Sequential Group Recommendation

PFA: Proportional Fairness Aggregation (NEW METHOD)

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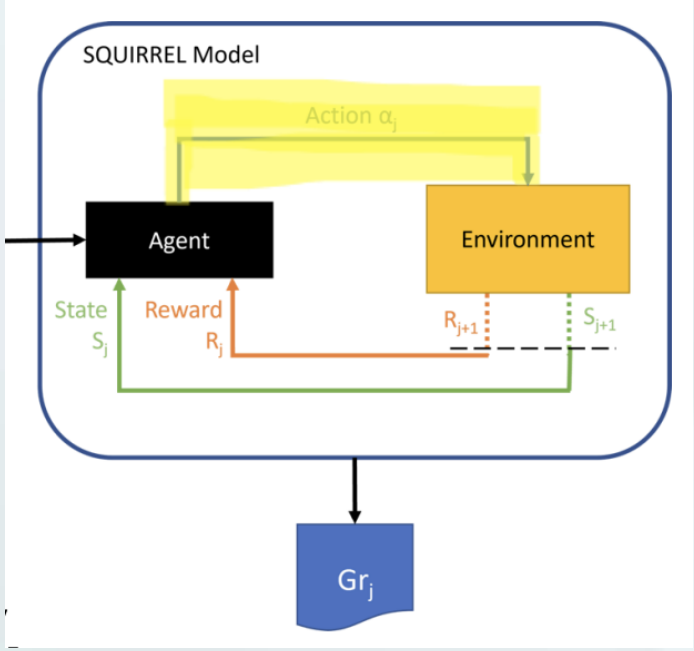

PFA Theory

Proportional Fairness Aggregation (PFA)

Core Innovation: Geometric Mean Aggregation

SDAA: score = $(w_1 \cdot p_1 + w_2 \cdot p_2 + w_3 \cdot p_3) / \Sigma w$ [Arithmetic]

aggregate preferences in a way that overlooks individual dissatisfaction.

PFA: score = $(p_1^{w_1} \times p_2^{w_2} \times p_3^{w_3})^{(1/\Sigma w)}$ [Geometric]

more sensitive to low values, reveal and address unfairness across group members.

Why Geometric Mean?

- Automatically penalizes imbalanced predictions
- Example: predictions [2.0, 5.0, 4.0]
  - Arithmetic: 3.67 (hides imbalance)
  - Geometric: 3.42 (lower score reflects unfairness)

Two-Phase Design:

1. Phase 1: Average aggregation filters top candidates (quality)
2. Phase 2: Geometric mean selects final items (fairness)

Dynamic Weights:

Gap-based adjustment: users below average get higher weights

Self-correcting over rounds : prevents cumulative unfairness

PFA Implementation : Sequential Workflow

Proportional Fairness Aggregation (PFA)

Round 1: Initialize

✓ All users start with equal weights [1.0, 1.0, 1.0]

✓ Predict ratings for all candidate movies. Recommend top-k items using PFA

Each Round (2-10): Four-Step Process

Step 1: Calculate Dynamic Weights

→ Check each user's cumulative satisfaction

→ Users below average → higher weights (e.g., 1.4)

→ Users above average → lower weights (e.g., 0.8)

Example: Round 2 weights = [0.43, 0.17, 0.40]. User 39 had highest satisfaction → lowest weight

Step 2: Phase 1 - Quality Filter

→ Use simple average: score = $(p_1 + p_2 + p_3) / 3$

→ Select top k×3 candidates (e.g., 15 movies). Ensures we only consider good items

Step 3: Phase 2 - Fairness Optimization

→ Use weighted geometric mean: $(p_1^{w_1} \times p_2^{w_2} \times p_3^{w_3})^{(1/\Sigma w)}$

→ Pick final k items (e.g., 5 movies)

→ Naturally prefers balanced items

Step 4: Update Satisfaction

→ Users watch recommended movies

→ Update cumulative satisfaction for next round.System self-corrects over time

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COMPARISON: Average vs Least Misery vs SDAA-Modified vs PFA
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Methods:
  1. Average
  2. Least Misery
  3. SDAA-Modified (original 1st submission)
  4. PFA (NEW - Proportional Fairness Aggregation)
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Average (baseline) | Group: [4, 39, 404]
================================================================
R 1 | Sat=[4:0.77, 39:0.93, 404:0.78] | Rew=0.835
R 2 | Sat=[4:0.81, 39:0.92, 404:0.81] | Rew=0.852
R 3 | Sat=[4:0.70, 39:0.84, 404:0.71] | Rew=0.834
R10 | Sat=[4:0.78, 39:0.91, 404:0.78] | Rew=0.839
----------------------------------------------------------------
Final | SatO=[4:0.77, 39:0.87, 404:0.76] | GroupSat=0.799 | Dis=0.117 | AvgRew=0.840

LeastMisery (baseline) | Group: [4, 39, 404]
================================================================
R 1 | Sat=[4:0.68, 39:0.85, 404:0.71] | Rew=0.789
R 2 | Sat=[4:0.83, 39:0.92, 404:0.80] | Rew=0.832
R 3 | Sat=[4:0.83, 39:0.91, 404:0.81] | Rew=0.846
R10 | Sat=[4:0.76, 39:0.91, 404:0.74] | Rew=0.838
----------------------------------------------------------------
Final | SatO=[4:0.76, 39:0.85, 404:0.75] | GroupSat=0.786 | Dis=0.103 | AvgRew=0.838

SDAA-Modified (original method) | Group: [4, 39, 404]
================================================================
R 1 | Sat=[4:0.69, 39:0.87, 404:0.73] | Rew=0.793
R 2 | Sat=[4:0.79, 39:0.90, 404:0.74] | Rew=0.820
R 3 | Sat=[4:0.80, 39:0.86, 404:0.79] | Rew=0.837
R10 | Sat=[4:0.81, 39:0.93, 404:0.76] | Rew=0.841
----------------------------------------------------------------
Final | SatO=[4:0.78, 39:0.87, 404:0.75] | GroupSat=0.800 | Dis=0.114 | AvgRew=0.834

PFA (IMPROVED) | expansion=3 | Group: [4, 39, 404]
================================================================
R 1 | Sat=[4:0.77, 39:0.93, 404:0.78] | MinSat=0.772 | W=[4:1.00, 39:1.00, 404:1.00] | Rew=0.835
R 2 | Sat=[4:0.75, 39:0.87, 404:0.77] | MinSat=0.754 | W=[4:0.43, 39:0.17, 404:0.40] | Rew=0.837
R 3 | Sat=[4:0.79, 39:0.90, 404:0.77] | MinSat=0.773 | W=[4:0.43, 39:0.17, 404:0.40] | Rew=0.844
R10 | Sat=[4:0.83, 39:0.92, 404:0.75] | MinSat=0.752 | W=[4:0.39, 39:0.18, 404:0.43] | Rew=0.840
----------------------------------------------------------------
Final | SatO=[4:0.77, 39:0.86, 404:0.75] | GroupSat=0.795 | Dis=0.111 | AvgRew=0.843
Fairness Range: 0.111
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Comparison results:

Test Group users: [4, 39, 404] - 10 Rounds

Average

Least Misery

SDAA

PFA

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FINAL COMPARISON
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Method                  GroupSat   GroupDis  AvgReward Status
----------------------------------------------------------------
Average                    0.799      0.117      0.840
LeastMisery                0.786      0.103      0.838
SDAA-Modified              0.800      0.114      0.834
PFA                        0.795      0.111      0.843 (NEW)

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KEY IMPROVEMENTS (PFA vs SDAA-Modified):
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Group Satisfaction: 0.800 → 0.795 (-0.005)
Group Disagreement: 0.114 → 0.111 (-0.004)
Average Reward:     0.834 → 0.843 (+0.009)

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✓ Best Reward:   PFA (AvgReward=0.843)
✓ Best Fairness: LeastMisery (Disagreement=0.103)
================================================================
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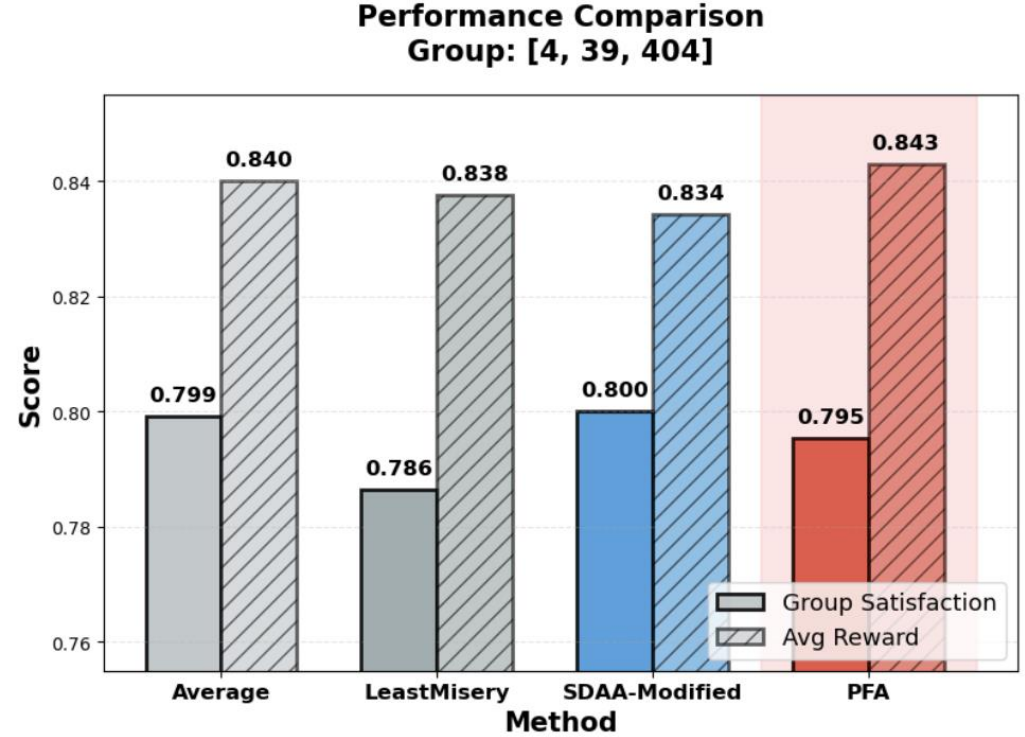
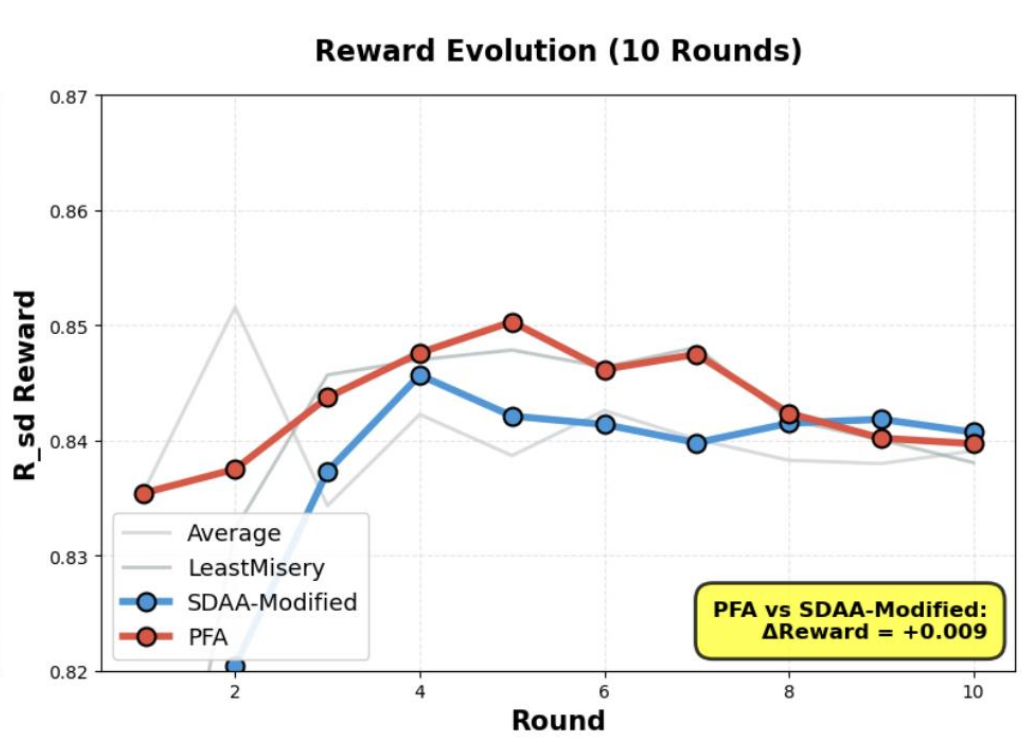
Results 1 - Single Group Test _ Visualization

Test Group users: [4, 39, 404] - 10 Rounds

Single Group Performance

Method	Group Satisfaction	Group Disagreement	Avg Reward (R_sd)
Average	0.799	0.117	0.840
Least Misery	0.786	0.103	0.838
SDAA-Modified	0.800	0.114	0.834
PFA (NEW)	**0.795**	**0.111**	**0.843**





Results 2 - Multi-Group Validation

Testing Across Diverse 10 Groups – also 10 Rounds

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MULTI-GROUP VALIDATION EXPERIMENTS
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Purpose: Validate PFA performance across diverse user groups
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[Step 1] Identifying candidate test groups...
✓ Found 329 active users with reasonable rating patterns
✓ Selected 10 test groups with varying diversity levels

Test Group Details:
--------------------------------------------------------------------------------
#    Users                Mean Ratings                Diversity
--------------------------------------------------------------------------------
1    [4, 312, 387]        [3.56, 3.71, 3.26]          0.450
2    [116, 377, 426]      [3.44, 3.91, 3.68]          0.475
3    [282, 425, 564]      [4.03, 3.53, 3.58]          0.501
4    [82, 303, 385]       [3.38, 3.91, 3.40]          0.525
5    [24, 151, 408]       [3.65, 3.54, 4.09]          0.550
6    [151, 200, 274]      [3.54, 3.81, 3.24]          0.574
7    [63, 361, 512]       [3.63, 3.18, 3.78]          0.600
8    [563, 594, 607]      [3.30, 3.92, 3.79]          0.624
9    [222, 476, 484]      [3.17, 3.68, 3.81]          0.649
10   [140, 166, 474]      [3.50, 4.07, 3.40]          0.675
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[Step 3] STATISTICAL ANALYSIS
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Descriptive Statistics:
--------------------------------------------------------------------------------
Method               Mean         Std Dev      Min        Max
--------------------------------------------------------------------------------
Average              0.8337       0.0280       0.7628     0.8595
LeastMisery          0.8352       0.0311       0.7561     0.8709
SDAA-Modified        0.8306       0.0210       0.7912     0.8613
PFA                  0.8354       0.0293       0.7616     0.8660

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PAIRED T-TESTS (PFA vs Baselines)
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Comparison                    Mean Δ       t-stat       p-value      Result
--------------------------------------------------------------------------------
PFA vs Average              0.0017       1.538        0.1584       Not sig.
PFA vs LeastMisery          0.0001       0.083        0.9356       Not sig.
PFA vs SDAA-Modified        0.0048       0.789        0.4501       Not sig.

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WIN RATE ANALYSIS (PFA victories per group)
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PFA vs Average           : 6W - 3T - 1L  (60.0% win rate)
PFA vs LeastMisery       : 5W - 1T - 4L  (50.0% win rate)
PFA vs SDAA-Modified     : 6W - 0T - 4L  (60.0% win rate)
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Robust performance: PFA outperforms baseline in 6 out of 10 diverse groups with +0.6% average improvement

Why It Works & Conclusion

1. Mathematical Innovation
   - SDAA uses arithmetic mean (addition) → averages hide problems
   - PFA uses geometric mean (multiplication) → naturally detects imbalance

2. Sequential Intelligence
   - SDAA: Fixed alpha parameter throughout all rounds
   - PFA: Dynamic gap-based weights that adapt each round
   - Unhappy users automatically get higher influence next round
   - System self-corrects → prevents cumulative unfairness

3. Quality-First Architecture
   - SDAA: Direct aggregation may sacrifice quality for fairness
   - PFA: Two-phase design ensures both objectives
     - Phase 1 filters high-quality candidates
     - Phase 2 selects fairest from quality pool
   - Result: Best reward AND better fairness