APPENDIX C
INTEGRAL IDENTITY

To derive the approximate cdf of $\gamma_{r_{m,b},d_m}$, we define the integral $\mathcal{G}(a,b;z)$ as

$$\mathcal{G}(a,b;z) \triangleq \int_0^\infty e^{-zy} y^{a-1}(1+y)^b dy, \qquad \Re(a)>0, \Re(z)>0. \tag{30}$$

Then, the integral $\mathcal{G}(a,b;z)$ can be rewritten as

$$\begin{aligned}\mathcal{G}(a,b;z) &= (a-1)!\Psi(a,a+b+1;z)\\ &\overset{(a)}{=} z^{-a}(a-1)!{}_2F_0(a,-b;-z^{-1})\\ &= z^{-a}(a-1)!\sum_{n=0}^{\infty}(a)_n(-b)_n\frac{(-z^{-1})^n}{n!}\\ &\overset{(a)}{=} z^{-a}(a-1)!\sum_{n=0}^{b}\frac{(a+n-1)!}{(a-1)!}\frac{(-1)^n b!}{(b-n)!}\frac{(-1)^n z^{-n}}{n!}\\ &= z^{-a}\sum_{n=0}^{b}\binom{b}{n}z^{-n}(a+n-1)! \end{aligned} \tag{31}$$

where $\Psi(\alpha,\gamma;z)$ is the confluent hypergeometric function of the second kind defined in [14, eq. (9.211.4)], ${}_2F_0(\alpha,\beta;z)$ is the hypergeometric function defined in [14, eq. (9.14.1)], and $(a)_n$ is a Pochhammer symbol defined as $(a)_n = a(a+1)\ldots(a+n-1) = \Gamma(a+n)/\Gamma(a)$ for nonnegative integer a and n. In (31), (a) follows from the identity ${}_2F_0(\alpha,\beta;-x^{-1}) = x^{\alpha}\Psi(\alpha,\alpha-\beta+1;x)$ in [15, eq. (6.6.3)], and (b) follows from the fact that

$$(-b)_n = \begin{cases}\frac{(-1)^n b!}{(b-n)!}, & 0 \le n \le b\\ 0, & n > b\end{cases} \tag{32}$$

for a nonnegative integer b.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their careful reading and insightful comments.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.
[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
[3] A. Nostratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74–80, Oct. 2004.
[4] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.
[5] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450–3460, Sep. 2007.
[6] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.
[7] G. Scutari and S. Barbarossa, "Distributed space-time coding for regenerative relay networks," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2387–2399, Sep. 2005.
[8] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.
[9] D. Lee and J. H. Lee, "Outage probability for opportunistic relaying on multicell environments," in *Proc. IEEE VTC 2009—Spring*, Barcelona, Spain, Apr. 2009, pp. 1–5.
[10] I. Krikidis, J. S. Thompson, S. McLaughlin, and N. Goertz, "Max-min relay selection for legacy amplify-and-forward systems with interference," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3016–3027, Jun. 2009.
[11] C. Zhong, S. Jin, and K.-K. Wong, "Dual-hop systems with noisy relay and interference-limited destination," *IEEE Trans. Commun.*, vol. 58, no. 3, pp. 764–768, Mar. 2010.
[12] J. Si, Z. Li, and Z. Liu, "Outage probability of opportunistic relaying in Rayleigh fading channels with multiple interferers," *IEEE Signal Process. Lett.*, vol. 17, no. 5, pp. 445–448, May 2010.
[13] H. Shin and M. Z. Win, "MIMO diversity in the presence of double scattering," *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 2976–2996, Jul. 2008.
[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. New York: Academic, 2007.
[15] A. Erdelyi, *Higher Transcendental Functions*, vol. 1. New York: McGraw-Hill, 1953.

Low-Complexity Groupwise OSIC-ZF Detection for $N \times N$ Spatial Multiplexing Systems

Yinman Lee, *Member, IEEE*, and Hong-Wei Shieh

***Abstract*—In this paper, we present a low-complexity groupwise ordered successive interference canceler (OSIC) with the zero-forcing (ZF) criterion for $N \times N$ spatial multiplexing systems. The proposed detection is composed of a number of processing stages, and at each stage, a fraction of the transmitted data streams are nulled to zero. In this way, the original matrix-inverse operations are replaced by a series of inversions with smaller sizes, and the computational complexity can then be largely reduced. We show that this groupwise OSIC-ZF approach can provide a good tradeoff between the computational complexity and the error-rate performance and, therefore, is very attractive as a new detection method standing between the linear ZF detection and the optimal OSIC-ZF detection.**

***Index Terms*—Groupwise ordered successive interference canceler (OSIC), multiple-input multiple-output (MIMO), spatial multiplexing, zero-forcing (ZF) criterion.**

I. INTRODUCTION

In recent years, much attention has been paid to the development of multiple-input–multiple-output (MIMO) systems for wireless communications. With multiple antennas at both the transmitter and the

Manuscript received August 9, 2010; revised November 23, 2010 and January 26, 2011; accepted February 9, 2011. Date of publication March 14, 2011; date of current version May 16, 2011. This work was supported by the National Science Council, Taiwan, under Grant NSC 99-2221-E-260-025. The review of this paper was coordinated by Dr. C. Cozzo.

The authors are with the Graduate Institute of Communication Engineering, National Chi Nan University, Nantou 54561, Taiwan (e-mail: ymlee@ncnu.edu.tw; s96325902@ncnu.edu.tw).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2011.2127499

receiver, pioneering work has indicated a remarkable improvement in terms of throughput capacity [1], [2]. To this end, spatial multiplexing has been proven to be a promising technique for approaching this theoretical limit [3], and different detection algorithms were then studied to provide different levels of system performance in the literature [4], [5]. One of the most commonly used strategies for extracting the spatially multiplexed data streams from different transmit antennas is to employ a matrix-inverse-based zero-forcing (ZF) detector at the receiver, which is attractive for its linear implementation, low computational complexity, and no need to estimate the average noise power. However, its error-rate performance is often not acceptable for many applications. To remedy this disadvantage, the Vertical-Bell Laboratories Layered Space-Time (V-BLAST) architecture has been proposed [6], in which the data streams are detected using nulling and successive interference cancelation (SIC) at each layer. To optimize the resultant error-rate performance, the SIC process should be ordered [7]. If the ZF criterion is used for nulling, the whole scheme will be referred to as the ordered SIC (OSIC)-ZF detection. Despite the error-rate improvement, it is well understood that the computational complexity required for the OSIC-ZF detection is rather high due to the successive matrix-inverse operations. To overcome this issue, several reduced-complexity OSIC schemes have recently been presented [8]–[12]. The primary focus of those works is on enhancing the calculation for the involved successive matrix inversions, and some complexity reduction is recorded.

In this paper, to take a different approach, we formulate a new interference cancelation method based on the ZF criterion. A specially designed groupwise OSIC-ZF detector for $N \times N$ systems with $N = 2^M$ and $M \geq 2$ is considered, which can provide a good tradeoff between the computational complexity and error-rate performance. Specifically, the detection is composed of a number of processing stages, and at each stage, a fraction of the transmitted data streams are nulled to zero. In this way, the original matrix-inverse operations are replaced by a series of inversions with smaller sizes, and the computational complexity can then be largely reduced. We show that the total number of floating-point operations (flops) required for the proposed detection is comparable with that for the linear ZF detection and is significantly lower than that for the optimal OSIC-ZF detection. At the same time, the bit error rate (BER) for the proposed detection is much smaller than that for the linear ZF detection and is essentially located in between that for the linear ZF detection and the optimal OSIC-ZF detection.

Throughout this correspondence, normal letters represent scalar quantities, boldface lowercase letters indicate vectors, and boldface uppercase letters designate matrices. In addition, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, and $(\cdot)^\dagger$ represent the transpose, conjugate transpose, matrix inverse, and pseudomatrix inverse, respectively.

II. Signal Model, Linear Zero-Forcing Detection, and Optimal Ordered Successive Interfence Cancellation–Zero-Forcing Detection

Consider a wireless communication system with N antennas at the transmitter and N antennas at the receiver, assuming $N = 2^M$ and $M \geq 2$. We define $h_{n_r,n_t}(k)$ to represent the (complex and possibly time varying) flat-fading channel response from transmit antenna n_t to receive antenna n_r at time instant k, with $n_t, n_r = 1, 2, \ldots, N$. Then, the complete $N \times N$ channel matrix can be presented as

$$\mathbf{H}(k) = \begin{bmatrix} h_{1,1}(k) & h_{1,2}(k) & \cdots & h_{1,N}(k) \\ h_{2,1}(k) & h_{2,2}(k) & \cdots & h_{2,N}(k) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N,1}(k) & h_{N,2}(k) & \cdots & h_{N,N}(k) \end{bmatrix}. \quad (1)$$

The spatially multiplexed data symbols from N transmit antennas can be collected into an $N \times 1$ complex-valued vector, which is denoted as $\mathbf{d}(k) = [d_1(k)\ d_2(k)\ \cdots\ d_N(k)]^T$. They are sent over the $N \times N$ channel environment. The equivalent baseband signals obtained at the receiver with multiple antennas also yield another $N \times 1$ complex-valued vector $\mathbf{r}(k) = [r_1(k)\ r_2(k)\ \cdots\ r_N(k)]^T$, which is given by

$$\mathbf{r}(k) = \mathbf{H}(k)\mathbf{d}(k) + \mathbf{z}(k) \quad (2)$$

where $\mathbf{z}(k)$ is an $N \times 1$ complex Gaussian noise vector with zero mean and equal variance in each dimension. In addition, all noise components are assumed to be independent.

The linear ZF detection is perhaps the simplest way to recover the transmitted data symbols in $\mathbf{d}(k)$. With the assumption of perfect channel state information at the receiving side, the inverse of the channel matrix, which is denoted as $(\mathbf{H}(k))^{-1}$, can be calculated, and the so-called "ZF" action can be utilized to obtain

$$\hat{\mathbf{d}}(k) = \mathcal{Q}\left[(\mathbf{H}(k))^{-1}\mathbf{r}(k)\right] \quad (3)$$

where $\mathcal{Q}[\cdot]$ accounts for the quantizer that maps its argument to the nearest signal point in the constellation, and $\hat{\mathbf{d}}(k)$ contains the output decisions for the original spatially multiplexed data symbols. For completeness, we also briefly review the V-BLAST architecture implemented by the optimal OSIC-ZF detection. The idea of this nonlinear interference-canceling approach is borrowed from the multiuser detection context [13]. It starts with nulling only one data stream by the ZF criterion. That particular data stream is selected since it owns the highest postdetection signal-to-noise ratio (SNR). Then, the effect of the detected data stream is subtracted from the received signal vector, resulting in a modified received signal vector with less of an "interferer." This process proceeds until all data streams are detected. Detailed procedure and performance of this optimal OSIC-ZF detection can be found in [6]. For comparison, the required computational complexity for both the linear ZF detection and the optimal OSIC-ZF detection will be revealed later in Section IV.

III. Proposed Low-Complexity Groupwise Ordered Successive Interfence Cancellation–Zero-Forcing Detection

In this section, we address a new groupwise OSIC-ZF detector that can provide a good compromise between the computational complexity and the error-rate performance. In the following derivation, we drop the index k in (1)–(3) to simplify the notation. Typically, in the $N \times N$ circumstance (again, with $N = 2^M$ and $M \geq 2$), the detection can be divided into M forward stages and M backward stages. The detailed operations of these stages are explained here.

In the beginning, the first forward stage comes. The original N-dimensional received signal vector in (2) is divided into two parts, i.e., the upper part and the lower part, each with length $N/2$ (or 2^{M-1}). The lower part is then multiplied by a set of canceling weights, and the last $N/2$ data symbols in the upper part of the received signal vector are canceled. Next, the equivalent channel responses are calculated, and a reduced-dimension linear system is reconstructed. This linear system is $N/2 \times N/2$ in dimension, which contains the transmitted data symbols $[d_1\ d_2\ \cdots\ d_{N/2}]^T$. The process of the first forward stage is shown in the left part of Fig. 1. To clarify the idea, we rewrite the received signal vector $\mathbf{r}$ in (2) as that in (4),

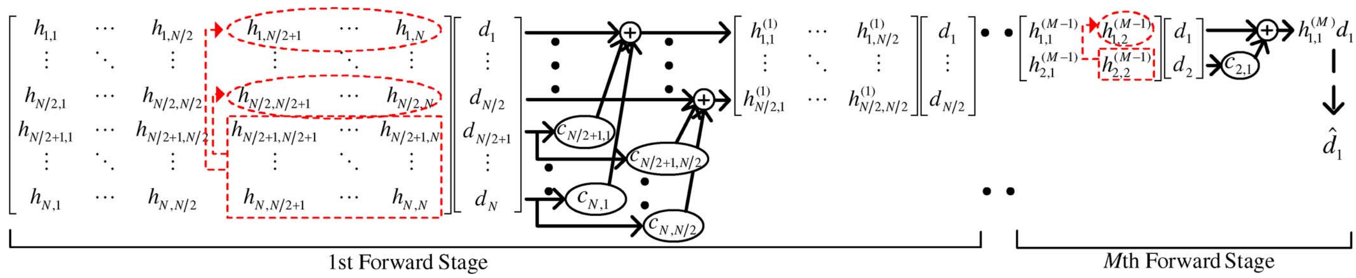

Fig. 1. M forward stages for the proposed groupwise OSIC-ZF detection.

shown at the bottom of the page. We introduce a set of canceling weights for $r_{N/2+1}$ to r_N in (4). The first part of these weights can be set to satisfy

$$\begin{bmatrix} h_{\frac{N}{2}+1,\frac{N}{2}+1} & \cdots & h_{N,\frac{N}{2}+1} \\ \vdots & \ddots & \vdots \\ h_{\frac{N}{2}+1,N} & \cdots & h_{N,N} \end{bmatrix} \begin{bmatrix} c_{\frac{N}{2}+1,1} \\ \vdots \\ c_{N,1} \end{bmatrix} = \begin{bmatrix} h_{1,\frac{N}{2}+1} \\ \vdots \\ h_{1,N} \end{bmatrix}. \quad (5)$$

It is not difficult to see that the weights $[c_{N/2+1,1} \ \cdots \ c_{N,1}]^T$ in (5) are designed to cancel $d_{N/2+1}-d_N$ in r_1, as partly illustrated in Fig. 1. Other parts of the weights can be set in a similar manner for the cancelation of $d_{N/2+1}-d_N$ in $r_2-r_{N/2}$, respectively. Combining all this kind of weights, we obtain the expression

$$\begin{bmatrix} h_{\frac{N}{2}+1,\frac{N}{2}+1} & \cdots & h_{N,\frac{N}{2}+1} \\ \vdots & \ddots & \vdots \\ h_{\frac{N}{2}+1,N} & \cdots & h_{N,N} \end{bmatrix} \begin{bmatrix} c_{\frac{N}{2}+1,1} & \cdots & c_{\frac{N}{2}+1,\frac{N}{2}} \\ \vdots & \ddots & \vdots \\ c_{N,1} & \cdots & c_{N,\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} h_{1,\frac{N}{2}+1} & \cdots & h_{\frac{N}{2},\frac{N}{2}+1} \\ \vdots & \ddots & \vdots \\ h_{1,N} & \cdots & h_{\frac{N}{2},N} \end{bmatrix}. \quad (6)$$

From (6), we see that these canceling weights can be determined through solving an $N/2 \times N/2$ linear system of equations with $N/2$ vectors of unknown variables. After finding the weights and performing the canceling action, we possess a new $N/2 \times N/2$ system involving only $d_1-d_{N/2}$ as

$$\begin{bmatrix} r_1^{(1)} \\ \vdots \\ r_{\frac{N}{2}}^{(1)} \end{bmatrix} = \begin{bmatrix} h_{1,1}^{(1)} & \cdots & h_{1,\frac{N}{2}}^{(1)} \\ \vdots & \ddots & \vdots \\ h_{\frac{N}{2},1}^{(1)} & \cdots & h_{\frac{N}{2},\frac{N}{2}}^{(1)} \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_{\frac{N}{2}} \end{bmatrix} + \begin{bmatrix} z_1^{(1)} \\ \vdots \\ z_{\frac{N}{2}}^{(1)} \end{bmatrix} \quad (7)$$

in which $r_1^{(1)}-r_{N/2}^{(1)}$ denote the new constructed signal components. The notation $(\cdot)^{(m)}$ is used to denote the signal obtained from the mth forward stage, with $m = 1, 2, \ldots, M$. In (7), also note that $h_{1,1}^{(1)}$ to $h_{N/2,1}^{(1)}$ to $h_{1,N/2}^{(1)}$ to $h_{N/2,N/2}^{(1)}$ are the reconstructed channel responses, and $z_1^{(1)}-z_{N/2}^{(1)}$ are the equivalent noise components. It is straightforward to express the channel responses in (7) as those in

$$\begin{bmatrix} h_{1,1}^{(1)} & \cdots & h_{1,\frac{N}{2}}^{(1)} \\ \vdots & \ddots & \vdots \\ h_{\frac{N}{2},1}^{(1)} & \cdots & h_{\frac{N}{2},\frac{N}{2}}^{(1)} \end{bmatrix} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,\frac{N}{2}} \\ \vdots & \ddots & \vdots \\ h_{\frac{N}{2},1} & \cdots & h_{\frac{N}{2},\frac{N}{2}} \end{bmatrix} - \begin{bmatrix} c_{\frac{N}{2}+1,1} & \cdots & c_{N,1} \\ \vdots & \ddots & \vdots \\ c_{\frac{N}{2}+1,\frac{N}{2}} & \cdots & c_{N,\frac{N}{2}} \end{bmatrix} \begin{bmatrix} h_{\frac{N}{2}+1,1} & \cdots & h_{\frac{N}{2}+1,\frac{N}{2}} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,\frac{N}{2}} \end{bmatrix}. \quad (8)$$

Likewise, we can conduct similar canceling and reconstructing actions for the new system in (7) as the second forward stage. Then, we have an equivalent $N/4 \times N/4$ system containing $[d_1 \ d_2 \ \cdots \ d_{N/4}]^T$. After the mth forward stage, the new reduced-dimension linear system can be represented by

$$\begin{bmatrix} r_1^{(m)} \\ \vdots \\ r_{2^{M-m}}^{(m)} \end{bmatrix} = \begin{bmatrix} h_{1,1}^{(m)} & \cdots & h_{1,2^{M-m}}^{(m)} \\ \vdots & \ddots & \vdots \\ h_{2^{M-m},1}^{(m)} & \cdots & h_{2^{M-m},2^{M-m}}^{(m)} \end{bmatrix} \times \begin{bmatrix} d_1 \\ \vdots \\ d_{2^{M-m}} \end{bmatrix} + \begin{bmatrix} z_1^{(m)} \\ \vdots \\ z_{2^{M-m}}^{(m)} \end{bmatrix}. \quad (9)$$

These processing stages are repeated again and again until we reach a 2×2 system with the transmitted data symbol d_1 and d_2 only. Accordingly, $M-1$ forward stages are achieved. For each of these forward stages, we summarize the operation steps here.

1) *Cancelation step:* In this step, the first task is to find the canceling weights, which is equivalent to solving a set of linear equations similar to that given in (6). Here, for numerical robustness, LU decomposition with forward/backward substitutions can be utilized [14]. For the mth forward stage, with $m = 1, 2, \ldots, M-1$, LU decomposition will be performed one

$$\begin{bmatrix} r_1 \\ \vdots \\ r_{\frac{N}{2}} \\ r_{\frac{N}{2}+1} \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,\frac{N}{2}} & h_{1,\frac{N}{2}+1} & \cdots & h_{1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{\frac{N}{2},1} & \cdots & h_{\frac{N}{2},\frac{N}{2}} & h_{\frac{N}{2},\frac{N}{2}+1} & \cdots & h_{\frac{N}{2},N} \\ h_{\frac{N}{2}+1,1} & \cdots & h_{\frac{N}{2}+1,\frac{N}{2}} & h_{\frac{N}{2}+1,\frac{N}{2}+1} & \cdots & h_{\frac{N}{2}+1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,\frac{N}{2}} & h_{N,\frac{N}{2}+1} & \cdots & h_{N,N} \end{bmatrix} \begin{bmatrix} d_1 \\ \vdots \\ d_{\frac{N}{2}} \\ d_{\frac{N}{2}+1} \\ \vdots \\ d_N \end{bmatrix} + \begin{bmatrix} z_1 \\ \vdots \\ z_{\frac{N}{2}} \\ z_{\frac{N}{2}+1} \\ \vdots \\ z_N \end{bmatrix} \quad (4)$$

time, and forward/backward substitutions will be performed 2^{M-m} times. After that, it is the multiplication of the received/constructed signals in the lower part and the canceling weights, and the subtraction of these resultant signals from the signals in the upper part.

2) *Reconstruction step:* This step contains the calculation for determining the reconstructed channel responses for the new reduced-dimension linear system, which is like that given in (8).

In a similar manner, the last forward stage is to solve the remaining 2×2 system, and the aforementioned cancelation/reconstruction steps or the simple Cramer's rule can be utilized. After all these M forward stages, we acquire the output decision $\hat{d}_1$, as depicted in Fig. 1.

Note that, in the application of the ordinary linear ZF detection for an $N \times N$ system, there exists only one possible way that can let each output decision keep a distortionless response to the corresponding detected data symbol with zero effect of interfering data symbols at the same time. From the preceding derivation, we see that the M forward stages can exactly null other data streams to zero while keeping the desired one undistorted, i.e., it does the same thing as that of "ZF." Therefore, we can assert that the output decision $\hat{d}_1$ obtained from this special formulation is exactly the same as that by the ordinary linear ZF detection.

After the finishing of all forward stages, M backward stages are immediately carried out. Each backward stage includes one cancelation step and one detection step as well. In the cancelation step of the first backward stage, assuming $\hat{d}_1 = d_1$, the effect of d_1 is subtracted from the reduced-dimension linear system after the $(M-1)$th forward stage, i.e., (9) with $m = M - 1$, as

$$\begin{bmatrix} r_1^{(M-1)} \\ r_2^{(M-1)} \end{bmatrix} - \begin{bmatrix} h_{1,1}^{(M-1)} \\ h_{2,1}^{(M-1)} \end{bmatrix} d_1 = \begin{bmatrix} h_{2,1}^{(M-1)} \\ h_{2,2}^{(M-1)} \end{bmatrix} d_2 + \begin{bmatrix} z_1^{(M-1)} \\ z_2^{(M-1)} \end{bmatrix}. \quad (10)$$

A maximal-ratio combiner can then be applied as the detection step to obtain the output decision $\hat{d}_2$. After having $\hat{d}_1$ and $\hat{d}_2$, the second backward stage is performed. Likewise, assuming both $\hat{d}_1$ and $\hat{d}_2$ are correct, the cancelation step for d_1 and d_2 can be similarly described as

$$\begin{bmatrix} r_1^{(M-2)} \\ r_2^{(M-2)} \\ r_3^{(M-2)} \\ r_4^{(M-2)} \end{bmatrix} - \begin{bmatrix} h_{1,1}^{(M-2)} & h_{1,2}^{(M-2)} \\ h_{2,1}^{(M-2)} & h_{2,2}^{(M-2)} \\ h_{3,1}^{(M-2)} & h_{3,2}^{(M-2)} \\ h_{4,1}^{(M-2)} & h_{4,2}^{(M-2)} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$
$$= \begin{bmatrix} h_{1,3}^{(M-2)} & h_{1,4}^{(M-2)} \\ h_{2,3}^{(M-2)} & h_{2,4}^{(M-2)} \\ h_{3,3}^{(M-2)} & h_{3,4}^{(M-2)} \\ h_{4,3}^{(M-2)} & h_{4,4}^{(M-2)} \end{bmatrix} \begin{bmatrix} d_3 \\ d_4 \end{bmatrix} + \begin{bmatrix} z_1^{(M-2)} \\ z_2^{(M-2)} \\ z_3^{(M-2)} \\ z_4^{(M-2)} \end{bmatrix}. \quad (11)$$

Next, in the detection step, we can employ the ZF detection to obtain $\hat{d}_3$ and $\hat{d}_4$, i.e.,

$$\begin{bmatrix} \hat{d}_3 \\ \hat{d}_4 \end{bmatrix} = \mathcal{Q}\left[\mathbf{H}_{3,4}^{\dagger}\mathbf{r}_{3,4}\right] = \mathcal{Q}\left[\left(\mathbf{H}_{3,4}^{H}\mathbf{H}_{3,4}\right)^{-1}\mathbf{H}_{3,4}^{H}\mathbf{r}_{3,4}\right] \quad (12)$$

in which

$$\mathbf{H}_{3,4} = \begin{bmatrix} h_{1,3}^{(M-2)} & h_{2,3}^{(M-2)} & h_{3,3}^{(M-2)} & h_{4,3}^{(M-2)} \\ h_{1,4}^{(M-2)} & h_{2,4}^{(M-2)} & h_{3,4}^{(M-2)} & h_{4,4}^{(M-2)} \end{bmatrix}^T \quad (13)$$

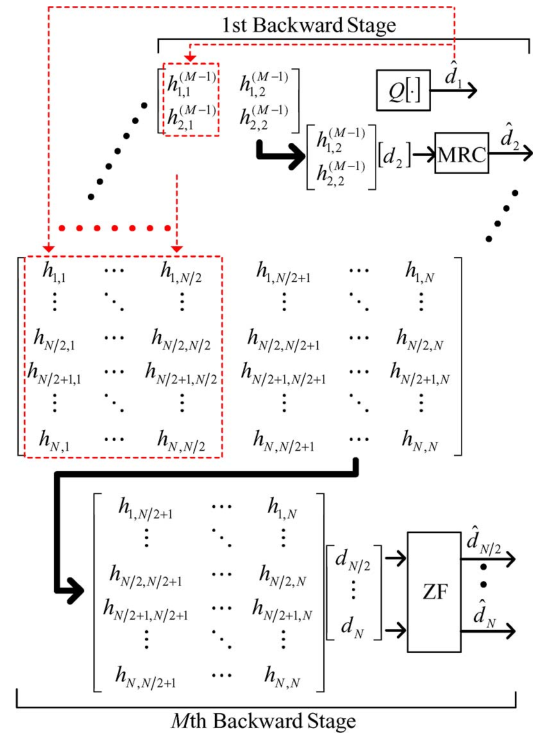

Fig. 2. M backward stages for the proposed groupwise OSIC-ZF detection.

and $\mathbf{r}_{3,4}$ denotes the result in (11). We see that a pseudoinverse of $\mathbf{H}_{3,4}$ is involved. Here, instead of finding the pseudoinverse of $\mathbf{H}_{3,4}$ in the conventional way, we take a slightly different approach to reduce the computational complexity for the ZF detection: We first calculate both $\mathbf{H}_{3,4}^H\mathbf{H}_{3,4}$ and $\mathbf{H}_{3,4}^H\mathbf{r}_{3,4}$ in (12) directly and then use Cholesky decomposition, together with forward/backward substitutions to acquire the output decisions $\hat{d}_3$ and $\hat{d}_4$, which is also a well-known approach for solving a system of linear equations with Hermitian structure [14]. With $\hat{d}_1$–$\hat{d}_4$, the third backward stage is to obtain $\hat{d}_5$–$\hat{d}_8$, and so on. Fig. 2 illustrates the whole picture of these M backward stages for the $N \times N$ scenario. In this way, $\hat{d}_1$–$\hat{d}_N$ are all obtained, and the so-called groupwise OSIC-ZF detection is accomplished. For clarity, we list the complete procedure of this proposed detection in Table I. The induced computational complexity will be derived and explained in detail in the next section.

One last thing to note is the detection order. Because of the successive cancelation, it is intuitively true that the correctness of the preceding output decisions is very important to the overall BER performance. Since the complete channel-matrix inverse is not available for the proposed groupwise OSIC-ZF detection, the ordering based on the postdetection SNR is not possible. Instead, the detection order can be determined based on the column-vector norms of the channel matrix, and the data symbol associated with the largest channel norm will be detected first. Although this kind of ordering is not optimal, the BER can still be largely enhanced, compared with that without ordering.

IV. Complexity Evaluation

In this section, we derive the required computational complexity for the linear ZF detection, optimal OSIC-ZF detection, and proposed groupwise OSIC-ZF detection. The computational complexity for implementing the linear ZF detection is first examined. Recall that $\mathbf{H}(k)$ given in (1) is an $N \times N$ complex-valued matrix without any special structure. Various techniques can be employed to calculate the inverse of $\mathbf{H}(k)$, and the most widely accepted one is through the

TABLE I
COMPLETE PROCEDURE FOR THE PROPOSED GROUPWISE OSIC-ZF DETECTION

Initialization	*Ordering:* Calculate the column-vector norms of the channel matrix and determine the detection order.
M **Forward stages**	**for** $m = 1 : M - 1$ *Cancelation:* Solve the $2^{M-m} \times 2^{M-m}$ system of linear equations via LU decomposition with forward/backward substitutions to obtain the canceling weights. Multiply the received/constructed signals in the lower part by the canceling weights and then subtract them from the signals in the upper part. *Reconstruction:* Calculate the equivalent channel responses, i.e., the coefficients for the new $2^{M-m} \times 2^{M-m}$ channel matrix. **end** Last forward stage: Solve the final 2×2 system and find the output decision $\hat{d}_1$.
M **Backward stages**	First backward stage: Cancel $\hat{d}_1$ from the 2×2 system and find the output decision $\hat{d}_2$. **for** $m = 1 : M - 1$ *Cancelation:* Cancel the previous detected 2^m data symbols from the $2^{m+1} \times 2^{m+1}$ system. *Detection:* Perform the specially-designed ZF detection for the $2^{m+1} \times 2^m$ system to obtain the new set of output decisions d_{2^m+1} to $d_{2^{m+1}}$. **end**

TABLE II
COMPLEX OPERATIONS REQUIRED FOR THE M FORWARD STAGES

First $M-1$ forward stages ($T = 2^{M-m}$ and $m = 1, 2, \cdots, M-1$)	**Operation**	**CM**	**CA**	**CD**
Cancelation step	LU decomposition	$T(T-1)(2T-1)/6$	$T(T-1)(2T-1)/6$	$T(T+1)/2$
	Forward substitution	$T^2(T-1)/2$	$T^2(T-1)/2$	/
	Backward substitution	$T^2(T-1)/2$	$T^2(T-1)/2$	T^2
	Weight multiplication	T^2	$T(T-1)$	/
	Signal subtraction	/	T	/
Reconstruction step	Channel response calculation	T^3	T^3	/
Last forward stage	**Operation**	**CM**	**CA**	**CD**
Cancelation step	Detection for $\hat{d}_1$	2	2	2

use of Gauss–Jordan elimination [14]. As shown in [15], computing such an inverse requires N^3 complex multiplications (CM)/complex divisions (CD) and $N(N-1)^2$ complex additions (CA). In addition, the multiplication of $(\mathbf{H}(k))^{-1}$ and $\mathbf{r}(k)$ requires N^2 CM and $N(N-1)$ CA. In fact, to perform the linear ZF detection, we do not really need to calculate the exact matrix inverse as that shown in (3). Ignoring the noise term $\mathbf{z}(k)$ in (2), we can apply any technique from solving a set of linear equations to obtain the output decisions for those transmitted data symbols. Again, as demonstrated in [14], an efficient and numerically robust approach to this end is to use LU decomposition, together with forward/backward substitutions. For an $N \times N$ linear system of equations, computing LU decomposition requires $N(N-1)(2N-1)/6$ CM, $N(N-1)(2N-1)/6$ CA, and $N(N+1)/2$ CD. Additionally, the forward substitution requires $N(N-1)/2$ CM and $N(N-1)/2$ CA, whereas the backward substitution requires $N(N-1)/2$ CM, $N(N-1)/2$ CA, and N CD. Therefore, the computational complexity in total for the linear ZF detection can be reduced to

$$\text{CM/CD}: \quad \frac{1}{3}N^3 + N^2 + \frac{2}{3}N \tag{14}$$

$$\text{CA}: \quad \frac{1}{3}N^3 + \frac{1}{2}N^2 - \frac{5}{6}N. \tag{15}$$

That is, only about one third of the computation load is needed, as compared with the implementation utilizing Gauss–Jordan elimination. When N is large, the saving is quite significant.

For the optimal OSIC-ZF detection, originally, the required computational complexity is proportional to N^4. As stated, some previous research lowers its computation load by a factor of N. For example, in [9], it was shown that, for an $N \times N$ spatial multiplexing system, the optimal OSIC-ZF detection can be accomplished with

$$\text{CM/CD}: \quad \frac{11}{3}N^3 + O(N^2) \tag{16}$$

$$\text{CA}: \quad 3N^3 + O(N^2) \tag{17}$$

where $O(\cdot)$ denotes the complexity order. As a result, the number of complex operations required for the optimal OSIC-ZF detection is notably decreased, particularly when N is large, but is still about 11 times larger than that for the linear ZF detection in terms of the required CM/CD operations.[1]

For the proposed groupwise OSIC-ZF detection, the operation steps and the related computational complexity for the M forward stages and M backward stages described in the previous section are listed in Tables II and III, respectively. In addition, note that the detection should be ordered based on the channel norms for better BER performance. This requires N^2 CM and $N(N-1)$ CA. In summary, the

[1]In fact, there exists other ways that can further reduce the computation load of the optimal OSIC-ZF detection [10]–[12]; however, the improvement is quite limited. For simplicity, we keep using this set of results for the complexity comparison in the succeeding sections.

TABLE III
COMPLEX OPERATIONS REQUIRED FOR THE M BACKWARD STAGES

First backward stage	**Operation**	**CM**	**CA**	**CD**
Cancelation step	Cancelation of $\hat{d}_1$	2	2	/
Detection step	MRC	4	2	1
Last $M-1$ backward stages ($U=2^m$ and $m=1,2,\cdots,M-1$)	**Operation**	**CM**	**CA**	**CD**
Cancelation step	Groupwise SIC	$2U^2$	$2U^2$	/
Detection step	Matching multiplication	$2U^2$	$U(2U-1)$	/
	Matrix multiplication	$U(U+1)2U/2$	$U(U+1)(2U-1)/2$	/
	Cholesky decomposition	$U(U+1)(U-1)/6$	$U(U+1)(U-1)/6$	$U(U+1)/2$
	Forward substitution	$U(U-1)/2$	$U(U-1)/2$	U
	Backward substitution	$U(U-1)/2$	$U(U-1)/2$	U

total computational complexity for the proposed groupwise OSIC-ZF detection for a general $N \times N$ system can be shown to be

$$\begin{aligned}\text{CM}: \quad & \sum_{m=1}^{M-1}\left(\frac{7}{3}T^3-\frac{1}{2}T^2+\frac{1}{6}T\right)+2 \\ & +\sum_{m=1}^{M-1}\left(\frac{7}{6}U^3+6U^2-\frac{7}{6}U\right)+6+N^2 \\ & =\frac{1}{2}N^3+O(N^2) \qquad (18)\end{aligned}$$

$$\begin{aligned}\text{CA}: \quad & \sum_{m=1}^{M-1}\left(\frac{7}{3}T^3-\frac{1}{2}T^2+\frac{1}{6}T\right)+2 \\ & +\sum_{m=1}^{M-1}\left(\frac{7}{6}U^3+\frac{11}{2}U^2-\frac{8}{3}U\right)+4+N(N-1) \\ & =\frac{1}{2}N^3+O(N^2) \qquad (19)\end{aligned}$$

$$\begin{aligned}\text{CD}: \quad & \sum_{m=1}^{M-1}\left(\frac{3}{2}T^2+\frac{1}{2}T\right)+2+\sum_{m=1}^{M-1}\left(\frac{1}{2}U^2+\frac{5}{2}U\right)+1 \\ & =\frac{2}{3}N^2+O(N) \qquad (20)\end{aligned}$$

once more, with $T=2^{M-m}$ and $U=2^m$, as indicated in Tables II and III. We can roughly see that the required computational complexity is slightly larger than that for the linear ZF detection with LU decomposition [as those in (14) and (15)] and is much smaller than that for the optimal OSIC-ZF detection [as those in (16) and (17)].

It is also interesting to observe that the derived computational complexity is for the detection under fast time-varying channel environments, i.e., the channel responses change quite frequently, and the detection should be recalculated at every time instant. If the channel environment changes slowly, for the linear ZF detection employing LU decomposition and forward/backward substitutions, the decomposition of the channel matrix obtained previously can be reused, and only forward/backward substitutions demanding N^2 CM/CD and N^2-N CA will be needed for processing. In addition, it can be shown that the optimal OSIC-ZF detection requires $2N^2-N$ CM and $3N^2-4N+1$ CA in this case. For the proposed groupwise OSIC-ZF detection, if the channel environment changes slowly, only "Weight multiplication" and "Signal subtraction" in the first $M-1$ forward stages of Table II, a fraction of the operations in the last forward stage of Table II, the operations in the first backward stage of Table III, and "Groupwise SIC," "Matching multiplication," "Forward substitution," and "Backward substitution" in the last $M-1$ backward stages of Table III will be processed. Thus, the required numbers of CM, CA, and CD are reduced to

$$\begin{aligned}\text{CM}: \quad & \sum_{m=1}^{M-1}T^2+5\sum_{m=1}^{M-1}U^2-\sum_{m=1}^{M-1}U+7 \\ & =2N^2+O(N) \qquad (21)\end{aligned}$$

$$\begin{aligned}\text{CA}: \quad & \sum_{m=1}^{M-1}T^2+5\sum_{m=1}^{M-1}U^2-2\sum_{m=1}^{M-1}U+5 \\ & =2N^2+O(N) \qquad (22)\end{aligned}$$

$$\text{CD}: \quad 2\sum_{m=1}^{M-1}U+2=2N-2. \qquad (23)$$

Again, we see that these values are larger than those for the linear ZF detection with LU decomposition but are smaller than those for the optimal OSIC-ZF detection when N is moderate to large. This means that, under slowly time-varying channel environments, the proposed groupwise OSIC-ZF detection still possesses a similar computational advantage.

V. PERFORMANCE

In this section, both the BER performance and computational complexity for the linear ZF detection, optimal OSIC-ZF detection, and the proposed groupwise OSIC-ZF detection with channel-norm ordering are examined. Here, one thing we need to note about is the nulling behavior of the proposed detection. As indicated in Section III, performing the forward nulling action is basically the same as performing ZF. This implies that the level of noise enhancement is the same as that from other ZF-based detection as well. We first check the BER for the three considered detection schemes. Fig. 3 is the error-rate simulation for 4×4 and 8×8 spatial multiplexing systems over flat Rayleigh fading channels. The BER for the OSIC-ZF detection with ordering based on channel norms is also included for completeness. From the figure, we see that the BER performance for the proposed groupwise OSIC-ZF detection is quite similar to that for the channel-norm-based OSIC-ZF detection. This means that the grouping in the detection does not affect the BER much. When comparing the proposed detection with the optimal OSIC-ZF detection, we see more BER degradation. From the preceding observation, we know that this degradation is

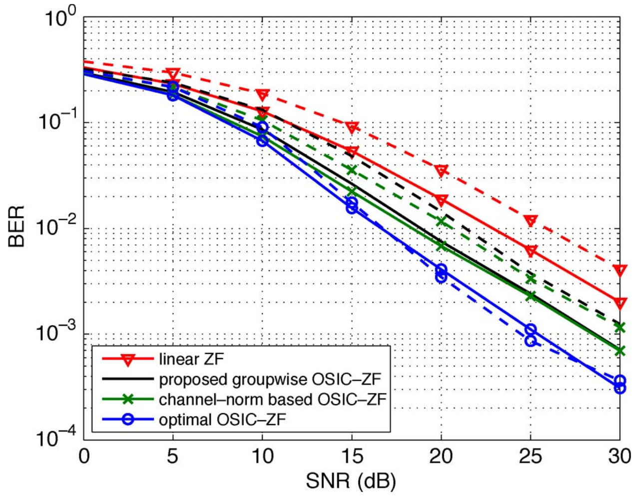

Fig. 3. BER for different detection schemes with quadrature phase-shift keying modulation. (Solid line) $N = 4$. (Dashed line) $N = 8$.

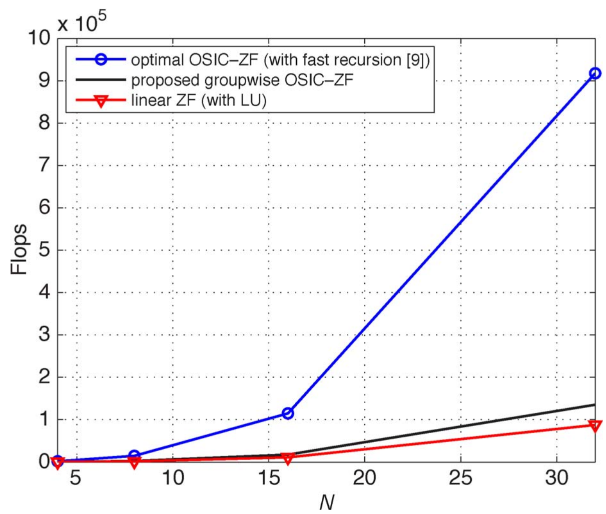

Fig. 4. Flop counts for different detection schemes for one complete detection.

mainly from ordering, which is much the same as the performance loss of the channel-norm-based OSIC-ZF detection, compared with the optimal OSIC-ZF detection. Importantly, the groupwise OSIC-ZF detection performs much better than the linear ZF detection. The BER curves for the proposed detection are essentially located at the middle of those for the linear ZF detection and the optimal OSIC-ZF detection for both 4×4 and 8×8 systems.

Next, we compare the required computational complexity. It is known that one CM/CD and one CA cost six and two flops, respectively [9]–[12]. According to the derivation for the numbers of CM, CA, and CD for the three considered detection schemes, we present the corresponding flop counts with different numbers of transmit/receive antennas in Fig. 4. In this set of results, the comparison is based on the calculation of one complete detection. As can be clearly seen, the linear ZF detection requires the least flops for all numbers of transmit/receive antennas, and the complexity difference between the groupwise OSIC-ZF detection and the linear one is not large. Interestingly, the proposed detection is significantly more efficient than the optimal OSIC-ZF detection, particularly when N goes large. To be more specific, comparing the proposed detection with the optimal OSIC-ZF detection, the complexity savings are 82.1%, 83.9%, 84.8%, and 85.3% for $N = 4, 8, 16, 32$, respectively. Together with the BER results shown earlier, we conclude that the groupwise OSIC-ZF detection is very attractive as a new approach standing between the linear ZF detection and the optimal OSIC-ZF detection. Its BER performance can be comparable with that of the channel-norm-based OSIC-ZF detection with very low implementation complexity.

VI. Conclusion and Discussions

In this paper, we have formulated a low-complexity detector for $N \times N$ spatial multiplexing systems. Our results show that this groupwise OSIC-ZF detection provides a good compromise between the error-rate performance and the computational complexity. When the number of transmit/receive antennas is moderate to large, it can significantly reduce the computation load while retaining the BER standing between the linear ZF detection and the optimal OSIC-ZF detection. In fact, this kind of detection can be extended to the scenario with any number of transmit/receive antennas. The only problem is that there is no unified way to characterize it and should be studied case by case. For example, when the number of transmit/receive antennas is odd, the idea of canceling half of the transmitted symbols cannot be achieved. Instead, we may cancel a smaller or larger number of these symbols. Consequently, different error-rate performance and computational complexity may be induced by different strategies. As a concluding remark, we also need to stress that the proposed detection method can be extended to other kinds of communication applications involving the idea of ZF or OSIC-ZF, e.g., as those in [13]. Based on the derivation and simulation given here, it is believed that good error-rate performance and noteworthy complexity saving can be obtained as well.

References

[1] G. J. Foschini and M. J. Gans, "On the limits of wireless communications in a fading environment when using multiple antennas," *Wireless. Per. Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[2] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecom.*, vol. 10, no. 6, pp. 585–596, Nov./Dec. 1999.

[3] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.

[4] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[5] S. Savazzi, M. Nicoli, and M. Sternad, "A comparative analysis of spatial multiplexing techqniues for outdoor MIMO-OFDM systems with a limited feedback constraint," *IEEE Trans. Veh. Technol.*, vol. 58, no. 1, pp. 218–230, Jan. 2009.

[6] G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *Electron. Lett.*, vol. 35, no. 1, pp. 14–16, Jan. 1999.

[7] S. Shao, Y. Tang, T. Kong, K. Deng, and Y. Shen, "Performance analysis of a modified V-BLAST system with delay offsets using zero-forcing detection," *IEEE Trans. Veh. Technol.*, vol. 56, no. 6, pp. 3827–3837, Nov. 2007.

[8] B. Hassibi, "A fast square-root implementation for BLAST," in *Conf. Rec. 34th Asilomar Conf. Signals, Syst. Comput.*, 2000, pp. 1255–1259.

[9] J. Benesty, Y. Huang, and J. Chen, "A fast recursive algorithm for optimum sequential signal detection in a BLAST system," *IEEE Trans. Signal Process.*, vol. 51, no. 7, pp. 1722–1730, Jul. 2003.

[10] H. Zhu, Z. Lei, and F. P. S. Chin, "An improved square-root algorithm for BLAST," *IEEE Signal Process. Lett.*, vol. 11, no. 9, pp. 772–775, Sep. 2004.

[11] T.-H. Liu and Y.-L. Liu, "Modified fast recursive algorithm for efficient MMSE-SIC detection of the V-BLAST system," *IEEE Trans. Wireless Commun.*, vol. 7, no. 10, pp. 3713–3717, Oct. 2008.

[12] Y. Shang and X.-G. Xia, "On fast recursive algorithm for V-BLAST with optimal ordered SIC detection," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2860–2865, Jun. 2009.

[13] S. Verdú, *Multiuser Detection*. Cambridge, U.K.: Cambridge Univ. Press, 1998.
[14] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing*. Cambridge, U.K.: Cambridge Univ. Press, 1992.
[15] L. N. Trefethen and D. Bau, III, *Numberical Linear Algebra*. Philadelphia, PA: SIAM, 1997.

Optimization of Time-Domain Spectrum Sensing for Cognitive Radio Systems

Sunyoung Lee and Seong-Lyun Kim, *Member, IEEE*

***Abstract*—Spectrum sensing is a critical issue in a cognitive radio (CR). In this paper, we propose a novel spectrum-sensing algorithm that exploits the time diversity of sensing results generated by a CR spectrum sensor. In the proposed algorithm, the CR sensor combines multiple sensing results obtained at different time points to make an optimal decision on the existence of spectrum access opportunity. It considers the traffic pattern of the primary user and maximizes the spectrum utilization of the CR, given the allowable interference level to the primary user. From our mathematical analysis, the principles of combining multiple sensing results in the time domain are revealed, and the detection and the false alarm probabilities are derived.**

***Index Terms*—Cognitive radio (CR), cooperative sensing, opportunistic spectrum access, time-combining spectrum sensing.**

I. Introduction

The cognitive radio (CR) system opportunistically accesses the frequency channel that primary users hold a license to use. The CR attempts to exploit as many spectrum opportunities as possible without interfering primary users beyond more than a certain tolerable level. To this end, the CR senses the presence of the primary user and decides whether to transmit a signal or not on the basis of the sensing result [1], [2]. One of the challenges in spectrum sensing is that the fading of a primary user signal leads to frequent sensing errors. To overcome this difficulty, a variety of cooperative sensing methods (e.g., [3]) have been proposed. The main idea of the cooperative sensing is that a fusion center collects multiple sensing results from multiple sensors at different locations to benefit from *spatial diversity*. Although it is shown that the cooperative sensing can greatly enhance the sensing performance, additional complexity and overhead are needed in the data collection and the fusion process [4], [5].

In this paper, we propose an alternative way to mitigate the deteriorating effect of channel fading. The proposed sensing method aims to reap a *time diversity gain*, rather than a spatial diversity gain, by combining multiple sensing results obtained by a single CR sensor at different time points. As a result, the CR sensor expects to have a similar diversity gain to the cooperative sensing without the overhead of the data collection process.

In designing the combining rule of time-domain sensing results, we need to answer a question, i.e., how to optimally combine the sensing results obtained at different time points. This is not a trivial problem for the following reason: The primary user alternates between active and inactive states over time. Therefore, the primary user state at a past time point, at which one of the sensing results is obtained, can be different from the current primary user state. This, in turn, means that the sensing result obtained a long time ago is less credible than the recently obtained one. Therefore, the sensing algorithm should take account of the difference in the credibility of each sensing result.

Our time-domain combining spectrum sensing (TDC-SS) algorithm is based on the Bayesian method and the Neyman–Pearson theorem. The Neyman–Pearson criterion maximizes the spectrum utilization while keeping the interference level less than a certain level. The Neyman–Pearson detection of a Markov process has been well studied in the literature [6], [7]. A simple Markov ON-OFF model has been assumed in CR research, e.g., spectrum sensing [8] and medium-access control for dynamic spectrum access [9]. In addition, it was proved that the simple Markov ON-OFF model strikes a balance between the accuracy and the complexity [10], [11]. In this paper, it is assumed that the state of the primary user evolves according to a Markov ON-OFF process. Considering the transition rate of the ON-OFF process, the TDC-SS algorithm sequentially updates the likelihood ratio of the primary user state by using the Bayesian method and decides the current state of the primary user from the Neyman–Pearson criterion. The resulting algorithm makes an optimal decision on the primary user state, in the sense that the spectrum utilization is maximized, by effectively combining sensing results with different credibilities.

We analyze the asymptotic behavior of the proposed TDC-SS algorithm. The log-likelihood ratio of the primary user state turns out to be a Markov process. We derive the limiting distribution of the log-likelihood ratio, from which we evaluate the performance measures. The analytic result clearly exhibits the impact of the transition rate of the primary user state on the performance of the TDC-SS algorithm. As the primary user state slowly alternates between ON and OFF, the TDC-SS algorithm combines more sensing results together and makes accurate detection of the primary user state. This is novel and generic in that the proposed TDC-SS algorithm can adjust itself to the transition rate of the primary user state. The TDC-SS algorithm improves the spectrum utilization up to about 4.3 times at the missed detection probability of 0.01, given that the transition rate is $2 \cdot 10^{-5}$ (times/ms).

The remainder of this paper is organized as follows: In Section II, we introduce our system model. In Section III, we propose the TDC-SS algorithm and analyze it. Some numerical results are presented in Section IV. A list of the key mathematical notations used in this paper is summarized in Table I.

II. System Model

A. CR and Primary User Model

Consider a CR system that shares a common frequency channel with a primary user. The channel bandwidth is denoted by W. Time is divided into frames, and it is synchronous between the primary user and the CR sensor. The length of a frame is denoted by T_F, and each frame is indexed by $t\ (= 1, 2, \ldots)$. A frame consists of a sensing duration, followed by a data transmission duration, whose lengths are T_S and T_D, respectively. The CR sensor senses the channel during T_S to determine the presence of the primary user. During the sensing duration in frame t, the CR sensor performs energy detection [12]

Manuscript received April 7, 2010; revised August 27, 2010, January 6, 2011, and February 16, 2011; accepted February 27, 2011. Date of current version May 16, 2011. This work was supported by the International Research and Development Program of the National Research Foundation of Korea, funded by the Ministry of Education, Science, and Technology of Korea under Grant K20901001406-09B1300-03110 (FY 2010). The review of this paper was coordinated by Prof. Y. Ma.

The authors are with the Radio Resource Management and Optimization Laboratory, School of Electrical and Electronic Engineering, Yonsei University, Seoul 120-749, Korea (e-mail: sunyoung@yonsei.ac.kr; slkim@yonsei.ac.kr).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2011.2132157