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CISC3220

String matching

The first solution for string match is called **Naïve**

Suppose we have a string A[A, S, D, F, G, H, J, K, L], the second string is B[D, F, G]

The idea of this algorithm is from beginning shift to the right until you match the string.

1. $n \leftarrow \text{length}[T]$
2. $m \leftarrow \text{length}[P]$
3. for $s \leftarrow 0$ to $n-m$ do
4. $j \leftarrow 1$
5. while $j \leq m$ and $T[s + j] = P[j]$ do
6. $j \leftarrow j + 1$
7. If $j > m$ then
8. return valid shift s
9. return no valid shift exist

the for-loop in line 3 is executed at most $n - m + 1$ times, and the while-loop in line 5 is executed at most m times, The run time for naïve string matching is $O((n - m + 1)m)$

Example for Naïve string-matching algorithm

SHIFT = 0

A= A. S. D. F. G. H, J, K, L

B= D, F, G

SHIFT=1

A= A. S. D. F. G. H, J, K, L

B= D, F, G

SHIFT=2

A= A. S. D. F. G. H, J, K, L

B= D, F, G

At this point the string is matched.

The second algorithm for string matching is **Rabin-Karp String Matching Algorithm**

Suppose we have a string S[1, 3, 4, 1, 4, 1, 2, 6]

Pattern P [4, 1, 4]

The basic idea is we consider P [4, 1, 4] as $4+1+4 = 9$, then we compare this value to every 3 elements in S string, until we find the matching string

To keep go on,

We take 1,2,4 from S, $1+2+4=7$ which is not equal to 9,

we shift to right 3,4,1, $3+4+1= 8$ which is not equal to 9

next we compare 4, 1 ,4 $4+1+4 =9$ valid match

as we keep go on, we can also find that $1+2+6$ this is also equal to 9, but the string does not match, we call this **spurious hit**.

To solve this problem we need more variables for this algorithm to work

Example,

we set $(4*10^2)+(1*10^1)+(4*10^0)$ this will give us $400+10+4=414$

what we do next is take 1, 3, 4, $(1*10^2)+(3*10^1)+(4*10^0) = 100+30+4=134$,

next is $(134 - 1*10^2)*10 + 1*10^0$ we minus $1*10^2$ is to remove 1 from previous high order, multiply by 10 is to move all power by 1 with the previous then add the new digit.

As we keep shifting, we will notice that there will be only one substring in S that is match to P.

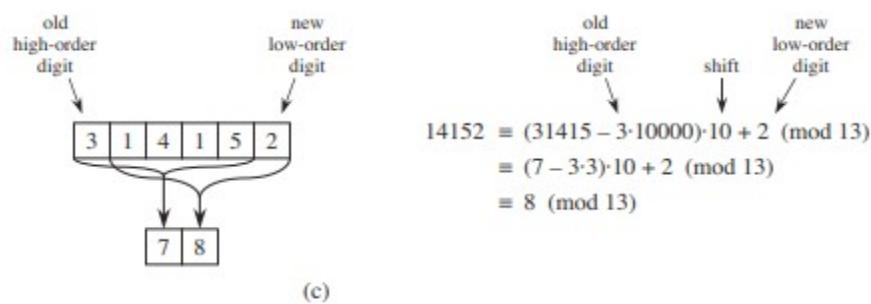
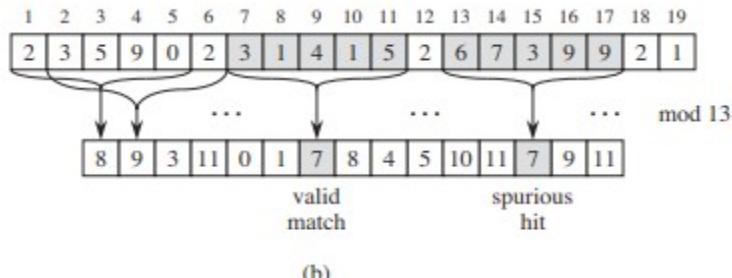
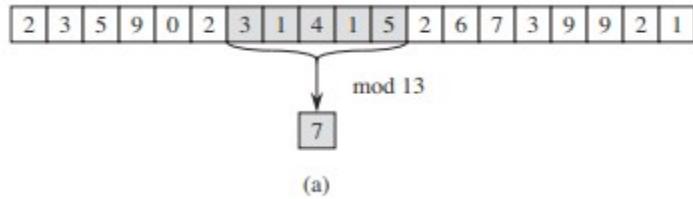
The book is giving different example that instead of adding all numbers, each character is converted in to decimal digit and strings of k-consecutive characters are represented as length-k decimal number, and find module for all numbers

Example: string 31415 is represented as 31,415

RABIN-KARP-MATCHER(T, P, d, q)

```

1   $n = T.length$ 
2   $m = P.length$ 
3   $h = d^{m-1} \bmod q$ 
4   $p = 0$ 
5   $t_0 = 0$ 
6  for  $i = 1$  to  $m$            // preprocessing
7       $p = (dp + P[i]) \bmod q$ 
8       $t_0 = (dt_0 + T[i]) \bmod q$ 
9  for  $s = 0$  to  $n - m$        // matching
10     if  $p == t_s$ 
11         if  $P[1..m] == T[s + 1..s + m]$ 
12             print "Pattern occurs with shift"  $s$ 
13     if  $s < n - m$ 
14          $t_{s+1} = (d(t_s - T[s + 1]h) + T[s + m + 1]) \bmod q$ 
```



Average run time $O(n + m)$.

String matching with finite automata

Example

String S [a, b, a, b, a, b, a, c, a, b, a]

Pattern P [a, b, a, b, a, c, a]

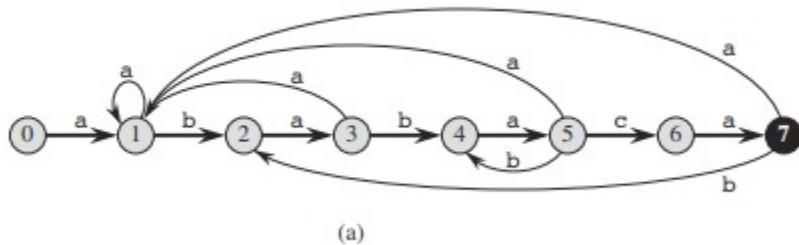
Numbers of type of element is a, b, c, d, 4 types

Total numbers of element of P is 6, the size of the table we will create is $6+1= 7$.

State	a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

a match with a therefore the first box we put 1, b and c does not match we put zero.

Next row, we have ab and aa , only we only a and a match, we put 1 in the first box in sec row, then ab and ab, we get 2 match, so we put 2, do the same to the rest



Every time that arrow turn back that means string does not matching

```

FINITE-AUTOMATON-MATCHER( $T, \delta, m$ )
1    $n = T.length$ 
2    $q = 0$ 
3   for  $i = 1$  to  $n$ 
4        $q = \delta(q, T[i])$ 
5       if  $q == m$ 
6           print "Pattern occurs with shift"  $i - m$ 

```

The Knuth-Morris-Pratt algorithm

Example

We have string S[a,b,a,b,a,b,a,b,a,c]

The pattern is P[a,b,c,d]

Let S has a pointer i and P has pointer j

S

a	b	a	b	a	b	a	b	a	b	c
1	2	3	4	5	6	7	8	9	10	11

P

	a	b	a	b	c
	0	0	1	2	0
0	1	2	3	4	5

The second row for P table is when a and a are matching in P we note that second a with 1 and b appears twice we note that 2. c only appears once, we put 0, we consider ab as prefix.

Now we start comparing, when i is 1, we have a , j +1 is a too, then we move to the next, b and b are match as well, then we move again.... We see that when i=5 is a, j+1= 5 is c, doesn't match what we do here is not go all the way back, we move j to the noted number 2 in the p box which is the first b. then we keep comparing, j will go back to 2 three time. Then we fine the matched string.

KMP-MATCHER(T, P)

```
1   $n = T.length$ 
2   $m = P.length$ 
3   $\pi = \text{COMPUTE-PREFIX-FUNCTION}(P)$ 
4   $q = 0$                                 // number of characters matched
5  for  $i = 1$  to  $n$                   // scan the text from left to right
6    while  $q > 0$  and  $P[q + 1] \neq T[i]$ 
7       $q = \pi[q]$                       // next character does not match
8      if  $P[q + 1] == T[i]$ 
9         $q = q + 1$                     // next character matches
10     if  $q == m$                       // is all of  $P$  matched?
11       print "Pattern occurs with shift"  $i - m$ 
12      $q = \pi[q]$                       // look for the next match
```

COMPUTE-PREFIX-FUNCTION(P)

```
1   $m = P.length$ 
2  let  $\pi[1..m]$  be a new array
3   $\pi[1] = 0$ 
4   $k = 0$ 
5  for  $q = 2$  to  $m$ 
6    while  $k > 0$  and  $P[k + 1] \neq P[q]$ 
7       $k = \pi[k]$ 
8      if  $P[k + 1] == P[q]$ 
9         $k = k + 1$ 
10        $\pi[q] = k$ 
11   return  $\pi$ 
```

Questions:

1. $T[a, a, a, a, a, a, a, b]$, $P[a, b]$ how many shift do we needed in naïve string matching algorithm.
Ans: 8-2=6 shift
2. Rabin Karp Algorithm makes use of elementary number theoretic notions.T or F
Ans: T
3. What is worst case run time for Rabin-Karp string matching algorithm
Ans: $O(n+m)$
4. What is average case run time for Rabin-Karp string matching algorithm
Ans: $O(nm)$
5. What is it called when if the rolling hash produces a candidate match due to this hash collision, which turns out not to be a string match?
Ans: spurious hit