

Signals and Systems

**Department of Electrical & Electronic Engineering
Southern University of Science and Technology**

Fall 2020



WANG Rui

- USTC - CSE – BEng (2000-2004)
- HKUST - ECE – PhD (2004-2009)
- Huawei - Senior Research Engineer (2009-2012)
- SUSTech – EEE - Associate Professor

Research Interests:

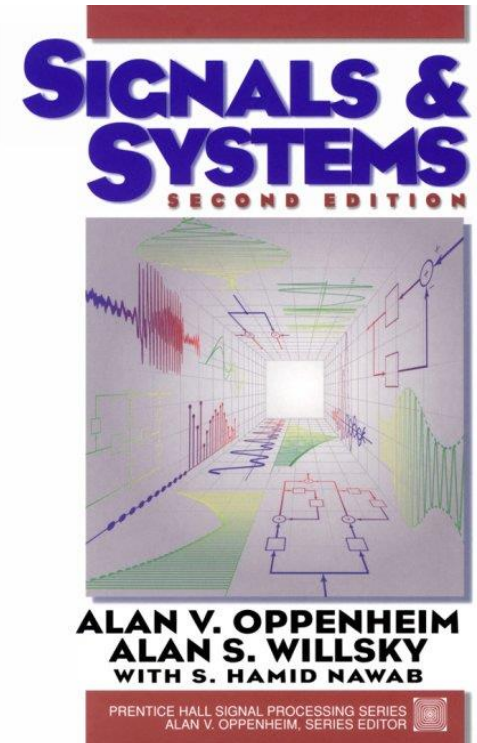
- Wireless communications: 5G/B5G/6G, VLC, mmWave and etc.
- Integrated sensing, computing and communications
- Stochastic optimization, Reinforcement learning, convex optimization and etc.

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Scope of Lecture

- “Signals and Systems”, Oppenheim, Willsky and Nawab, 2nd Edition, 1997, Prentice-Hall.
- This course teaches **Chapters 1 to 8**.
 - ◆ Roughly two weeks for one chapter
 - ◆ Middle-term exam for **Chapters 1 to 4**
 - ◆ Final exam for **all**



Textbook reading is crucial, as I cannot cover every detail in slides

Three Pillars

**Lectures
(Tutorial)**

Matlab Labs

**YOU
100%**

Assignment/Quiz 20%

Mid-term Exam 25%

Final Exam 25%

Lab Reports

***Project Report &
Presentation***

30%

Class Schedules

- Lab Session – **Starts from the first week**
- Instructor: Dr. Guang Wu (吴光)
- Tutorials – **From Mon. to Thur., TBD**
- Every week (**except week 1**)
- Assignment: Every week (**except week 1**)
- Submit assignment in **softcopy to Blackboard system**
- Deadline: Next Wednesday, 12pm.



群名称: Signals and Systems, 2020...
群 号: 883107947

Signals and Systems

- **Signals:** everything which carries information
- **Systems:** everything which processes input signal and generate output signal

Communication Signals & Systems



Can you find any example of signals and systems when making a phone call?

- Transmitter, channel and receiver are all systems.
- Each system has one input signal and one output signal.

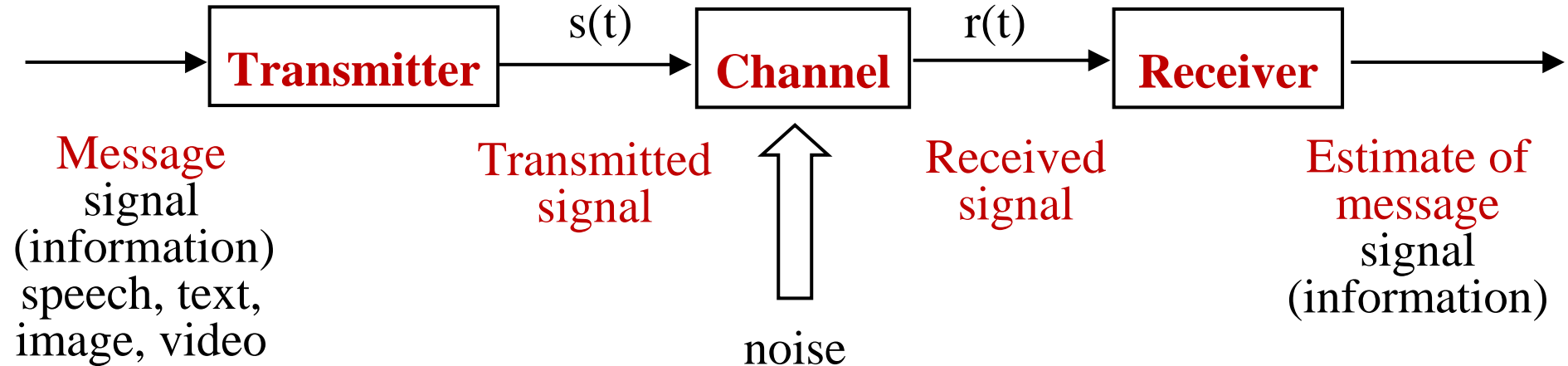
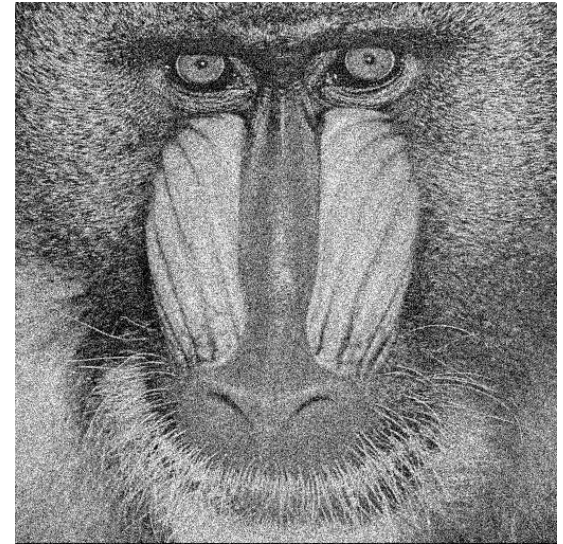
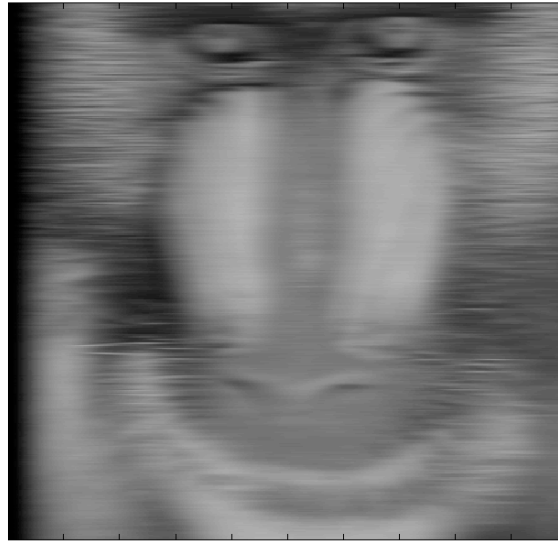
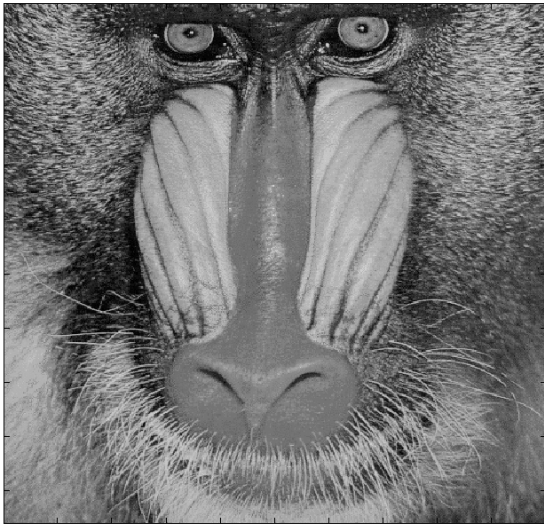


Image Processing



More examples of signals

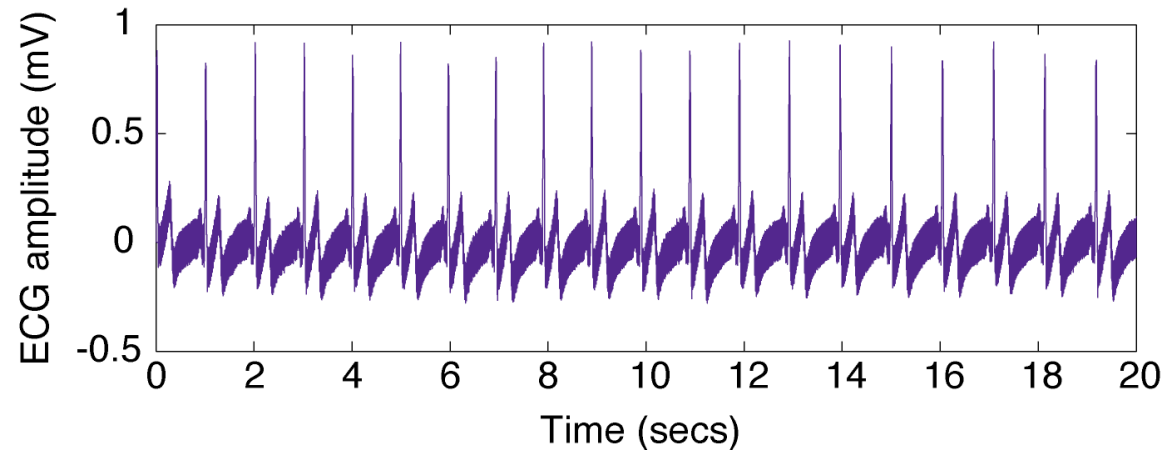
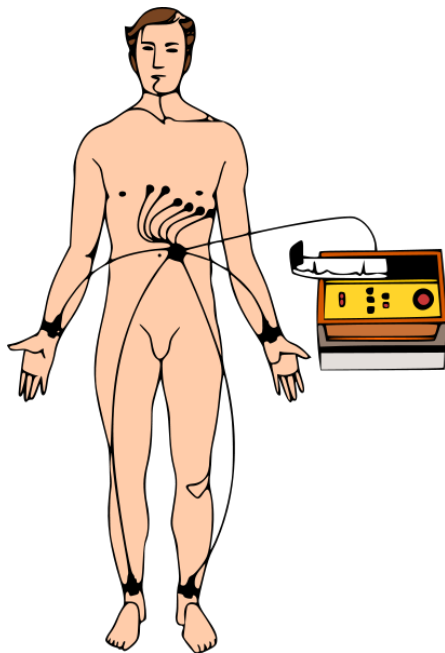
- Electrical signals – voltages and currents in a circuit
- Acoustic signals – audio or speech signals
- Video signals – movie
- Biological signals – sequence of bases in a gene
- We will treat **noise** as unwanted signals.

Signals and Systems from Our Point of View

- **Signals** are variables that carry information, like **function**.
- **Systems** process input signals to produce output signals.
- The course is about using **mathematical** techniques to analyze and synthesize systems which process signals.

Independent Variable of Signals

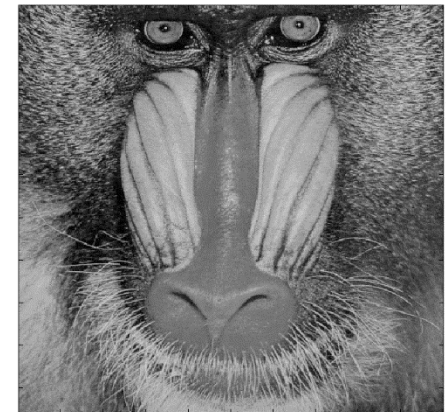
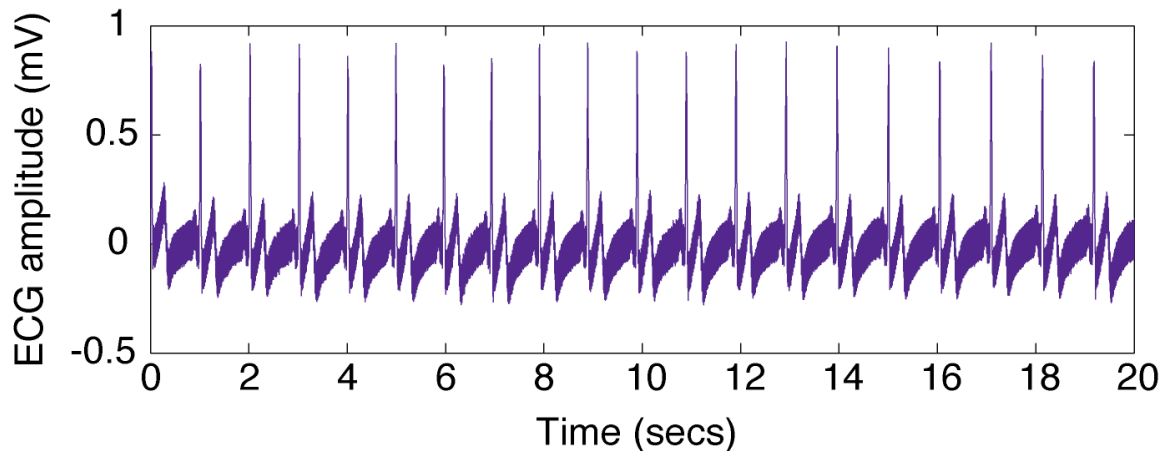
- **Time** is often the independent variable.
- Example: the electrical activity of the heart recorded with chest electrodes — the electrocardiogram (ECG).



Signal Classification 1:

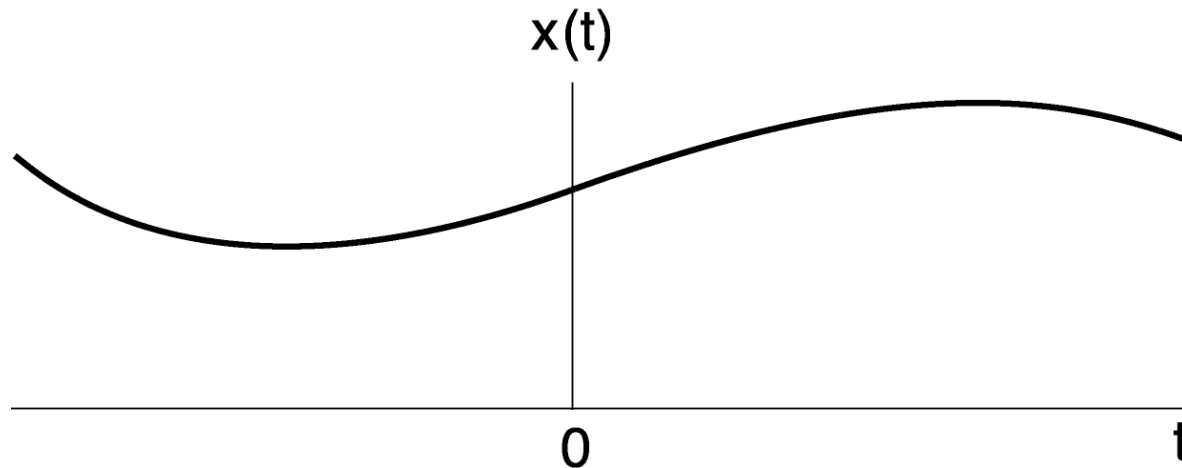
Dimension of Independent Variable

- An independent variable can be 1-D (t in the ECG), 2-D (x, y in an image), or 3-D (x, y, t in an video).



- We focus on 1-D for mathematical simplicity but the results can be extended to 2-D or even higher dimensions.

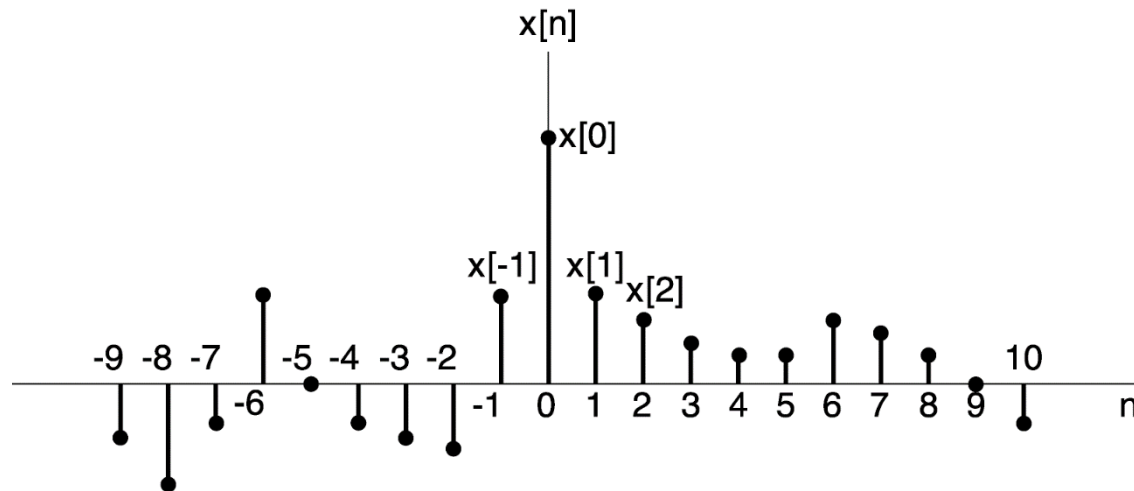
Signal Classification 2: Continuous-time (CT) Signals



- Independent variable is continuous
- Most of the signals in the physical world are CT signals.
- E.g. voltage & current, pressure, temperature, velocity, etc.

Notation: $x(t)$

Discrete-time (DT) Signals



- Independent variable is integer
- Examples of DT signals: DNA sequence, population of the n -th generation of certain species

Notation: $x[n]$

Many Human-made Signals are DT



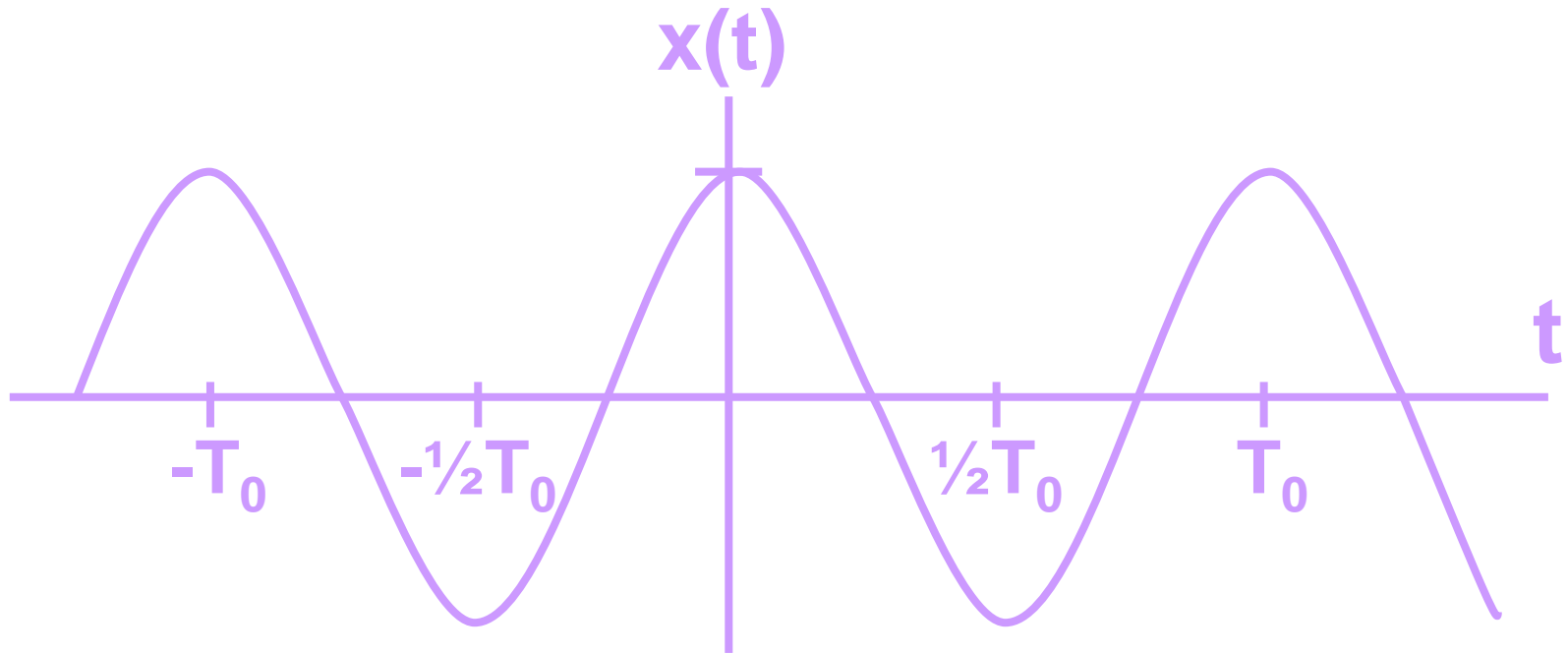
*Weekly Dow-Jones
industrial average*



Digital image

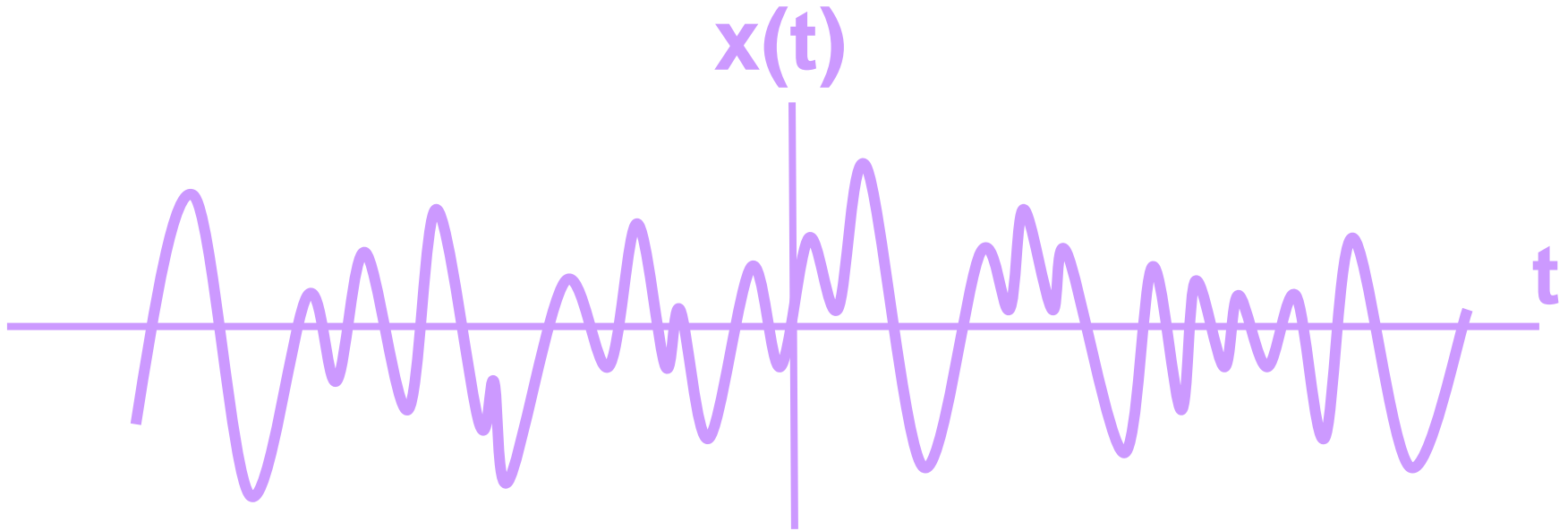
- Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

Signal Classification 3: Deterministic Signal



- Each value of the signal is fixed, and can be determined by a mathematical expression, rule, or table.

Signal Classification 3: Random Signal



- Signal value at any time instance is a random variable.

Signal Classification 4:

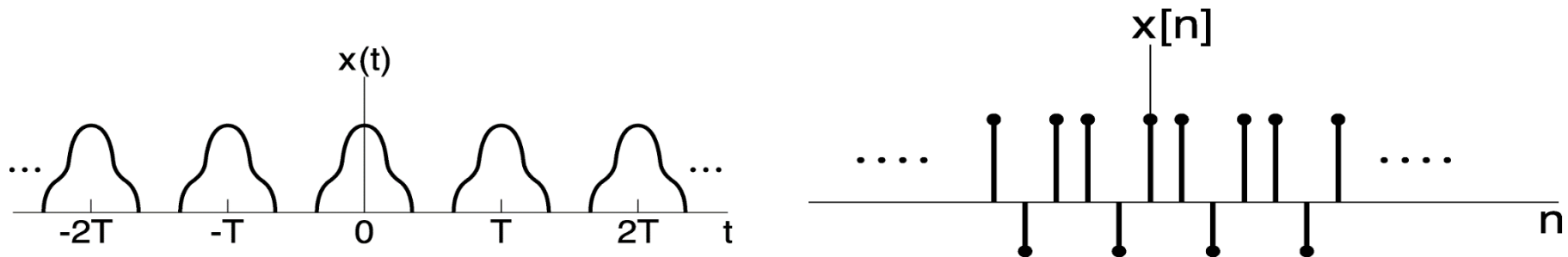
Periodic / Aperiodic

- **Periodic** Signals

CT: $x(t) = x(t + T)$, T : period

$x(t) = x(t + mT)$, m : integer

DT: $x[n] = x[n + N] = x[n + mN]$, N : period

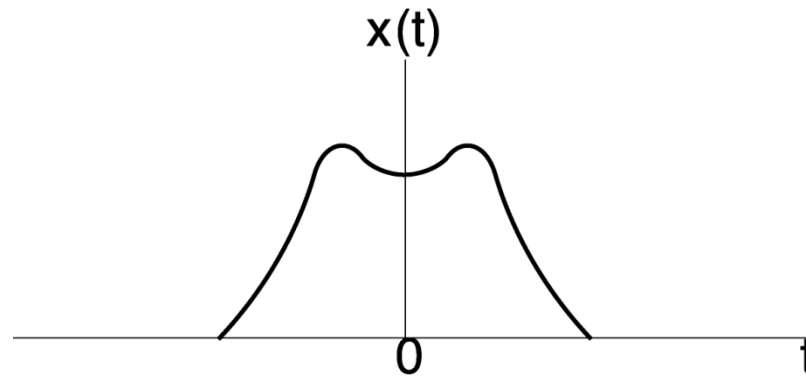


- **Fundamental period**: the smallest positive period
- **Aperiodic**: NOT period

Signal Classification 5: Even / Odd

- Even and Odd Signals

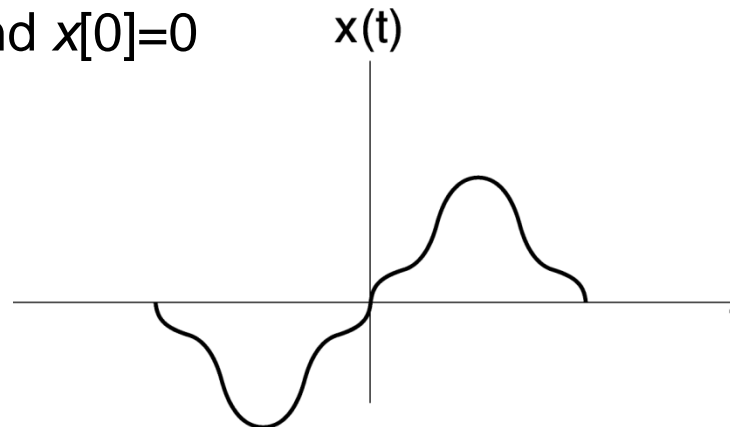
Even $x(t) = x(-t)$ or $x[n] = x[-n]$



Example: $\cos(t)$

Odd $x(t) = -x(-t)$ or $x[n] = -x[-n]$

$x(0)=0$, and $x[0]=0$



Example: $\sin(t)$

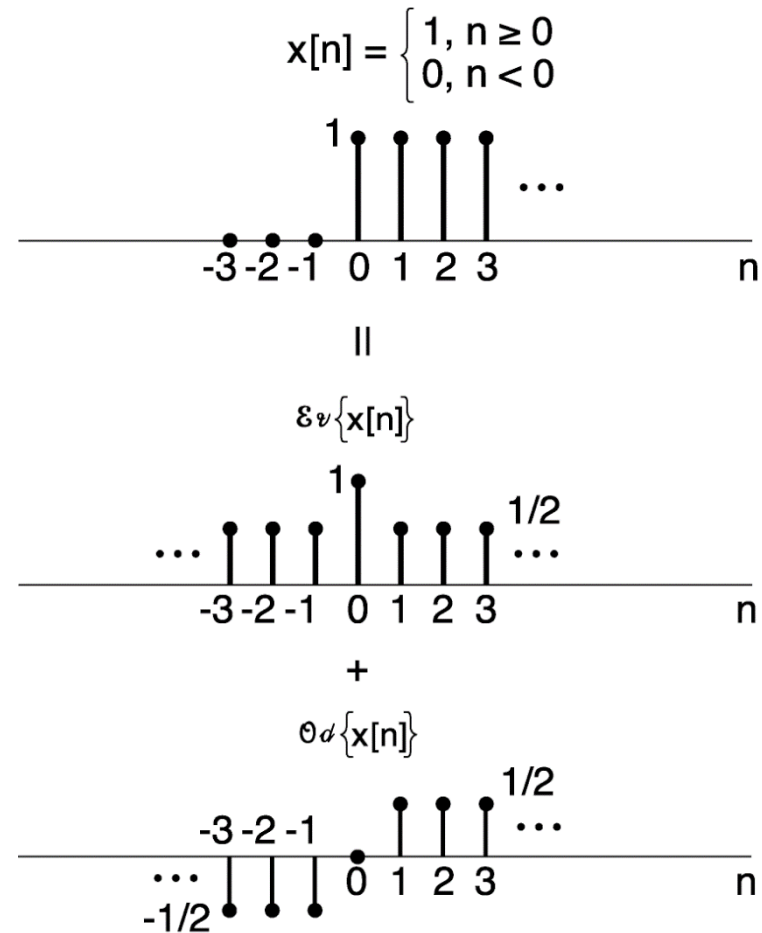
- Any signals can be expressed as **a sum of Even and Odd** signals. That is:

$$x(t) = x_{\text{even}}(t) + x_{\text{odd}}(t),$$

where:

$$x_{\text{even}}(t) = [x(t) + x(-t)]/2,$$

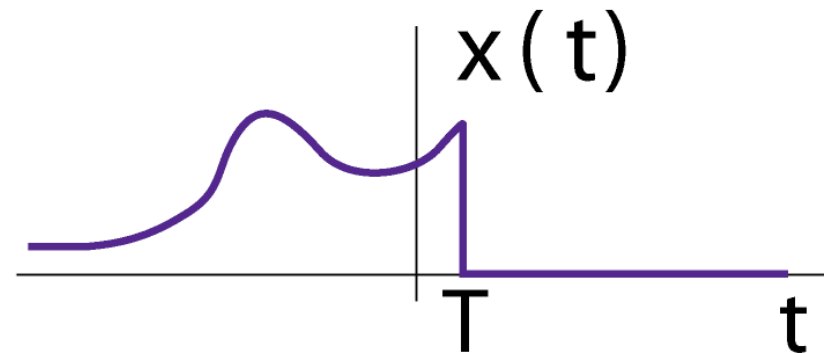
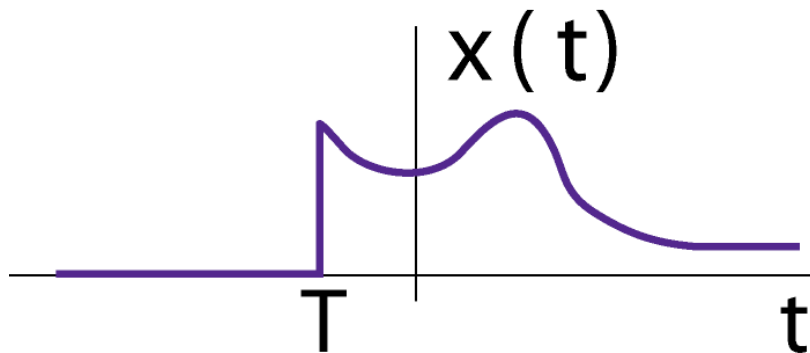
$$x_{\text{odd}}(t) = [x(t) - x(-t)]/2.$$



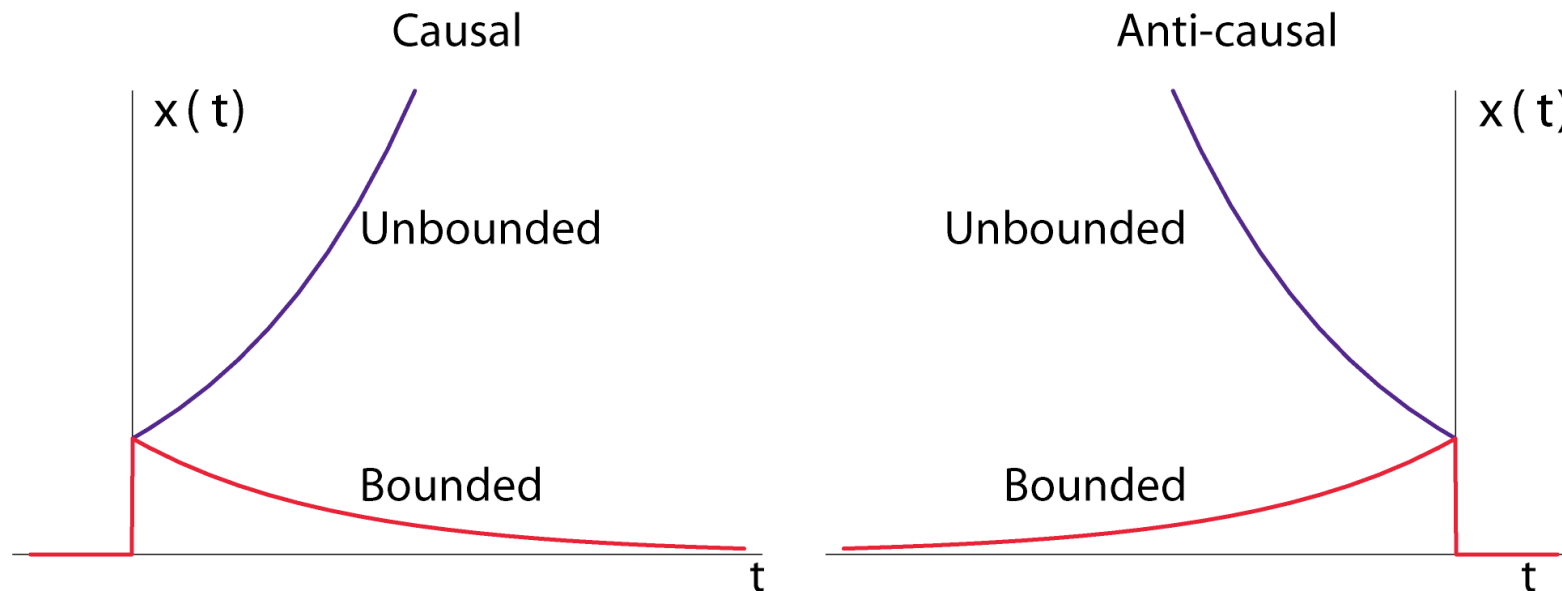
Signal Classification 6:

Right- and Left-Sided

- A right-sided signal is zero for $t < T$, and
- A left-sided signal is zero for $t > T$, where T can be positive or negative.



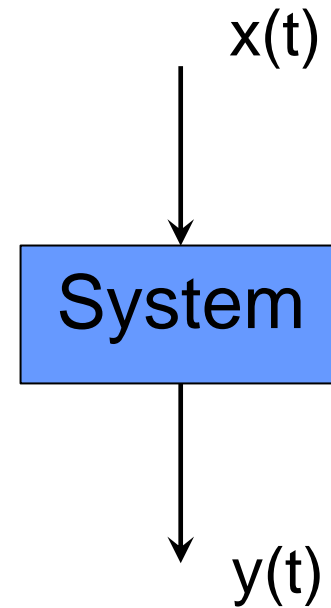
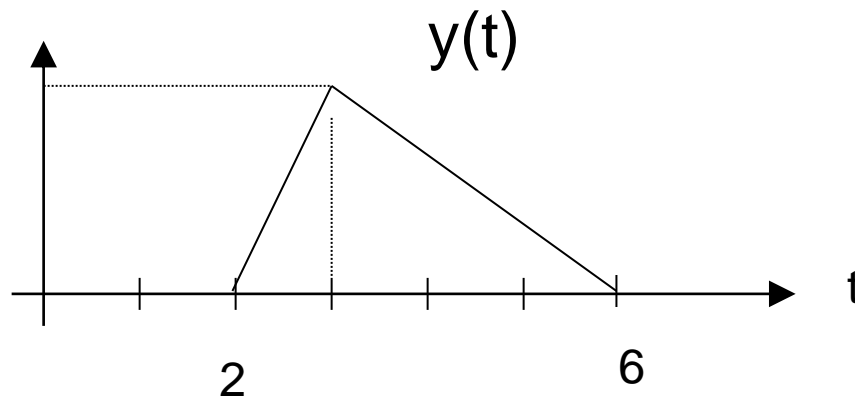
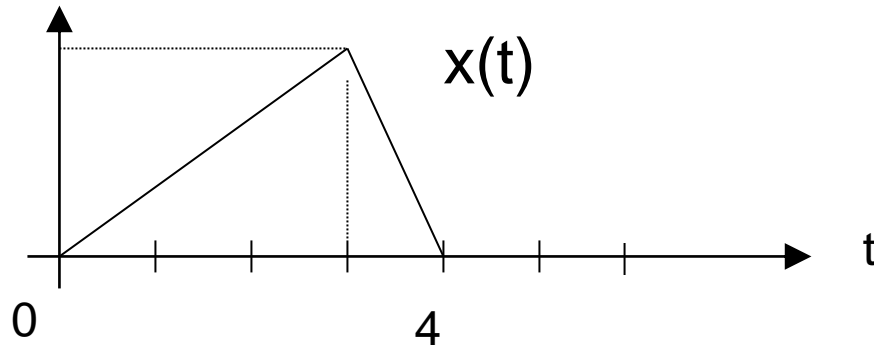
Classification 7: Bounded and Unbounded



- Bounded signal: the absolute value of signal is bounded.
- Unbounded signal: otherwise

$$\exists C, |x(t)| \leq C \forall t$$

Transformation of a Signal



Transformation of a Signal

- Time Shift

$$x(t) \rightarrow x(t - t_0) \quad , \quad x[n] \rightarrow x[n - n_0]$$

- Time Reversal

$$x(t) \rightarrow x(-t) \quad , \quad x[n] \rightarrow x[-n]$$

- Time Scaling

$$x(t) \rightarrow x(at) \quad , \quad x[n] \rightarrow ?$$

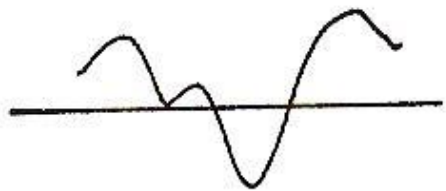
- Combination

$$x(t) \rightarrow x(at + b) \quad , \quad x[n] \rightarrow ?$$

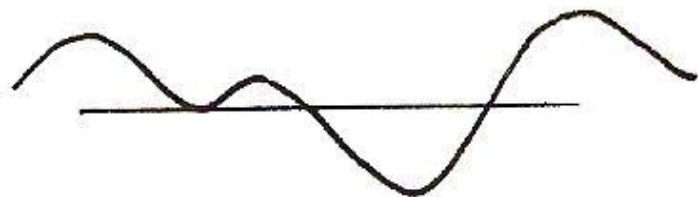
Transformation of a Signal

Time Scaling

$x(t)$



$x(at), a < 1$



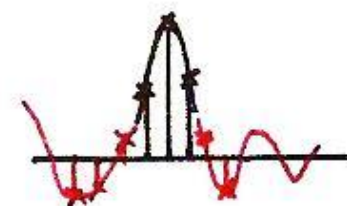
$x(at), a > 1$



$x[n]$

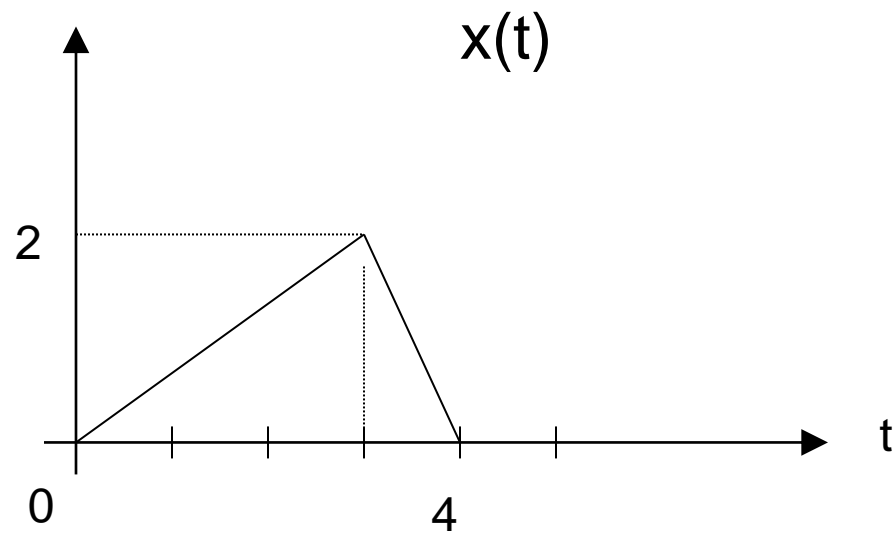


?



?

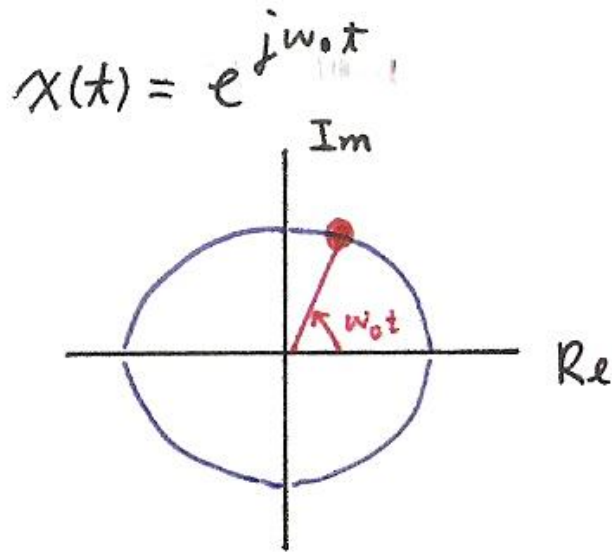
Class problem



$x(-2t+2)$?

Exponential Signals

- A **very important** class of signals is presented as:
CT signals of the form $x(t) = e^{j\omega t}$
DT signals of the form $x[n] = e^{j\omega n}$
- For both *exponential* CT and DT signals, x is a complex quantity and has:
a real and imaginary part [i.e., *Cartesian form*], or equivalently
a magnitude and a phase angle [i.e., *polar form*].
- We will use whichever form that is convenient.

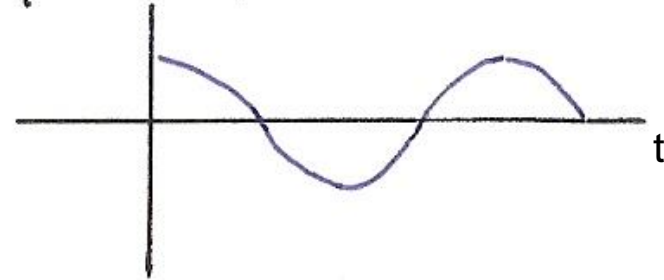


Euler's relation

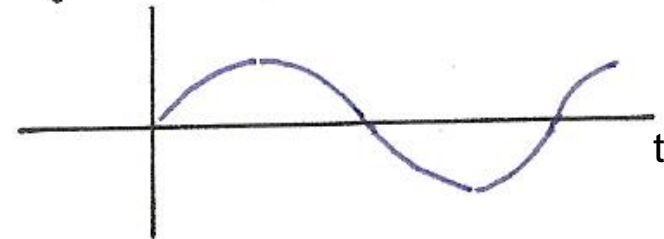
$$e^{jx} = \cos x + j \sin x$$

$\omega_0 t$ is defined as phase

$$\operatorname{Re} \{ e^{j\omega_0 t} \} = \cos \omega_0 t$$



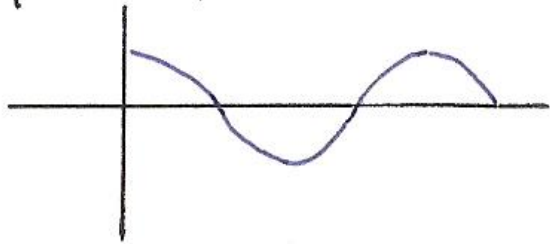
$$\operatorname{Im} \{ e^{j\omega_0 t} \} = \sin \omega_0 t$$



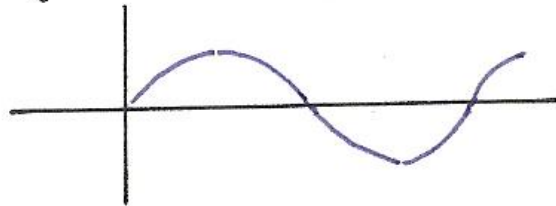
Real and imaginary parts are periodic signals with the same period, but **out of phase** (90° phase difference)

$$x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j \sin(\omega_0 t)$$

$$\operatorname{Re} \{ e^{j\omega_0 t} \} = \cos \omega_0 t$$



$$\operatorname{Im} \{ e^{j\omega_0 t} \} = \sin \omega_0 t$$



-Fundamental (angular) frequency: ω_0

-Fundamental period: $T_0 = \frac{2\pi}{\omega_0}$

-In CT, $e^{j\omega_0 t}$ **always** periodic

-larger $\omega_0 \Rightarrow$ higher frequency

$$x[n] = e^{j\omega_0 n} = \cos\omega_0 n + j \sin\omega_0 n$$

Is it periodic?

Larger $\omega_0 \Rightarrow$ higher frequency?

$$e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$

Periodicity Properties of DT Complex Exponentials

Important difference between $e^{j\omega_0 n}$ and $e^{j\omega_0 t}$:

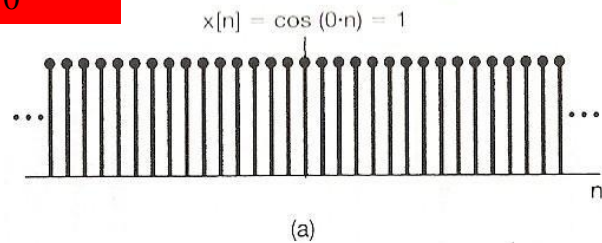
- $e^{j\omega_0 n}$ is periodic w.r.t. ω_0

$$e^{j(\omega_0 + m \cdot 2\pi)n} = e^{j\omega_0 n} \cdot e^{jm \cdot 2\pi n} = e^{j\omega_0 n}$$

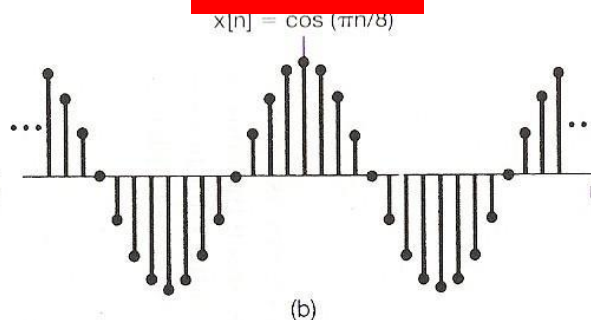
- However, $e^{j\omega_0 t}$ is aperiodic w.r.t. ω_0

$$\forall x \neq 0, e^{j(\omega_0 + x)t} = e^{j\omega_0 t} e^{jxt} \neq e^{j\omega_0 t}$$

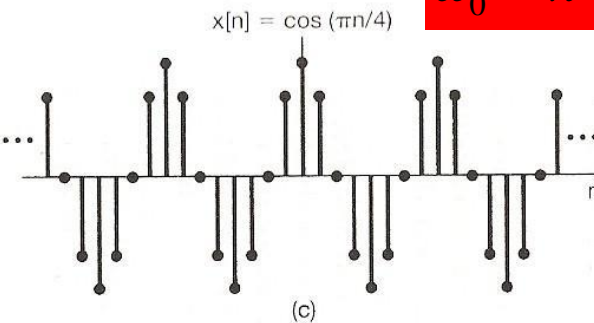
$$\omega_0 = 0$$



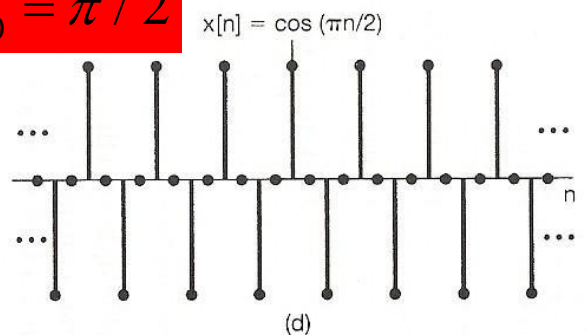
$$\omega_0 = \pi / 8$$



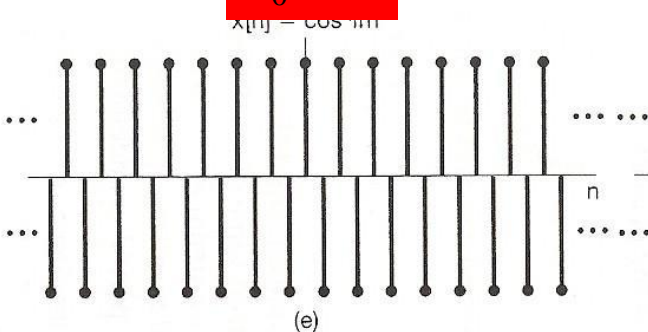
$$\omega_0 = \pi / 4$$



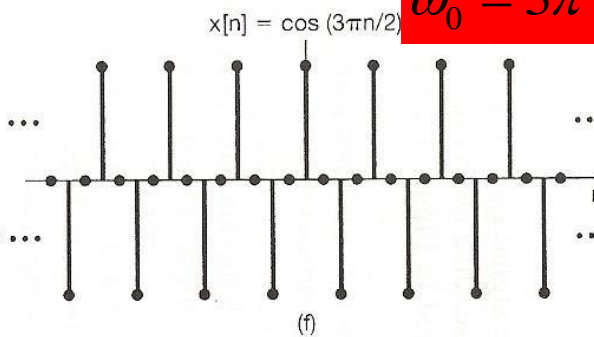
$$\omega_0 = \pi / 2$$



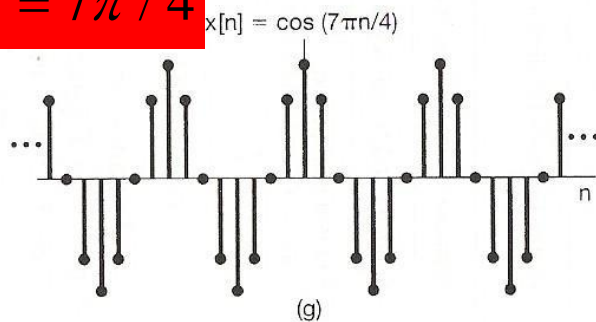
$$\omega_0 = \pi$$



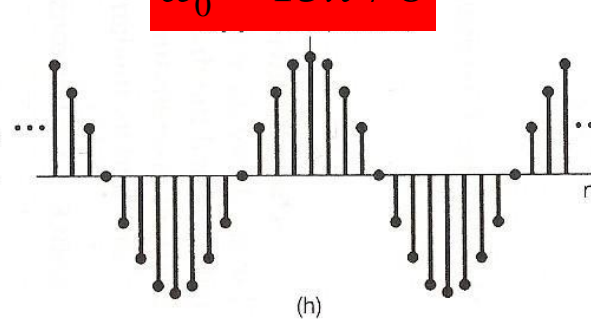
$$\omega_0 = 3\pi / 2$$



$$\omega_0 = 7\pi / 4$$



$$\omega_0 = 15\pi / 8$$



$$\omega_0 = 2\pi$$

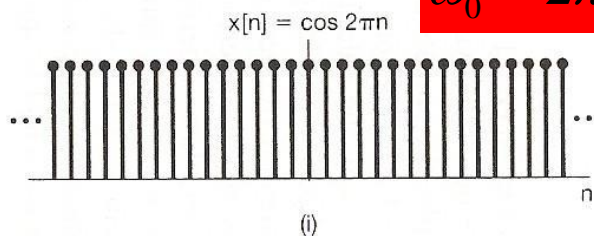


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

- We need **only consider a frequency interval of length 2π** , and on most cases, we use the interval: $0 \leq \omega_0 < 2\pi$, or $-\pi \leq \omega_0 < \pi$
- $e^{j\omega_0 n}$ does **not** have a continually increasing rate of oscillation as ω_0 is increased.

lowest-frequency (slowly varying): ω_0 near 0, 2π , ..., or $2k \cdot \pi$

highest-frequency (rapid variation): ω_0 near $\pm \pi$, ..., or $(2k+1) \cdot \pi$

$$e^{j(2k+1)\pi n} = e^{j\pi n} = (e^{j\pi})^n = (-1)^n$$

$$e^{j2\pi n} = (e^{j2\pi})^n = (1)^n = 1$$

Harmonically Related Signal Sets

- A set of periodic exponentials which have a **common period**.

$$\{\phi_k(t) = e^{jk\omega_0 t}, k = 0, \pm 1, \pm 2, \dots\}$$

Fundamental (Angular) Frequency : $|k\omega_0|$

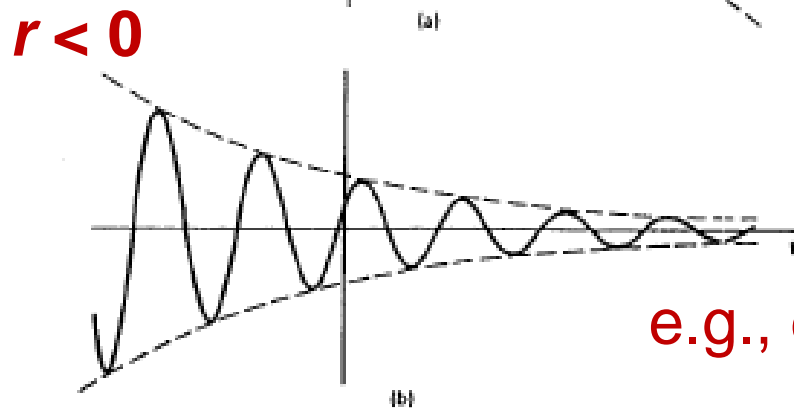
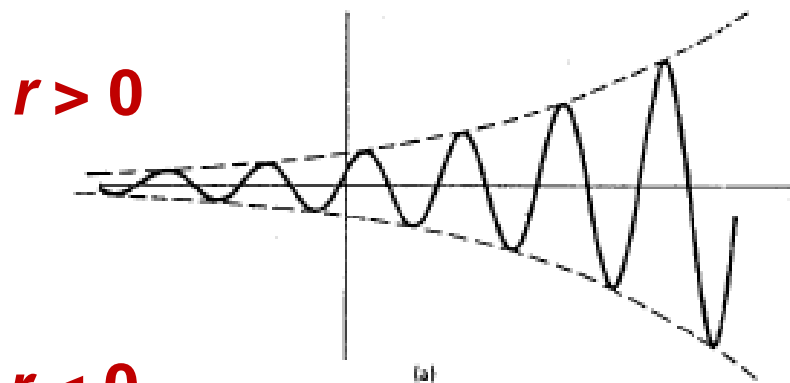
Fundamental Period: $\frac{2\pi}{|k\omega_0|}$

Common Period: $\frac{2\pi}{|\omega_0|}$

General Complex Exponential Signals - CT

- General format (C and a are complex numbers)

$$x(t) = Ce^{at} = |C| e^{j\theta} \cdot e^{(r+j\omega_0)t} = |C| e^{rt} \cdot e^{j(\omega_0 t + \theta)}$$



e.g., damped sinusoids

General Complex Exponential Signals - DT

- General format (C and α are complex numbers)

$$x[n] = C\alpha^n = |C|e^{j\vartheta} \cdot |\alpha|^n e^{j\omega_0 n} = |C||\alpha|^n e^{j(\omega_0 n + \vartheta)}$$

