

1. 小孔成像

2.

$$\begin{aligned} c' &= \frac{c}{n} \\ c_{\text{水}} &= \frac{c}{1.33} \\ c_{\text{冕牌玻璃}} &= \frac{c}{1.51} \\ c_{\text{火石玻璃}} &= \frac{c}{1.65} \\ c_{\text{加拿大树胶}} &= \frac{c}{1.526} \\ c_{\text{金刚石}} &= \frac{c}{2.417} \end{aligned}$$

3. 求像距  $s'_1$

$$\begin{aligned} \frac{h}{s} &= \frac{h'_1}{s'_1} = \frac{h'_2}{s'_2} \\ h'_1 &= 60\text{mm}, h'_2 = 70\text{mm}, s'_2 = s'_1 + 50\text{mm} \\ s'_1 &= \frac{h'_1 \times s'_2}{h'_2} = \frac{60\text{mm} \times (s'_1 + 50\text{mm})}{70\text{mm}} \\ s'_1 &= 300\text{mm} \end{aligned}$$

4. 因为全反射的条件是入射角大于临界角，所以

$$\begin{aligned} n \sin(\theta) &= n' \sin(\theta') \\ n = 1.5, n' = 1.0, \theta' &= \frac{\pi}{2} \\ \sin(\theta) &= \frac{2}{3} \\ w_{max} &= w + 2h \tan(\theta) = 1\text{mm} + 2 \times 200\text{mm} \tan(\arcsin(\frac{2}{3})) = 358.77\text{mm} \end{aligned}$$

5. 光疏到光密的界面，无法全反射

6. 设入射角为  $\theta$ ，入射介质折射率为  $n$ ，出射介质折射率为  $n'$ ，则有

$$n \sin(\theta) = n' \sin(\theta')$$

则出射平板的角也为  $\theta$ ，可知入射光线与出射光线平行。

7.

$$n \sin(\alpha) = \sin(\delta)$$

8. 光纤内需全反射，

$$\begin{aligned} \frac{n_2}{n_1} &= \sin(\theta_{min}) \\ n_0 \sin(I_1) &= n_1 \sin(\theta_{min}) = n_2 \end{aligned}$$

9.

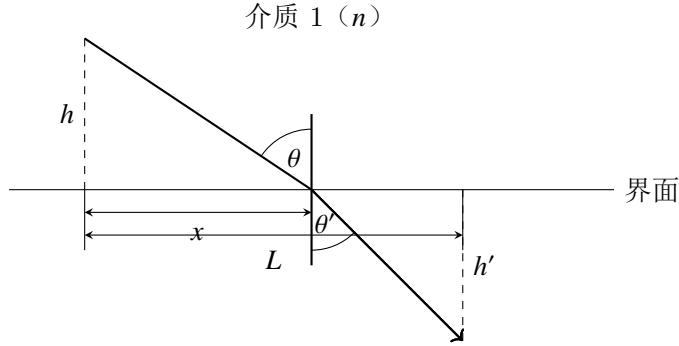
$$\sin(\theta_0) = n \sin(\theta'_0)$$

$$\frac{\pi}{4} = \theta'_0 + \theta_1$$

$$n \sin(\theta_1) \geq 1$$

$$0 \leq \theta_0 \leq 0.099$$

10. 费马证明折射



根据几何关系，有光程

$$\begin{aligned} S_1 &= n\sqrt{h^2 + x^2} \\ S_2 &= n'\sqrt{h'^2 + (L-x)^2} \\ S &= S_1 + S_2 = n\sqrt{h^2 + x^2} + n'\sqrt{h'^2 + (L-x)^2} \end{aligned}$$

由费马原理， $S$  取极值时，光线实际走过的路径。对  $x$  求导，有

$$\frac{dS}{dx} = \frac{nx}{\sqrt{h^2 + x^2}} - \frac{n'(L-x)}{\sqrt{h'^2 + (L-x)^2}} = 0$$

即

$$\begin{aligned} \frac{nx}{\sqrt{h^2 + x^2}} &= \frac{n'(L-x)}{\sqrt{h'^2 + (L-x)^2}} \\ n \sin(\theta) &= n \frac{x}{\sqrt{h^2 + x^2}} \\ n' \sin(\theta') &= n' \frac{L-x}{\sqrt{h'^2 + (L-x)^2}} \\ n \sin(\theta) &= n' \sin(\theta') \end{aligned}$$

11. 考虑光程差存在于反射段则

$$\begin{aligned} \tan(\theta) &= \frac{dy}{dx} \\ f &= y + \frac{x}{\tan(\theta)} \\ \frac{df}{dx} = 0 &= \frac{1+y'^2}{2y'} - \frac{xy''(1+y'^2)}{2y'^2} \end{aligned}$$

化简为

$$xy'' - y'^2 = 0$$

解得

$$y = C_1x^2 + C_2$$

即抛物线

12. 考虑光程和

$$c = n\sqrt{(x-l)^2 + y^2} + n'\sqrt{(x-l')^2 + y^2}$$

即笛卡尔卵形线

13.

$$\beta = \frac{y'}{y} = \frac{nl'}{n'l}$$

$$\begin{aligned}\frac{n'_k = n_{k+1}}{\beta = \frac{y'_1}{y_1} \cdot \frac{y'_2}{y_2} = \frac{n_1 l'_1}{n'_1 l_1} \cdot \frac{n_2 l'_2}{n'_2 l_2}} \\ \beta = \frac{n_1 l'_1 l'_2}{n'_2 l_1 l_2}\end{aligned}$$

14. 画图可知

$$2\theta = \theta' \quad (1)$$

所以考虑小角即  $\sin(\theta) \approx \theta$ , 有

$$n \sin(\theta) = \sin(\theta')$$

$$n\theta = 2\theta$$

$$n = 2$$

15. 入射和出射光线平行

16.

$$l_1 = -\infty, r = 30mm, n'_1 = 1.5, n_1 = 1.0$$

$$\begin{aligned}\frac{n'_1}{l'_1} - \frac{n_1}{l_1} &= \frac{n'_1 - n_1}{r} \\ l'_1 &= 90mm\end{aligned}$$

$$l_2 = l'_1 - 2r = 30mm, n_2 = n'_1, n'_2 = n_1$$

$$\begin{aligned}\frac{n'_2}{l'_2} - \frac{n_2}{l_2} &= \frac{n'_2 - n_2}{-r} \\ l'_2 &= 15mm \\ \beta &= \frac{l'_1 l'_2}{l_1 l_2} < 0\end{aligned}$$

为实像

$$l = -\infty, r = 30mm$$

$$\begin{aligned}\frac{1}{l'} + \frac{1}{l} &= \frac{2}{r} \\ l' &= 15mm\end{aligned}$$

为虚像

由第一个公式可知,  $l_2 = 30mm$

$$\begin{aligned}\frac{1}{l'_2} + \frac{1}{l_2} &= \frac{2}{-r} \\ l'_2 &= -10mm\end{aligned}$$

$$l_3 = l'_2 + 2r = 50mm, n_3 = n_2 = 1.5, n'_3 = n'_2 = 1$$

$$\begin{aligned}\frac{n'_3}{l'_3} - \frac{n_3}{l_3} &= \frac{n'_3 - n_3}{r} \\ l'_3 &= 75mm\end{aligned}$$

17.

$$\beta = \frac{nl'}{n'l}$$

$$r = 150mm, n = 1, n' = 1.5$$

$$n \left( \frac{1}{r} - \frac{1}{l} \right) = n' \left( \frac{1}{r} - \frac{1}{l'} \right)$$

$$\beta = \frac{2r}{2r+l}$$

$$l = -\infty, l' = 450mm, \beta = 0$$

$$l = -1000mm, l' = 642.8571mm, \beta = 0.23$$

$$l = -100mm, l' = -225mm, \beta = 0.75$$

$$l = 0mm, l' = 0mm, \beta = 1$$

$$l = 150mm, l' = 150mm, \beta = 2$$

$$l = 200mm, l' = 180mm, \beta = 3$$

## 18. 中心气泡

$$l = r = -200mm, n = 1.5, n' = 1$$

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n' - n}{r}$$

$$l' = -200mm$$

1/2 半径气泡

$$l = -300mm, r = -200mm, n = 1.5, n' = 1$$

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n' - n}{r}$$

$$l' = -400mm$$

换一边观察

$$l = -100mm, r = -200mm, n = 1.5, n' = 1$$

$$l' = -80mm$$

## 19.

$$l_1 = -\infty, r_1 = 100mm, r_2 = \infty, n_1 = 1, n'_1 = n_2 = 1.5, n'_2 = 1$$

$$\frac{n'_1}{l'_1} - \frac{n_1}{l_1} = \frac{n'_1 - n_1}{r_1} = \frac{n'_1}{l'_1}$$

$$l'_1 = 300mm, l_2 = l'_1 - d = 0mm$$

$$l'_2 = 0mm$$

在第二面的十字线上

$$l_1 = -d = -300mm, r = -100mm, n = 1.5, n' = 1$$

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n' - n}{r}$$

$$l' = \infty$$

 $h = 10mm$  时不能认为是高斯成像

$$\sin(\theta_1) = \frac{10mm}{300mm}$$

$$n_1 \sin(\theta_1) = n'_1 \sin(\theta'_1)$$

$$\frac{\sin(\theta'_1 - \theta_1)}{r} = \frac{\sin(\theta'_1)}{l'_1 - r}$$

$$\theta_2 = \theta'_1 - \theta_1$$

$$n'_1 \sin(\theta_2) = n_1 \sin(\theta'_2)$$

$$l_2 = l'_1 - d$$

$$h_2 = l_2 \tan(-\theta_2)$$

$$l'_2 = \frac{h_2}{\tan(-\theta'_2)}$$

20.

$$r = -100mm, \beta = -\frac{l'}{l}, \frac{1}{l'} + \frac{1}{l} = \frac{2}{r}$$

$$\beta = 0, l = -\infty, l' = -50mm$$

$$\beta = -0.1, l = -550mm, l' = -55mm$$

$$\beta = -0.2, l = -300mm, l' = -60mm$$

$$\beta = -1, l = -100mm, l' = -100mm$$

$$\beta = 1, l = 0mm, l' = 0mm$$

$$\beta = 5, l = -40mm, l' = 200mm$$

$$\beta = 10, l = -40mm, l' = 450mm$$

$$\beta = \infty, l = -50mm, l' = \infty$$

21.  $\beta < 0$  时虚实相同,  $\beta > 0$  时虚实相反

$$\beta = -4, l = \frac{8}{5r}, l' = \frac{5}{2r}$$

$$\beta = 4, l = \frac{8}{3r}, l' = -\frac{3}{2r}$$

$$\beta = -\frac{1}{4}, l = \frac{5}{2r}, l' = \frac{8}{5r}$$

$$\beta = \frac{1}{4}, l = -\frac{3}{2r}, l' = \frac{3}{8r}$$

22. 考虑几何关系, 只有入射第一面的像成像于反射面上, 出射光线与入射光线平行

$$l_1 = -\infty, l'_1 = 2r$$

$$\frac{n}{l'_1} - \frac{1}{l_1} = \frac{n-1}{r}$$

$$n = 2$$

23.

$$\theta = (2n-1)\alpha \quad (2)$$