Applied Machine Learning

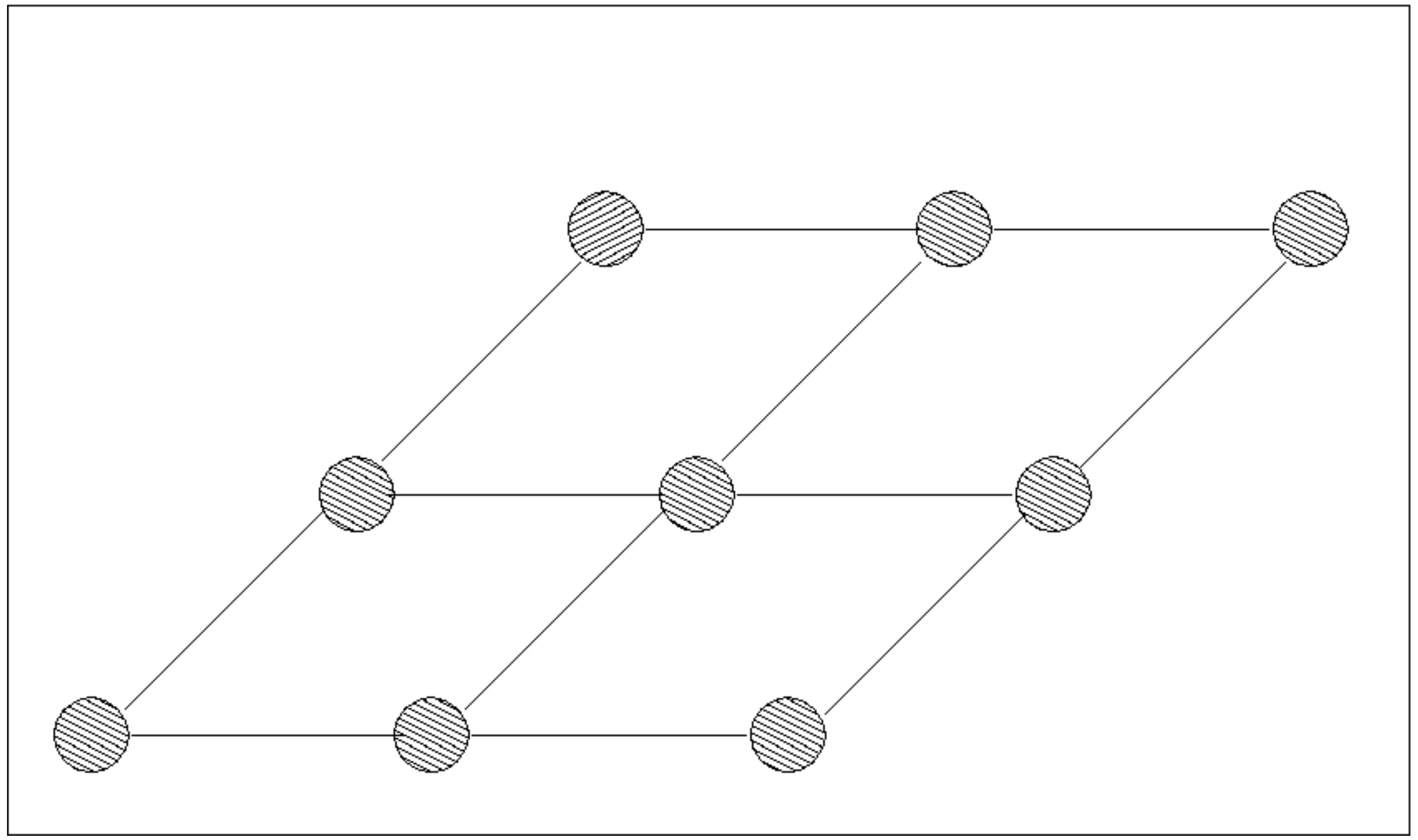
Mean Field Inference for Image Denoising

Mean Field Inference for Image Denoising

- Image Denoising as a Boltzmann Machine
- Optimization Problem
- Mean Field Inference

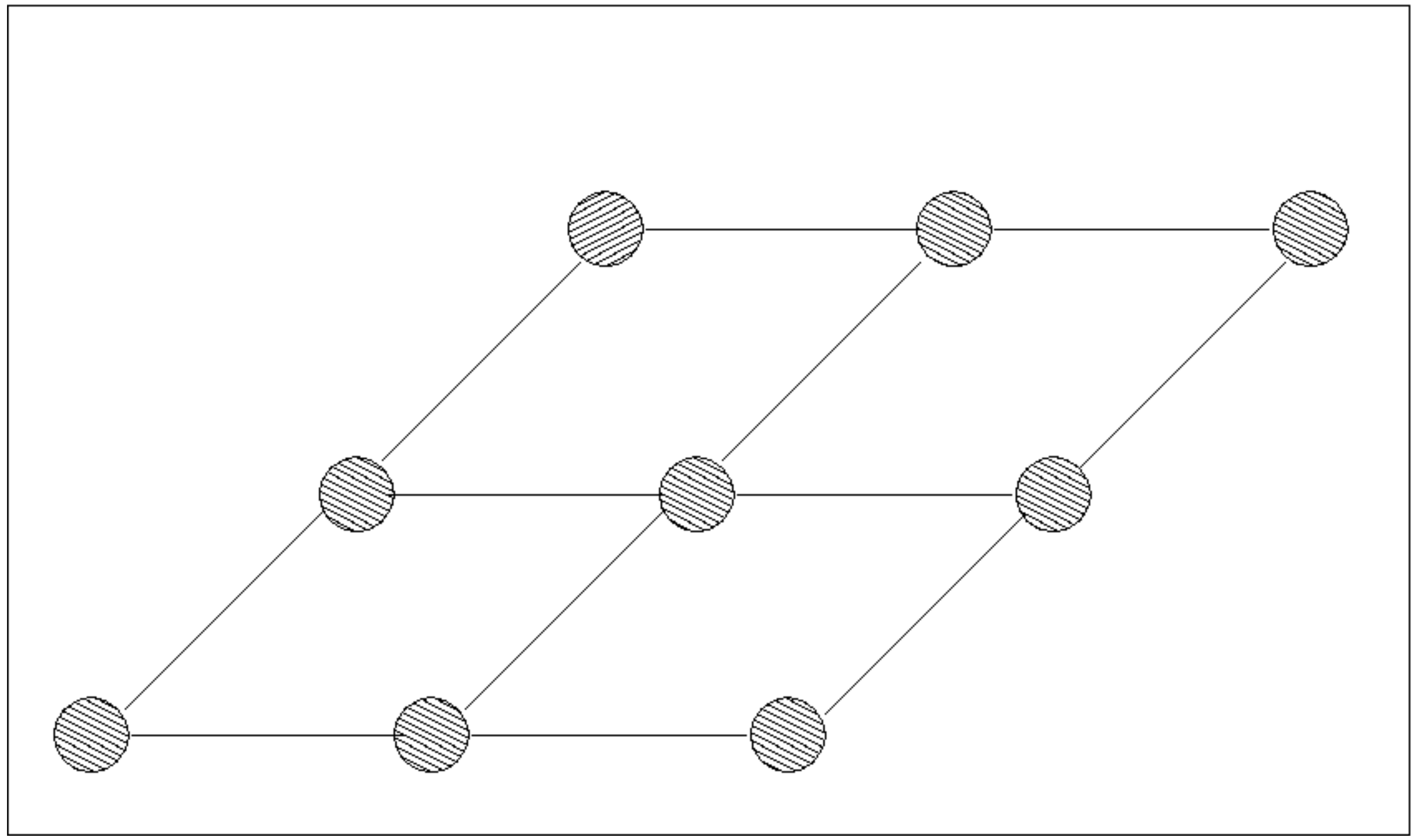
Binary Image

- Binary image
    - True pixel values [on,off]
        - Hidden nodes
        - Connections to adjacent nodes
            - local dependencies around each node
    - Noisy pixel values [on,off]
        - Observed nodes
        - Connections to associated true pixel node
            - Noise may flip the true pixel value



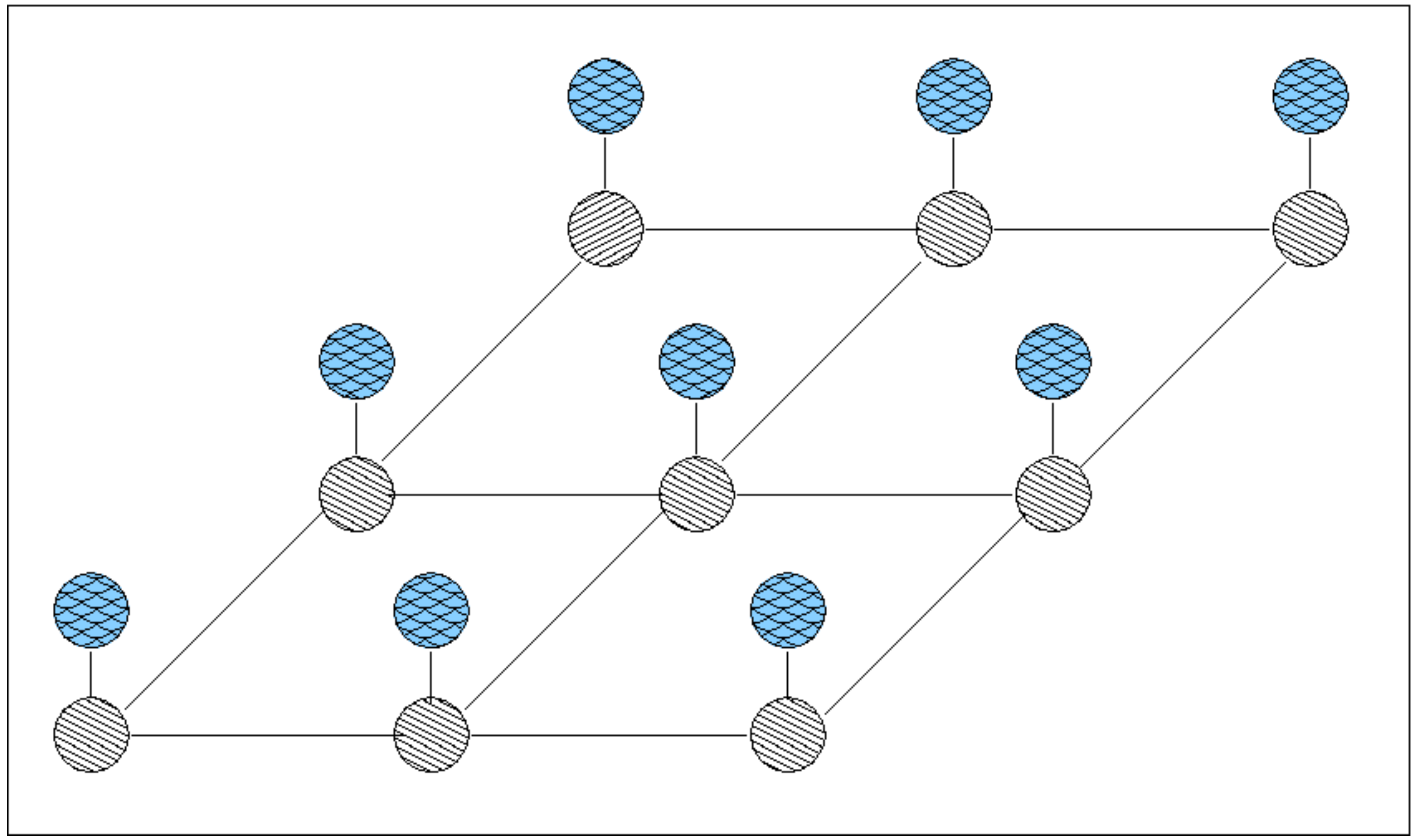
Boltzmann Machine for Denoising Images

- U: Set of nodes
  - Values $\{\text{off}, \text{on}\} \mapsto \{-1,1\}$
  - H: hidden nodes
    - H_i: true value of pixel i
    - Local connections to adjacent nodes
  - X: observed nodes
    - X_i: value of pixel i affected by noise
    - Only connected to H_i
- θ: Weights in the connections



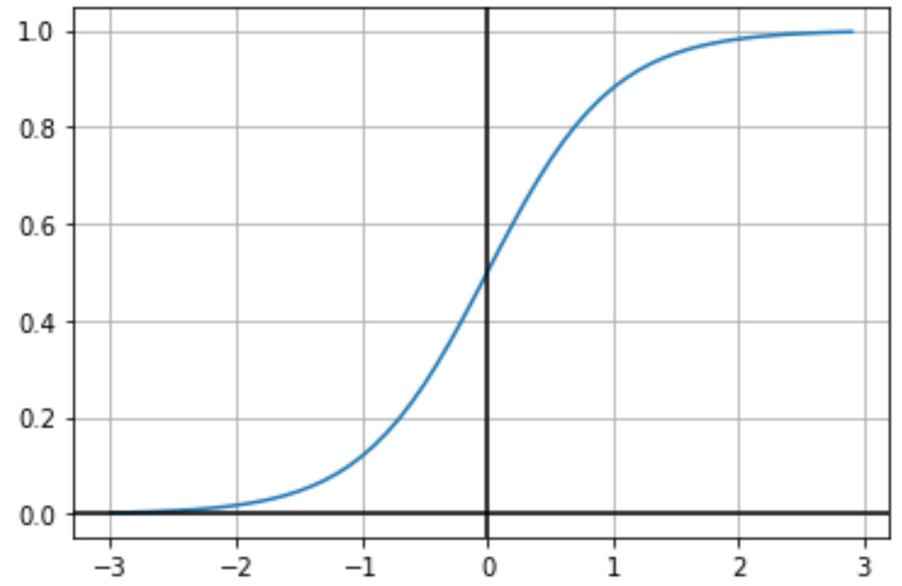
Boltzmann Machine for Denoising Images

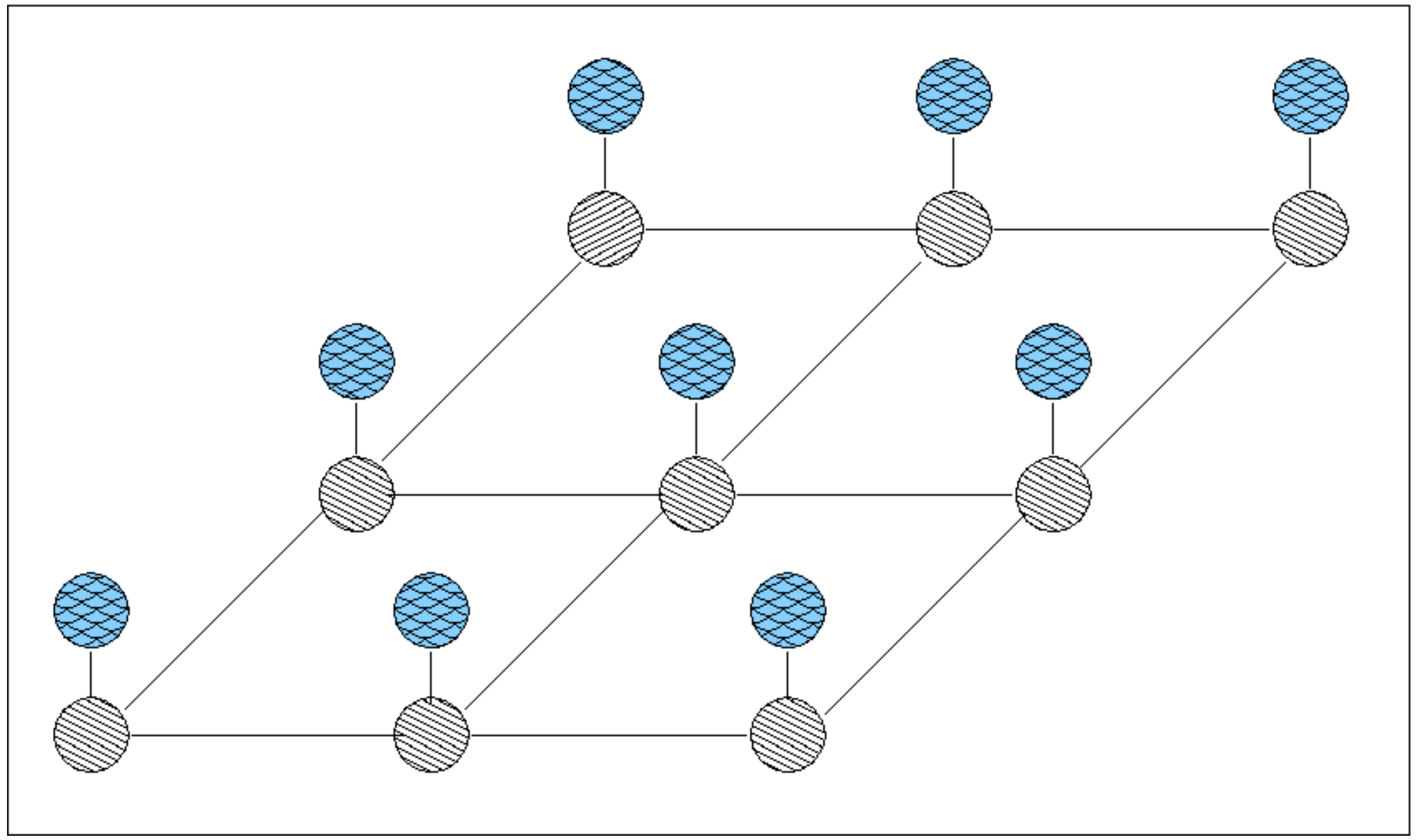
- U: Set of nodes $\{-1,1\}$
  - H: hidden nodes X: observed nodes
- θ: Weights in the connections
- Goal: find the values in hidden nodes that maximize
  - Maximum A Posteriori Inference
    - MAP Inference
    - $\log P(H|X,\theta) = -E(H,X|\theta) - \log \sum_{H} e^{-E(H,X|\theta)}$
  - Variational Inference
    - tractable distribution $Q(H;\theta)$
      - close to $P(H|X)$



Mean Field Inference

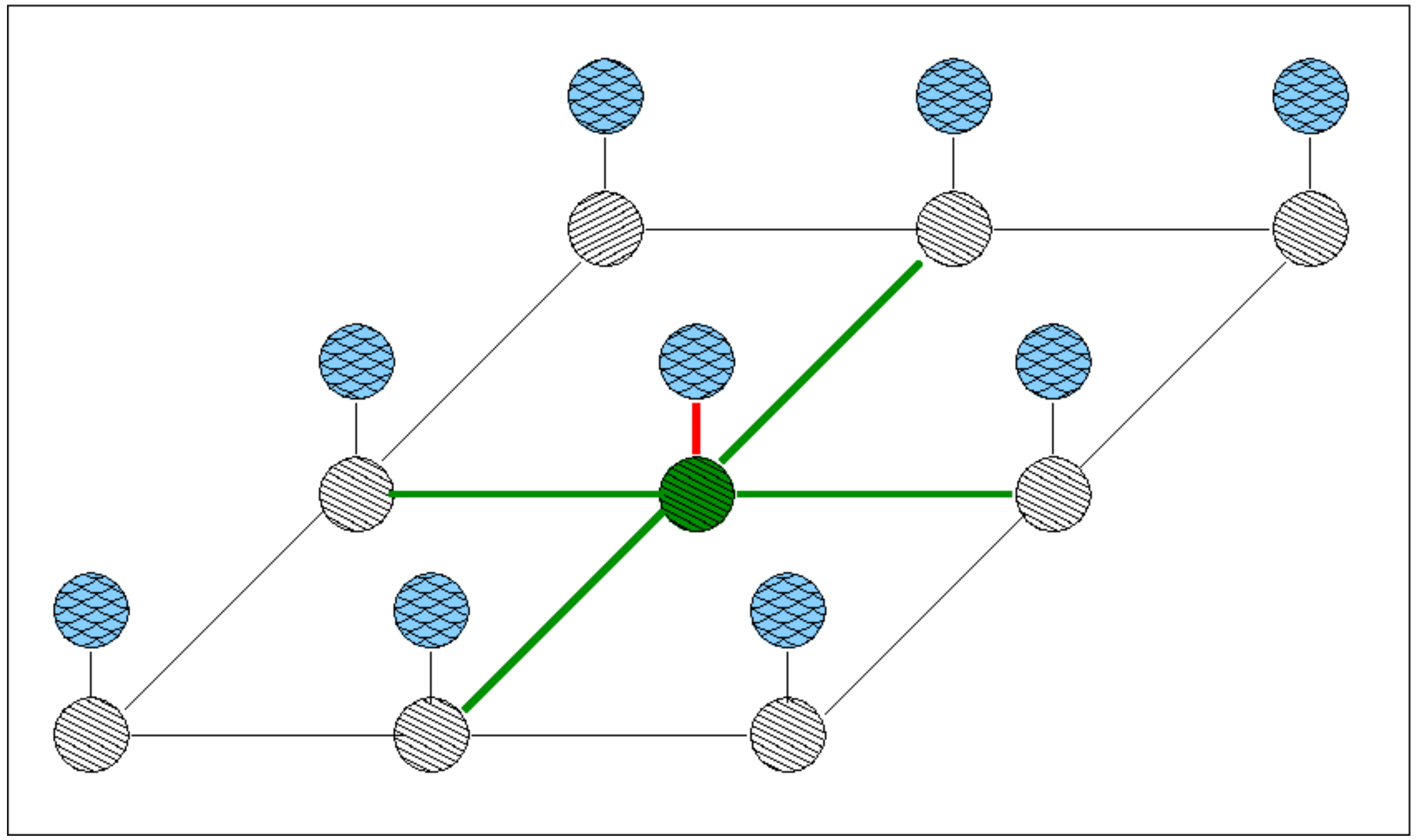
- U: Set of nodes $\{-1,1\}$
  - H: hidden nodes, X: observed nodes
- θ: Weights in the connections
- Goal: find the values in hidden nodes that maximize
  - $p_i = \sum_{j \in N(i) \cap H} \theta_{i,j}(2\pi_j - 1) + \theta_i^X X_j$
  - $\pi_i = \dfrac{e^{p_i}}{e^{p_i} + e^{-p_i}}$
  - $\theta_{i,j} = k$: positive constant to favor agreement between adjacent nodes
  - θ_i^X: based on noise. Probability of flipped pixel value:
    $p_{flip} = \dfrac{e^{-\theta_i^X}}{e^{-\theta_i^X} + e^{\theta_i^X}}$





Mean Field Inference for Denoising Images

- U: Set of nodes $\{-1,1\}$
  - H: hidden nodes, X: observed nodes
- θ: Weights in the connections
- Variational Inference. Mean Field Algorithm
  - Initialize π_i to random uniform distribution [0,1]
  - While (π_i's change more than some ϵ)
    - update each $\pi_i = \dfrac{e^{p_i}}{e^{p_i} + e^{-p_i}}$
      - with $p_i = \sum_{j \in N(i) \cap H} \theta_{i,j}(2\pi_j - 1) + \theta_i^X X_j$
  - Estimated $H_i = \begin{cases} 1 & \pi_i >= 0.5 \\ -1 & \pi_i < 0.5 \end{cases}$



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