3DGS-based SLAM

Research Notes & Literature Review

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July 1, 2024

Outline

Overview

Timeline

11/2023 - 12/2024

SplaTAM [1]

GS-SLAM [2]

Gaussian-SLAM [3]

MonoGS [4]

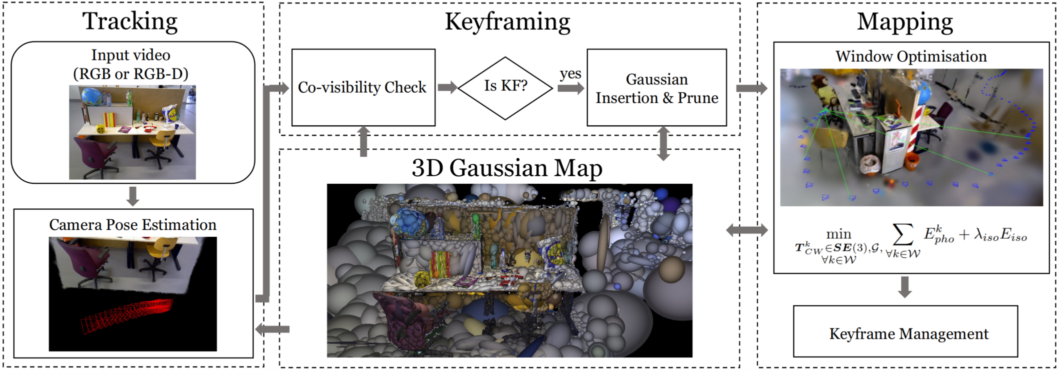
01/2024 - 04/2024

05/2024 - 06/2024

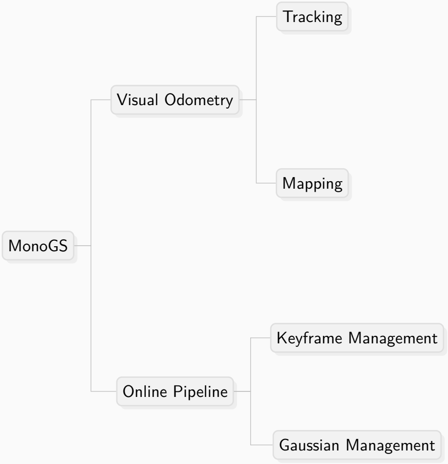
accepted to top conferences

MonoGS

Overview i

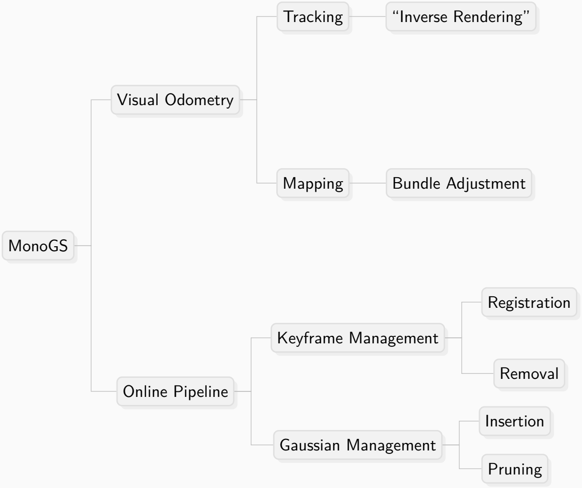


(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM



(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Taxonomy



(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Taxonomy

- MonoGS
  - Visual Odometry
    - Tracking
      - "Inverse Rendering"
        - Analytical Jacobians Derivation
        - Photometric RGB & Depth Loss
        - Optimizable Exposure
        - Penalization on non-edge/low-opacity pixels
    - Mapping
      - Bundle Adjustment
        - Photometric RGB & Depth Loss
        - Isotropic Regularization
        - Random Recall
  - Online Pipeline
    - Keyframe Management
      - Registration
        - Gaussian Covisibility
        - Relative Translation
      - Removal
        - Gaussian Overlap Coefficient
    - Gaussian Management
      - Insertion
        - Keyframing
      - Pruning
        - Gaussian Covisibility

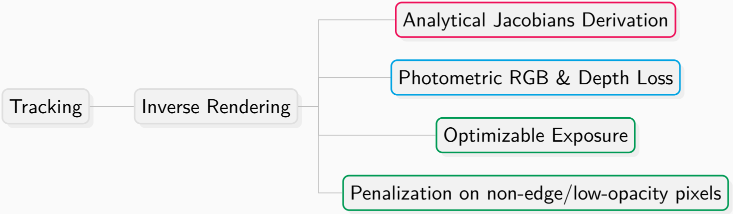
key method

trick

convention

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Tracking: Overview



Track camera poses by inverse rendering,

key method | trick | convention

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Tracking: Overview

- Tracking
  - Inverse Rendering
    - Analytical Jacobians Derivation
    - Photometric RGB & Depth Loss
    - Optimizable Exposure
    - Penalization on non-edge/low-opacity pixels

Track camera poses by inverse rendering,

- through the extended differentiable rendering pipeline,

key method trick convention

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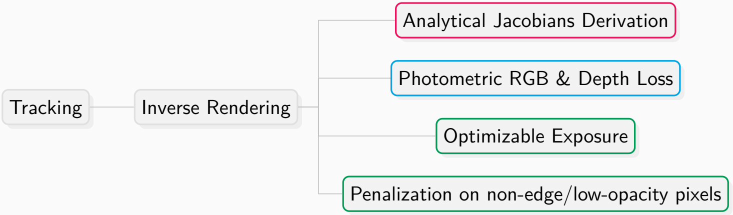


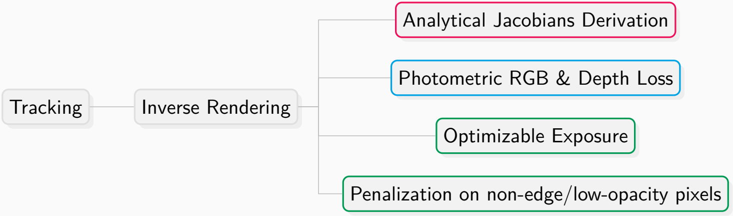
Track camera poses by inverse rendering,

- through the extended differentiable rendering pipeline,
- by a direct optimization against fixed 3D Gaussians,

key method trick convention

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Tracking: Overview



Track camera poses by inverse rendering,

- through the extended differentiable rendering pipeline,
- by a direct optimization against fixed 3D Gaussians,
- with some tricks to be more adaptive to brightness and more robust to noise.

key method trick convention

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Tracking: Derivation of Jacobians i

Firstly, let's review the projection of 3D Gaussians.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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$$\mathcal{N}\left(\mu_w, \Sigma_w\right) \overset{\pi}{\mapsto} \mathcal{N}\left(\mu_i, \Sigma_i\right) \tag{1}$$

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is achieved by

$$\mu_i = \pi\left(\mathbf{T}_{cw} \cdot \mu_w\right) \tag{2}$$

$$\Sigma_i = \mathbf{J}_\pi \mathbf{R}_{cw} \Sigma_w \mathbf{R}_{cw}^{\mathrm{T}} \mathbf{J}_\pi^{\mathrm{T}} \tag{3}$$

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$$\mathcal{N}\left(\mu_w, \Sigma_w\right) \xmapsto{\pi} \mathcal{N}\left(\mu_i, \Sigma_i\right) \tag{1}$$

is achieved by

$$\underbrace{\mu_i}_{\in \mathbb{P}^2,\ \text{2D(image) mean}} = \underbrace{\pi}_{\text{projection}} \left(\underbrace{\mathbf{T}_{cw}}_{\in \mathrm{SE}(3),\ \text{camera pose}} \cdot \underbrace{\mu_w}_{\in \mathbb{P}^3,\ \text{3D(world) mean}} \right) \tag{2}$$

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$\in \mathrm{SE}(3)$, camera pose

projection

$\in \mathbb{P}^2$, 2D(image) mean

$$\Sigma_i = \mathbf{J}_\pi \mathbf{R}_{cw} \Sigma_w \mathbf{R}_{cw}^{\mathrm{T}} \mathbf{J}_\pi^{\mathrm{T}} \tag{3}$$

$\in \mathbb{R}^{3\times 3}$, 3D(world) covariance

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$\in \mathbb{R}^{3\times 3}$, 3D(world) covariance

$\in \mathrm{SO}(3)$, rotation component of $\mathbf{T}_{cw}$

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- μ_w: $\in \mathbb{P}^3$, 3D(world) mean
- $\mathbf{T}_{cw}$: $\in \mathrm{SE}(3)$, camera pose
- π: projection
- μ_i: $\in \mathbb{P}^2$, 2D(image) mean

$$\Sigma_i = \mathbf{J}_\pi \mathbf{R}_{cw} \Sigma_w \mathbf{R}_{cw}^{\mathrm{T}} \mathbf{J}_\pi^{\mathrm{T}} \tag{3}$$

- Σ_w: $\in \mathbb{R}^{3\times3}$, 3D(world) covariance
- $\mathbf{R}_{cw}$: $\in \mathrm{SO}(3)$, rotation component of $\mathbf{T}_{cw}$
- $\mathbf{J}_\pi$: $\in \mathbb{R}^{2\times3}$, Jacobian of the linear approximation of π
- Σ_i: $\in \mathbb{R}^{2\times2}$, 2D(image) covariance

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Tracking: Derivation of Jacobians ii

The chain rule,

$$\frac{\partial \mu_i}{\partial \mathbf{T}_{cw}} = \frac{\partial \mu_i}{\partial \mu_c} \frac{\partial \mu_c}{\partial \mathbf{T}_{cw}} \tag{4}$$

$$\frac{\partial \Sigma_i}{\partial \mathbf{T}_{cw}} = \frac{\partial \Sigma_i}{\partial \mathbf{J}_\pi} \frac{\partial \mathbf{J}_\pi}{\partial \mu_c} \frac{\partial \mu_c}{\partial \mathbf{T}_{cw}} + \frac{\partial \Sigma_i}{\partial \mathbf{R}_{cw}} \frac{\partial \mathbf{R}_{cw}}{\partial \mathbf{T}_{cw}} \tag{5}$$

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Tracking: Derivation of Jacobians ii

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The Lie Algebra,

$$\frac{\partial \mu_c}{\partial \mathbf{T}_{cw}} = \begin{bmatrix} \mathbf{I} & -\mu_c^{\times} \end{bmatrix} \tag{6}$$

$$\frac{\partial \mathbf{R}_{cw}}{\partial \mathbf{T}_{cw}} = \begin{bmatrix} \mathbf{0} & -\mathbf{R}_{cw}^{\times}(:,1) \\ \mathbf{0} & -\mathbf{R}_{cw}^{\times}(:,2) \\ \mathbf{0} & -\mathbf{R}_{cw}^{\times}(:,3) \end{bmatrix} \tag{7}$$

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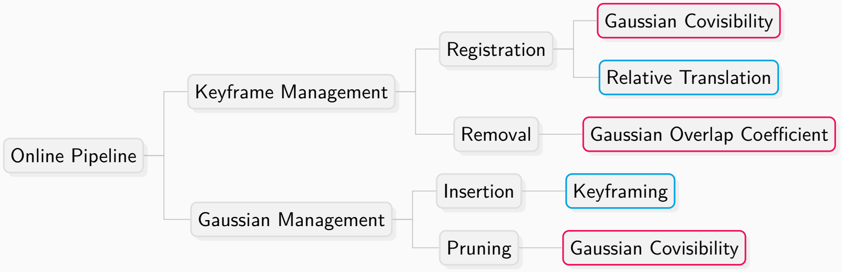

Keyframe Management:

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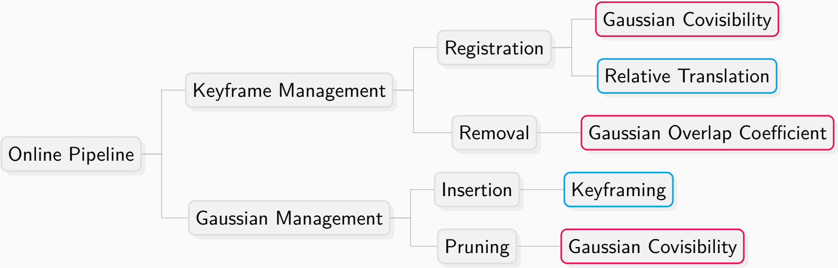
key method | trick | convention

(arXiv, 2016) DSO: Direct Sparse Odometry

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Online Pipeline: Overview



Keyframe Management:

- Classic strategies, e.g. covisibility & overlap, from DSO [5].

key method trick convention

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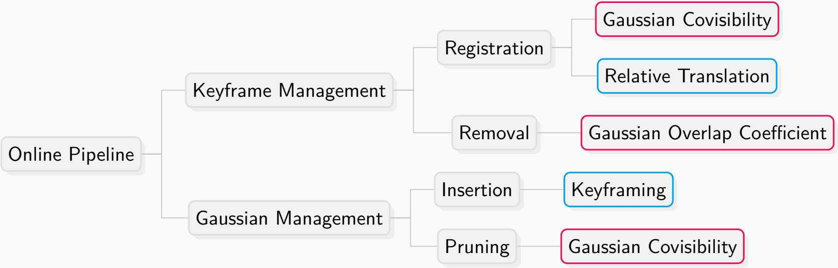

Keyframe Management:

- Classic strategies, e.g. covisibility & overlap, from DSO [5].
- Off-the-shelf occlusion-aware Gaussian visibility is leveraged to construct metrics.

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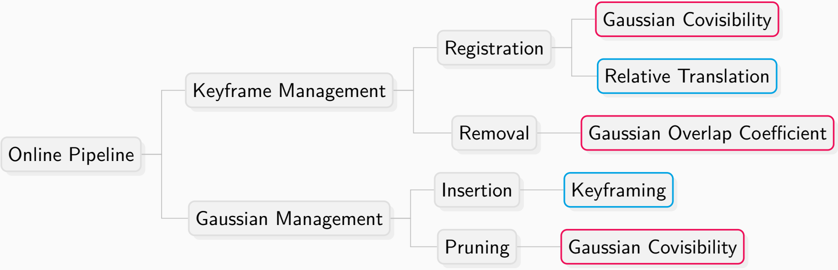

Gaussian Management:

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key method   trick   convention

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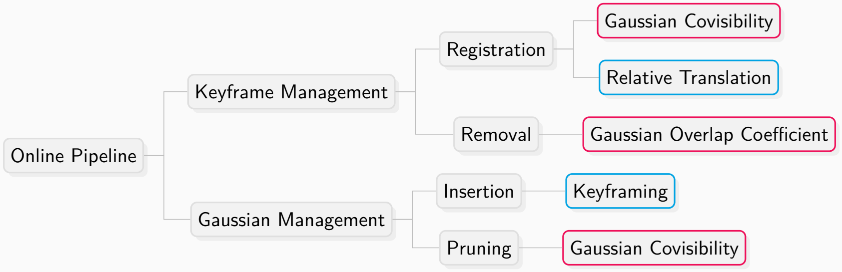

Gaussian Management:

- Insertion: triggered by keyframing, followed by Gaussian initialization.

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key method   trick   convention

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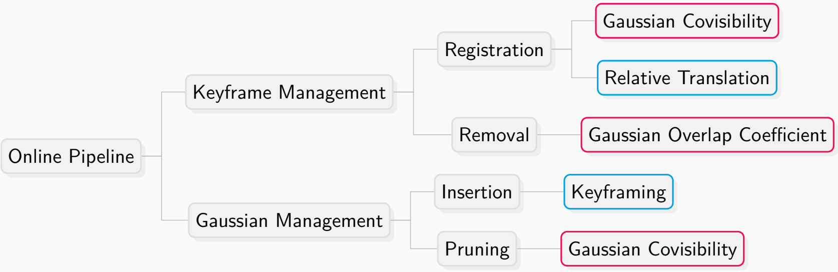

Gaussian Management:

- Insertion: triggered by keyframing, followed by Gaussian initialization.
- Pruning: to remove unstable/incorrect Gaussians by covisibility in a monocular setting.

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key method | trick | convention

(arXiv, 2016) DSO: Direct Sparse Odometry

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Keyframe Management: Prerequistes

1. What is keyframing or keyframe management?

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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   - Infeasible to optimize jointly on all frames online.

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     (a trade-off between efficiency and accuracy/robustness/...)

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3. How should we select keyframes?
   - non-redundant and observing the same area.
   - spanning a wide baseline for better multi-view constraints.

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Keyframe Management: Registration i

If any of the following conditions is true...

In practice, $\tau_1 = 0.95$.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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Small Gaussian Covisibility | **Condition i, Keyframe Registration**

Gaussian covisibility between the current frame and the previous keyframe drops below a threshold.

$$\frac{\left|\mathrm{v}\left(\mathcal{G}, \mathcal{F}_i\right) \cap \mathrm{v}\left(\mathcal{G}, \mathcal{F}_j\right)\right|}{\left|\mathrm{v}\left(\mathcal{G}, \mathcal{F}_i\right) \cup \mathrm{v}\left(\mathcal{G}, \mathcal{F}_j\right)\right|} < \tau_1 \tag{8}$$

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Keyframe Management: Registration ii

Large Relative Translation — **Condition ii, Keyframe Registration**

Translation from the previous keyframe w.r.t. to the median depth reaches a threshold.

$$\frac{\left\|\mathbf{t}_{\mathcal{F}_i\mathcal{F}_j}\right\|_2}{\bar{D}_{\mathcal{F}_i\mathcal{F}_j}} > \tau_2, \quad \bar{D}_{\mathcal{F}_i\mathcal{F}_j} = \frac{1}{2HW} \sum^{\{\mathcal{F}_i,\mathcal{F}_j\}} \sum_{h=0}^{H} \sum_{w=0}^{W} d(h, w) \tag{9}$$

In practice, $\tau_2 = 0.04$. Additionally, evaluate the Gaussian covisibility only if the relative translation is not too small (> 0.02) for efficiency.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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Keyframe Management: Registration ii

Large Relative Translation **Condition ii, Keyframe Registration**

Translation from the previous keyframe w.r.t. to the median depth reaches a threshold.

$$\frac{\left\| \overbrace{\mathbf{t}_{\mathcal{F}_i \mathcal{F}_j}}^{\in \mathbb{R}^3\text{, translation from } \mathcal{F}_i \text{ to } \mathcal{F}_j} \right\|_2}{\underbrace{\bar{D}_{\mathcal{F}_i \mathcal{F}_j}}_{\in \mathbb{R}\text{, the median depth}}} > \tau_2, \quad \bar{D}_{\mathcal{F}_i \mathcal{F}_j} = \frac{1}{2\,\underbrace{H}_{\text{image height}}\,W} \sum^{\{\mathcal{F}_i, \mathcal{F}_j\}} \sum_{h=0}^{H} \sum_{w=0}^{W} \underbrace{d(h, w)}_{\text{depth of pixel } (h, w)} \tag{9}$$

In practice, $\tau_2 = 0.04$. Additionally, evaluate the Gaussian covisibility only if the relative translation is not too small (> 0.02) for efficiency.
(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Registration ii

Large Relative Translation **Condition ii, Keyframe Registration**

Translation from the previous keyframe w.r.t. to the median depth reaches a threshold.

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In practice, $\tau_2 = 0.04$. Additionally, evaluate the Gaussian covisibility only if the relative translation is not too small (> 0.02) for efficiency.
(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal i

If any of the following conditions is true...

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal i

If any of the following conditions is true...

Beyond Window Capacity	Condition i, Keyframe Removal
Remove one of previous keyframes	

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal i

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Beyond Window Capacity **Condition i, Keyframe Removal**

Remove one of previous keyframes

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal i

If any of the following conditions is true...

Beyond Window Capacity — **Condition i, Keyframe Removal**

Remove one of previous keyframes that minimize the impact on the overall baseline length.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal i

If any of the following conditions is true...

Beyond Window Capacity | **Condition i, Keyframe Removal**

Remove one of previous keyframes that minimize the impact on the overall baseline length.

$$\mathcal{F}^* = \arg\max_{\mathcal{F} \in \mathcal{W}} \, \mathrm{l}\left(\mathcal{W} \setminus \{\mathcal{F}\}\right) \tag{10}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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$$\mathcal{F}^* = \underset{\mathcal{F}\in\mathcal{W}}{\arg\max}\ \mathrm{l}\left(\mathcal{W} \setminus \{\mathcal{F}\}\right), \quad \mathrm{l}\left(\mathcal{W}\right) = \sum_{i=1}^{|\mathcal{W}|} \sum_{j=1}^{i} \left\| \mathbf{t}_{\mathcal{F}_i \mathcal{F}_j} \right\| \tag{10}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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Remark: for the best multi-view constraints.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal ii

Low Gaussian Overlap Coefficient — **Condition ii, Keyframe Removal**

Remove multiple previous keyframes if the “Gaussian overlap coefficient” drops below a threshold.

Szymkiewicz–Simpson coefficient

In practice, $\tau_4 = 0.4$.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal ii

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Remove multiple previous keyframes if the "Gaussian overlap coefficient" drops below a threshold.

$$\frac{|\,\mathrm{v}\left(\mathcal{G}, \mathcal{F}_i\right) \cap \mathrm{v}\left(\mathcal{G}, \mathcal{F}_j\right)|}{\min\left(|\,\mathrm{v}\left(\mathcal{G}, \mathcal{F}_i\right)|, |\,\mathrm{v}\left(\mathcal{G}, \mathcal{F}_j\right)|\right)} < \tau_4 \tag{11}$$

Szymkiewicz–Simpson coefficient

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(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Keyframe Management: Removal ii

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Remark: not observing the same area.

Szymkiewicz–Simpson coefficient

In practice, $\tau_4 = 0.4$.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Insertion

- Why do we need "Gaussian insertion"?

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Insertion

- Why do we need "Gaussian insertion"?
  - SLAM is for robotic exploration.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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- When do we need "Gaussian insertion"?

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Insertion

- Why do we need "Gaussian insertion"?
  - SLAM is for robotic exploration.
- When do we need "Gaussian insertion"?

Keyframing — **Condition i, Gaussian Insertion**

Insertion is triggered for every new keyframe.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Initialization

- How do we insert Gaussians?

In practice, “low”: 0.2σ; “high”: 0.5σ, where σ is the standard deviation of the rendered depth map.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Initialization

- How do we insert Gaussians?
  - Gaussian insertion is Gaussian initialization.

In practice, “low”: 0.2σ; “high”: 0.5σ, where σ is the standard deviation of the rendered depth map.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Initialization

- How do we insert Gaussians?
  - Gaussian insertion is Gaussian initialization.

If Depth Available **Gaussian Initialization**

Back-project in a per-pixel, per-Gaussian approach.

In practice, “low”: 0.2σ; “high”: 0.5σ, where σ is the standard deviation of the rendered depth map.

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Gaussian Management: Initialization

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  - Gaussian insertion is Gaussian initialization.

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Back-project in a per-pixel, per-Gaussian approach.

If Depth Unavailable — **Gaussian Initialization**

Leverage the rendered depth map.

In practice, "low": 0.2σ; "high": 0.5σ, where σ is the standard deviation of the rendered depth map.

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Gaussian Management: Initialization

- How do we insert Gaussians?
  - Gaussian insertion is Gaussian initialization.

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Back-project in a per-pixel, per-Gaussian approach.

If Depth Unavailable — **Gaussian Initialization**

Leverage the rendered depth map.

- for pixels with depth: use the rendered depth and assign a “low” covariance.

In practice, “low”: 0.2σ; “high”: 0.5σ, where σ is the standard deviation of the rendered depth map.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Initialization

- How do we insert Gaussians?
  - Gaussian insertion is Gaussian initialization.

If Depth Available — **Gaussian Initialization**

Back-project in a per-pixel, per-Gaussian approach.

If Depth Unavailable — **Gaussian Initialization**

Leverage the rendered depth map.

- for pixels with depth: use the rendered depth and assign a “low” covariance.
- for pixels w/o depth: use the median of rendered depth and assign a “high” covariance.

In practice, “low”: 0.2σ; “high”: 0.5σ, where σ is the standard deviation of the rendered depth map.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Pruning

- Why do we need "Gaussian Pruning"?

If no pruning, although the majority of incorrect Gaussians vanish quickly in following optimization, there are some survivals.

In practice, $\tau_{\alpha} = 0.7$.

In practice, the pruned Gaussians are inserted in the last 3 keyframes and unobserved by any other 3 keyframes in the sliding window.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Pruning

- Why do we need "Gaussian Pruning"?
  - if depth unavailable, too many incorrect newly inserted Gaussians.

If no pruning, although the majority of incorrect Gaussians vanish quickly in following optimization, there are some survivals.

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(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Gaussian Management: Pruning

- Why do we need "Gaussian Pruning"?
    - if depth unavailable, too many incorrect newly inserted Gaussians.

Low Gaussian Opacity — **Condition i, Gaussian Pruning**

Low opacity Gaussians are pruned.

$$\{\mathcal{G}_i \in \mathcal{G} \mid \alpha(\mathcal{G}_i) < \tau_\alpha\} \tag{12}$$

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Low opacity Gaussians are pruned.

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Low Gaussian Covisibility — **Condition ii, Gaussian Pruning**

For "just" inserted Gaussians but unobserved by "some other" keyframes, are pruned out.

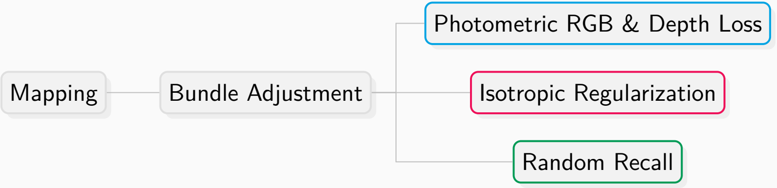
If no pruning, although the majority of incorrect Gaussians vanish quickly in following optimization, there are some survivals.

In practice, $\tau_\alpha = 0.7$.

In practice, the pruned Gaussians are inserted in the last 3 keyframes and unobserved by any other 3 keyframes in the sliding window.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

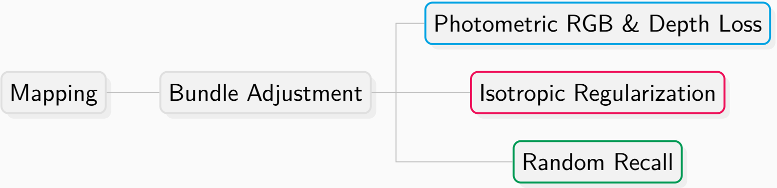
Mapping: Overview



key method trick convention

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Overview

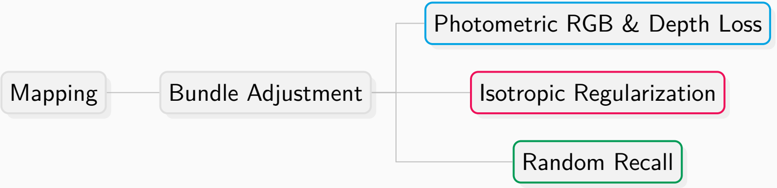


- Why do we need mapping in **3DGS** SLAM?

key method trick convention

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Overview



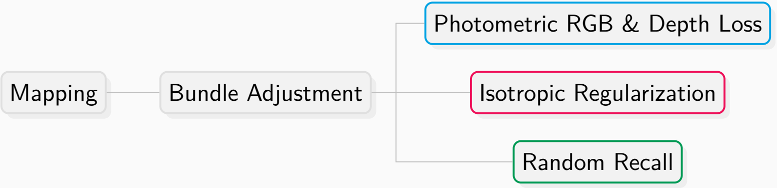
- Why do we need mapping in **3DGS** SLAM?
  - Local: Optimize newly inserted 3D Gaussians.

---

key method   trick   convention

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Overview



- Why do we need mapping in **3DGS** SLAM?
  - Local: Optimize newly inserted 3D Gaussians.
  - Global: Reconstruct a globally 3D-coherent structure.

key method trick convention

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Bundle Adjustment & Random Recall

Bundle Adjustment

$$\underset{\mathcal{G},\{\mathbf{T}_{cw}(\mathcal{F}_k)|\mathcal{F}_k\in\mathcal{W}\}}{\operatorname{argmin}} \sum_{\mathcal{F}_k}^{\mathcal{W}} \mathcal{L}_{pho}\left(\mathcal{F}_k\right) \tag{13}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Bundle Adjustment & Random Recall

Bundle Adjustment

$$\underset{\boxed{\mathcal{G}},\{\mathbf{T}_{cw}(\mathcal{F}_k)|\mathcal{F}_k\in\mathcal{W}\}}{\operatorname{argmin}} \sum_{\mathcal{F}_k}^{\mathcal{W}} \mathcal{L}_{pho}\left(\mathcal{F}_k\right) \tag{13}$$

3D Gaussians

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Bundle Adjustment

$$\underset{\mathcal{G},\ \{\mathbf{T}_{cw}(\mathcal{F}_k)|\mathcal{F}_k \in \mathcal{W}\}}{\operatorname{argmin}} \sum_{\mathcal{F}_k}^{\mathcal{W}} \mathcal{L}_{pho}(\mathcal{F}_k) \tag{13}$$

3D Gaussians

camera poses of keyframes in the sliding window

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Isotropic Regularization

- Why do we need “isotropic regularization”?

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Isotropic Regularization

- Why do we need "isotropic regularization"?
    - Observation: isotropic Gaussians behave better than anisotrophic.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Isotropic Regularization

- Why do we need "isotropic regularization"?
  - Observation: isotropic Gaussians behave better than anisotrophic.
  - Analysis: no constraints on the elongation along the viewing ray direction, **even with depth**.

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Isotropic Regularization

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Isotropic Regularization

$$\mathcal{L}_{iso} = \sum_{i=1}^{|\mathcal{G}|} \left\| \mathrm{s}(\mathcal{G}_i) - \bar{\mathrm{s}}(\mathcal{G}_i) \right\|_1, \tag{14}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

- Why do we need "isotropic regularization"?
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Isotropic Regularization

$\in \mathbb{N}$, total number of Gaussians

$$\mathcal{L}_{iso} = \sum_{i=1}^{\boxed{|\mathcal{G}|}} \|\mathrm{s}(\mathcal{G}_i) - \bar{\mathrm{s}}(\mathcal{G}_i)\|_1, \tag{14}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

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Isotropic Regularization

$$\mathcal{L}_{iso} = \sum_{i=1}^{\overbrace{|\mathcal{G}|}^{\in \mathbb{N}\text{, total number of Gaussians}}} \| \overbrace{\mathbf{s}(\mathcal{G}_i)}^{\in \mathbb{R}^3\text{, scale of } i\text{-th Gaussian}} - \bar{\mathbf{s}}(\mathcal{G}_i) \|_1 , \tag{14}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Isotropic Regularization

- Why do we need "isotropic regularization"?
  - Observation: isotropic Gaussians behave better than anisotrophic.
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Isotropic Regularization

$\in \mathbb{N}$, total number of Gaussians

$\in \mathbb{R}^3$, scale of i-th Gaussian

$\in \mathbb{R}^3$, averaged scale of i-th Gaussian

$$\mathcal{L}_{iso} = \sum_{i=1}^{|\mathcal{G}|} \| \mathbf{s}(\mathcal{G}_i) - \bar{\mathbf{s}}(\mathcal{G}_i) \|_1, \quad \bar{\mathbf{s}}(\mathcal{G}_i) = \begin{bmatrix} \left(s(\mathcal{G}_i)^x + s(\mathcal{G}_i)^y + s(\mathcal{G}_i)^z\right)/3 \\ \left(s(\mathcal{G}_i)^x + s(\mathcal{G}_i)^y + s(\mathcal{G}_i)^z\right)/3 \\ \left(s(\mathcal{G}_i)^x + s(\mathcal{G}_i)^y + s(\mathcal{G}_i)^z\right)/3 \end{bmatrix} \tag{14}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Mapping: Conclusion

The Overall Optimization for Mapping

$$\operatorname*{argmin}_{\mathcal{G},\{\mathbf{T}_{cw}(\mathcal{F}_k)|\mathcal{F}_k\in\mathcal{W}^+\}} \sum_{\mathcal{F}_k}^{\mathcal{W}^+} \mathcal{L}_{pho}\left(\mathcal{F}_k\right) + \lambda_{iso}\mathcal{L}_{iso} \tag{15}$$

(CVPR Highlight, 2024) MonoGS: Gaussian Splatting SLAM

Appendix

References i

[1] N. Keetha, J. Karhade, K. M. Jatavallabhula, *et al.*, *SplaTAM: Splat, track & map 3d gaussians for dense RGB-d SLAM*, Apr. 16, 2024. arXiv: 2312.02126[cs]. [Online]. Available: http://arxiv.org/abs/2312.02126 (visited on 05/20/2024) (cit. on p. iv).

[2] C. Yan, D. Qu, D. Wang, *et al.*, *GS-SLAM: Dense visual SLAM with 3d gaussian splatting*, Nov. 21, 2023. arXiv: 2311.11700[cs]. [Online]. Available: http://arxiv.org/abs/2311.11700 (visited on 12/26/2023) (cit. on p. iv).

[3] V. Yugay, Y. Li, T. Gevers, and M. R. Oswald, *Gaussian-SLAM: Photo-realistic dense SLAM with gaussian splatting*, Mar. 22, 2024. arXiv: 2312.10070[cs]. [Online]. Available: http://arxiv.org/abs/2312.10070 (visited on 03/27/2024) (cit. on p. iv).

[4] H. Matsuki, R. Murai, P. H. J. Kelly, and A. J. Davison, *Gaussian splatting SLAM*, Apr. 14, 2024. arXiv: 2312.06741[cs]. [Online]. Available: http://arxiv.org/abs/2312.06741 (visited on 05/20/2024) (cit. on p. iv).

[5] J. Engel, V. Koltun, and D. Cremers, “Direct sparse odometry,” in *arXiv:1607.02565*, Jul. 2016 (cit. on pp. xxvii–xxix).