

Analysis of Strategic Behavior in School Choice Mechanisms

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1 Results for 3 Students and 3 Schools Scenario

In this section, I present my analysis of strategic behavior in a 3x3 school choice mechanism implemented over two rounds. My simulation framework systematically explores the possibility of beneficial preference manipulation.

1.1 Simulation Framework

My code implements a comprehensive search for strategic manipulation opportunities with the following structure:

- Student s_1 's true preference is fixed as: $c_1 \succ c_2 \succ c_3$
- For all other agents (3 students and 3 schools), each has 6 possible preference orderings
- This creates a total of $6^5 = 7776$ possible preference combinations to examine

1.2 Strategic Analysis Methodology

For each preference profile, I investigate whether s_1 can benefit from strategic manipulation through:

1. First round: s_1 misreports preferences while others report truthfully
2. Second round:
 - Other students update their preferences based on first-round outcomes
 - s_1 reverts to truthful reporting
3. Comparison: I compare s_1 's final outcome under strategic behavior versus truthful reporting

1.3 Key Findings

My exhaustive simulation revealed:

- Among the 6^5 total cases examined, 144 cases demonstrated successful strategic manipulation
- In these cases, s_1 's strategic misreporting in the first round, followed by truthful reporting in the second round, led to a strictly better outcome compared to being truthful throughout
- This suggests that even in a relatively simple two-round matching mechanism, there exist significant opportunities for beneficial strategic behavior

1.4 Detailed Case Study of Beneficial Strategic Behavior

Let's examine a specific case (Case ID 1044) that demonstrates how strategic manipulation can lead to better outcomes:

1.4.1 Initial Preference Setup

- True Student Preferences:
 - $s_1: c_1 \succ c_2 \succ c_3$
 - $s_2: c_1 \succ c_2 \succ c_3$
 - $s_3: c_3 \succ c_1 \succ c_2$
- School Preferences:
 - $c_1: s_3 \succ s_2 \succ s_1$
 - $c_2: s_1 \succ s_2 \succ s_3$
 - $c_3: s_1 \succ s_2 \succ s_3$

1.4.2 Truthful Reporting Scenario

When all students report preferences honestly:

- First round matching: $\mu_1 = \{(s_1, c_2), (s_2, c_1), (s_3, c_3)\}$
- s_1 receives their second choice c_2

1.4.3 Strategic Manipulation

s_1 employs the following strategy:

1. Misreports first-round preference as: $c_1 \succ c_3 \succ c_2$
2. First round matching results: $\mu'_1 = \{(s_1, c_3), (s_2, c_2), (s_3, c_1)\}$
3. Second round preference updates:

- s_2 updates to: $c_2 \succ c_1 \succ c_3$
 - s_3 maintains: $c_3 \succ c_1 \succ c_2$
4. s_1 reverts to truthful reporting in second round
 5. Final matching: $\mu'_2 = \{(s_1, c_1), (s_2, c_2), (s_3, c_3)\}$

Through this strategic manipulation, s_1 ultimately obtains their top choice c_1 , a clear improvement over c_2 received under truthful reporting.

2 Results for 4 Students and 4 Schools Scenario

3 Intuition Behind First-Round Manipulation