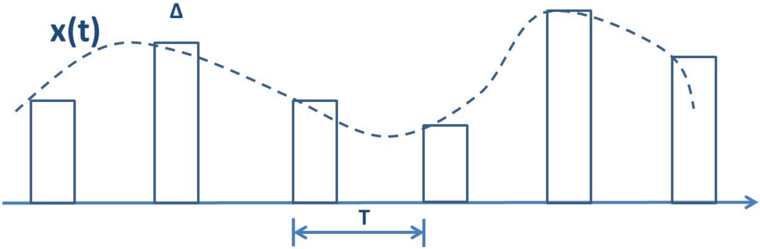
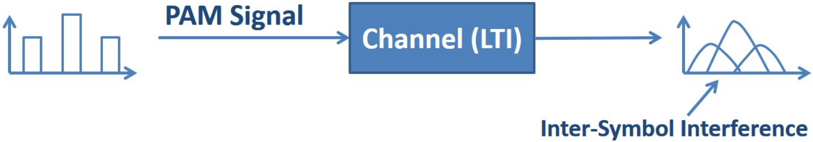
Recap: Pulse-Amplitude Modulation (PAM)



- Pulse-Amplitude Modulation (PAM): sample with period T and hold for duration Δ
- Questions:
  - Without channel distortion, how to demodulate?
  - Can we use pulse with other shape in PAM?

Recap: Inter-Symbol Interference (ISI)

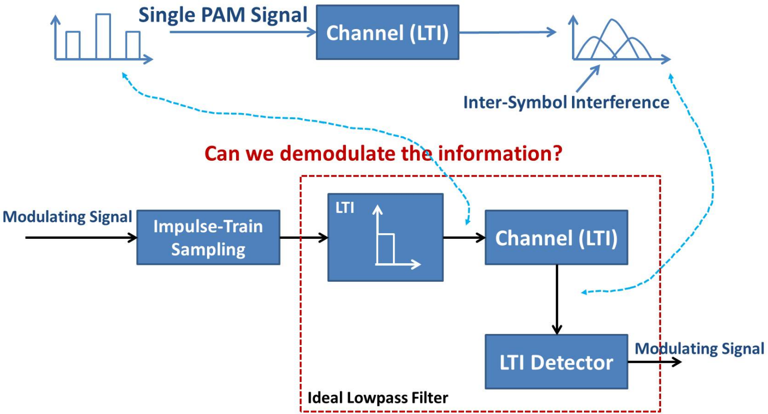
- Wired communication channels are usually low-pass filters
- E.g., optical fiber, twisted pair
- Rectangular wave will disperse and overlap with each other



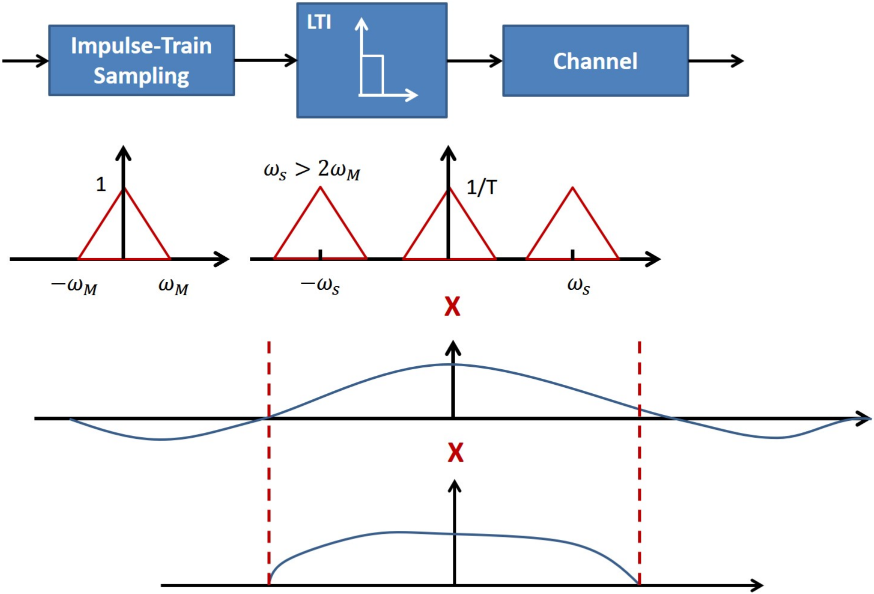
- Wireless channel has multipath effect

Handle ISI at Rx

- Receiver recovers the modulating signal with an LTI system



- When using rectangular wave in PAM, the bandwidth is infinity

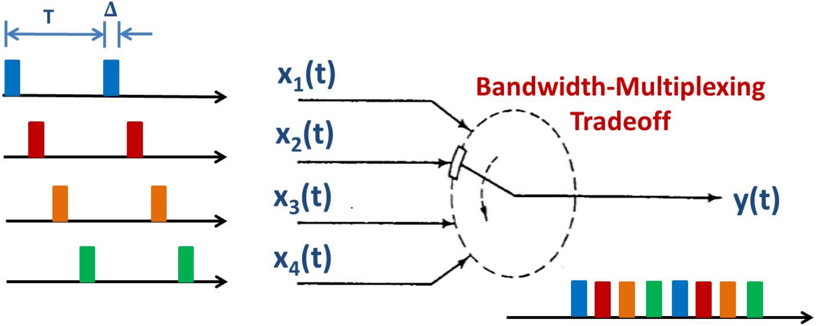


Summary on PAM

- Modulation $\leftrightarrow$ Sampling (zero-order hold); Demodulation $\leftrightarrow$ Recovery
- In additional to rectangular wave, other pulses can be used for PAM
- Demodulation without channel distortion
- Demodulation with channel distortion (ISI)
  - Recover modulating signal: make sure pulse shaping + channel + receiver's detector = ideal low-pass filter

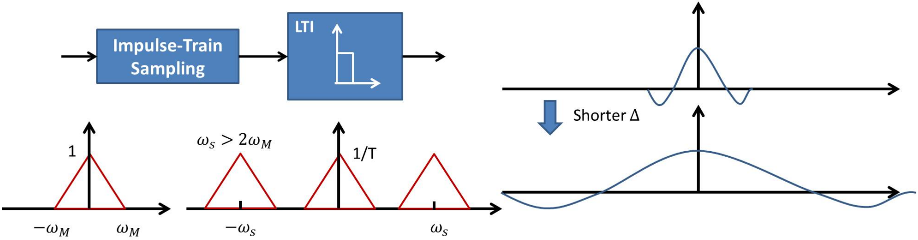
Time-Division Multiplexing

AM signals with pulse-train carrier or PAM signals can be multiplexed in time domain



Discussion on TDM

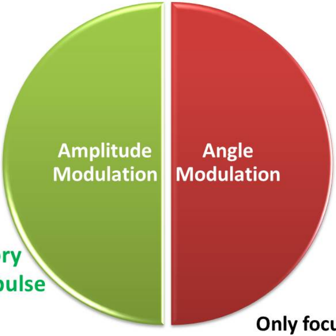
- The number of multiplexed signals is determined by T and Δ
- Can T be as large as we want?
  - T is the sampling period
  - Let ω_M be the bandwidth of modulating signal
  - $\frac{2\pi}{T} > 2\omega_M \Rightarrow T < \frac{\pi}{\omega_M}$
- Can Δ be as short as we want?
  - Shorter pulse $\Rightarrow$ larger bandwidth consumption



What is more ...

Amplitude of carrier contains information

Angle of carrier contains information



- Sinusoid carrier
  - FDM
- Pulse train carrier
- PAM
  - Demod. without ISI
  - Demod. With ISI
    1. Sampling theory
    2. Band-limited pulse
  - TDM

- Phase modulation
- Frequency modulation
  - Narrowband
  - Wideband

Only focus on some basic properties
The demodulator will be introduced in next semester

Angle Modulation

- Angle Modulation: Information is carried by the angle of the carrier

$$y(t) = A\cos(\underbrace{\omega_c}_{\text{Carrier Frequency}} t + \underbrace{\theta_c(t)}_{\text{Information}})$$

- Phase Modulation (PM):

$$\theta_c(t) = \theta_0 + k_p \underbrace{x(t)}_{\text{Modulating Signal}}$$

- Frequency Modulation (FM):

$$\frac{d\theta_c(t)}{dt} = k_f x(t) \text{ or } \theta_c(t) = k_f \int_0^t x(t)dt$$

- Instantaneous frequency: $\omega_i(t) = \omega_c + \frac{d\theta_c(t)}{dt}$

Narrowband FM

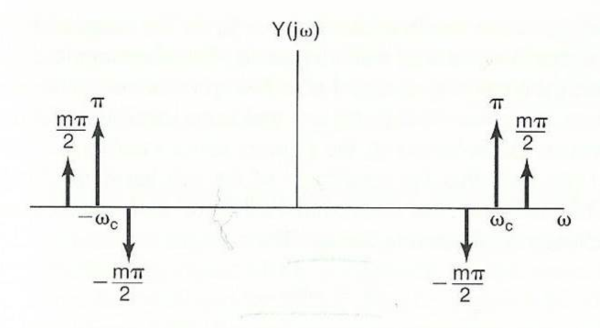
- Modulating signal: $x(t) = As(t)$, where $max_t|s(t)| = 1$
- Narrowband condition: $|\theta_c(t)| = |k_f A \int_0^t s(t)dt| \to 0$
- Modulated signal:

$$
\begin{aligned}
y(t) &= cos\left[\omega_c t + k_f A \int_0^t s(t)dt\right] \\
&= cos\omega_c t \; cos\left(k_f A \int_0^t s(t)dt\right) - sin\omega_c t \; sin\left(k_f A \int_0^t s(t)dt\right) \\
&\approx cos\omega_c t - \left(k_f A \int_0^t s(t)dt\right) sin\omega_c t
\end{aligned}
$$

- Observation: Narrowband FM is similar to AM
- How is the power efficiency of narrowband FM?

Narrowband FM Example

- Let $x(t) = Acos\omega_m t$, $\theta_c(t) = \frac{k_f A}{\omega_m}(sin\omega_m t)$
- Narrowband Condition: modulation index $m = \frac{k_f A}{\omega_m} \to 0$
- $y(t) \approx cos\omega_c t - m(sin\omega_m t)(sin\omega_c t)$
- Observation: Bandwidth ($2\omega_m$) of narrowband FM signal is determined by the bandwidth of modulating signal, independent of its amplitude



Wideband FM

- Example: $x(t) = Acos\omega_m t$
- Modulated signal: $y(t) = cos(\omega_c t + \frac{k_f A}{\omega_m} sin\omega_m t) = cos(\omega_c t + msin\omega_m t)$
- Using Bessel functions J_n, we have

$$
\begin{aligned}
cos(\omega_c t + msin\omega_m t) &= J_0(m)cos\omega_c t \\
&\quad -J_1(m)(cos(\omega_c - \omega_m)t - cos(\omega_c + \omega_m)t) \\
&\quad +J_2(m)(cos(\omega_c - 2\omega_m)t + cos(\omega_c + 2\omega_m)t) \\
&\quad -\dots \\
&= \sum_{n=-\infty}^{\infty} J_n(m)cos(\omega_c t + n\omega_m t)
\end{aligned}
$$

Bandwidth of Wideband FM

m	J0	J1	J2	J3	J4	J5	J6	J7	J8
0	1								
0.25	0.98	0.12							
0.5	0.94	0.24	0.03						
1.0	0.77	0.44	0.11	0.02					
2.0	0.22	0.58	0.35	0.13	0.03				
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01		
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02

- If $m = 0.25$, the bandwidth is $2\omega_m$
- If $m = 0.5$, the bandwidth is $4\omega_m$
- Carson's rule: the bandwidth of FM signal is approximated to be $2(m+1)\omega_m$