Maximum Entropy Classifier

Supervised Machine Learning

CSE538 - Spring 2024

Topics we will cover

- *Supervised classification (open-vocabulary)*
  - Goal of logistic regression
  - The “loss function” -- what logistic regression tries to optimize
  - Logistic regression with multiple features
  - How to evaluation: Training and test datasets
  - Overfitting: role of regularization

Text Classification

The Buccaneers win it!

President Biden vetoed bill

Twitter to be acquired by Apple

She <u>will</u> drive to the office, to make sure the lawyer gives the <u>will</u> to the family.

<u>will.n</u> or *<u>will.v</u>* ?

noun or verb

I like the the movie.

The movie is like terrible.

Supervised Classification

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X - features of N observations (i.e. words)

Y - class of each of N observations

GOAL: Produce a *model* that outputs the most likely class y_i, given features x_i.

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Some function or rules to go from X to Y, as close as possible.

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Supervised Classification

Supervised Machine Learning: Build a model with examples of outcomes (i.e. Y) that one is trying to predict. (The alternative, *unsupervised* machine learning, tries to learn with only an X).

Classification: The outcome (Y) is a discrete class.

for example: $y \in$ {not-noun, noun}

$y \in$ {noun, verb, adjective, adverb}

$y \in$ {positive_sentiment, negative_sentiment}).

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Binary classification goal: Build a model that can estimate $P(A=1|B=?)$

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Example: Y: 1 if target is verb, 0 otherwise;
X: 1 if "was" occurs before target; 0 otherwise

I was reading for NLP. *We were fine.* *I am good.*

The cat was very happy. *We enjoyed the reading material.* *I was good.*

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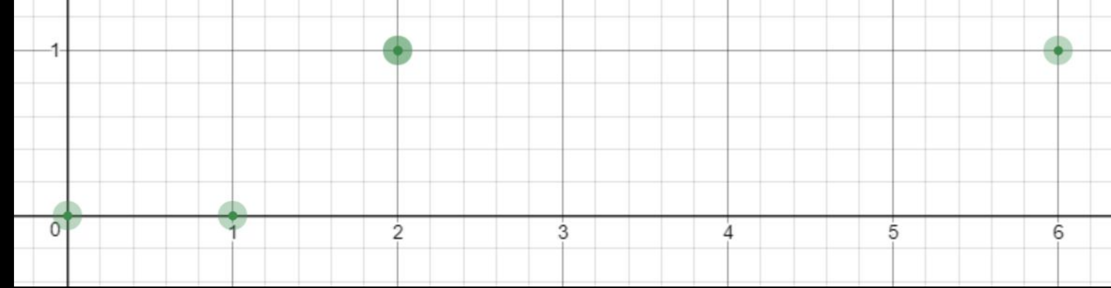
Logistic Regression



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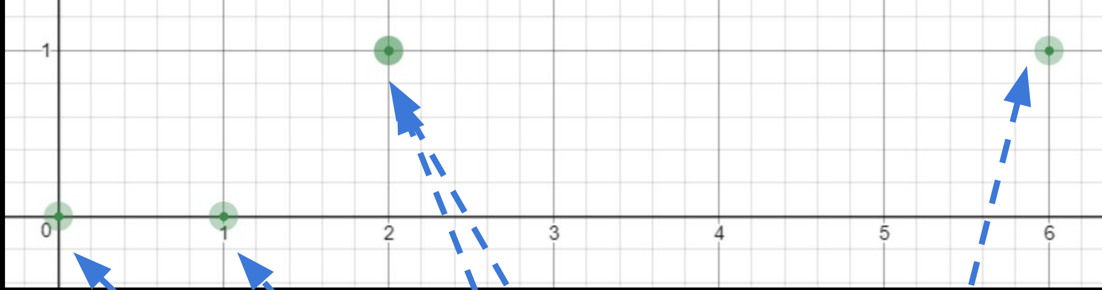

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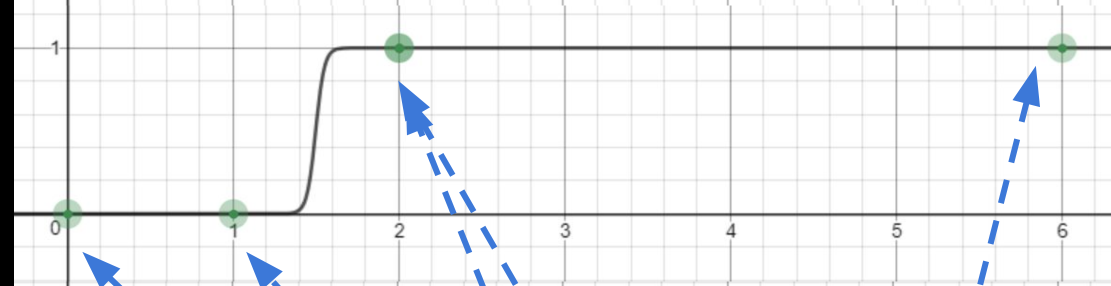

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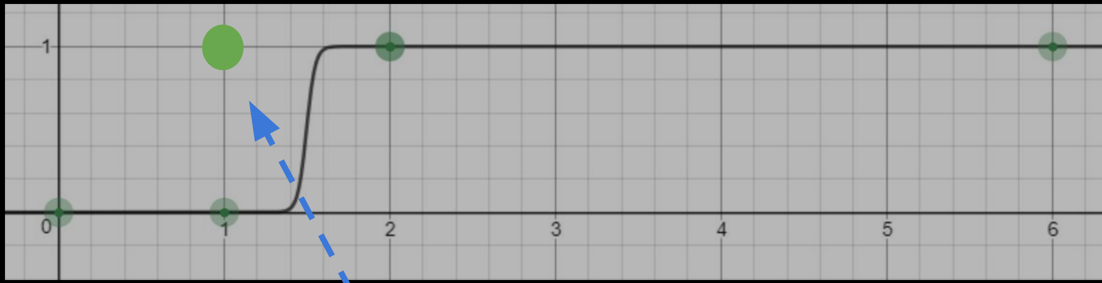

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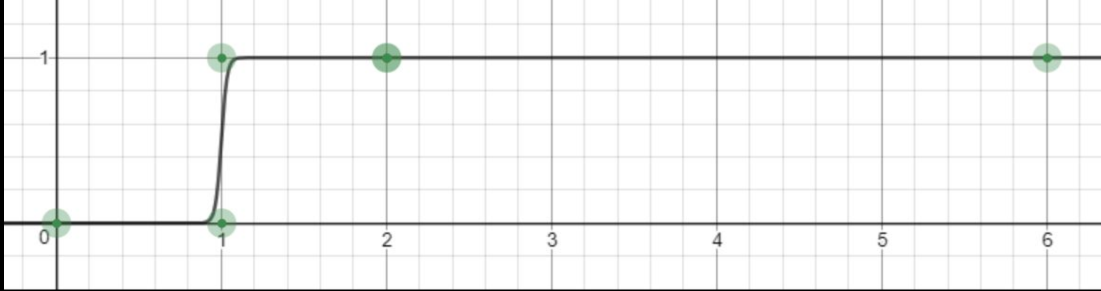

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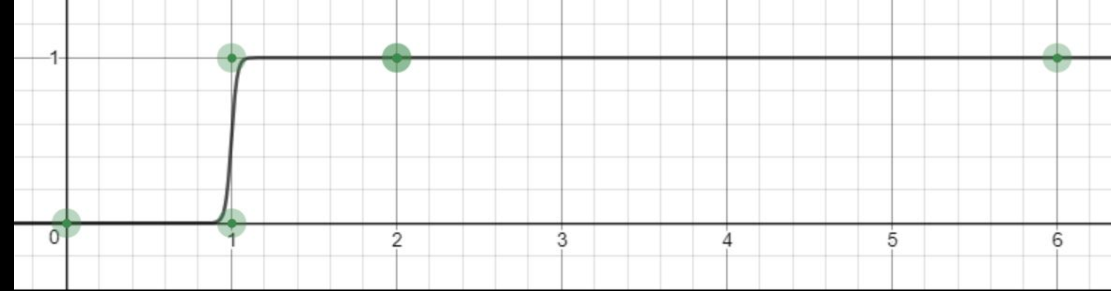

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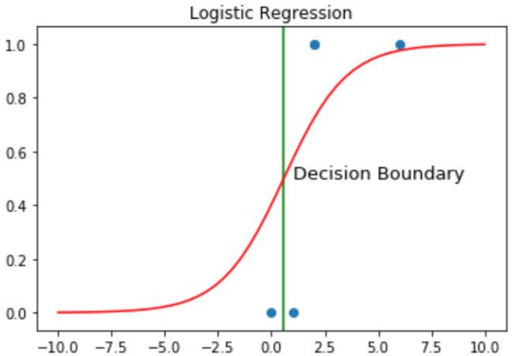


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In [78]:  1  -b_0/b_1
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In [28]:  1  logisticRegr.predict(x)
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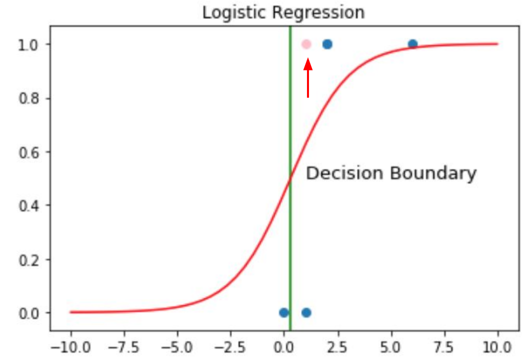


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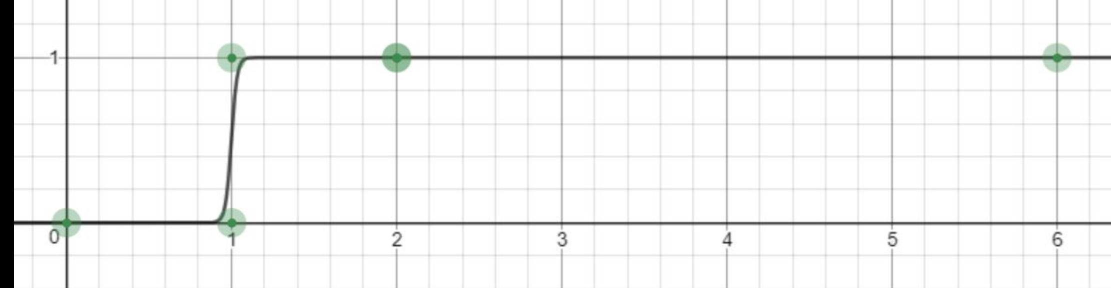

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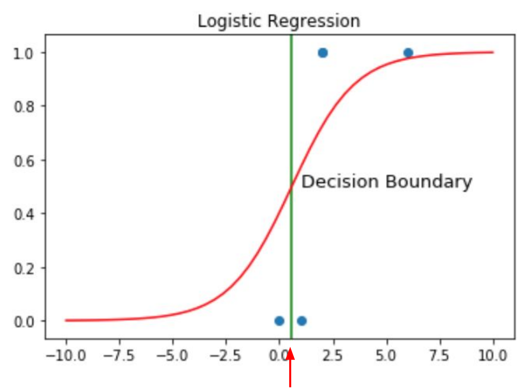


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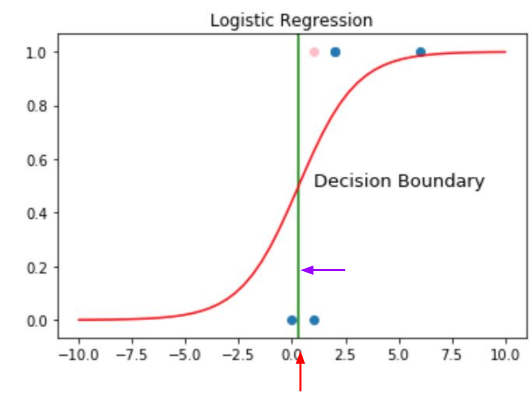


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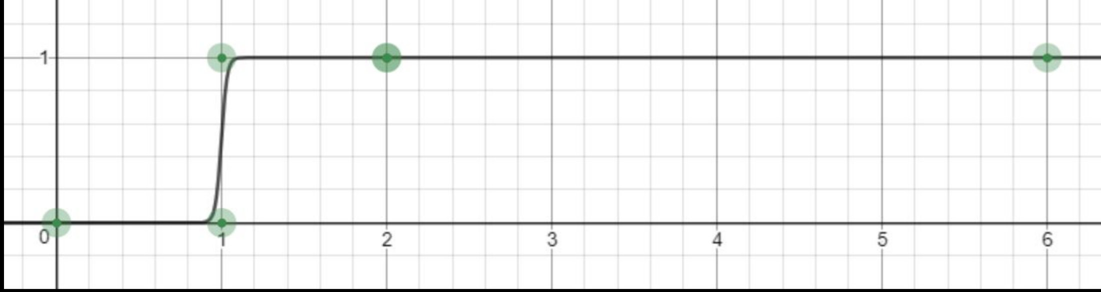

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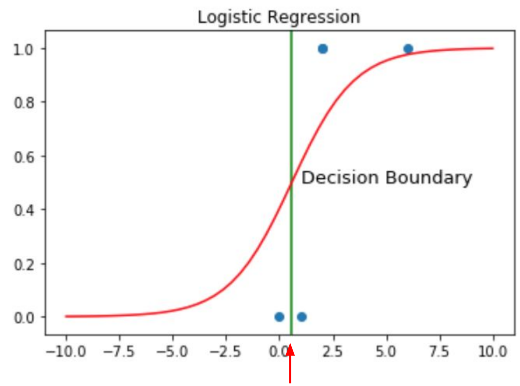


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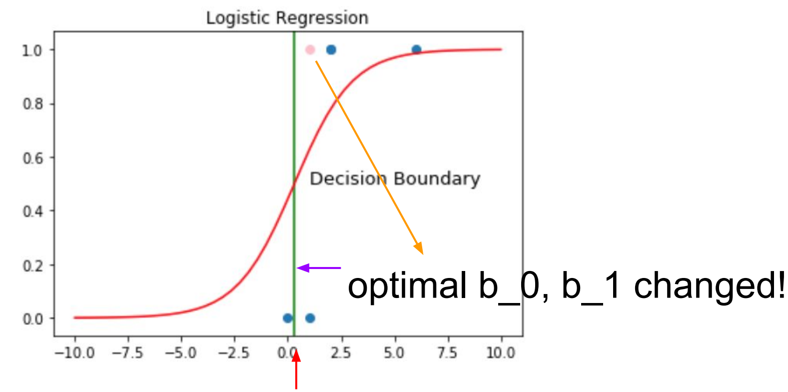


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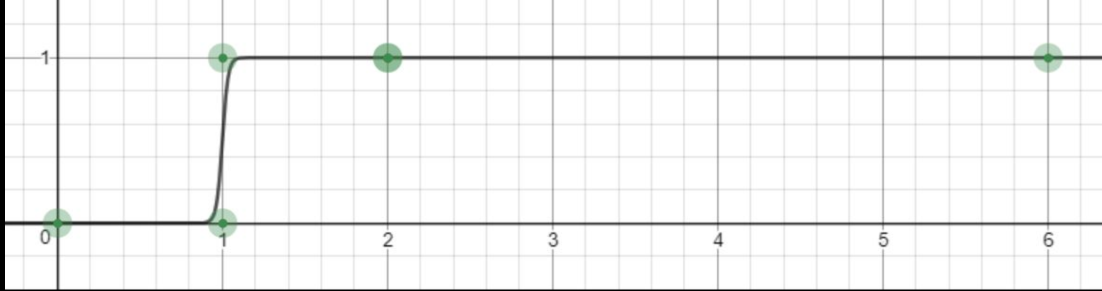

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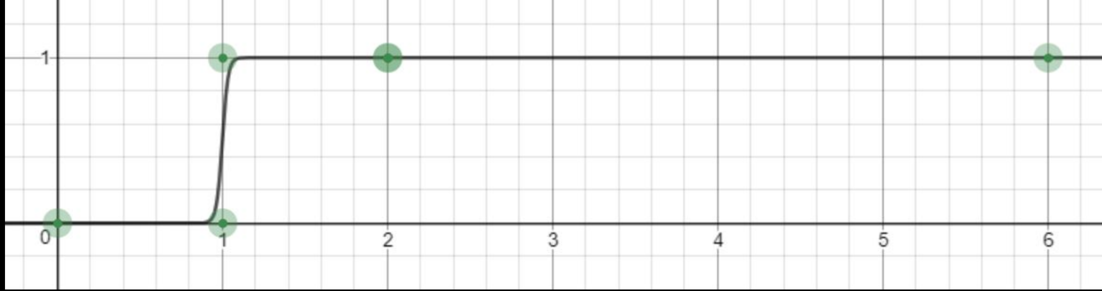

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X1: number of capital letters in target and surrounding words.

Let's add a feature! X2: does the target word start with a capital letter?

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x2	x1	y
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0	1	0
0	0	0
1	6	1
1	2	1
1	1	1

Logistic Regression on features (*x*)

$Y_i \in \{0, 1\}$; X is a **single value** and can be anything numeric.

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<u>Vector notation</u>

β and x_i are vectors of size m

first feature is intercept:
$x_{*,0} = [1, 1 ..., 1]_N$

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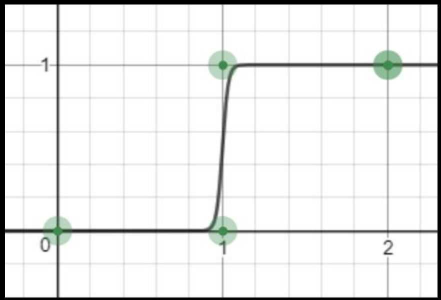

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“best fit” : whatever maximizes the likelihood function:

$$\boxed{L(\beta|X,Y) = \prod_{i=1}^{n} P(Y_i = 1|x_i)^{y_i}(1 - P(Y_i = 1|x_i))^{1-y_i}}$$

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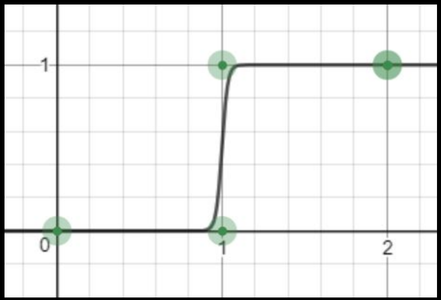

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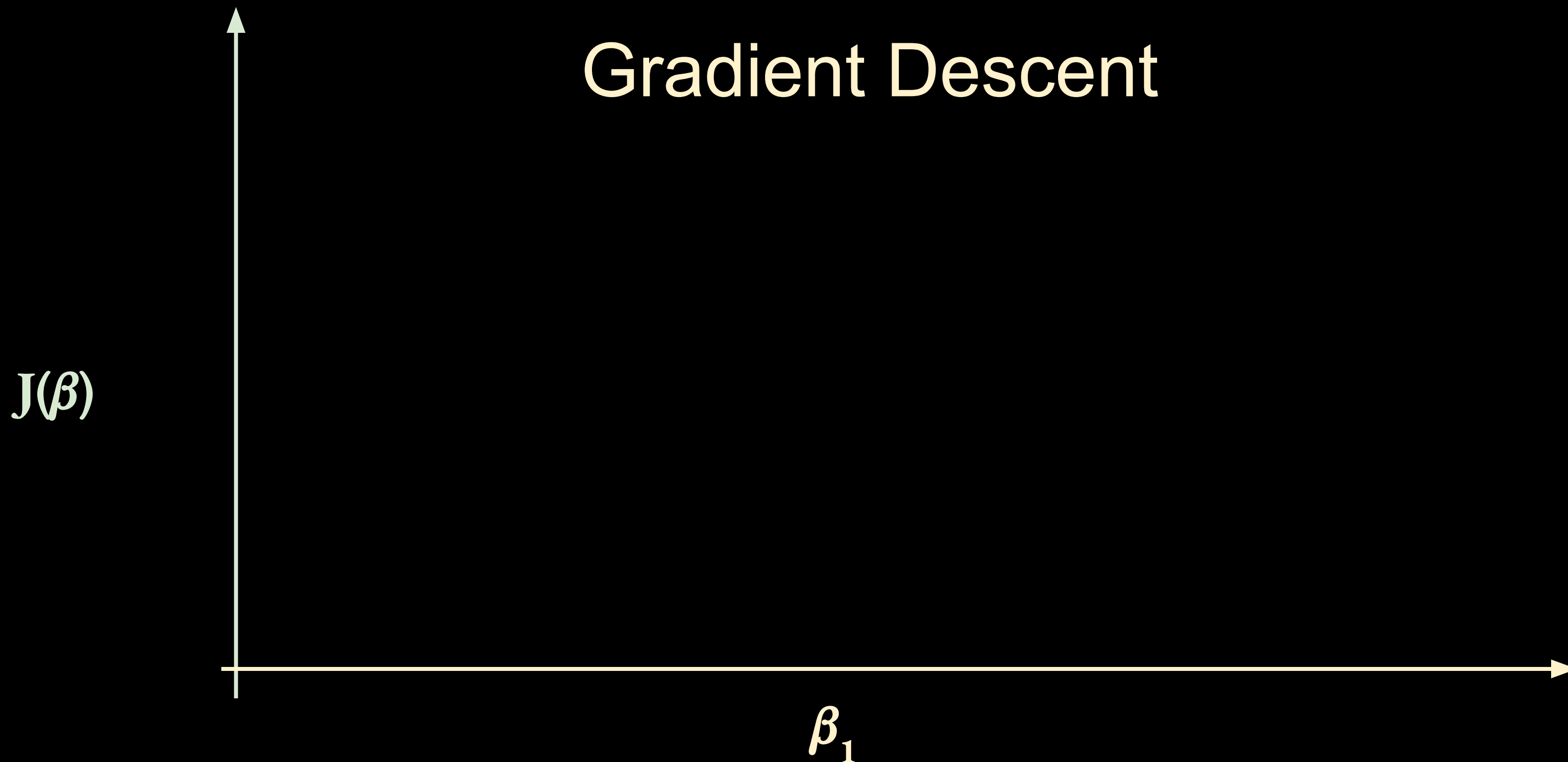
Gradient Descent



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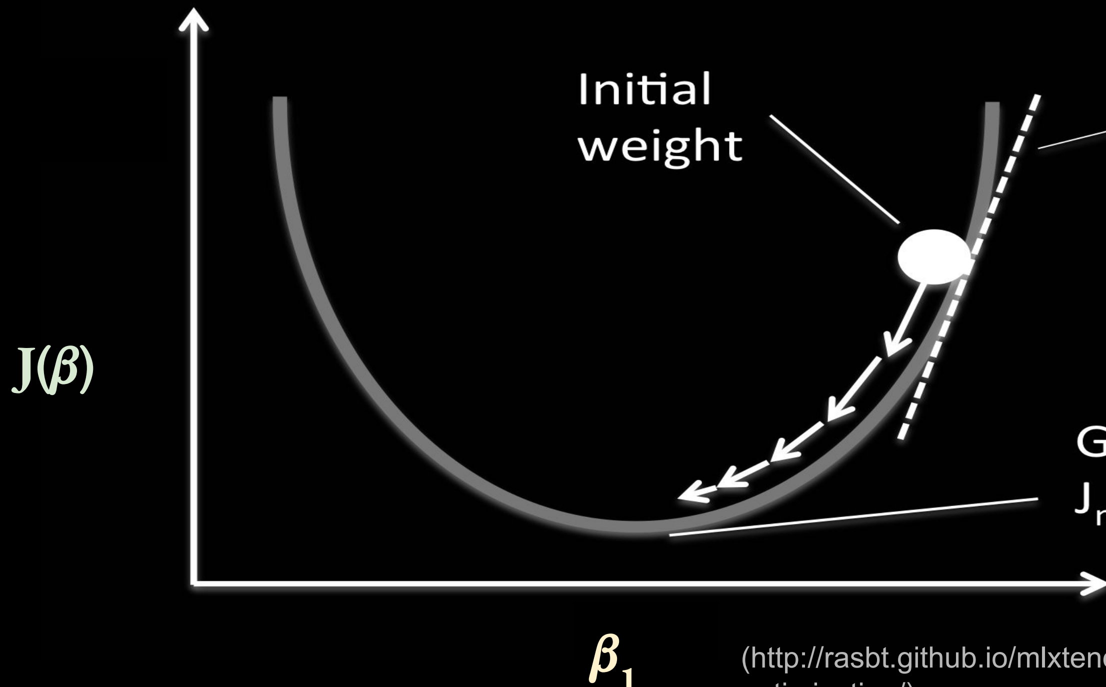

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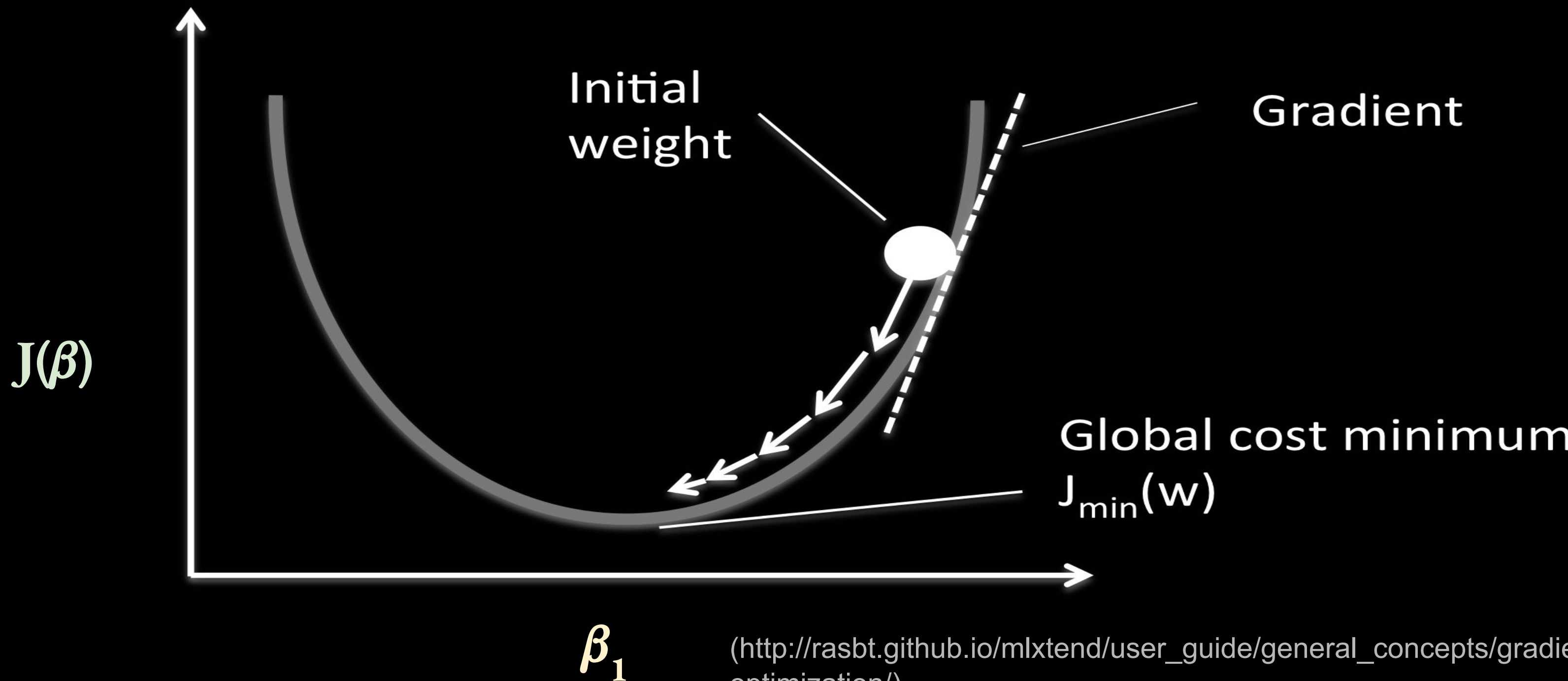

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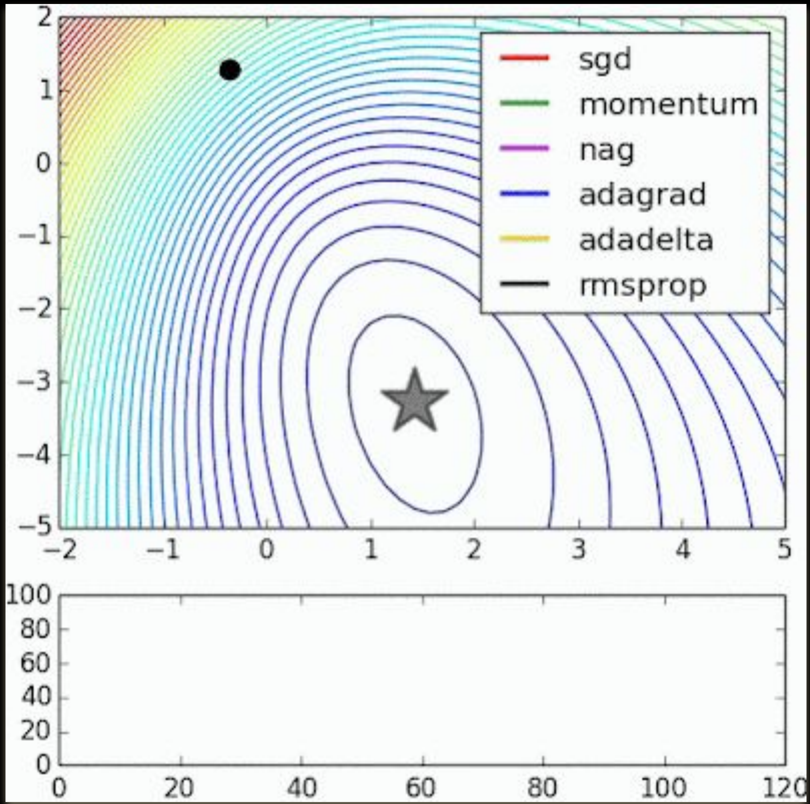

(Animation: Alec Radford, 2018)

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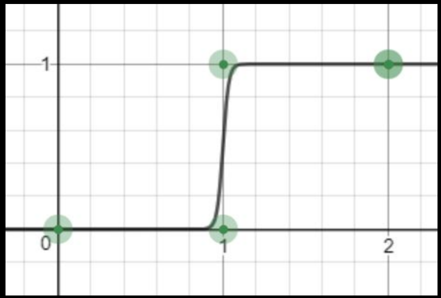

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- Numb
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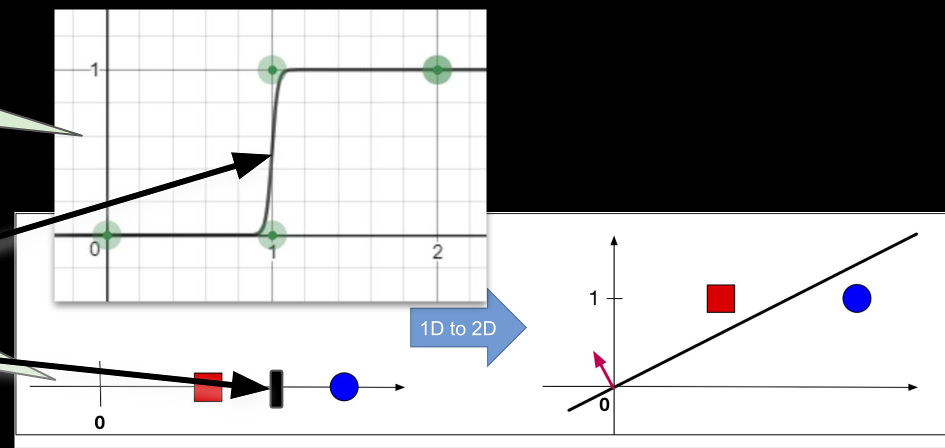
- Prec

Y-axis is Y (i.e. 1 or 0)

To make room for multiple Xs, let's get rid of y-axis. Instead, show **decision point**.



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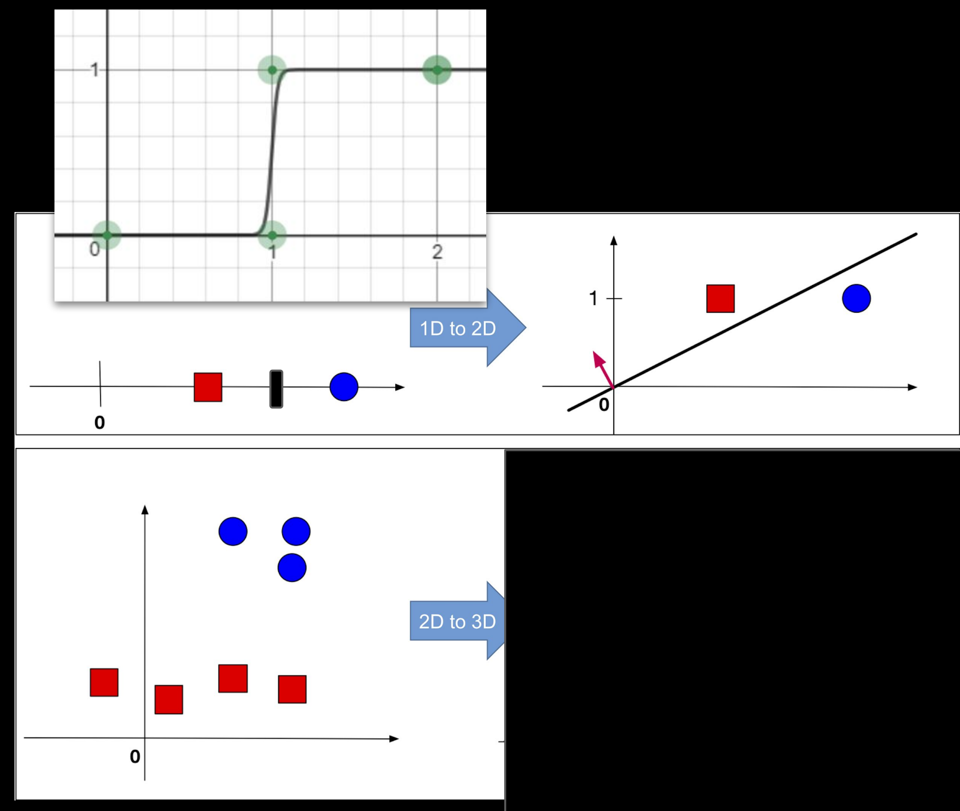

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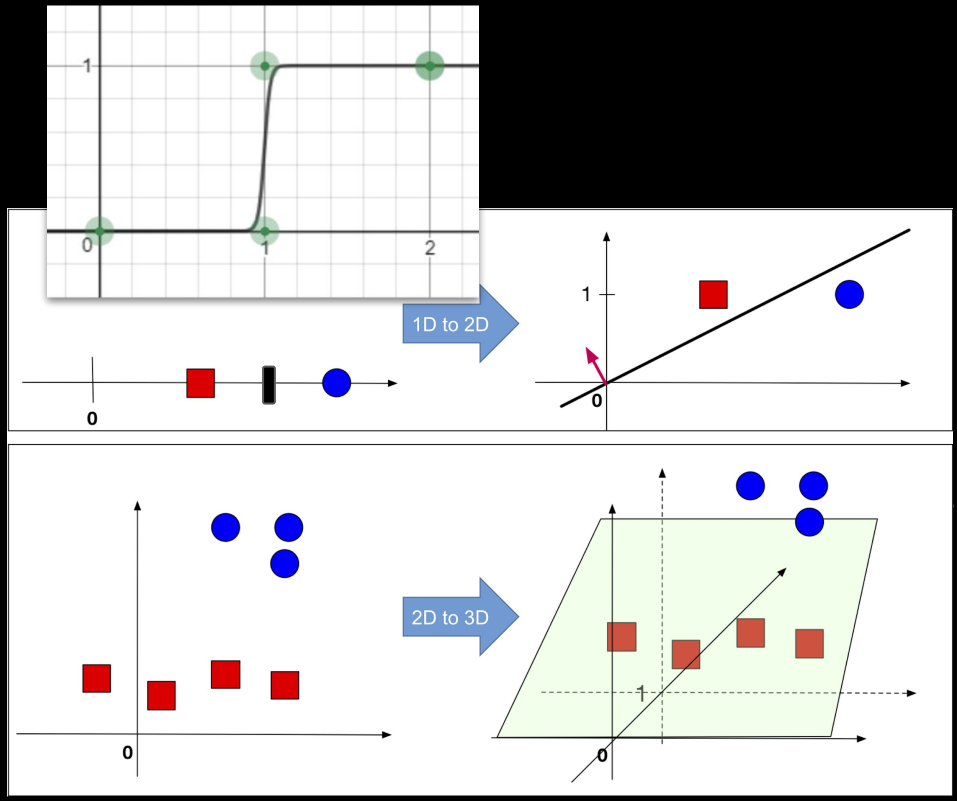
- Number of capital letters surrounding: integer
- Begins with capital letter: {0, 1}
- Preceded by “the”? {0, 1}



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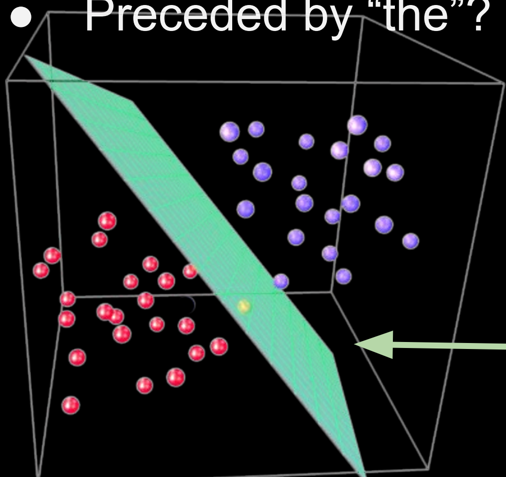


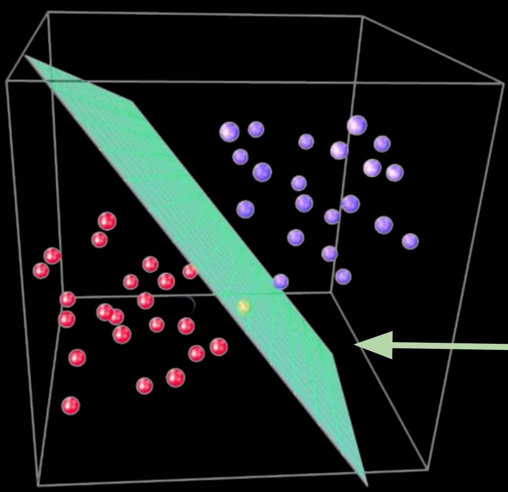
We’re learning a linear (i.e. flat) *separating hyperplane*, but fitting it to a *logit* outcome.

(https://www.linkedin.com/pulse/predicting-outcomes-probabilities-logistic-regression-konstantinidis/)

Logistic Regression

$Y_i \in \{0, 1\}$; X can be anything numeric.

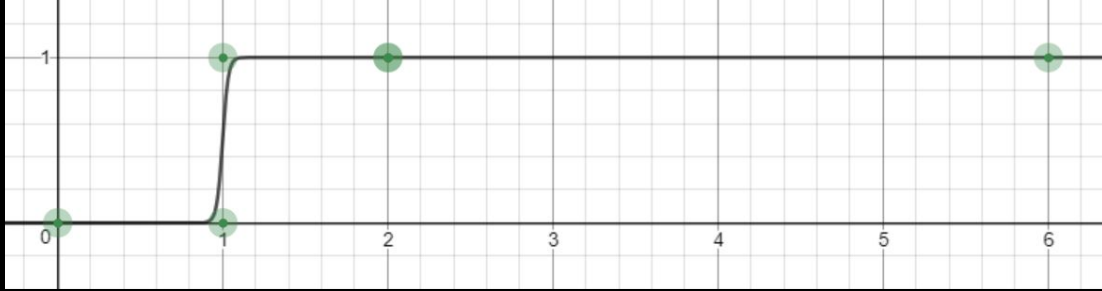
$$logit(p_i) = log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \sum_{j=1}^{m} \boxed{\beta_j x_{ij}} = 0$$



We're still learning a linear *separating hyperplane*, but fitting it to a *logit* outcome.

(https://www.linkedin.com/pulse/predicting-outcomes-probabilities-logistic-regression-konstantinidis/)

Logistic Regression



Example: Y: 1 if target is a part of a proper noun, 0 otherwise;
X: number of capital letters in target and surrounding words.

They attend Stony Brook University. *Next to the brook Gandalf lay thinking.*

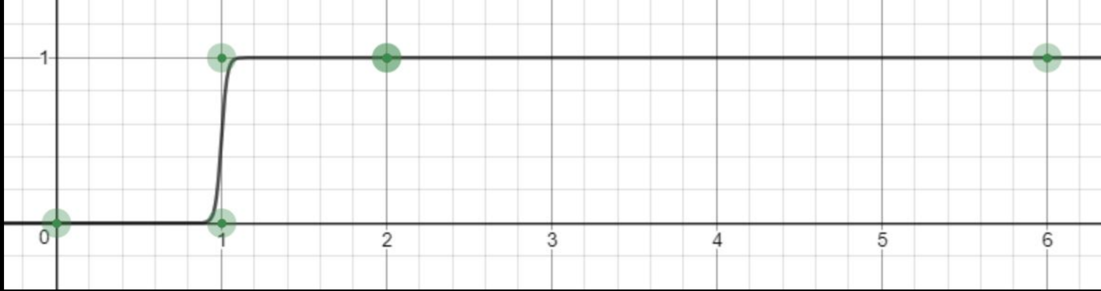
The trail was very stony. *Her degree is from SUNY Stony Brook.*

The Taylor Series was first described by Brook Taylor, the mathematician.

They attend Binghamton.

x	y
2	1
1	0
0	0
6	1
2	1
1	1

Logistic Regression



Example: Y: 1 if target is a part of a proper noun, 0 otherwise;

X1: number of capital letters in target and surrounding words.

Let's add a feature! X2: does the target word start with a capital letter?

They attend Stony Brook University. *Next to the brook Gandalf lay thinking.*

The trail was very stony. *Her degree is from SUNY Stony Brook.*

The Taylor Series was first described by Brook Taylor, the mathematician.

They attend Binghamton.

x2	x1	y
1	2	1
0	1	0
0	0	0
1	6	1
1	2	1
1	1	1

Terminology

β ≈ weight ≈ coefficient ≈ parameters ≈ θ

Logistic Regression ≈ Maximum Entropy Classifier

loss function ≈ cost function

PyTorch Intro: Logistic Regression

1. Tensors
2. Numeric functions as a graph/network (forward pass)
3. Loss function (training loop)
4. Autograd (backward pass)

PyTorch Intro: Logistic Regression

1. Tensors
2. Numeric functions as a graph/network (forward pass)

   `nn.module` object maps *X* to *y_pred*

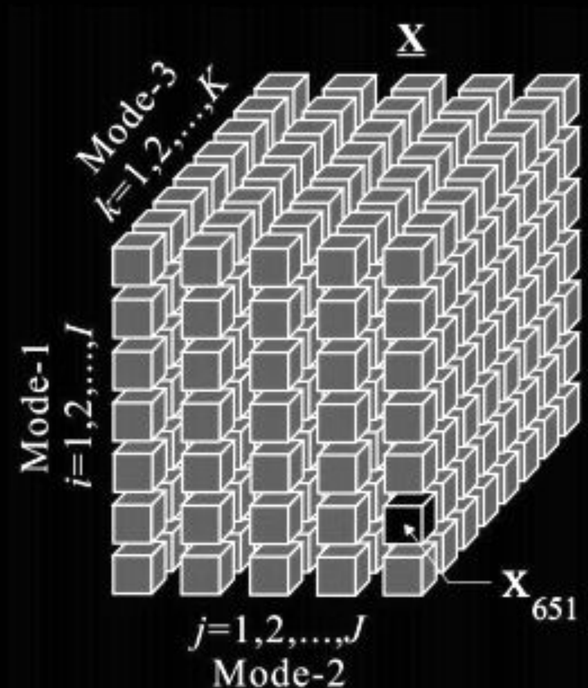
3. Loss function (training loop)

   `loop` that evaluates *ypred* versus *y*

4. Autograd (backward pass)

   `torch computation` that updates the parameters

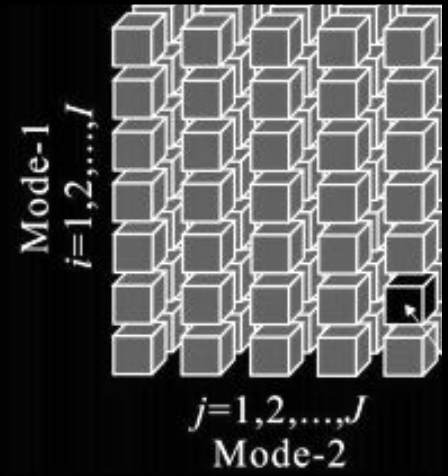
PyTorch: 1. Tenors



A multi-dimensional matrix

(i.stack.imgur.com)

PyTorch: 1. Tenors



(i.stack.imgur.com)

- A multi-dimensional matrix

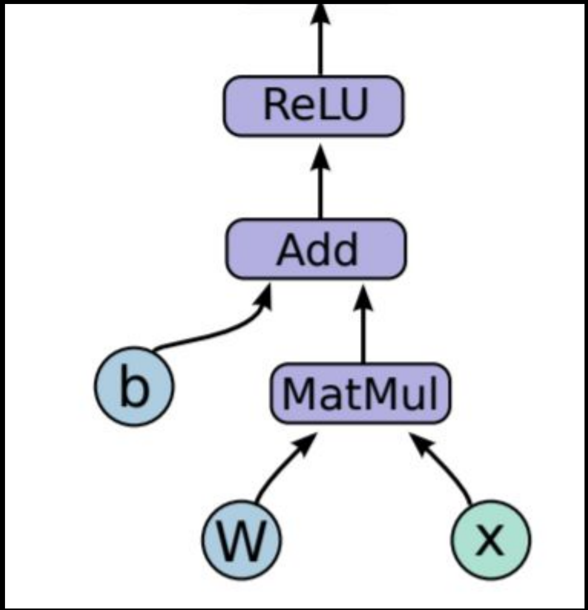
A 2-d tensor is just a matrix.
1-d: vector
0-d: a constant / scalar

Note: Linguistic ambiguity:
Dimensions of a Tensor =/=
Dimensions of a Matrix

PyTorch: 2. Numeric functions as a graph/network (forward pass)

Efficient, high-level built-in **linear algebra** for neural network operations.

Can be conceptualized as a graph of operations on tensors (matrices):



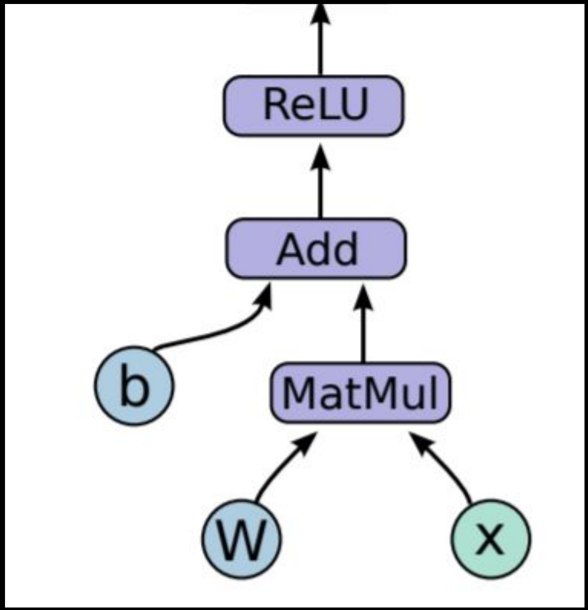
PyTorch: 2. Numeric functions as a graph/network (forward pass)

Efficient, high-level built-in **linear algebra** for neural network operations.

Can be conceptualized as a graph of operations on tensors (matrices):

```
import torch
from torch import nn #predefined nodes

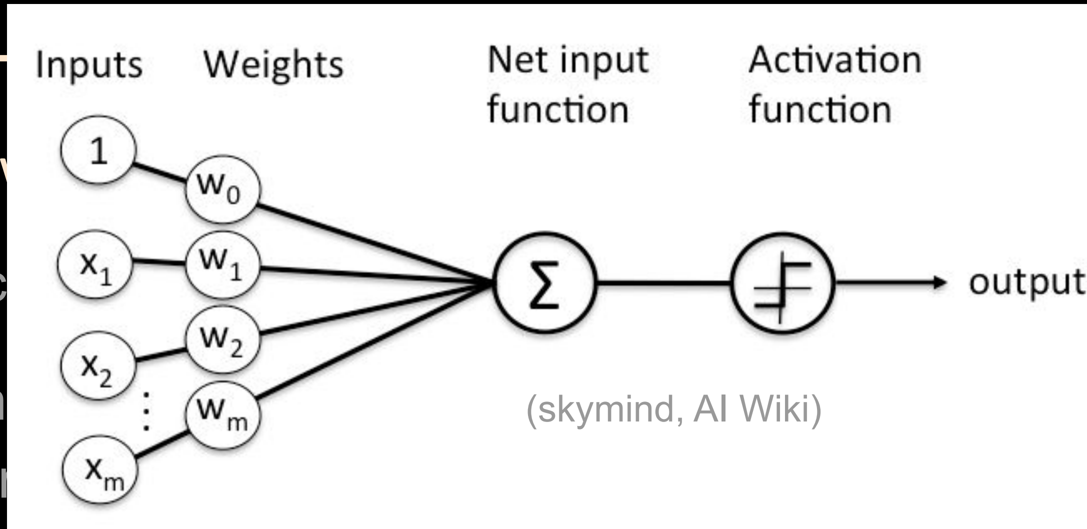
x = torch.Tensor(input)
w= torch.random.randn(X.shape, 1) #weights
z = torch.matmul(x, beta)
yhat = nn.functional.relu(z)
loss = nn.MSELoss(yhat, torch.Tensor(y))
```



Py… network
(for…

Effic… al network operations.

Can … oper…



(skymind, AI Wiki)

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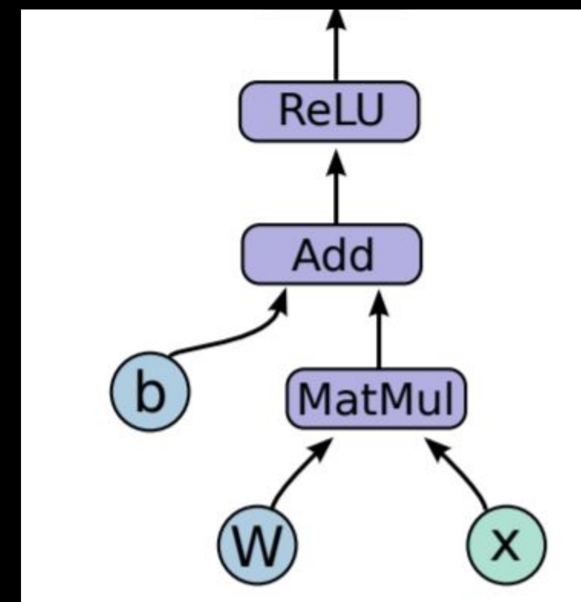

PyTorch: 2. Numeric functions as a graph/network
(forward pass: defined in "forward" method of nn.Module)

```
class LogReg(nn.Module):
    ...

    def forward(self, X):
        #This is where the model itself is defined.
        #For logistic regression the model takes in X and returns
        #the results of a decision function

        newX = torch.cat((X, torch.ones(X.shape[0], 1)), 1) #add intercept

        return 1/(1 + torch.exp(-self.linear(newX)))
                                 #logistic function on the linear output
```

PyTorch: 2. Numeric functions as a graph/network

(forward pass: defined in "forward" method of nn.Module)

```
class LogReg(nn.Module):
    def __init__(self, num_feats, num_classes,
                 learn_rate = 0.01, device = torch.device("cpu") ):
        #the constructor; define any layer objects (e.g. Linear)
        super(LogReg, self).__init__()
        self.linear = nn.Linear(num_feats+1, num_classes)

    def forward(self, X):
        #This is where the model itself is defined.
        #For logistic regression the model takes in X and returns
        #the results of a decision function

        newX = torch.cat((X, torch.ones(X.shape[0], 1)), 1)
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        return 1/(1 + torch.exp(-self.linear(newX)))
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```

PyTorch: 3. Loss Function (training loop)

```
#runs the training loop of pytorch model:
sgd = torch.optim.SGD(model.parameters(), lr=learning_rate)
loss_func = torch.mean(-torch.sum(y*torch.log(y_pred))

#training loop:
for i in range(epochs):
    model.train()
    sgd.zero_grad()
    #forward pass:
    ypred = model(X)
    loss = loss_func(ypred, y)
    #backward: /(applies gradient descent)
    loss.backward()
    sgd.step()

    if i % 20 == 0:
        print("  epoch: %d, loss: %.5f" %(i, loss.item()))
```

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```

To Optimize Betas (all weights/parameters within the neural net):

Stochastic Gradient Descent (SGD)

-- optimize over one sample each iteration

Mini-Batch SDG:

--optimize over *b* samples each iteration

PyTorch: 3. Loss Function (training loop)

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#runs the training loop of pytorch model:
sgd = torch.optim.SGD(model.parameters(), lr=learning_rate)
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PyTorch: 4. Autograd (backward pass)

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    #backward: /(applies gradient descent)
    loss.backward()
    sgd.step()

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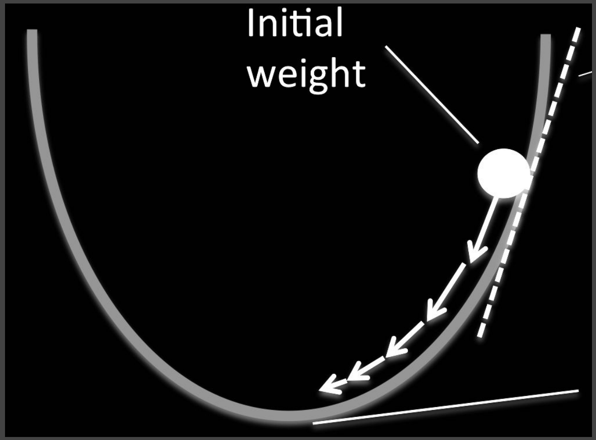
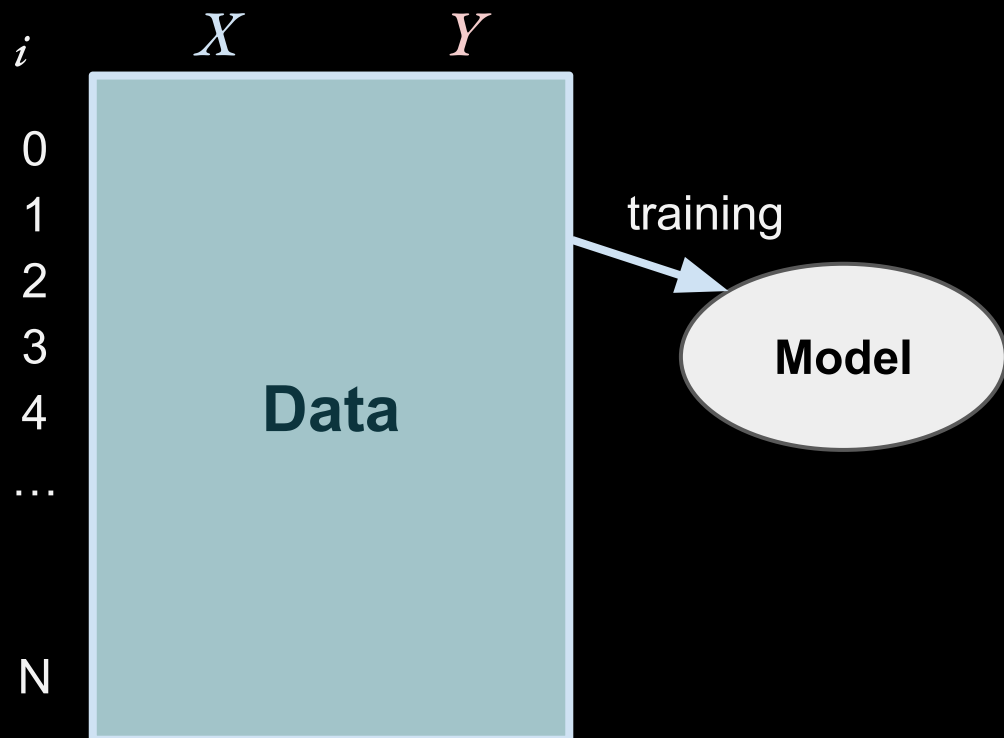
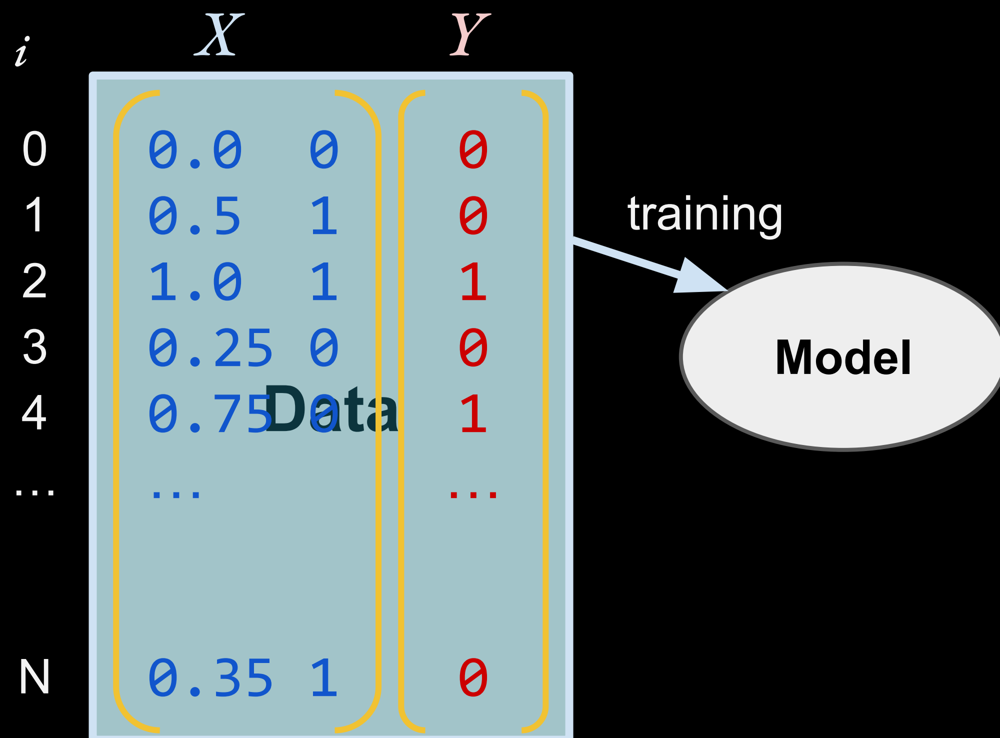


Machine Learning: How to setup data



Machine Learning: How to setup data



Machine Learning: How to setup data

“Corpus”

raw data: sequences of characters

i	X		Y
0	0.0	0	0
1	0.5	1	0
2	1.0	1	1
3	0.25	0	0
4	0.75	0	1
…	…		…
N	0.35	1	0

Data

training

Machine Learning: How to setup data

Feature Extraction

"Corpus"

raw data:
sequences of
characters

--pull out *observations* and *feature vector* per observation.

i	X		Y
0	0.0	0	0
1	0.5	1	0
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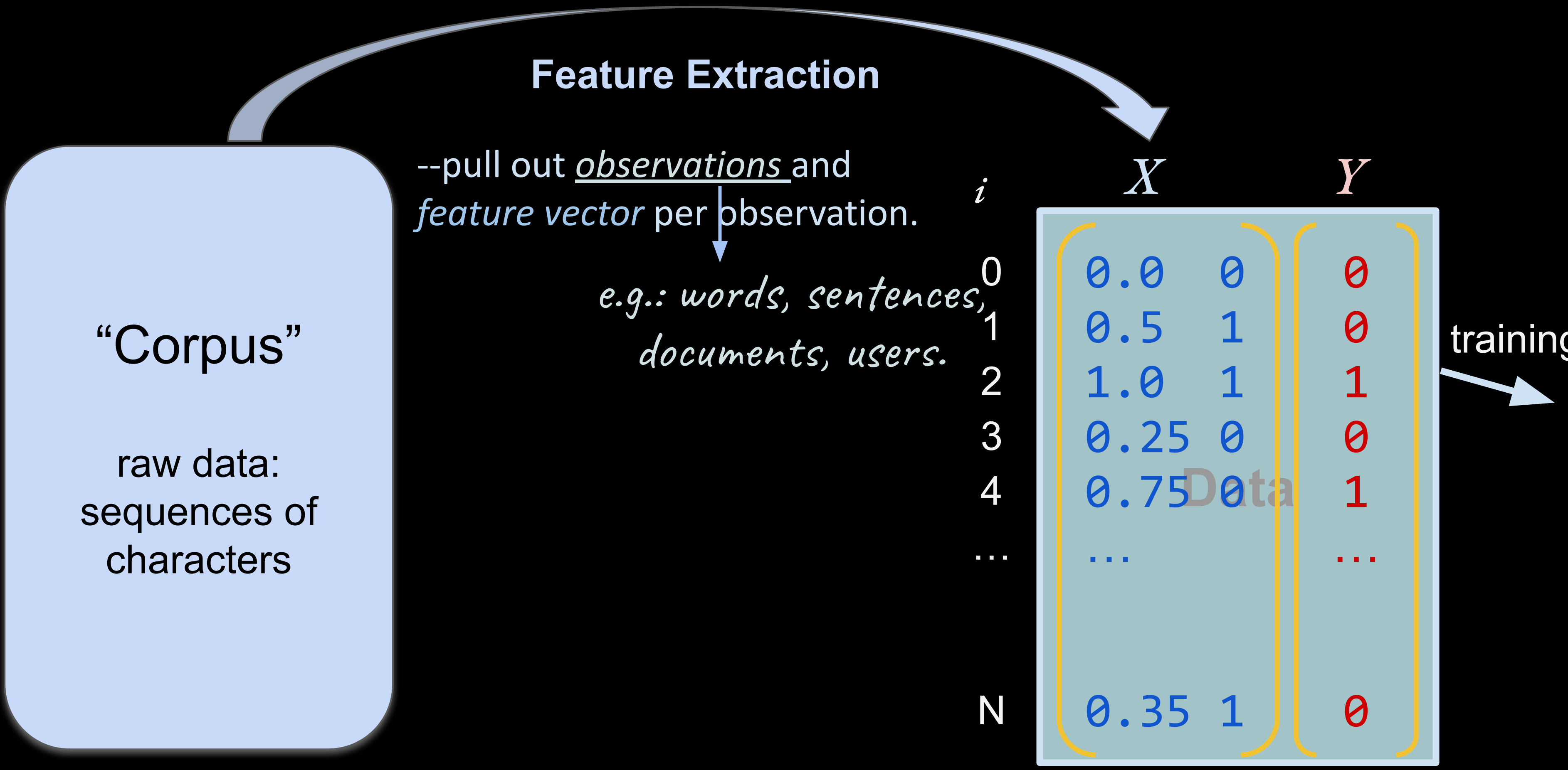

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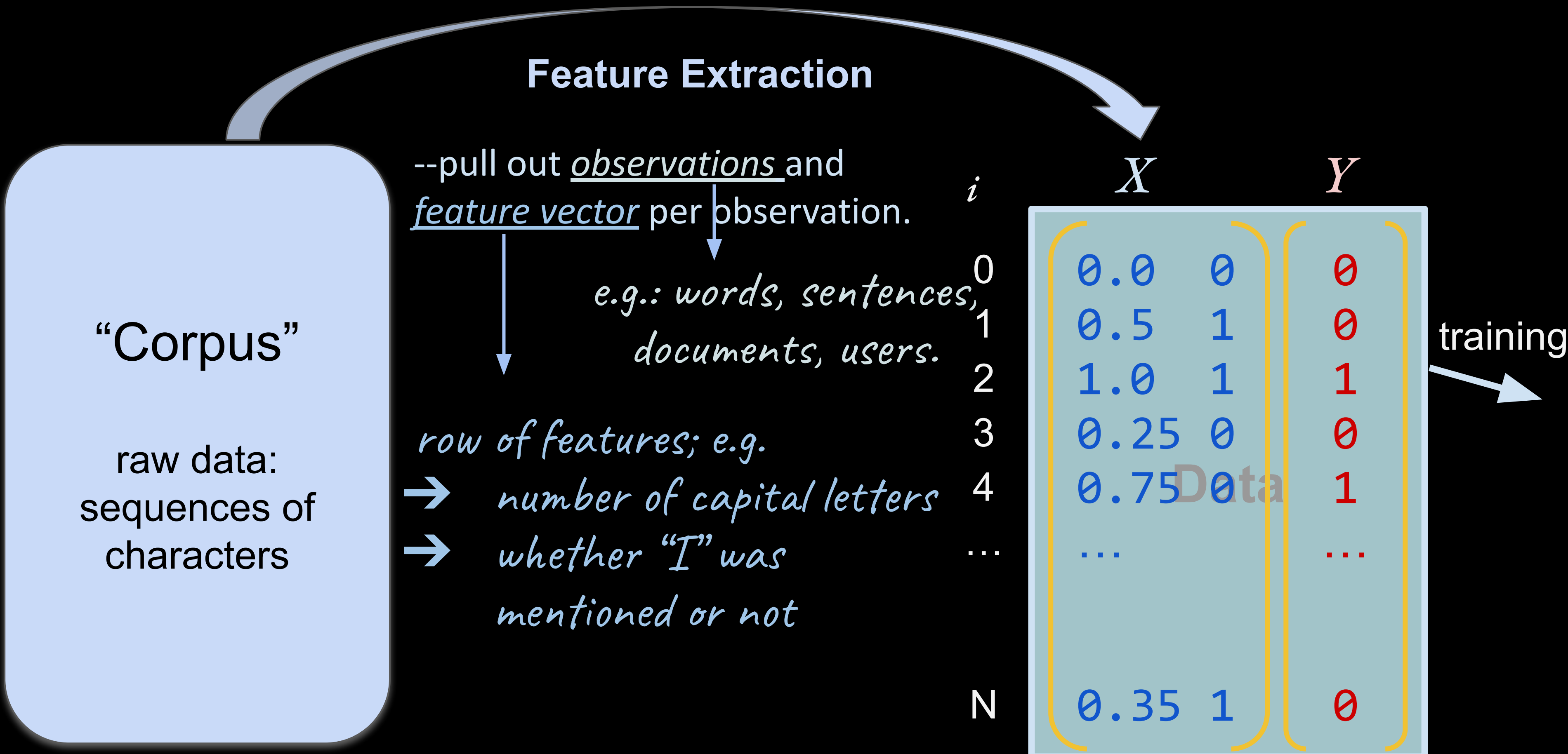

Machine Learning: How to setup data

"Corpus"

raw data: sequences of characters

Feature Extraction

--pull out *observations* and *feature vector* per observation.

e.g.: words, sentences, documents, users.

row of features; e.g.

- *number of capital letters*
- *whether "I" was mentioned or not*
- *k features indicating whether k words were mentioned or not*

Data

i	X		Y
0	0.0	0	0
1	0.5	1	0
2	1.0	1	1
3	0.25	0	0
4	0.75	0	1
…	…		…
N	0.35	1	0

training

Machine Learning: How to setup data

Feature Extraction

X Y

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

“Corpus”

raw data: sequences of characters

of features; e.g.

→ *number of capital letters*

→ *whether “I” was mentioned or not*

→ ***k features indicating whether k words were mentioned or not***

Data

Machine Learning: How to setup data

Feature Extraction

X Y

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is word present in document?

The book was interesting so I was happy .

"Corpus"

raw data: sequences of characters

of features; e.g.

→ number of capital letters

→ whether "I" was mentioned or not

→ *k* features indicating whether *k* words were mentioned or not

Data

Machine Learning: How to setup data

Feature Extraction

X Y

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- Each word gets an index in the vector
- 1 if present; 0 if not

“Corpus”

Feature example: is word present in document?

The book was interesting so I was happy .

raw data: sequences of characters

`[0, 1, 1, 0, 1, …, 1, 0, 1, 1, 0, 1, …, 1]`^{k}

→ *k features indicating whether k words were mentioned or not*

Data

Machine Learning: How to setup data

Feature Extraction

X Y

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is word present in document

"Corpus"

raw data: sequences of characters

Data

The book was interesting so I was happy .

[0, 1, 1, 0, 1, …, 1, 0, 1, 1, 0, 1, …, 1]^k

a

→ *k features indicating whether k words were mentioned or not*

sad

Machine Learning: How to setup data

Feature Extraction

X Y

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is previous word "the"?

"Corpus"

raw data: sequences of characters

The ***book*** *was interesting so I was happy .*

[0, 1, 1, 0, 1, …, 1, 0, 1, 1, 0, 1, …, 1]^k

Data

→ *k features indicating whether k words were mentioned or not*

Machine Learning: How to setup data

Feature Extraction

"Corpus"

raw data: sequences of characters

X Y

Data

Multi-hot Encoding

- Each word gets an index in the vector
- 1 if present; 0 if not

Feature example: is previous word "the"?

The ***book*** *was interesting so I was happy .*

~~[0, 1, 1, 0, 1, …, 1, 0, 1, 1, 0, 1, …, 1]~~^{k}

→ k features indicating whether k words were mentioned or not

Machine Learning: How to setup data

Feature Extraction

X Y

"Corpus"

raw data: sequences of characters

Data

One-hot Encoding

- Each word gets an index in the vector
- All indices 0 except present word:

Feature example: is previous word "the"?

The ***book*** *was interesting so I was happy .*

`[0, 1, 0, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]`^{k}

→ *k features indicating whether k words were mentioned or not*

Machine Learning: How to setup data

Feature Extraction

X Y

One-hot Encoding

- Each word gets an index in the vector
- All indices 0 except present word:

Feature example: which is previous word?

The book was interesting so I was happy .

[0, 1, 0, 0, 0, ..., 0, 0, 0, 0, 0, 0, ..., 0]^k

[0, 0, 1, 0, 0, ..., 0, 0, 0, 0, 0, 0, ..., 0]^k

"Corpus"

raw data: sequences of characters

Data

Machine Learning: How to setup data

Feature Extraction

X Y

One-hot Encoding

- Each word gets an index in the vector
- All indices 0 except present word:

Feature example: which is previous word?

“Corpus”

raw data: sequences of characters

The book was interesting so I was happy .

Data

`[0, 1, 0, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]`^{k}

`[0, 0, 1, 0, 0, …, 0, 0, 0, 0, 0, 0, …, 0]`^{k}

Machine Learning: How to setup data

Feature Extraction

X Y

Multiple One-hot encodings for one observation

(1) word before; (2) word after

"Corpus"

*The book was **interesting** so I was happy .*

raw data: sequences of characters

`[0, 0, 0, 0, 1, 0, …, 0]`^{k} `[0, …, 0, 1, 0, …, 0]`^{k}

Data

Machine Learning: How to setup data

Feature Extraction

X Y

Multiple One-hot encodings for one observation

(1) word before; (2) word after

"Corpus"

*The book was **interesting** so I was happy .*

raw data: sequences of characters

Data

[0, 0, 0, 0, 1, 0, …, 0]^k [0, …, 0, 1, 0, …, 0]^k

=

[0, 0, 0, 0, 1, 0, …, 0, 0, …, 0, 1, 0, …, 0]^{2k}

Machine Learning: How to setup data

Feature Extraction

X Y

Multiple One-hot encodings for one observation

(1) word before; (2) word after; (3) percent capitals

*The book was **Interesting** so I was happy .*

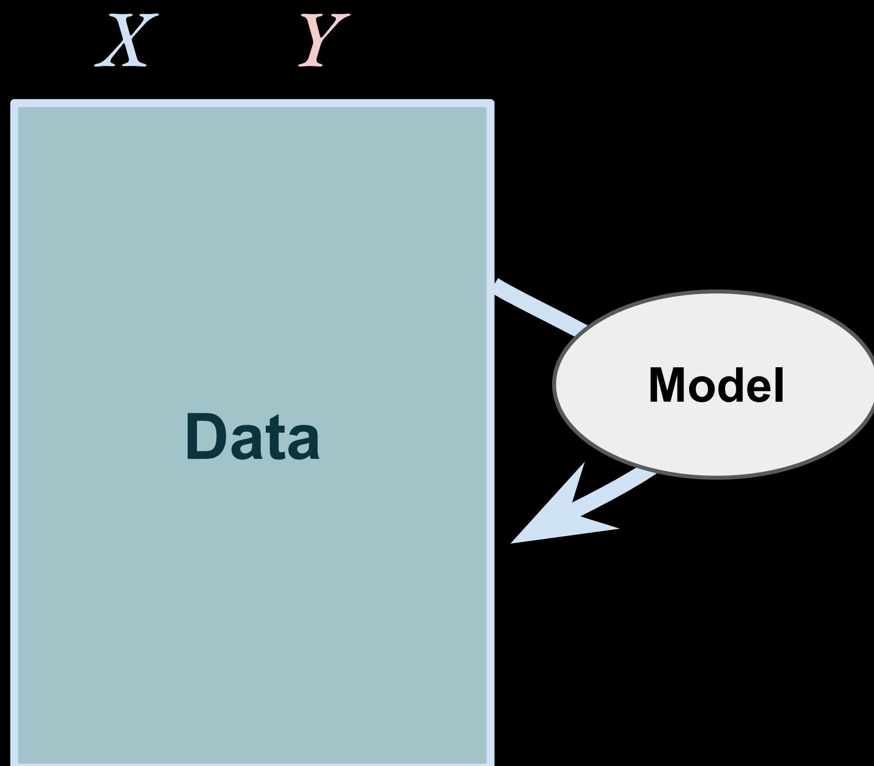
[0, 0, 0, 0, 1, 0, …, 0]^k [0, …, 0, 1, 0, …, 0]^k

=

[0, 0, 0, 0, 1, 0, …, 0, 0, …, 0, 1, 0, …, 0]^{2k}

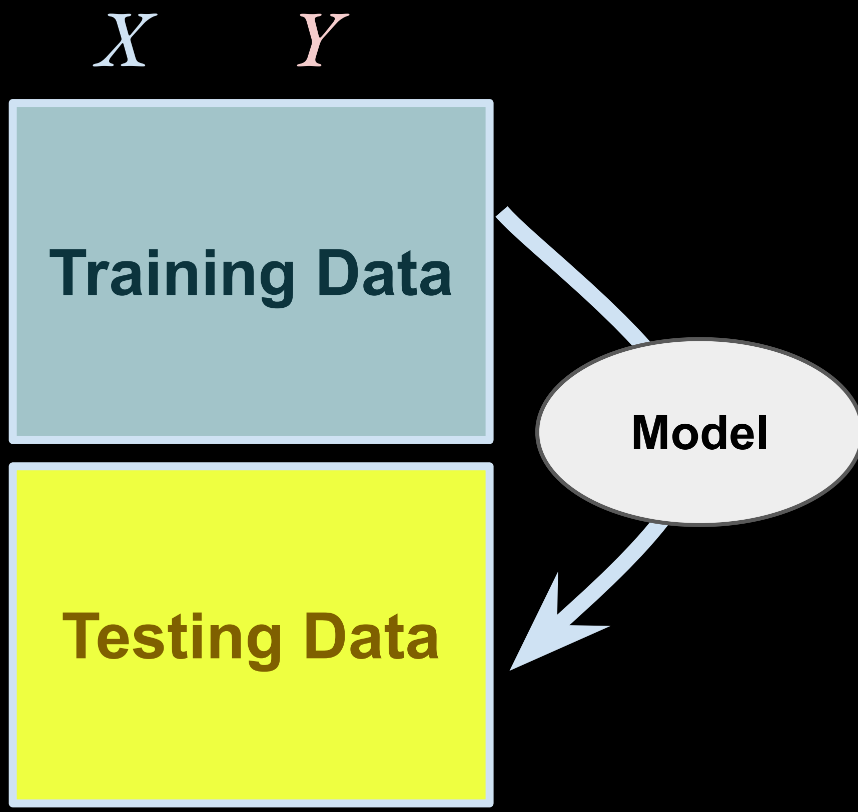
[0, 0, 0, 0, 1, 0, …, 0, 0, …, 0, 1, 0, …, 0, 0.09]$^{2k+1}$

Machine Learning: How to setup data



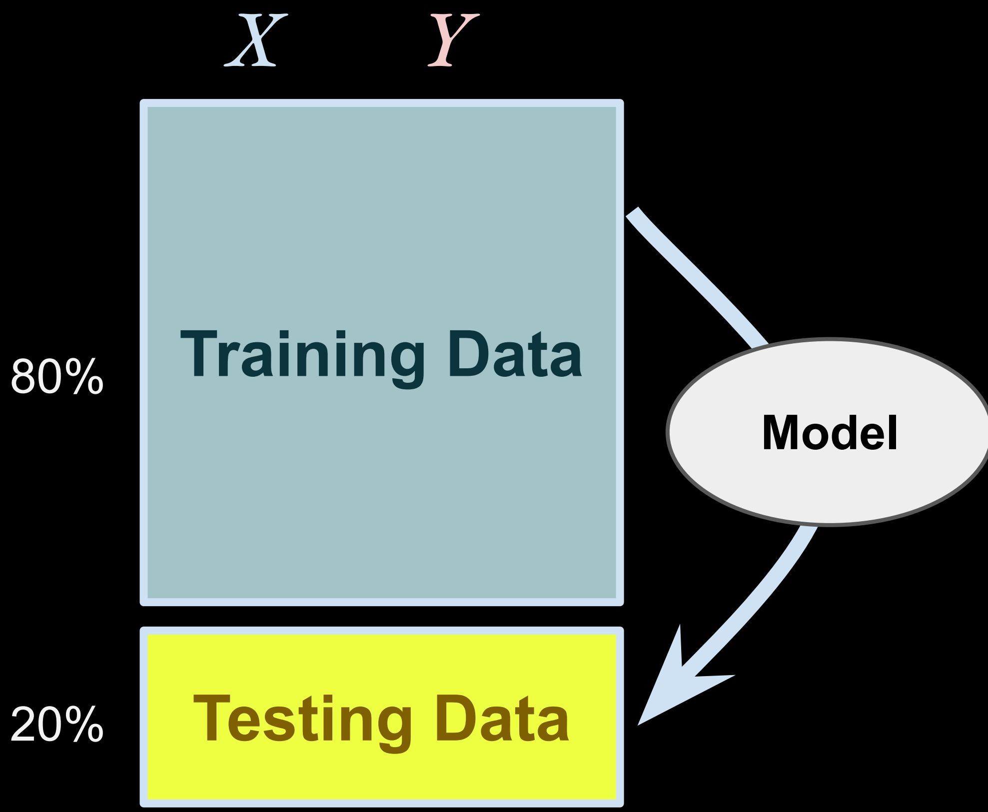
Does the model hold up?

Machine Learning Goal: Generalize to new data



Does the
model hold up?

Machine Learning Goal: Generalize to new data



Does the model hold up?

Logistic Regression - Regularization

$$X = Y$$

0.5	0	0.6	1	0	0.25		1
0	0.5	0.3	0	0	0		1
0	0	1	1	1	0.5		0
0	0	0	0	1	1		0
0.25	1	1.25	1	0.1	2		1

Logistic Regression - Regularization

$X \quad = \quad Y$

0.5	0	0.6	1	0	0.25	1
0	0.5	0.3	0	0	0	1
0	0	1	1	1	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

Logistic Regression - Regularization

$$X = Y$$

x_1	x_2	...				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	0.3	0	0	0	1
0	0	1	1	1	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

$$1.2 + -63 * x_1 + 179 * x_2 + 71 * x_3 + 18 * x_4 + -59 * x_5 + 19 * x_6 = logit(Y)$$

Logistic Regression - Regularization

$$X = Y$$

x_1	x_2	…				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	0.3	0	0	0	1
0	0	1	1	1	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

$$1.2 + -63*x_1 + 179*x_2 + 71*x_3 + 18*x_4 + -59*x_5 + 19*x_6 = logit(Y)$$

Logistic Regression - Regularization

X $=$ Y

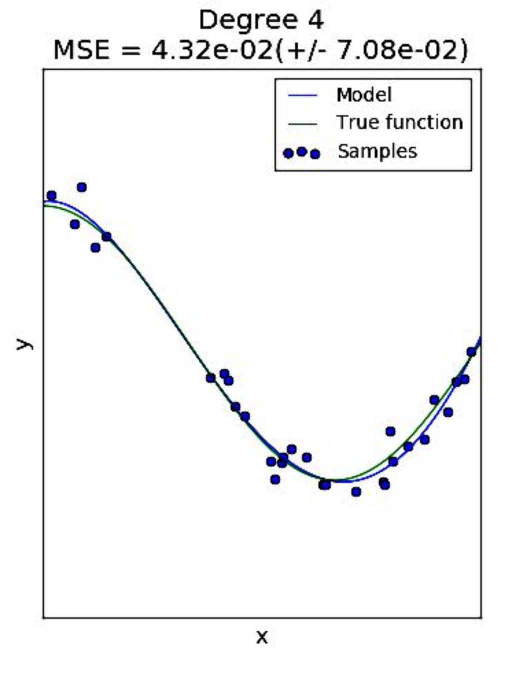
x_1	x_2	...				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	[illegible]	[illegible]	[illegible]	0	1
0	0	[illegible]	[illegible]	[illegible]	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

“overfitting”

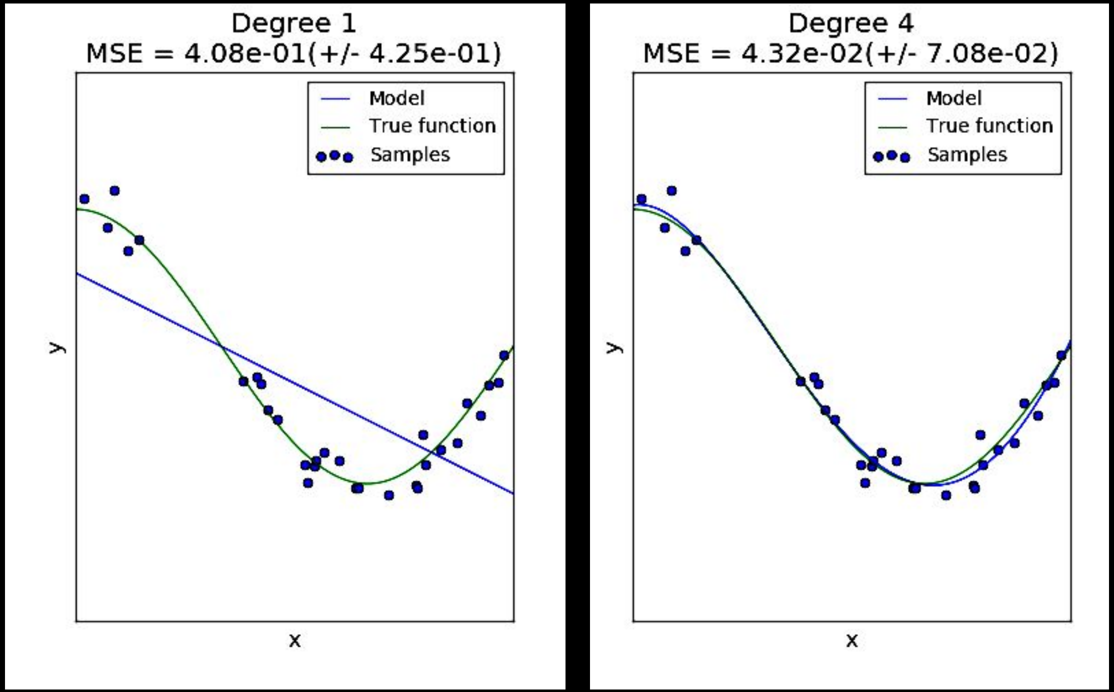
$$1.2 + -63*x_1 + \boxed{179*x_2} + 71*x_3 + 18*x_4 + -59*x_5 + 19*x_6 = logit(Y)$$

Python Example

Overfitting (1-d non-linear example)



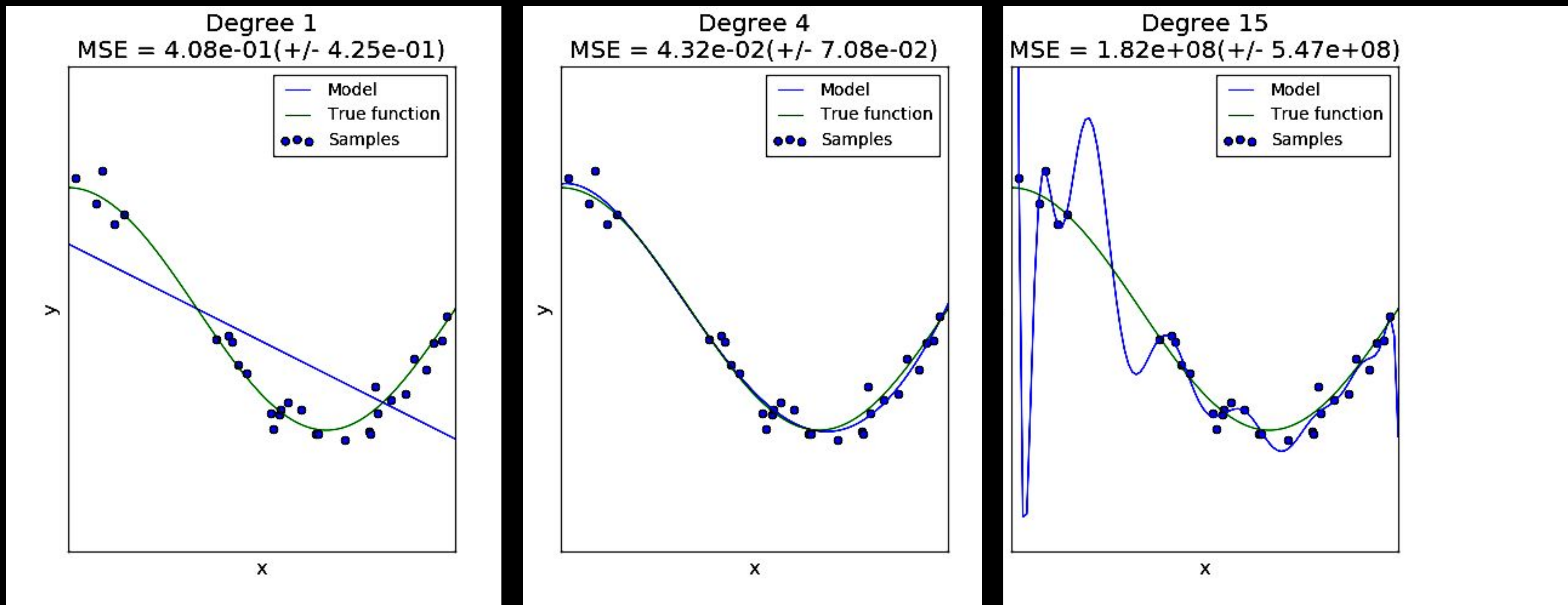
Overfitting (1-d non-linear example)



Underfit

(image credit: Scikit-learn; in practice data are rarely this clear)

Overfitting (1-d non-linear example)



Underfit

Overfit

(image credit: Scikit-learn; in practice data are rarely this clear)

Logistic Regression - Regularization

$$X \qquad = \qquad Y$$

x_1	x_2	…				Y
0.5	0	0.6	1	0	0.25	1
0	0.5	0.[illegible]	0	[illegible]	0	1
0	0	[illegible]	[illegible]	[illegible]	0.5	0
0	0	0	0	1	1	0
0.25	1	1.25	1	0.1	2	1

“overfitting”

$$1.2 + -63 * x_1 + 179 * x_2 + 71 * x_3 + 18 * x_4 + \boxed{-59 * x_5} + 19 * x_6 = logit(Y)$$

Logistic Regression - Regularization

X = Y

x_1	x_2	...				Y
0.5	0	0.6	[illegible]	[illegible]	[illegible]	1
0	0.5	[illegible]	[illegible]	[illegible]	[illegible]	1
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
0.25	[illegible]	[illegible]	[illegible]	0.1	2	1

"overfitting": generally due to trying to fit too many features given the number of observations.

1.2 + $-63 * x_1$ + 17[illegible]x_2 + $71 * x_3$ + $18 * x_4$ + $-59 * x_5$ + $19 * x_6$ = $logit(Y)$

Logistic Regression - Regularization

X $=$ Y

x_1	x_2	Y
0.5	0	1
0	0.5	1
0	0	0
0	0	0
0.25	1	1

What if only 2 predictors?

Logistic Regression - Regularization

X $= Y$

x_1	x_2	Y
0.5	0	1
0	0.5	1
0	0	0
0	0	0
0.25	1	1

What if only 2 predictors?
A: better fit

$$0 + 2*x_1 + 2*x_2 = logit(Y)$$

Logistic Regression - Regularization

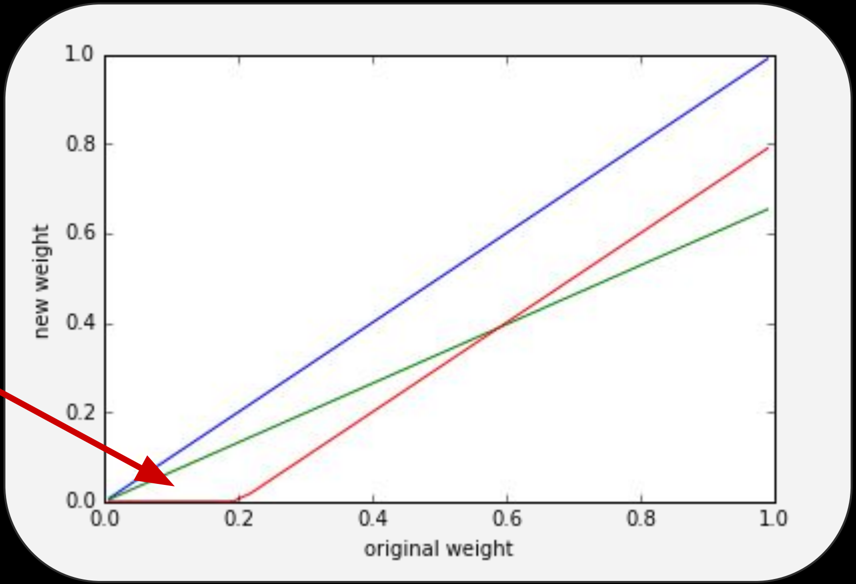
L1 Regularization - “The Lasso”

Zeros out features by adding values that keep from perfectly fitting the data.

Logistic Regression - Regularization

L1 Regularization - “The Lasso”

Zeros out features by adding values that keep from perfectly fitting the data.



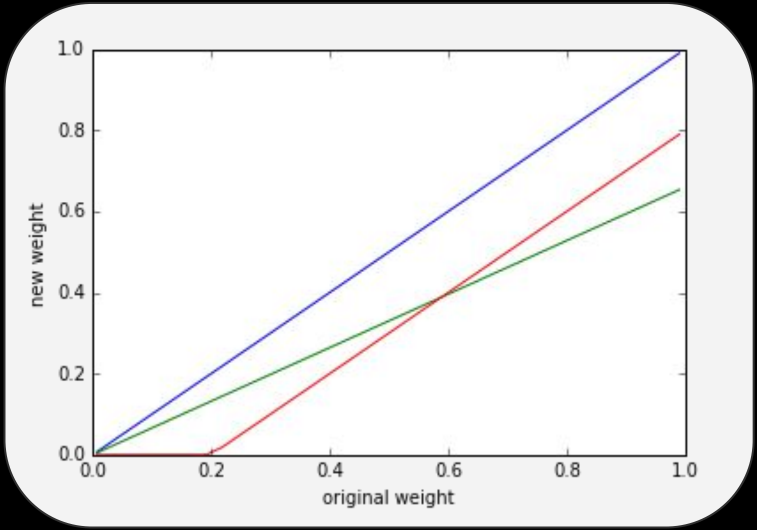
Logistic Regression - Regularization

L1 Regularization - "The Lasso"

Zeros out features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, ..., \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

set betas that maximize L



Logistic Regression - Regularization

L1 Regularization - “The Lasso”

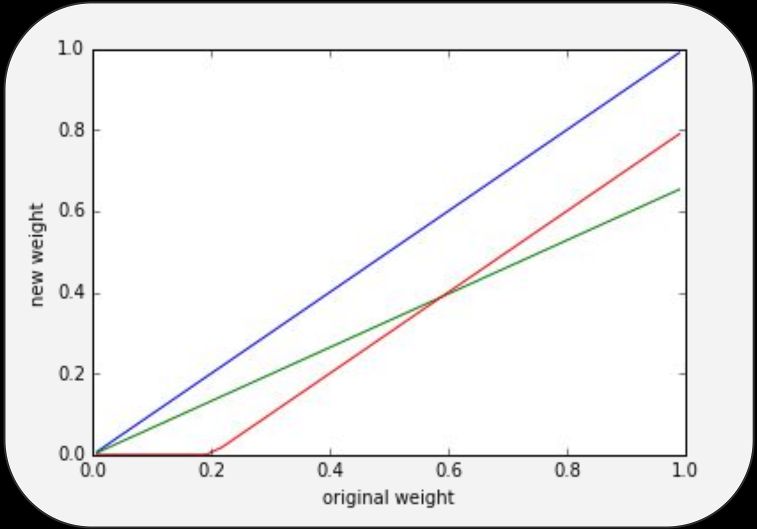
Zeros out features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, ..., \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i} - \frac{1}{C}\sum_{j=1}^{m}|\beta_j|$$

set betas that maximize *penalized* L

This is for likelihood

for log loss, would add the penalty



Logistic Regression - Regularization

Sometimes written as:

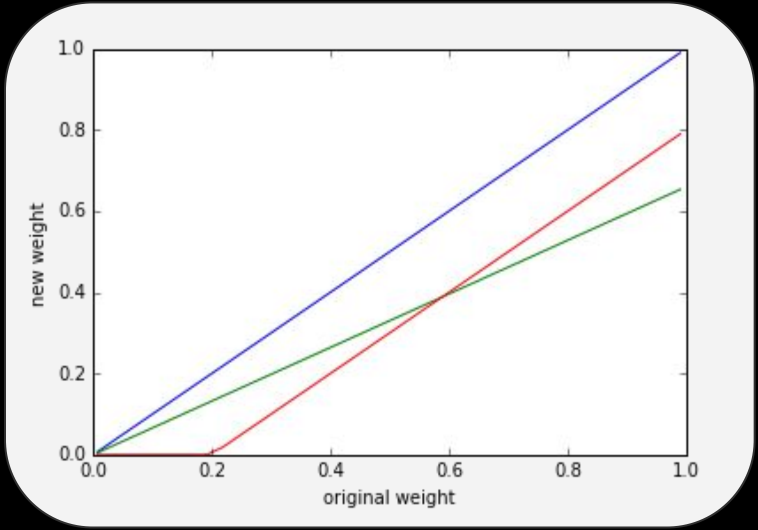
$$||\beta||_1$$

L1 Regularization - “The Lasso”

Zeros out features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, \ldots, \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i} - \frac{1}{C}\overbrace{\sum_{j=1}^{m} |\beta_j|}$$

set betas that maximize *penalized* L



Logistic Regression - Regularization

Sometimes written as:

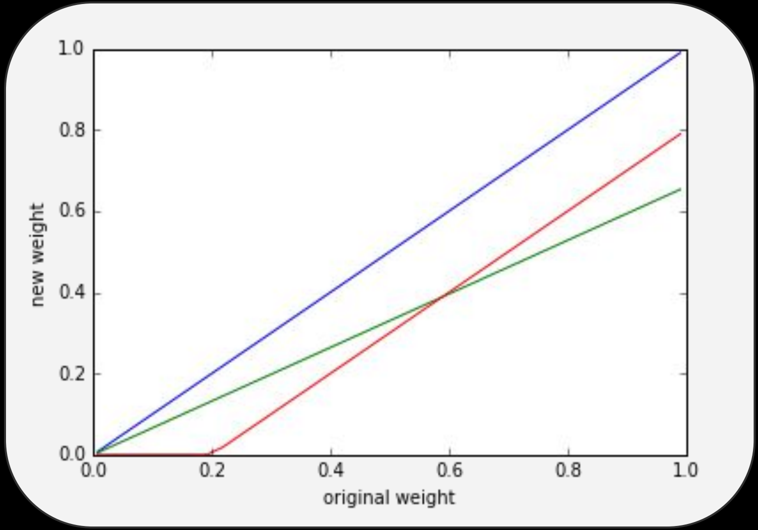
$$||\beta_j||_2^2$$

L2 Regularization - "Ridge"

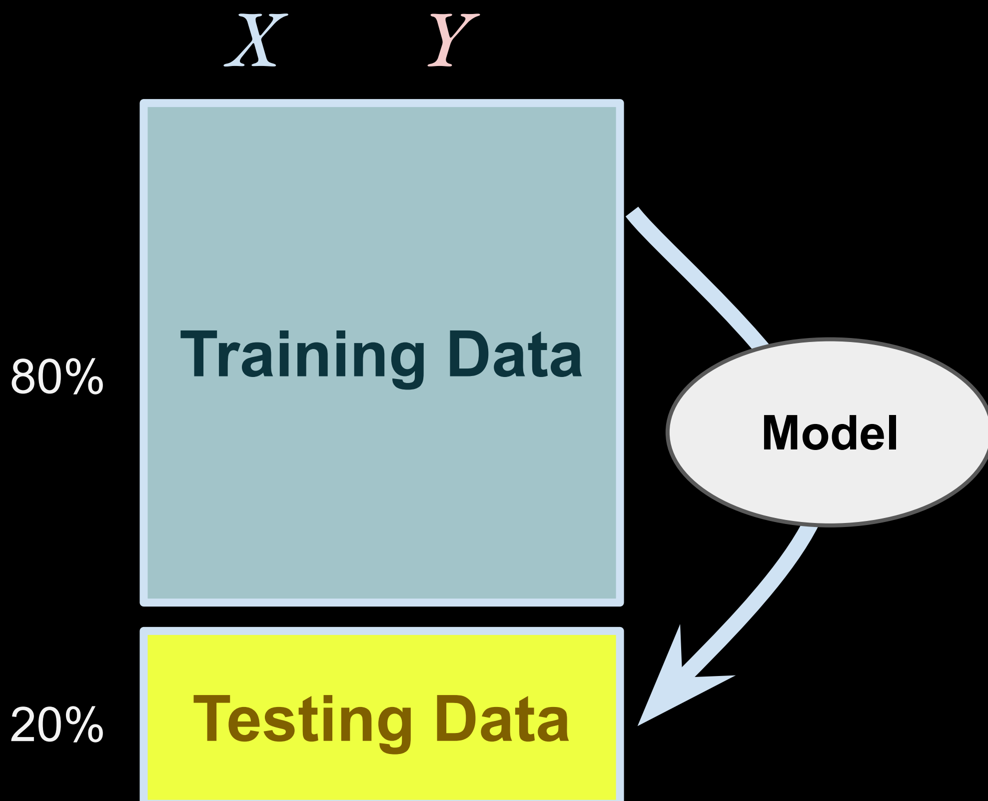
Shrinks features by adding values that keep from perfectly fitting the data.

$$L(\beta_0, \beta_1, \ldots, \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i} - \frac{1}{C}\sum_{j=1}^{m} \beta_j^2$$

set betas that maximize *penalized* L

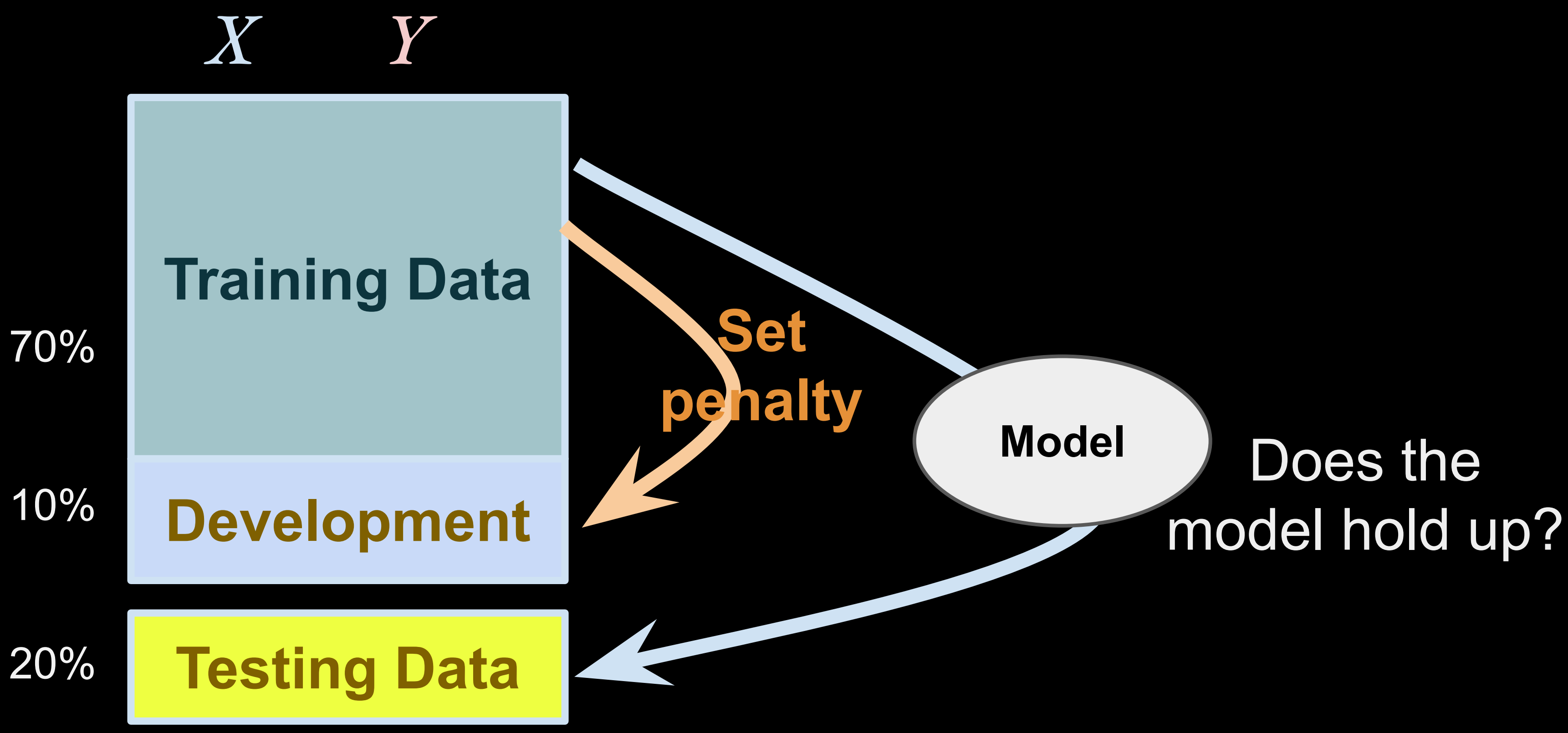


Machine Learning Goal: Generalize to new data



Does the model hold up?

Machine Learning Goal: Generalize to new data



Logistic Regression - Review

- Probabilistic Classification: *P(Y | X)*
- Learn logistic curve based on example data
  - training + development + testing data
- Set betas based on maximizing the *likelihood* (or based on minimizing *log loss*)
  - “shifts” and “twists” the logistic curve
  - separation represented by hyperplane at 0.50
- Multivariate features: Multi-, One-hot encodings
- Overfitting and Regularization

Extra Material

Alternative to gradient descent:

$$L(\beta_0, \beta_1, ..., \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

$$p_i \equiv P(Y_i = 1 | X_i = x) = \frac{e^{\boxed{\beta_0} + \boxed{\beta_1} x_i}}{1 + e^{\boxed{\beta_0} + \boxed{\beta_1} x_i}}$$

To estimate β, one can use *reweighted least squares:*

(Wasserman, 2005; Li, 2010)

set $\hat{\beta}_0 = ... = \hat{\beta}_m = 0$ (remember to include an intercept)

1. Calculate p_i and let W be a diagonal matrix where element$(i,i) = p_i(1-p_i)$.
2. Set $z_i = logit(p_i) + \frac{Y_i - p_i}{p_i(1-p_i)} = X\hat{\beta} + \frac{Y_i - p_i}{p_i(1-p_i)}$
3. Set $\hat{\beta} = (X^T W X)^{-1} X^T W z$ //weighted lin. reg. of Z on Y.
4. Repeat from 1 until $\hat{\beta}$ converges.

Alternative to gradient descent:

$$L(\beta_0, \beta_1, \ldots, \beta_k | X, Y) = \prod_{i=1}^{n} p(x_i)^{y_i} (1 - p(x_i))^{1-y_i}$$

This is just one way of finding the betas that maximize the likelihood function. In practice, we will use existing libraries that are fast and support additional useful steps like **regularization**..

To estimate β , one can use *reweighted least squares:*

set $\hat{\beta}_0 = \ldots = \hat{\beta}_m = 0$ (remember to include an intercept)

1. Calculate p_i and let W be a diagonal matrix where element$(i, i) = p_i(1 - p_i)$.
2. Set $z_i = logit(p_i) + \frac{Y_i - p_i}{p_i(1 - p_i)} = X\hat{\beta} + \frac{Y_i - p_i}{p_i(1 - p_i)}$
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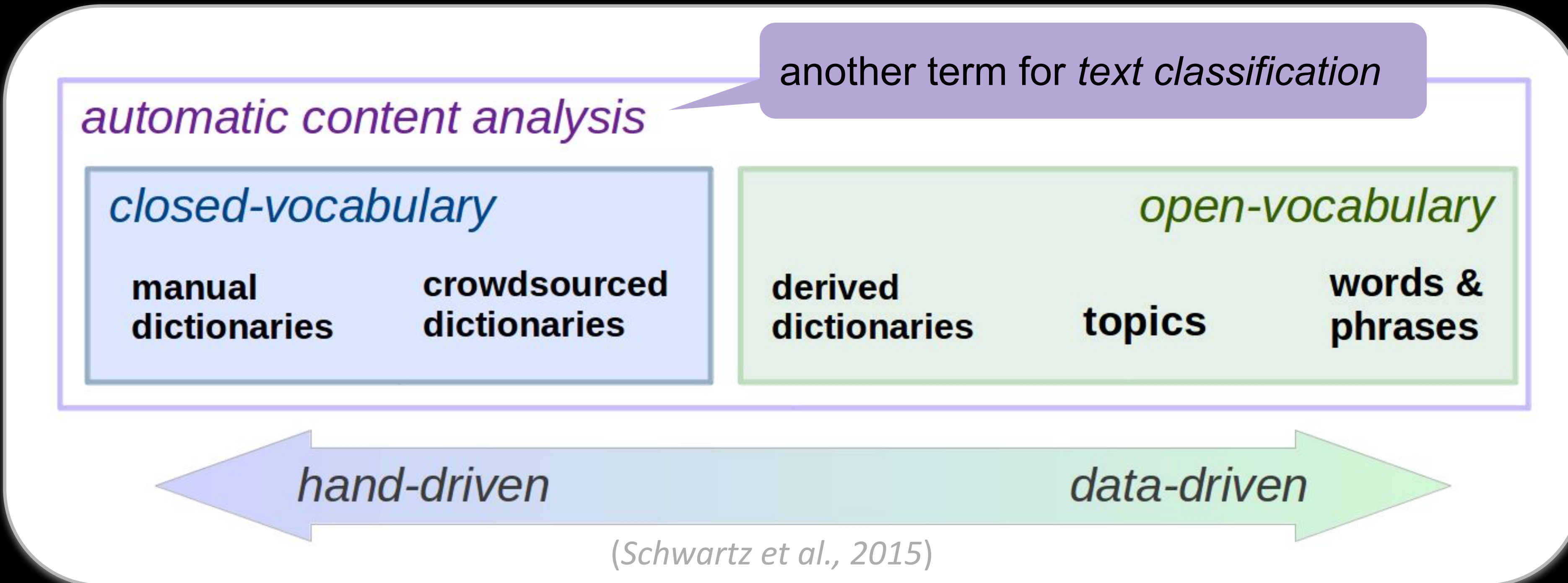
(Wasserman, 2005; Li, 2010)



	Proposed word lists	Accuracy	Ties
Human 3 + stats	positive: *love, wonderful, best, great, superb, still, beautiful* negative: *bad, worst, stupid, waste, boring, ?, !*	69%	16%

Figure 2: Results for baseline using introspection and simple statistics of the data (including *test* data).

PyTorch: 2. Numeric functions as a graph/network

(forward pass: defined in "forward" method of nn.Module)

```
class MultiClassLogReg(nn.Module):
    def __init__(self, num_feats, num_classes,
                 learn_rate = 0.01, device = torch.device("cpu") ):
        #the constructor; define any layer objects (e.g. Linear)
        super(MultiClassLogReg, self).__init__()
        self.linear = nn.Linear(num_feats+1, num_classes)

    def forward(self, X):
        #This is where the model itself is defined.
        #For logistic regression the model takes in X and returns
        #the results of a decision function

        newX = torch.cat((X, torch.ones(X.shape[0], 1)), 1) #add intercept

        return 1/(1 + torch.exp(-self.linear(newX)))
                                          #logistic function on the linear output

        return self.linear(newX) #only use linear if using cross-entropy loss
```

Two equivalent options for multi-class:

option 1 (what the previous slides covered)

```
#in model/forward:
        return self.linear(newX) #only use linear if using cross-entropy loss

#in loss/train:
        loss_func = nn.CrossEntropyLoss() #includes log softmax
                #alternative: nn.NLLLoss() #negative log likelikelihood loss
```

option 2

```
#in model/forward:
        return nn.log_softmax(self.linear(newX)) #log softmax is multiclass

#in loss/train:
        loss_func = nn.NLLLoss() #negative log likelikelihood loss
```