1. 小孔成像
2.

$$c' = \frac{c}{n}$$
$$c_{水} = \frac{c}{1.33}$$
$$c_{冕牌玻璃} = \frac{c}{1.51}$$
$$c_{火石玻璃} = \frac{c}{1.65}$$
$$c_{加拿大树胶} = \frac{c}{1.526}$$
$$c_{金刚石} = \frac{c}{2.417}$$

3. 求像距 $s_1^{'}$

$$\frac{h}{s} = \frac{h_1^{'}}{s_1^{'}} = \frac{h_2^{'}}{s_2^{'}}$$
$$h_1^{'} = 60mm, h_2^{'} = 70mm, s_2^{'} = s_1^{'} + 50mm$$
$$s_1^{'} = \frac{h_1^{'} \times s_2^{'}}{h_2^{'}} = \frac{60mm \times (s_1^{'} + 50mm)}{70mm}$$
$$s_1^{'} = 300mm$$

4. 因为全反射的条件是入射角大于临界角，所以

$$n\sin(\theta) = n'\sin(\theta')$$
$$n = 1.5, n' = 1.0, \theta' = \frac{\pi}{2}$$
$$\sin(\theta) = \frac{2}{3}$$
$$w_{max} = w + 2h\tan(\theta) = 1mm + 2 \times 200mm\tan(\arcsin(\frac{2}{3})) = 358.77mm$$

5. 光疏到光密的界面，无法全反射
6. 设入射角为 θ，入射介质折射率为 n，出射介质折射率为 n'，则有

$$n\sin(\theta) = n'\sin(\theta')$$

则出射平板的角也为 θ，可知入射光线与出射光线平行。

7.

$$n\sin(\alpha) = \sin(\beta)$$
$$\beta - \alpha = \delta \approx (n-1)\alpha$$

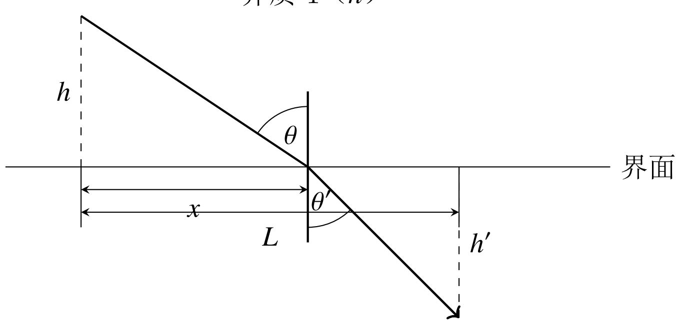
8. 光纤内需全反射，

$$n_0\sin(I_1) = n_1\sin(I_2)$$
$$n_1\sin(\frac{\pi}{2} - I_2) = n_2$$
$$n_0\sin(I_1) = \sqrt{n_1^2 - n_2^2}$$

9.

$$\sin(\theta_0) = n\sin(\theta_0^{'})$$
$$\frac{\pi}{4} = \theta_0^{'} + \theta_1$$
$$n\sin(\theta_1) \geq 1$$
$$0 \leq \theta_0 \leq 0.099$$

10. 费马证明折射



根据几何关系，有光程

$$S_1 = n\sqrt{h^2 + x^2}$$
$$S_2 = n'\sqrt{h'^2 + (L - x)^2}$$
$$S = S_1 + S_2 = n\sqrt{h^2 + x^2} + n'\sqrt{h'^2 + (L - x)^2}$$

由费马原理，S 取极值时，光线实际走过的路径。对 x 求导，有

$$\frac{dS}{dx} = \frac{nx}{\sqrt{h^2 + x^2}} - \frac{n'(L - x)}{\sqrt{h'^2 + (L - x)^2}} = 0$$

即

$$\frac{nx}{\sqrt{h^2 + x^2}} = \frac{n'(L - x)}{\sqrt{h'^2 + (L - x)^2}}$$
$$n\sin(\theta) = n\frac{x}{\sqrt{h^2 + x^2}}$$
$$n'\sin(\theta') = n'\frac{L - x}{\sqrt{h'^2 + (L - x)^2}}$$
$$n\sin(\theta) = n'\sin(\theta')$$

11. 考虑光程差存在于反射段则

$$\tan(\theta) = \frac{dy}{dx}$$
$$f = y + \frac{x}{\tan(\theta)}$$
$$\frac{df}{dx} = 0 = \frac{1 + y'^2}{2y'} - \frac{xy''(1 + y'^2)}{2y'^2}$$

化简为

$$xy'' - y'^2 = 0$$

解得

$$y = C_1 x^2 + C_2$$

即抛物线

12. 考虑光程和

$$c = n\sqrt{(x - l)^2 + y^2} + n'\sqrt{(x - l')^2 + y^2}$$

即笛卡尔卵形线

13.

$$\beta = \frac{y'}{y} = \frac{nl'}{n'l}$$

$$n'_k = n_{k+1}$$
$$\beta = \frac{y'_1}{y_1} \cdot \frac{y'_2}{y_2} = \frac{n_1 l'_1}{n'_1 l_1} \cdot \frac{n_2 l'_2}{n'_2 l_2}$$
$$\beta = \frac{n_1 l'_1 l'_2}{n'_2 l_1 l_2}$$

14. 画图可知

$$2\theta = \theta' \tag{1}$$

所以考虑小角即 $\sin(\theta) \approx \theta$，有

$$n \sin(\theta) = \sin(\theta')$$
$$n\theta = 2\theta$$
$$n = 2$$

15. 入射和出射光线平行

16.

$$l_1 = -\infty, r = 30mm, n'_1 = 1.5, n_1 = 1.0$$
$$\frac{n'_1}{l'_1} - \frac{n_1}{l_1} = \frac{n'_1 - n_1}{r}$$
$$l'_1 = 90mm$$
$$l_2 = l'_1 - 2r = 30mm, n_2 = n'_1, n'_2 = n_1$$
$$\frac{n'_2}{l'_2} - \frac{n_2}{l_2} = \frac{n'_2 - n_2}{-r}$$
$$l'_2 = 15mm$$
$$\beta = \frac{l'_1 l'_2}{l_1 l_2} < 0$$

为实像

$$l = -\infty, r = 30mm$$
$$\frac{1}{l'} + \frac{1}{l} = \frac{2}{r}$$
$$l' = 15mm$$

为虚像

由第一个公式可知，$l_2 = 30mm$

$$\frac{1}{l'_2} + \frac{1}{l_2} = \frac{2}{-r}$$
$$l'_2 = -10mm$$
$$l_3 = l'_2 + 2r = 50mm, n_3 = n_2 = 1.5, n'_3 = n'_2 = 1$$
$$\frac{n'_3}{l'_3} - \frac{n_3}{l_3} = \frac{n'_3 - n_3}{r}$$
$$l'_3 = 75mm$$

17.

$$\beta = \frac{nl'}{n'l}$$
$$r = 150mm, n = 1, n' = 1.5$$
$$n\left(\frac{1}{r} - \frac{1}{l}\right) = n'\left(\frac{1}{r} - \frac{1}{l'}\right)$$

$$\beta = \frac{2r}{2r+l}$$

$$l = -\infty, l' = 450mm, \beta = 0$$

$$l = -1000mm, l' = 642.8571mm, \beta = 0.23$$

$$l = -100mm, l' = -225mm, \beta = 0.75$$

$$l = 0mm, l' = 0mm, \beta = 1$$

$$l = 150mm, l' = 150mm, \beta = 2$$

$$l = 200mm, l' = 180mm, \beta = 3$$

18. 中心气泡

$$l = r = -200mm, n = 1.5, n' = 1$$

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{r}$$

$$l' = -200mm$$

1/2 半径气泡

$$l = -300mm，r = -200mm, n = 1.5, n' = 1$$

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{r}$$

$$l' = -400mm$$

换一边观察

$$l = -100mm, r = -200mm, n = 1.5, n' = 1$$

$$l' = -80mm$$

19.

$$l_1 = -\infty, r_1 = 100mm, r_2 = \infty, n_1 = 1, n_1' = n_2 = 1.5, n_2' = 1$$

$$\frac{n_1'}{l_1'} - \frac{n_1}{l_1} = \frac{n_1'-n_1}{r_1} = \frac{n_1'}{l_1'}$$

$$l_1' = 300mm, l_2 = l_1' - d = 0mm$$

$$l_2' = 0mm$$

在第二面的十字线上

$$l_1 = -d = -300mm, r = -100mm, n = 1.5, n' = 1$$

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{r}$$

$$l' = \infty$$

$h = 10mm$ 时不能认为是高斯成像

$$\sin(\theta_1) = \frac{10mm}{300mm}$$

$$n_1 \sin(\theta_1) = n_1' \sin(\theta_1')$$

$$\frac{\sin(\theta_1' - \theta_1)}{r} = \frac{\sin(\theta_1')}{l_1' - r}$$

$$\theta_2 = \theta_1' - \theta_1$$

$$n_1' \sin(\theta_2) = n_1 \sin(\theta_2')$$

$$l_2 = l_1' - d$$

$$h_2 = l_2 \tan(-\theta_2)$$

$$l_2' = \frac{h_2}{\tan(-\theta_2')}$$

20.

$$r = -100mm, \beta = -\frac{l'}{l}, \frac{1}{l'} + \frac{1}{l} = \frac{2}{r}$$

$$\beta = 0, l = -\infty, l' = -50mm$$

$$\beta = -0.1, l = -550mm, l' = -55mm$$

$$\beta = -0.2, l = -300mm, l' = -60mm$$

$$\beta = -1, l = -100mm, l' = -100mm$$

$$\beta = 1, l = 0mm, l' = 0mm$$

$$\beta = 5, l = -40mm, l' = 200mm$$

$$\beta = 10, l = -40mm, l' = 450mm$$

$$\beta = \infty, l = -50mm, l' = \infty$$

21. $\beta < 0$ 时虚实相同，$\beta > 0$ 时虚实相反

$$\beta = -4, l = \frac{5}{8r}, l' = \frac{5}{2r}$$

$$\beta = 4, l = \frac{3}{8r}, l' = -\frac{3}{2r}$$

$$\beta = -\frac{1}{4}, l = \frac{5}{2r}, l' = \frac{8}{5r}$$

$$\beta = \frac{1}{4}, l = -\frac{3}{2r}, l' = \frac{3}{8r}$$

22. 考虑几何关系，只有入射第一面的像成像于反射面上，出射光线与入射光线平行

$$l_1 = -\infty, l_1' = 2r$$

$$\frac{n}{l_1'} - \frac{1}{l_1} = \frac{n-1}{r}$$

$$n = 2$$

23.

$$\theta = (2n-1)\alpha \tag{2}$$