound for $P(|D_N|' \geq \epsilon')$.**
4. **Experiments were repeated using 50 different samples of random test strategies S_N (Note: no strategy is obtained more than once).**

Computational Studies: Observations

1. **The empirical value for $P(|D_N|' \geq \epsilon')$ is less than the theoretical value given by Chebyshev's bound.**

2. **The empirical distribution of $G(i,j) - G(i)$ for the two definitions of generalisation performance is quite similar for different sizes of S_N.**

   - **The estimation from using a smaller S_N is more similar compared to the estimation from using larger S_Ns.**
   - **The estimations are stable in terms of varying sample sizes starting from a small S_N.**

Computational Studies: Results I

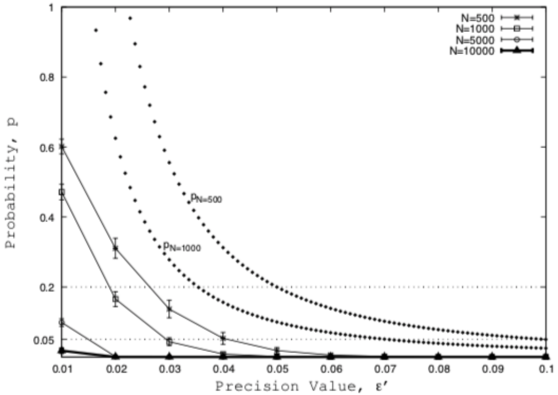


Figure 13: Values of $P(|D_N|' \geq \epsilon')$ for ϵ' in $[0.01, 0.1]$ and different sizes of S_N used to compute $\hat{G}_W(i)$. Each curve is obtained by averaging over results from 50 independent samples S_N (error bars representing 95% confidence level). P_N gives the curves for the Chebyshev's bounds.

Computational Studies: Results II

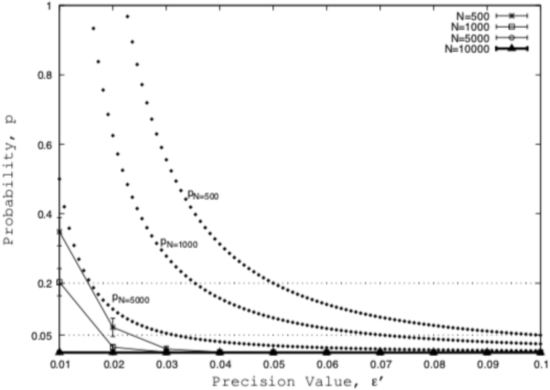


Figure 14: Values of $P(|D_N|' \geq \epsilon')$ for ϵ' in $[0.01, 0.1]$ and different sizes of S_N used to compute $\hat{G}_A(i)$. Each curve is obtained by averaging over results from 50 independent samples S_N (error bars representing 95% confidence level). P_N gives the curves for the Chebyshev's bounds.

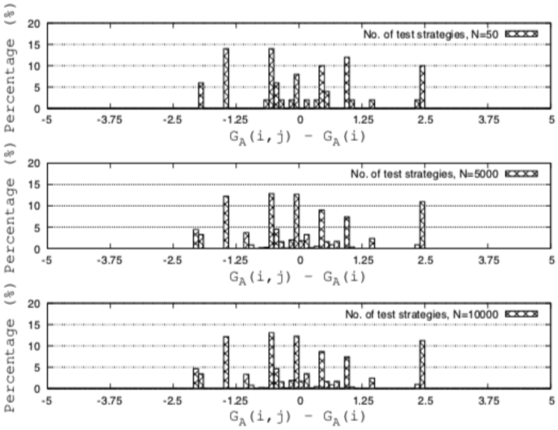


Figure 15: Empirical distributions of $G_A(i,j) - G_A(i)$ for $N = 50, 5000, 10000$.

Computational Studies on Tighter Bounds

1. **Use similar experimental setup as before.**
2. **Still use three-choice IPD since we can compute the true variance of $G_i(j)$, σ_i^2.**

Computational Studies on Tighter Bounds: Results I

N	# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8
10	2500	2400	2500	2400	2400	2100	0	2400
50	2464	2496	2244	1476	2496	1924	1204	2356
100	2304	2484	2059	1344	2464	1924	1275	2436
500	2436	2500	2224	1539	2491	1895	1317	2477
1000	2419	2498	2131	1470	2484	1875	1218	2482
5000	2432	2492	2108	1481	2490	1849	1341	2476
10000	2434	2497	2118	1515	2493	1816	1321	2473
TRUE	**2436**	**2496**	**2126**	**1475**	**2491**	**1816**	**1293**	**2477**

Figure 16: Comparison of $\hat{\sigma}_N^2$ for various N with the true value σ^2 for 8 strategies for $G_W(i)$ where $\sigma^2 \in [0, 2500]$.

Computational Studies on Tighter Bounds: Results II

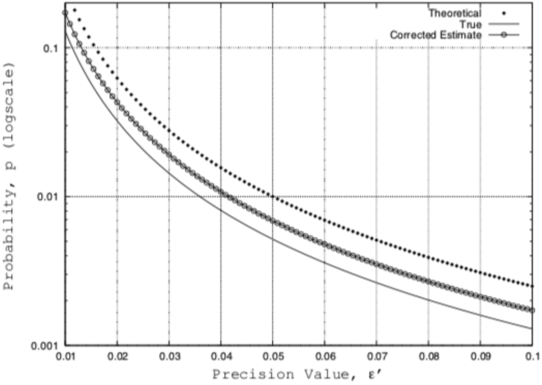


Figure 17: The curve "Theoretical" gives the original Chebyshev's bound when σ^2_{MAX} is used. The curve "True" gives the actual bound when σ^2 is used. The curve "Corrected Estimate" gives the tighter Chebyshev's bound when $\hat{\sigma}^2_{N,U} + \delta$ is used. All curves were obtained for $N = 10000$.

Biased vs. Unbiased Samples

1. **So far, measurements are made with respect to an unbiased sample, e.g., consider uniform distribution of test strategies in the space.**

2. **However, for some problems, one may be more interested with a biased sample.**

3. **For actual games, the majority of strategies in the space may be poor or mediocre. Rather than consider each strategy having equal chances of being met in some tournament, we may be more interested in a small number of strong-performing strategies.**

4. **The question now is how we can sample these strategies. We can't sample strong-performing strategies with higher probabilities because we don't usually have a metric strategy space.**

An Approximate Solution: Partial Enumerative Search

1. **Randomly sample a large number of test strategies and choose the best performing strategies as our biased test sample.**

2. **Since the sample is much larger than what co-evolution can possibly search, we hope that some of these test strategies are good (beats a lot of other strategies) and unseen (co-evolved strategies never met them).**

However,

1. **A sample of obtained from a single partial enumerative search may not be sufficiently diversed as these strategies might be behaviorally similar (i.e., having the same weakness that can be exploited).**

2. **We address this problem by repeating the procedure several times, i.e., multiple partial enumerative search.**

Multiple Partial Enumerative Search

1. $r = 1$: **Sample** PS **strategies,** Q_i **,** $i = 1, 2, \ldots, PS$**, randomly.**

2. **Every strategy competes with all other strategies in the sample, including itself (i.e., each strategy competes a total of** PS **games).**

3. **Detect the strategy index** $s \in \{1, 2, \ldots, PS\}$ **so that** Q_s **is yielding the highest total payoff. Let** $Q^{(r)} := Q_s$**.** $r = r + 1$**.**

4. **Repeat steps one to three** PE **times to obtain** PE**-sized biased sample of test strategies,** $Q^{(r)}$**,** $r = 1, 2, \ldots, PE$**.**

Outline of This Lecture

Conclusions

1. **We have presented a theoretical framework for measuring generalisation performance rigorously in co-evolutionary learning. For the first time, quantitative analysis of generalisation performance of any co-evolutionary learning system can be performed.**

2. **The framework is extremely general and independent of any games, distributions and algorithms.**

3. **The theoretical framework can be applied to concrete games and algorithms, and estimate the generalisation performances.**

4. **Empirical results show that a small sample is usually good enough in estimating the generalisation performance.**

5. **More details:**
   Siang Yew Chong, Peter Tino, and Xin Yao. "Measuring generalization performance in coevolutionary learning". In: *IEEE Transactions on Evolutionary Computation* 12.4 (2008), pp. 479–505

Outline of This Lecture

References for This Lecture I

[1] Siang Yew Chong, Peter Tino, and Xin Yao. "Measuring generalization performance in coevolutionary learning". In: *IEEE Transactions on Evolutionary Computation* 12.4 (2008), pp. 479–505.

[2] Andrew M Colman. *Game theory and experimental games: The study of strategic interaction*. Elsevier, 2016.

[3] Paul J Darwen and Xin Yao. "On evolving robust strategies for iterated prisoner's dilemma". In: *Progress in Evolutionary Computation*. Springer, 1993, pp. 276–292.

[4] James H Davis, Patrick R Laughlin, and Samuel S Komorita. "The social psychology of small groups: Cooperative and mixed-motive interaction". In: *Annual review of Psychology* 27.1 (1976), pp. 501–541.

[5] Natalie S Glance and Bernardo A Huberman. "The dynamics of social dilemmas". In: *Scientific American* 270.3 (1994), pp. 76–81.

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[7] Boris V Gnedenko. *Theory of probability*. Routledge, 2017.

[8] Harold I Jacobson. "The maximum variance of restricted unimodal distributions". In: *The annals of mathematical statistics* 40.5 (1969), pp. 1746–1752.

[9] Xin Yao and Paul J Darwen. "An experimental study of N-person iterated prisoner's dilemma games". In: *Progress in evolutionary computation*. Springer, 1993, pp. 90–108.

Reading List for Next Lecture

1. **H. Wang, L. Jiao, and X. Yao, "Two arch2: An improved two-archive algorithm for many-objective optimization,"** ***IEEE Transactions on Evolutionary Computation*****, vol. 19, no. 4, pp. 524–541, 2015.**
2. **H. Chen and X. Yao, "Multiobjective neural network ensembles based on regularized negative correlation learning,"** ***IEEE Transactions on Knowledge and Data Engineering*****, vol. 22, no. 12, pp. 1738–1751, 2010.**