第一部分 选择题（每题 4 分，总共 20 分）

Part One Select one from the given four options (4 marks each question, in total 20 marks):

1. 若随机事件 ***A*** 与 ***B*** 相互独立，则 $P(A\cup B)=$

   A. $P(A)+P(B)$
   B. $P(A)+P(B)-P(A)P(B)$
   C. $P(A)P(B)$
   D. $P(\bar{A})+P(\bar{B})$

   Assume the events ***A*** and ***B*** are independent to each other, then $P(A\cup B)=$

   A. $P(A)+P(B)$
   B. $P(A)+P(B)-P(A)P(B)$
   C. $P(A)P(B)$
   D. $P(\bar{A})+P(\bar{B})$

2. 设$F(x,y)$是二维随机变量(X,Y)的分布函数，下面四个结论中错误的是：

   A. $F(+\infty,+\infty)=1$
   B. $F(-\infty,-\infty)=0$
   C. $F(+\infty,y)=1$
   D. $F(x,-\infty)=0$

   Assume $F(x,y)$ is the distribution function of a two-dimensional random variable $(\boldsymbol{X},\boldsymbol{Y})$**,** which conclusion is wrong:

   A. $F(+\infty,+\infty)=1$
   B. $F(-\infty,-\infty)=0$
   C. $F(+\infty,y)=1$
   D. $F(x,-\infty)=0$

3. 设随机变量$X\sim b\,(2,p),Y\sim b\,(3,p)$，若$P(X\geq 1)=\frac{5}{9}$，则$P(Y\geq 1)=$_____ .

   A. $\frac{8}{27}$ B. $\frac{19}{27}$ C. $\frac{5}{9}$ D. $\frac{4}{9}$.

Assume the random variables $X \sim b\,(2,p), Y \sim b\,(3,p)$. If $P(X \geq 1) = \frac{5}{9}$, $P(Y \geq 1) =$____ .

A. $\frac{8}{27}$　　B. $\frac{19}{27}$　　C. $\frac{5}{9}$　　D. $\frac{4}{9}$.

4. 设$X_i(i=1,2,3)$为三个正态随机变量，且$X_1 \sim N\,(0,1), X_2 \sim N\,(0,2^2), X_3 \sim N\,(5,3^2)$，记$p_i = P(-2 < X_i < 2),\ i = 1,2,3$，则____

A. $p_1 > p_2 > p_3$
B. $p_3 > p_1 > p_2$
C. $p_2 > p_1 > p_3$
D. $p_1 > p_3 > p_2$.

Let $X_i(i=1,2,3)$ be three normal distributed random variables $X_1 \sim N\,(0,1),\ X_2 \sim N\,(0,2^2),\ X_3 \sim N\,(5,3^2)$. Let $p_i = P(-2 < X_i < 2),\ i = 1,2,3$, then_____

A. $p_1 > p_2 > p_3$
B. $p_3 > p_1 > p_2$
C. $p_2 > p_1 > p_3$
D. $p_1 > p_3 > p_2$.

5. 设 X 与 Y 为二随机变量，下面叙述正确的是 ___

A. 若 X 与 Y 均为一维正态随机变量，则(X,Y)是二维正态随机向量；
B. 若 X 与 Y 均为一维均匀随机变量，则(X,Y)是二维均匀随机向量；
C. 若(X,Y)是二维正态随机向量，则 X 与 Y 均为一维正态随机变量；
D. 若(X,Y)是二维均匀随机向量，则 X 与 Y 均为一维均匀随机变量.

Let X *and* Y be two random variables, which statement of the following is true _____

A. If X *and* Y are both one-dimensional normal distributions, (X,Y) is a bivariate normal distribution;
B. If X *and* Y are both one-dimensional uniform distributions，(X,Y) is a two-dimensional uniform distribution；
C. If (X,Y) is a bivariate normal distribution, X and Y are both one-dimensional normal distributions；
D. If (X,Y) is a two-dimensional uniform distribution, X and Y are both one-dimensional uniform distributions

第二部分 填空题（每题 2 分，总共 20 分）

Part Two Fill in the boxes for each Question (2 marks each box, in total 20 marks)

1. 随机地把C, S, S, T, U五个字母排成一排，计算得到SUSTC的概率_______.

   Line up five letters C, S, S, T, U randomly, the probability you get SUSTC is________.

2. A、B、C 3 位同学同时独立参加数学补考考试，不及格的概率分别为 0.4, 0.3, 0.5. 恰有 2 位同学不及格的概率是________. 如果已经知道这 3 位同学中有 2 位不及格，那么其中 1 位是 B 同学不及格的概率是_________。

   Students A, B, C independently attend the mathematics resit examination at the same time. The probability of their failure is 0.4, 0.3, 0.5. The probability that exactly two students fail is ______. If it is known that two out of three students fail, the probability of student B fails is ______.

3. 在一场五局三胜制的游戏中，双方每局的胜率分别是60%和40%，且每局之间相互独立。则游戏结束时每边至少赢了一局的概率是______.

   In the best three-out-of-five games, the probability that each side wins a game is 60%and40%respectively, and all games are independent. The probability that each side wins at least one game before the whole games end is _____.

4. 设事件A和B满足$P(A) = P(B) = \frac{2}{3}$, $P(A \cup B) = 1$，则$P(\bar{A} \cup \bar{B}) =$_______.

   Suppose two events A and B satisfies $P(A) = P(B) = \frac{2}{3}$, $P(A \cup B) = 1$, then $P(\bar{A} \cup \bar{B}) =$_______.

5. 设随机变量$X \sim EXP(\lambda)$服从指数分布。则$P(4 > X > 3 | X > 2) =$________. 当参数

$\lambda =$____时这个概率取到最大值。

Suppose random variable X satisfies exponential distribution $X \sim EXP(\lambda)$. $P(4 > X > 3|X > 2) =$_______. When the parameter $\lambda =$____, this probability reaches its maximum.

6. 设随机变量X和Y独立，且均匀分布在$[1,3]$，则$P(\max(X, Y) > 2) =$_____.

   Suppose two random variables X and Y independently uniformly distributed on $[1,3]$. $P(\max(X, Y) > 2) =$_____.

7. 设$X \sim N(\mu, \sigma^2)$, $Y \sim N(2\mu, \sigma^2)$是相互独立的正态分布的随机变量且$P(X - Y \geq 2) = \frac{1}{2}$。则$\mu =$_____.

   Let $X \sim N(\mu, \sigma^2)$, $Y \sim N(2\mu, \sigma^2)$ be two independent normal distributed random variables and $P(X - Y \geq 2) = \frac{1}{2}$. Then $\mu =$_____.

8. 设$X \sim P(\lambda)$服从参数为λ的泊松分布，则$Y = X^3$的频率函数为_____.

   Suppose $X \sim P(\lambda)$ has a Poisson distribution with parameter λ. Then the frequency function for $Y = X^3$ is _____.

第三部分 问答题（每题 10 分，总共 60 分）

Part Three Questions and Answers (10 marks each question, in total 60 marks)

1. 设随机变量X表示某个人打靶的准心情况，其概率分布密度函数

$$f(x) = \begin{cases} 3x^2 & x \in (0,1) \\ 0 & x \notin (0,1) \end{cases}$$

设定事件$\left\{X \leq \frac{2}{3}\right\}$发生为打靶成功。另 Y 代表三次打靶成功的次数，求刚好成功一次的概率。

Let X stand for the result of shooting target practice from someone. The density function of X is

$$f(x) = \begin{cases} 3x^2 & x \in (0,1) \\ 0 & x \notin (0,1) \end{cases}$$

Assume $\left\{X \leq \frac{2}{3}\right\}$ as a successful practice. Let Y stand for the number of successful practice in total 3 shots. Find the probability when Y=1.

2. (X, Y) 是两个离散随机变量，有如下联合概率分布

Y \ X	1	2	3
0	1/6	a	1/18
1	1/3	2/9	b

求：（1）**a** 与 **b** 存在的关系 ；（2）若 **X** 与 **Y** 独立，求 **a** 与 **b** 的值。

Suppose (X,Y) be a two-dimensional discrete random variable with the following distribution

Y \ X	1	2	3
0	1/6	a	1/18
1	1/3	2/9	b

(1) find the relationship between **a** and **b**; (2) given **X** and **Y** independent, find **a** and **b**.

3. 设二维随机变量(X, Y)的联合概率密度为

$$f(x,y)=\begin{cases} cx^2y & x^2 \le y \le 1, \\ 0 & \text{其他} \end{cases}$$

求：（1）常数 c ；　　（2）求$P\{X > Y\}$.

Let (X, Y) be a two-dimensional distribution with the joint density function

$$f(x,y)=\begin{cases} cx^2y & x^2 \le y \le 1, \\ 0 & \text{otherwise} \end{cases}$$

(1) find the constant c; (2) compute P(X>Y).

4. 某地区 18 岁女青年的血压（收缩压，以 mmHg 计）服从$N(110,\ 12^2)$，在该地区任选一 18 岁女青年，测量她的血压X. 确定最小的x, 使得$P\{X > x\} \le 0.05$.（可能用到的参数：$\Phi(1.645) = 0.95$）

Suppose the blood pressure (systolic pressure, measured in mmHg) of 18 years old women somewhere has a normal distribution $N(110,\ 12^2)$. Randomly select a 18 years old woman and measure her systolic pressure X. Find the smallest x so that $P\{X > x\} \le 0.05$. (it might be used $\Phi(1.645) = 0.95$)

5. 设随机变量(X, Y)在区域 G 上服从均匀分布，其中 G 由$x - y = 0,\ x + y = 2$ 与 $y = 0$ 围成.

(1) 求边际密度$f_X(x)$.
(2) 求条件密度$f_{X|Y}(x|y)$.

Suppose a two-dimensional random variable (X, Y) is uniformly distributed in the region G where G is formed by the three lines $x - y = 0,\ x + y = 2\ and\ y = 0$.

(1) Find the marginal density function $f_X(x)$
(2) Find the conditional density function $f_{X|Y}(x|y)$

6. 设随机变量X的概率密度为

$$f(x)=\begin{cases}\frac{1}{9}x^2, & 0<x<3,\\ 0, & \text{其它}.\end{cases}$$

令随机变量
$$Y=\begin{cases}2 & X\le 1,\\ X & 1<X<2,\\ 1 & X\ge 2.\end{cases}$$

(1) 求Y的累积分布函数；

(2) 求概率$P\{X\le Y\}$.

Let X be a random variable with the density function

$$f(x)=\begin{cases}\frac{1}{9}x^2, & 0<x<3,\\ 0, & \text{others}\end{cases}$$

Define
$$Y=\begin{cases}2 & X\le 1,\\ X & 1<X<2,\\ 1 & X\ge 2.\end{cases}$$

(1) Find the cumulative distribution function of Y.
(2) Compute $P\{X\le Y\}$.