An Integrated Framework for Elevator Traffic Design under General Traffic Conditions Using Origin Destination Matrices, Virtual Interval and the Monte Carlo Simulation Method

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考虑使用了od矩阵、交通模式和蒙特卡罗抽样法，但考虑的情况较为复杂，为大楼的楼层进行了分类，分成了可进入层和居住层。

An Integrated Framework for Elevator Traffic Design under General Traffic Conditions Using Origin Destination Matrices, Virtual Interval and the Monte Carlo Simulation Method

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Abstract

The conventional design methodology for elevator traffic analysis has been applied to the case of up peak traffic (or incoming traffic conditions). The only user requirement are usually the expected arrival rate (*AR%*) expressed as a percentage of the building population requesting service in the peak five minutes, and the target interval. The interval as classically used will be referred to as the physical interval in this paper as it is only relevant for the case of a single entrance and incoming traffic conditions.

This paper presents an integrated methodology for the design of elevator traffic systems for the general case of mixed traffic conditions. It presents a fully integrated framework that covers the steps from user requirements to the selection of the number of required elevators.

The user requirements describing the traffic conditions can be specified by the user, expressed as the percentage arrival rate (*AR%*), the mix of incoming traffic, outgoing traffic and inter-floor traffic. This paper derives equations that can be used to combine the mix of traffic, the floor arrival percentages and the floor population percentages into an origin-destination (*OD*) matrix. The origin destination matrix is then adjusted and normalised in order to account for rational passenger behaviour (i.e., a passenger will not travel to the same floor that he/she is at). A method is presented for the random generation of passenger origin-destination pairs using the origin-destination matrix (which is necessary when using the Monte Carlo Simulation (*MCS*) method to calculate the round trip time).

A novel equation for evaluating the round trip time under the assumption of equal floor heights and top speed attained in one floor journey is derived and used. The equation is derived using a stepwise derivation and verification process. The verification is carried out against the Monte Carlo simulation method for finding the value of the round trip time.

The concept of a virtual interval (as opposed to the conventionally used physical interval usually used in elevator traffic system design) is introduced in order to allow the selection of the number elevators to be carried out. The virtual interval is the average value of the time between the consecutive reversals of the elevators in the group.

Keywords: Elevator; lift, round trip time; physical interval; virtual interval; incoming traffic; outgoing traffic; inter-floor traffic; general traffic conditions; Monte Carlo Simulation; origin-destination matrix.

Nomenclature

a the rated acceleration in m/s^2

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d_f the height of a floor in m
E is the number of elevators in the group
H is the highest reversal floor in a round trip having units of floors
ic is the percentage of incoming traffic under the traffic mix
if is the percentage of inter-floor traffic under the traffic mix
j the rated jerk in m/s^3
L is the lowest reversal floor in a round trip having units of floors
N the total number of floors in the building
og is the percentage of outgoing traffic under the traffic mix
P the number of passengers served in one round trip
$P_{arr}(i)$ the percentage arrival from the i^{th} floor
p_{ij} is the passenger transition probability from the i^{th} floor to the j^{th} floor
RTT the round trip time in s
S is the expected number of stops in a round trip
S_D is the average number of down stops in a round trip
S_{LCS} is the average number of Lower Coincidental Stops in a round trip
S_U is the average number of up stops in a round trip
S_{UCS} is the average number of Upper Coincidental Stops in a round trip
t_{dc} the door closing time in s
t_{do} the door opening time in s
t_{pi} the passenger boarding time in s
t_{po} the passenger alighting time in s
U is the total building population
$U(i)$ the building population on the i^{th} floor
v the rated speed in m/s

1. INTRODUCTION

The conventional elevator traffic design methodology relies on designing the elevator traffic system assuming 100% incoming traffic. Under incoming traffic conditions, passengers arrive at the building entrances (e.g., lobby and car park floors) and travel to occupant floor up in the building. Designing based on 100% incoming traffic conditions is a reflection of the morning up-peak where the majority of passengers are entering the building. It has always assumed that an elevator traffic system designed to meet the morning up-peak demand will cope comfortably with the demand at any other time of the day.

However, recent work has shown that modern changes in working practices are changing the peak passenger demand. It is likely that in some buildings the peak passenger demand is the lunchtime peak where some passenger are leaving the building for lunch (outgoing traffic), some passengers are entering the building after returning from lunch (incoming traffic), and other passengers are moving within the building (i.e., inter-floor traffic). Examples of traffic mix conditions that are believed to be representative of the lunch-time peak traffic conditions in many modern office buildings include: 40%:40%:20% [1]; 45%:45%:10% [2]; 42%, 42%, 16% [3]; incoming, outgoing and inter-floor traffic, respectively.

In addition, it is possible to calculate the round trip time for general traffic conditions under the Poisson arrival process assumption ([4], [5]) or the plentiful-passenger-supply assumption [6].

Based on these last two points, this paper thus presents an integrated framework for carrying out an elevator traffic design under general traffic conditions (i.e., any traffic mix of incoming traffic, outgoing traffic and inter-floor traffic). It presents a procedure starting from the user requirements of the arrival rate (*AR%*) and the target interval (int_{tar}) as well as the specific traffic conditions (i.e., traffic mix) and ending with the required number of elevators and the actual interval and the handling capacity (*HC%*). In doing this it heavily relies on the concept of origin-destination matrix as an intermediate step for calculating the round trip time, as well as the new concept of the virtual interval (as opposed to the physical interval) which is a more appropriate variable to use under general traffic conditions (as opposed to the 100% incoming traffic conditions traditionally used).

The framework presented in this paper can then be combined with the *HARint* plane [7] or the *HARint* space [8] methodologies in order to be used as a comprehensive universal elevator traffic system design tool, whereby the average passenger waiting time and the average passenger travelling times can also be added as user requirements, and outputs such as the car capacity, the elevator speeds can be provided as outputs of the design.

Section 2 outlines conventions and terminology for describing the types of floors in a building and types of traffic and presents the procedure for converting the floor arrival percentages, the floor percentage populations and the traffic mix into an origin-destination matrix. Section 3 presents a procedure for generating passenger origin-destination pairs, a step that is critical for evaluating the round trip time using the Monte Carlo simulation method. A numerical example on generating passenger origin-destination pairs is also presented in section 3. A novel round trip time equation is derived in section 4 for the case of general traffic conditions assuming that top speed is attained in one floor journey and that floor heights are equal. It has a form that is very similar to the conventional round trip time equation for the case of incoming traffic conditions, and thus be very insightful.

Once general traffic conditions have been introduced, the physical interval ceases to be suitable and has to be re-defined to suit the case of general traffic conditions. The virtual interval is introduced as an alternative to the physical interval and is defined in section 5. The virtual interval can be used with the round trip time in order to find the required number of elevators.

A complete numerical example is introduced in section 6 to illustrate how the integrated framework can be used to first evaluate the origin-destination matrix and then calculate the round trip time using the equation derived in section 4 and then finding the required number of elevators, the actual interval and the handling capacity. Conclusions are drawn in section 7.

2. DESCRIBING TRAFFIC IN A BUILDING

This section develops a systematic methodology for describing the traffic in a building, by converting the traffic mix and the floor arrival and population percentages into an origin-destination matrix.

2.1 Types of Floors

This section looks at the classification of floors. Any floor in a building can either be described as an entrance floor or an occupant floor. An entrance floor (referred to in short as *ent* floor) is a floor through which passengers can either enter or exit the building. An entrance floor can also be referred to an entrance/exit floor, as it becomes an exit floor when the traffic is out-going.

An occupant floor is a floor through which passengers cannot enter or exit the building, but where they will reside during their stay in the building (referred to in short as an *occ* floor). Based on the assumption above, all floors in the building would be denoted as either entrance floors or occupant floors, but not both. This is true in most buildings, although there might be some rare exceptions to this rule, where a floor could simultaneously be an entrance floor and an occupant floor.

An example of a building that is represented in this form is shown below in Table 1. Each floor is either an entrance/exit floor or an occupant floor. The percentage of passengers entering the building via an entrance floor is denoted as the percentage arrival rate ($Pr_{arr}(i)$). The population of a floor expressed as a percentage of the total building population is denoted as the percentage population (($U(i)/U$)).

Where a floor is an entrance floor, it has a non-zero value for its percentage arrival rate and a zero percentage population. Where a floor is an occupant floor, it has a non-zero percentage population but a zero percentage arrival rate.

It is customary for the entrance floors to be contiguous and for the occupant floors to be contiguous, but this is not necessary. An example of a non-contiguous floor is where a restaurant is located on the top floor of a building and is classified as an entrance/exit floor (as it is functionally external to the building albeit not physically external). The design methodology presented later in this paper can cope with the general case where the entrance floors are non-contiguous and the occupant floors are non-contiguous.

Table 1 shows a building with two entrances with unequal percentage arrival rates (0.8 ground floor denoted as *G*; 0.2 from the basement denoted as *B*). There are five occupant floor denoted as *L1* to *L5*. They have unequal population percentages.

Table 1: Representation of traffic in a building.

Floor notation	#	Floor name	Entrance/Exit floor or Occupant floor	Percentage arrival rate (%)	Population percentage
N	7	*L5*	*occ*	0	80/480=0.1667
N-1	6	*L4*	*occ*	0	80/480=0.1667
...	5	*L3*	*occ*	0	80/480=0.1667
...	4	*L2*	*occ*	0	120/480=0.25
...	3	*L1*	*occ*	0	120/480=0.25
2	2	*G*	*ent*	0.8	0
1	1	*B*	*ent*	0.2	0

The lowest floor in the building is denoted as floor 1 and the topmost floor as floor *N*. The following convention is followed in describing the values of arrival percentages and populations for the floors:

> $Pr_{arr}(i)$ is used to denote the arrival percentage of the i^{th} floor, where *i* runs from 1 to *N*
>
> $U(i)/U$ is used to denote the percentage population of the i^{th} floor, where *i* runs from 1 to *N,* where $U(i)$ is the population of the i^{th} floor and *U* is the total building population.

The representation of traffic in a building as shown in Table 1 is the default format and represents pure incoming traffic into the building. Under such a traffic condition, all passengers would be entering the building form the entrance floors and heading to the occupant floors.

A general format for representing the traffic in a building is shown in Table 2 below. As a generalisation, the term arrival can be extended to arrival/departure to cover both passengers entering the building under incoming traffic conditions and passengers leaving the building under out-going traffic conditions. By setting a population percentage for a floor to zero, it is an indication that it is an entrance floor; by setting an arrival percentage for a floor to zero, it is an indication that it is an occupant floor, as shown in Table 3.

Table 2: General representation format for a building.

Floor	Percentage Arrival/Departure	Percentage Population
N	$Pr_{arr}(N)$	$(U(N)/U)$
N-1	$Pr_{arr}(N-1)$	$(U(N-1)/U)$
…	…	…
…	…	…
…	…	…
2	$Pr_{arr}(2)$	$(U(2)/U)$
1	$Pr_{arr}(1)$	$(U(1)/U)$

It is worth noting that the summation of the percentage arrival/departure rates is 1 and the summation of all the building percentage populations is 1 as shown in equations (1) and (2) below.

$$\sum_{i=1}^{N} \Pr_{arr}(i) = 1 \qquad ....(1)$$

$$\sum_{i=1}^{N} \left(\frac{U(i)}{U} \right) = 1 \qquad ....(2)$$

The floor percentage (R(i)) is the arrival percentage for an entrance floor or the population percentage for an occupant floor, as shown in Table 3.

Table 3: General representation format for a building.

Type	#	Percentage arrival rate	Population percentage	Floor percentage (***R(i)***)
Occupant Floors	*N*	-	$(U(N)/U)$	$(U(N)/U)$
	N-1	-	$(U(N-1)/U)$	$(U(N-1)/U)$
	...	-	…	…
	...	-	…	…
	i+2	-	$(U(i+2)/U)$	$(U(i+2)/U)$
	i+1	-	$(U(i+1)/U)$	$(U(i+1)/U)$
Entrance Floors	*i*	$Pr_{arr}(i)$	-	$Pr_{arr}(i)$
	i-1	$Pr_{arr}(i-1)$	-	$Pr_{arr}(i-1)$
	...	…	-	…
	...	…	-	…
	2	$Pr_{arr}(2)$	-	$Pr_{arr}(2)$
	1	$Pr_{arr}(1)$	-	$Pr_{arr}(1)$

Neither the occupant floors nor the entrance/exit floors need to be contiguous. They are shown as contiguous in Table 2 and Table 3 for convenience and ease of understanding.

2.2 Types of Traffic

This sub-section classifies the possible types of journeys and thus the possible type of traffic. Every passenger journey must logically have an origin and a destination. Considering that any floor can either be an entrance/exit floor or an occupant floor, there can exist in theory four types of journeys depending on the classification of the origin and destination floors for each journey.

A journey that starts from an entrance/exit floor and terminates at an occupant floor is denoted as an incoming traffic journey. A journey that starts from an occupant floor and terminates at an entrance/exit floor is denoted as an outgoing traffic journey. A journey that starts from an occupant floor and terminates at an occupant floor is denoted as an inter-floor journey. A journey that starts from an entrance/exit floor and terminates at an entrance/exit floor is denoted as an inter-entrance journey and will be discounted in this paper. These four types of traffic are listed in Table 4 below.

Table 4: Types of traffic.

Start floor (origin)	End floor (destination)	Type of traffic	Description
Entrance/exit	Occupant	Incoming traffic	Passengers arriving into the building
Occupant	Entrance/exit	Outgoing traffic	Passengers leaving the building
Occupant	Occupant	Inter-floor traffic	Passengers moving within the building (restaurants, meeting rooms)
Entrance/exit	Entrance/exit	Inter-entrance traffic	Could only take place in very rare special situations.

2.3 Description of the Traffic in a Building

It has become customary to describe the prevailing traffic in a building at any one point in time as a mixture of the three types of traffic described in the previous sub-section.

The percentage of the traffic that is incoming at any one point in time is denoted as ic; the percentage of the traffic that is outgoing at any one point in time is denoted as og; and the traffic that is inter-floor at any one point in time is denoted as if. The combination of these three numbers can be used to describe the traffic mix as shown below (where any of these parameters can vary between 0 and 1):

$$ic:og:if$$

As expected the sum of all three numbers should add up to 1 as shown in equation (3) below. Thus, assigning values for two of these numbers automatically sets the value of the third parameter.

$$ic + og + if = 1 \qquad \text{..........(3)}$$

As an example, one suggested composition of the lunchtime traffic conditions can be described by the following representation [2]:

ic:*og*:*if* as 0.45:0.45:0.10 respectively

2.4 Origin-Destination (*OD*) Matrix

The origin-destination matrix is a concise compact format that can be used to fully describe the traffic in a building. It is an *N*-dimensional matrix that contains the probabilities of a passenger choosing to go from an origin floor to a destination floor in a given round trip. The row index represents the origin floor; the column index represents the destination matrix. The sum of all terms in the *OD* matrix must add up to 1. It is worth nothing all the events in the *OD* matrix are mutually exclusive (i.e., if one of them takes place in a round trip, the others cannot take place in the same round trip). Thus if a passenger chooses to go from the third floor to the 7th floor in a round trip, he/she cannot also go from the 8th floor to the 2nd floor in the same round trip.

The general format for an *OD* matrix is shown in equation (4) below. All the diagonal elements are equal to zero, as it is assumed that passengers are rational and would not travel from a floor to the same floor.

$$OD_{adj} = \begin{bmatrix} 0 & p_{1\cdot 2} & .. & .. & p_{1\cdot N-1} & p_{1\cdot N} \\ p_{2\cdot 1} & 0 & .. & .. & p_{2\cdot N-1} & p_{2\cdot N} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ p_{N-1\cdot 1} & p_{N-1\cdot 2} & .. & .. & 0 & p_{N-1\cdot N} \\ p_{N\cdot 1} & p_{N\cdot 2} & .. & .. & p_{N\cdot N-1} & 0 \end{bmatrix} \qquad ....(4)$$

The origin-destination matrix is critical for the elevator traffic design process, specifically for the following two functions:

1. The origin-destination matrix is used to generate passengers with origin and destination pairs to be used in finding the round trip time using Monte Carlo simulation.

2. The origin-destination matrix can also be used to derive the probabilities of any type of event taking place in a round trip (e.g., the probability of a journey taking place between the third and sixth floors without stopping at the fourth and fifth floors). Deriving these probabilities is critical to deriving equations for evaluating the round trip time.

2.5 Converting the Traffic Mix into an Origin Destination Matrix

Elevator traffic design has usually been carried out under up peak (incoming) traffic conditions. However, this paper attempts to deal with the elevator traffic design under general traffic conditions (which can comprise a mixture of incoming, outgoing and inter-floor traffic). For this reason and in preparation for the complete design process presented in future sections, this section provides a procedure for converting the arrival-population-table to an origin-destination matrix.

The first step is to compile the floor percentages into a column matrix and in a row matrix and multiply them, as shown in equations (5) and (6). The floor percentage, denoted as *R(i),* is the percentage arrival for the i^{th} floor when the floor is an entrance/exit floor or the percentage population of the i^{th} floor when the floor is an occupant floor.

$$R(i) = \begin{cases} \Pr_{arr}(i) & \text{for entrance floors} \\ \\ \dfrac{U(i)}{U} & \text{for occupant floors} \end{cases} \qquad \text{..........(5)}$$

The initial origin-destination matrix can be evaluated as shown in equation (6) below:

$$OD_{ini} = \begin{bmatrix} R(1) \\ R(2) \\ .. \\ .. \\ .. \\ .. \\ R(N-1) \\ R(N) \end{bmatrix} \cdot \begin{bmatrix} R(1) & R(2) & .. & .. & .. & .. & R(N-1) & R(N) \end{bmatrix} \qquad \text{..........(6)}$$

The result is an *N* by *N* matrix that is denoted as the initial origin-destination matrix (OD_{ini}), shown below in equation (7):

$$OD_{ini} = \begin{bmatrix} R(1)\cdot R(1) & R(1)\cdot R(2) & .. & .. & R(1)\cdot R(N-1) & R(1)\cdot R(N) \\ R(2)\cdot R(1) & R(2)\cdot R(2) & .. & .. & R(2)\cdot R(N-1) & R(2)\cdot R(N) \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ R(N-1)\cdot R(1) & R(N-1)\cdot R(2) & .. & .. & R(N-1)\cdot R(N-1) & R(N-1)\cdot R(N) \\ R(N)\cdot R(1) & R(N)\cdot R(2) & .. & .. & R(N)\cdot R(N-1) & R(N)\cdot R(N) \end{bmatrix} \qquad \text{..........(7)}$$

The next step is to convert the initial OD matrix as shown above by splitting it into four areas according to the type of traffic, as shown in Figure 1 below.

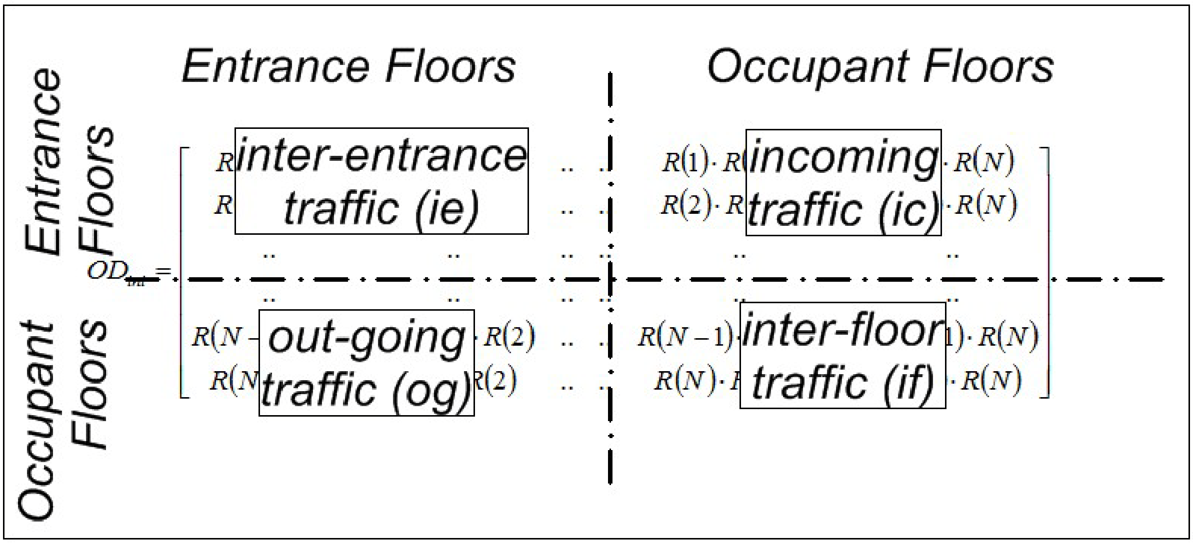

Figure 1: Split of the *OD* matrix into four traffic areas.

Figure 1 assumes that the entrance floors are contiguous and that all the occupant floors are contiguous. However, the approach shown in this paper does not necessarily require that to be the case and can deal with the general case where entrance floors are non-contiguous and where occupant floors are non-contiguous. At this stage, the sum of all the elements of the matrix at this stage is 4 (1 from each of the traffic mode areas).

It is assumed as a general case that the percentage of inter-entrance traffic is 0%, where it has been assumed to represent rare cases as discussed in an earlier sub-section. Thus all the elements in inter-entrance traffic area of the matrix will be zeroed. All the elements in the out-going traffic matrix will be multiplied by the percentage outgoing traffic (denoted as *og*); all the elements in the incoming traffic area of the matrix will be multiplied by the percentage incoming traffic (denoted as *ic*); all the element in the inter-floor traffic area of the matrix will be multiplied by the percentage inter-floor traffic (denoted as *if*).

Once this step has been carried out, then the sum of all the elements in the original destination matrix will add up to 1. More specifically, the sum of all the elements in the inter-entrance areas will be equal to 0; the sum of all the elements in the incoming traffic area will be equal to *ic*; the sum of all the elements in the outgoing traffic area will be equal to *og*; and the sum of all the elements in the inter-floor traffic areas will be equal to *if*.

The last step is to zero the non-zero diagonal elements in order to reflect passenger rational behaviour. These element will be in the inter-floor area (as the inter-entrance area has already been zeroed; and the incoming and outgoing areas do no cover the diagonal). This is shown in equation (8) below.

$$p_{ii} = 0 \quad for \quad i = j \qquad \text{..........(8)}$$

But as the diagonal items have been zeroed, the sum of all the elements in the matrix no longer adds up to 1. More specifically, the sum of all the elements in the inter-floor area does not add up to the value of *if* any more. In order to correct this, the matrix has to be adjusted by dividing all the elements in the inter-floor area by an adjusting factor, *M*, that can be evaluated as shown in equation (9) below.

$$M = 1 - \left(\frac{\sum_{i=1}^{N} p_{ii}}{if} \right) \qquad \text{..........(9)}$$

...where i runs for all the indices for the occupant floors. The final origin-destination matrix (denoted as OD_{fin}) is show in equation (10) below, following all the amendments and adjustments. The format assumes that the upper floors are occupant floors and that the lower floors are entrance/exit floors. This is not necessarily always the case and the notation can be altered to suit each building as appropriate.

$$OD_{fin} = \left[\begin{array}{ccc|ccc} 0 & 0 & .. & .. & ic \cdot \Pr_{arr}(1) \cdot \frac{U(N-1)}{U} & ic \cdot \Pr_{arr}(1) \cdot \frac{U(N)}{U} \\ 0 & 0 & .. & .. & ic \cdot \Pr_{arr}(2) \cdot \frac{U(N-1)}{U} & ic \cdot \Pr_{arr}(1) \cdot \frac{U(N)}{U} \\ .. & .. & .. & .. & .. & .. \\ \hline .. & .. & .. & .. & .. & .. \\ og \cdot \frac{U(N-1)}{U} \cdot \Pr_{arr}(1) & og \cdot \frac{U(N-1)}{U} \cdot \Pr_{arr}(2) & .. & .. & 0 & if \cdot \frac{U(N-1) \cdot U(N)}{M \cdot U^2} \\ og \cdot \frac{U(N)}{U} \cdot \Pr_{arr}(1) & og \cdot \frac{U(N)}{U} \cdot \Pr_{arr}(2) & .. & .. & if \cdot \frac{U(N) \cdot U(N-1)}{M \cdot U^2} & 0 \end{array} \right] \qquad \text{..........(10)}$$

The matrix in this form can now be used in order to generate random passenger origin-destination pairs for evaluating the round trip time using the Monte Carlo Simulation (*MCS*) method. It can also be used to evaluate the round trip time using formula by calculation, as shown in section 4 later in this paper.

2.6 Rules of the Final Origin-Destination Matrix

The final origin-destination matrix must obey a number of rules, listed below:

1. The sum of all the elements in the *OD* matrix should add up to 1.
2. All the elements of the diagonal of the matrix should be equal to zero.
3. The sum of all the elements in the incoming traffic area of the *OD* matrix should add up to the percentage incoming traffic (i.e., *ic*).
4. The sum of the all the elements in the out-going traffic area should add up to the percentage outgoing traffic (i.e., *og*).
5. The sum of all the elements in the inter-floor traffic area in the *OD* matrix should add up to the percentage inter-floor traffic (i.e., *if*).
6. The elements in the inter-entrance traffic area in the *OD* matrix should be zero.

It is worth noting that work has been carried out in trying to estimate the origin-destination traffic from elevator movements in the building ([9], [10], [11] and [12]). The outcome can be used in a number of ways, including generating virtual passenger traffic [13] or deciding on the suitable group control algorithm to adopt during that period of time [14].

3. RANDOM GENERATION OF PASSENGERS USING THE OD MATRIX

This section examines in detail the method of generating individual passenger origin-destination pairs using the origin-destination matrix. This is necessary for the evaluation of the round trip time using the Monte Carlo simulation (*MCS*) method.

The following is a complete set of steps that are used to generate origin-destination pairs for passengers:

1. The initial origin-destination matrix is generated using the floor arrival percentages and the floor percentage populations, by multiplying them and populating the matrix.
2. The four areas in the matrix are delineated in accordance with the expected three types of traffic: incoming, outgoing and inter-floor.
3. The inter-entrance area of the matrix is zeroed.
4. The elements of each part of the matrix are multiplied by the corresponding percentage traffic (i.e., the incoming traffic elements are multiplied by *ic*; the outgoing traffic elements are multiplied by *og*; the inter-floor traffic elements are multiplied by *if*).
5. The remaining non-zero diagonal elements (which are expected to be in the inter-floor area) should be zeroed. All the remaining non-zero elements in the inter-floor area are divided by the adjusting factor *M*, in order to restore the sum of the elements in the inter-floor traffic area back to the value of *if*.
6. The final form of the OD matrix is ready at this stage. A quick check at this stage can be carried out at this stage to ensure that the sum of all the elements in the OD matrix is 1, and that the sum of the elements in each area of the matrix corresponds to the percentage traffic type (*ic*; *og*; *if*).
7. A probability density function (pdf) is generated from the matrix. The number of possible values for the random variables is N^2. Each random variable is an origin/destination pair.
8. The probability density function (*pdf*) is converted to a cumulative distribution function (*cdf*) by the use of integration/summation.
9. Random numbers (between 0 and 1) are generated and applied to the cdf in order to randomly generate passengers with origin-destination pairs.

A numerical example is presented below that illustrates the procedure of randomly generating passenger origin-destination pairs.

3.1 Numerical Example on Developing the Origin-Destination Matrix and Using it to Randomly Generate Passengers

This section presents a practical numerical example on developing the original-destination matrix and using it to generate passenger origin-destination pairs.

A building has two entrance floors (*B* and *G*) and four occupant floors (1, 2, 3 and 4). The percentage arrival rates from the two entrances are 20% and 80% from *B* and *G* respectively. The percentage populations of the occupant floors are: 40%, 30%, 20% and 10% for the occupant floors 1, 2, 3 and 4 respectively. It is required that passenger origin-destination pairs are randomly generated based on a traffic mix of 40%:50%:10% of incoming, outgoing and inter-floor traffic respectively.

The first step is to compile the initial origin-destination matrix, by multiplying the percentage arrival rates and the percentage floor populations.

Table 5: Generalised representation of traffic for a building depending on the traffic mix.

Type	#	Percentage arrival	Percentage population
Occupant Floors	4	0	0.1
	3	0	0.2
	2	0	0.3
	1	0	0.4
Entrance Floors	G	0.8	0
	B	0.2	0

By multiplying the floor percentage *R(i)* for each floor by the floor percentage for each floor, the initial *OD* matrix can be populated, as shown in Table 6 below.

Table 6: The initial origin-destination matrix derived from percentage arrivals and percentage populations.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0.04	0.16	0.08	0.06	0.04	0.02
G	0.8	0.16	0.64	0.32	0.24	0.16	0.08
1	0.4	0.08	0.32	0.16	0.12	0.08	0.04
2	0.3	0.06	0.24	0.12	0.09	0.06	0.03
3	0.2	0.04	0.16	0.08	0.06	0.04	0.02
4	0.1	0.02	0.08	0.04	0.03	0.02	0.01

It is worth noting that the initial origin-destination matrix as it is currently shown has four distinct areas that are outlined in thick lines. The first area (top left) represents the inter-entrance traffic, which has been so far assumed to be rare. The second area (top right area) represents incoming traffic. The third area (bottom left area) represents outgoing traffic. The fourth area (bottom right area) represents inter-floor traffic.

As it has been assumed that the inter-entrance traffic percentage is zero, the corresponding area in the matrix will be zeroed. In addition, the various traffic areas are multiplied by their corresponding traffic percentage. The resultant matrix is shown in Table 7, below.

Table 7: The initial origin-destination matrix adjusted to suit the traffic mix.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	0.032	0.024	0.016	0.008
G	0.8	0	0	0.128	0.096	0.064	0.032
1	0.4	0.04	0.16	0.016	0.012	0.008	0.004
2	0.3	0.03	0.12	0.012	0.009	0.006	0.003
3	0.2	0.02	0.08	0.008	0.006	0.004	0.002
4	0.1	0.01	0.04	0.004	0.003	0.002	0.001

The next step is to zero the diagonal element, and then adjust the inter-floor elements by the value of *M*. The value of *M* is calculated below:

$$M = 1 - \left(\frac{\sum_{i=1}^{N} \left(\Pr_{arr}(i)\% \cdot \frac{U(i)}{U}\% \right)}{if} \right) = 1 - \frac{0.016 + 0.009 + 0.004 + 0.001}{0.1} = 0.7 \qquad ....(11)$$

Following the zeroing of the diagonal and the division of the all the inter-floor elements by *M* (i.e., 0.7 in this case), the final *OD* matrix can be seen in Table 8.

Table 8: The final origin-destination matrix after the zeroing of the diagonal of the inter-floor area.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	0.032	0.024	0.016	0.008
G	0.8	0	0	0.128	0.096	0.064	0.032
1	0.4	0.04	0.16	0	12/700	8/700	4/700
2	0.3	0.03	0.12	12/700	0	6/700	3/700
3	0.2	0.02	0.08	8/700	6/700	0	2/700
4	0.1	0.01	0.04	4/700	3/700	2/700	0

The final *OD* matrix shown in Table 8 is in fact the probability density function (*pdf*) matrix. In order for it to be used to generate passenger origin-destination pairs, it must be converted to a cumulative distribution function (*cdf*) matrix. This has been done in Table 9, where each element is equal to the sum of itself and all the preceding elements. As shown by the arrows superimposed on the table, the direction of adding the terms to produce the *CDF* terms is one row at a time, moving from left to right and then down to the next row.

Table 9: Converting the *PDF OD* matrix into a *CDF OD* matrix by progressively adding the terms.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	4/125	7/125	9/125	2/25
G	0.8	2/25	2/25	26/125	38/125	46/125	2/5
1	0.4	11/25	3/5	3/5	108/175	22/35	111/175
2	0.3	93/140	549/700	561/700	561/700	81/100	57/70
3	0.2	146/175	32/35	162/175	327/350	327/350	164/175
4	0.1	663/700	691/700	139/140	349/350	1	1

The values have been shown as fractions in Table 9 and as decimals to four decimal places in Table 10, in order to make it easier to generate random passenger origin-destination pairs.

Table 10: The final origin-destination matrix in the form of a *cdf* expressed to four decimal places.

	Floor	B	G	1	2	3	4
Floor	Arrival percentage/ population percentage	0.2	0.8	0.4	0.3	0.2	0.1
B	0.2	0	0	0.0320	0.0560	0.0720	0.0800
G	0.8	0.0800	0.0800	0.2080	0.3040	0.3680	0.4000
1	0.4	0.4400	0.6000	0.6000	0.6171	0.6286	0.6343
2	0.3	0.6643	0.7843	0.8014	0.8014	0.8100	0.8143
3	0.2	0.8343	0.9143	0.9257	0.9343	0.9343	0.9371
4	0.1	0.9471	0.9871	0.9929	0.9971	1	1

Using the *CDF*, it is now possible to generate some passengers. In order to generate three passengers, three random numbers between 0 and 1 are generated, giving the following three random numbers: 0.814, 0.308 and 0.407.

1. Taking the first random number (0.814) it lies between 0.8100 and 0.8143, and is thus assigned to 0.8143, and would represent an up passenger journey from floor 2 to floor 4.

2. The second random number (0.308) lies between 0.3040 and 0.3680 and is assigned to 0.3680 and thus represents an up passenger journey from floor *G* to floor 3.

3. The third random number (0.407) lies between 0.4000 and 0.4400, and is thus assigned to 0.4400. It represents a down passenger journey from floor 1 to floor *B*.

4. DERIVATION OF A ROUND TRIP TIME EQUATION FOR GENERAL TRAFFIC CONDITIONS

Equations have been previously derived for the generalised case with the following conditions [6]:

1. Unequal floor heights.
2. Top speed not attained in one floor journey.
3. Unequal floor populations.
4. Multiple entrances.
5. General traffic conditions (i.e., traffic consisting of a mixture of incoming traffic, out-going traffic and inter-floor traffic).

However, the calculations involved become too complicated and are impractical unless programmed in software. This section derives an equation for the round trip time for the following conditions (note that the first two conditions are different from the general case in [6] and the last two conditions are the same as the general case in [6]):

1. Equal floor heights.
2. Top speed attained in one floor journey.
3. Multiple entrances.
4. General traffic conditions (i.e., traffic consisting of a mixture of incoming traffic, out-going traffic and inter-floor traffic).

The set of equations that will be derived apply to general traffic conditions and multiple entrances, but assume that the floor heights are equal and that the top speed is attained in one floor journey.

As will be seen later, the resulting round trip time equation is much simpler than that derived in [6]. Moreover, it is also very *intuitive and informative* as the final form of the equation bears significant resemblance to the classical equation for the round trip time for the case of incoming traffic only (still assuming unequal floor heights and top speed not attained in one floor journey).

As shown in Figure 2, the elevator movement can be visualised as a continuous ring around the building, and not necessarily attaining the highest floor in the building in each round trip, or attaining the lowest floor in the building in each round trip. The average value of the highest attained floor in a round trip will be denoted as *H* (also known as the highest reversal floor) and has units of floors. The average value of the lowest attained floor in a round trip will be denoted as *L* and will also have units of floors.

It is acknowledged that the actual values of *H* and *L* will vary from one round trip to the next, but it is the average values over a large number of round trips that will be evaluated and used in the round trip time equation. In the next two sub-sections, formulae are derived for *H* and *L* as a function of the origin-destination matrix and the number of passengers.

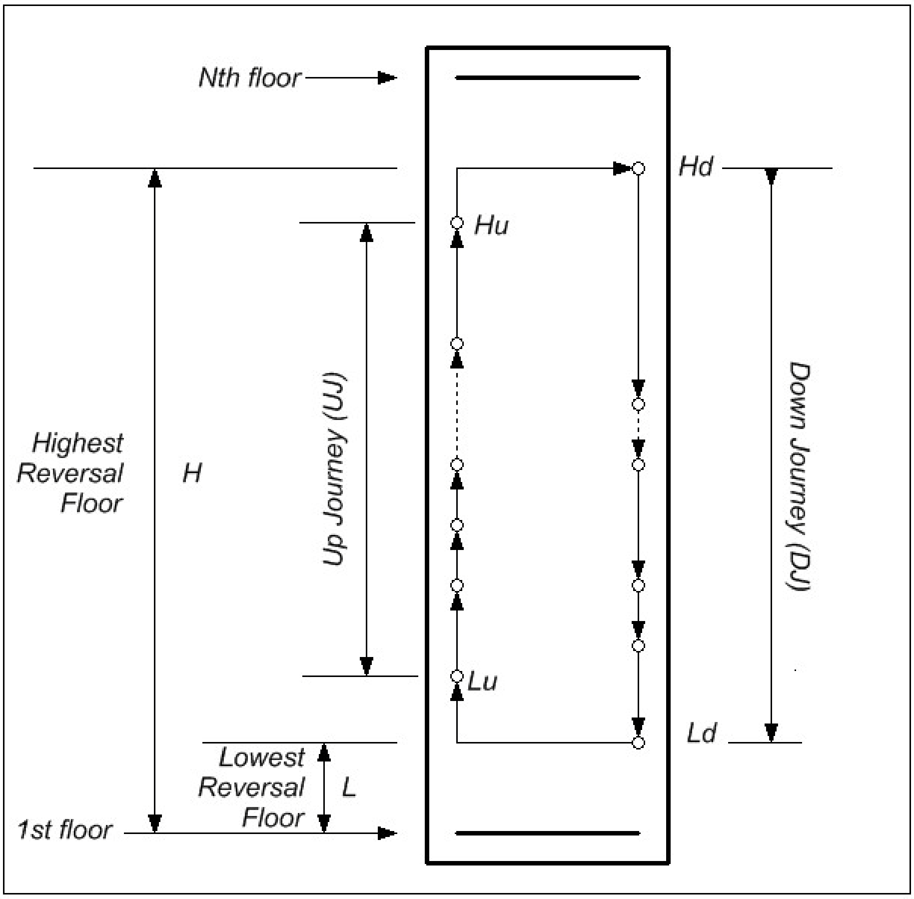

Figure 2: General overview showing the lowest reversal floor (*L*) and the highest reversal floor (*H*).

All the formulae for the probabilities for the various events will be based on the origin-destination matrix. It is worth noting that all probabilities related to up travelling passenger can be found in the upper triangle of the origin-destination matrix (i.e., all entries that are above the diagonal), and that all probabilities related to down travelling passengers can be found in the lower triangle of the origin-destination matrix (i.e., all entries that are below the diagonal). The upper triangle of the matrix is that which is above the diagonal, and the lower triangle of the matrix is that which is below the diagonal as shown in Figure 3.

$$
\begin{matrix}
0 & p_{12} & p_{13} & \cdots & p_{1j} & \cdots & \cdots & \cdots & \cdots & p_{1N} \\
p_{21} & 0 & p_{23} & \cdots & p_{2j} & \cdots & \cdots & \cdots & \cdots & p_{2N} \\
p_{31} & p_{32} & 0 & \cdots & p_{3j} & \cdots & \cdots & \cdots & \cdots & p_{3N} \\
\cdots & \cdots & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p_{i1} & p_{i2} & p_{i3} & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & p_{iN} \\
\cdots & \cdots & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \cdots \\
p_{N1} & p_{N2} & p_{N3} & \cdots & p_{Nj} & \cdots & \cdots & \cdots & \cdots & 0
\end{matrix}
$$

Figure 3: upper triangle and lower triangle in the origin destination matrix.

4.1 Derivation of formulae for *H and L*

During any round trip the elevator will attain the highest floor, denoted as *H*. The highest reversal in any round trip is decided by the highest destination of the up travelling passengers or the highest origin of the down travelling passenger whichever is higher. It has unit of floors. This is consistent with the general approach, as it has been assumed that the floor heights are equal.

In order to derive a formula for the expected value of the highest reversal floor, it is necessary to derive formulae for the following two events:

The probability of the elevator not travelling to any floor above the i^{th} floor in a round trip

The probability of the elevator not travelling to any floor above the $(i-1)^{th}$ floor in a round trip.

The difference between the probabilities of these two events is the probability of the i^{th} floor being the highest reversal floor in a round trip.

As the traffic is general traffic in this case, there are two conditions that need to be true in a round trip for the elevator not to travel above the i^{th} floor, and these are:

No up travelling passengers destined to (i.e., travelling to) a floor that is above the i^{th} floor
AND
No down travelling passengers originating from (i.e., travelling from) a floor that is above the i^{th} floor

The next step is to derive a formula for these two events. Thus, the probability of the first event can be found by excluding all the probabilities of an up travelling passenger having as a destination any of the floors above the i^{th} floor. These probabilities should reside in the upper triangle (as they relate to up travelling passengers) and should be in columns with a value higher than i. Such an area has been denoted as A and shown as hatched in Figure 4.

As for the down travelling passengers not originating at any floors above the i^{th} floor, these probabilities would reside in the lower triangle of the origin-destination matrix (as they are down travelling passengers) and should be in rows with a value higher than i. Such an area has been denoted as area B and shown as hatched in Figure 4.

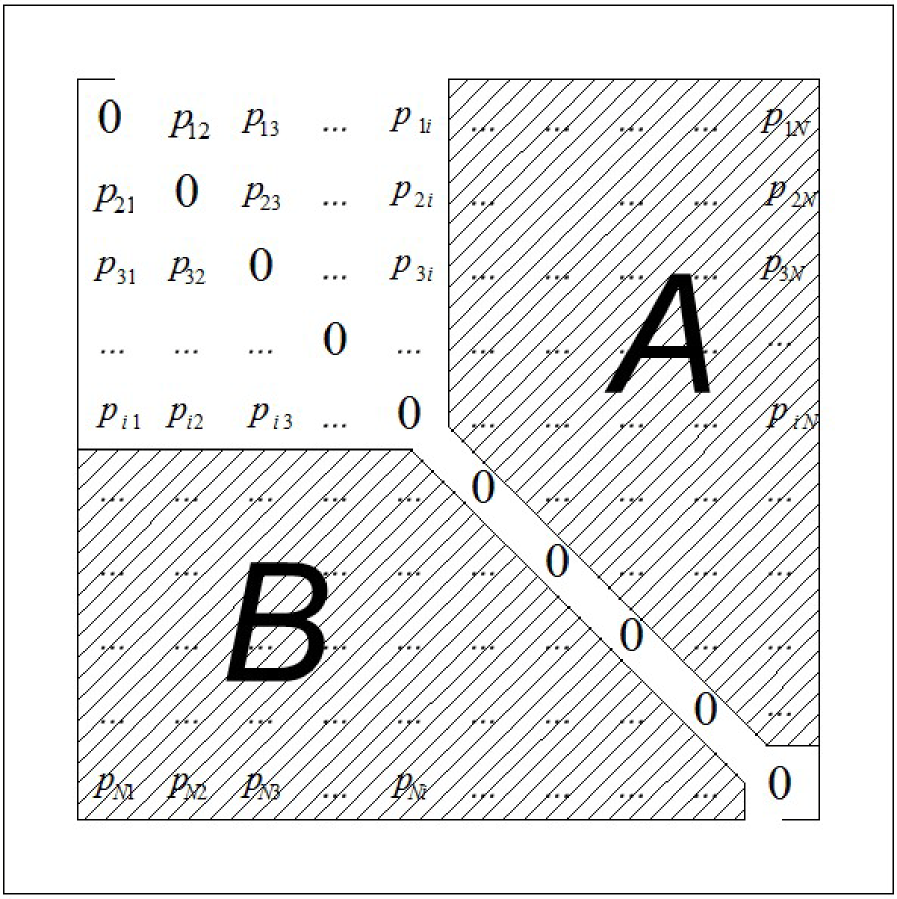

Figure 4: The shaded area *A* that represents the probability of an up travelling passenger heading for a floor above the i^{th} floor.

The event of a passenger not travelling upwards to a floor above the i^{th} floor in a round trip and not travelling downwards from a floor above the i^{th} floor in the same round trip shall be denoted as *Q*. The probability of the event *Q* can thus be found by subtracting the sum of the probabilities in areas *A* and *B* from 1:

$$\Pr(Q) = 1 - \sum\sum(A) - \sum\sum(B) \qquad \text{……….(12)}$$

Adding the limits of summation:

$$\Pr(Q) = 1 - \left(\sum_{k=1}^{i} \sum_{j=i+1}^{N} \left(p_{kj}\right) + \sum_{k=i+1}^{N-1} \sum_{j=k+1}^{N} \left(p_{kj}\right) \right) - \left(\sum_{k=i+1}^{N} \sum_{j=1}^{i} \left(p_{kj}\right) + \sum_{k=i+2}^{N} \sum_{j=i+1}^{k} \left(p_{kj}\right) \right) \qquad \text{……….(13)}$$

As the passenger decisions are assumed to be independent, then the probability that none of the *P* passengers will upwards to a floor above the i^{th} floor in a round trip and that none of the *P* passengers will travel downwards from a floor above the i^{th} floor in the same round trip can be found by raising the value found in (13) to the power *P*. Such an event is in fact the same as the highest reversal floor being smaller than *i*:

$$\Pr(H \le i) = \left(1 - \left(\sum_{k=1}^{i} \sum_{j=i+1}^{N} (p_{kj}) + \sum_{k=i+1}^{N-1} \sum_{j=k+1}^{N} (p_{kj})\right) - \left(\sum_{k=i+1}^{N} \sum_{j=1}^{i} (p_{kj}) + \sum_{k=i+2}^{N} \sum_{j=i+1}^{k} (p_{kj})\right)\right)^{P} \quad \text{……….(14)}$$

A similar approach can be followed to find the probability of the highest reversal floor being smaller than *i*-1, by using areas *C* and *D* that are shaded in Figure 5.

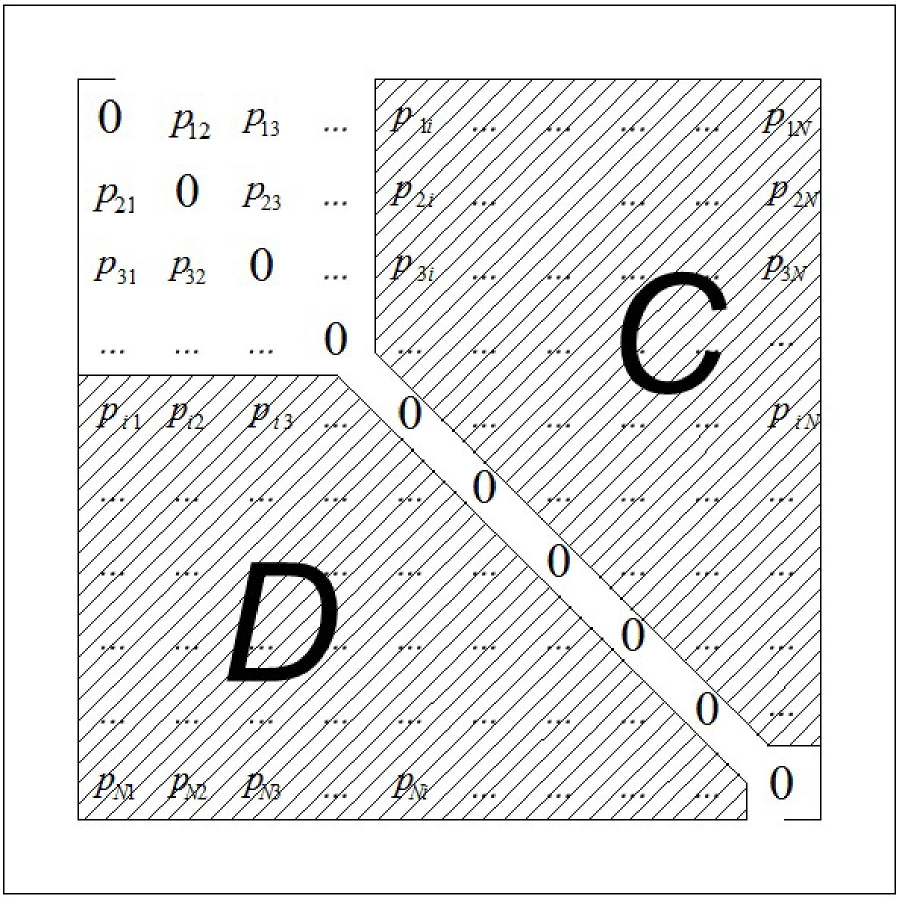

Figure 5: Areas *C* and *D* used in deriving the formula for the probability of the highest reversal floor being smaller than *i*-1.

Substituting i-1 in place of i in equation (14) gives:

$$\Pr(H \le i-1) = \left(1 - \left(\sum_{k=1}^{i-1} \sum_{j=i}^{N} (p_{kj}) + \sum_{k=i}^{N-1} \sum_{j=k+1}^{N} (p_{kj})\right) - \left(\sum_{k=i}^{N} \sum_{j=1}^{i-1} (p_{kj}) + \sum_{k=i+1}^{N} \sum_{j=i}^{k} (p_{kj})\right)\right)^{P} \quad \text{……….(15)}$$

Using the results from (14) and (15), the probability of the highest reversal floor being equal to *i* can be found as shown in (16) below:

$$\Pr(H = i) = \Pr(H \le i) - \Pr(H \le i-1) \quad \text{……….(16)}$$

In a similar way, the probability of the lowest reversal floor, *L*, being larger than or equal to *i* can be found by finding a formula for the probability of no down passengers travelling to a floor below the i^{th} floor in a round trip and no up passengers travelling from a floor below the i^{th} floor in a round trip (using areas *E* and *F* shown in Figure 6):

$$\Pr(L \geq i) = \left(1 - \sum\sum(E) - \sum\sum(F)\right)^{P} \qquad \text{……….(17)}$$

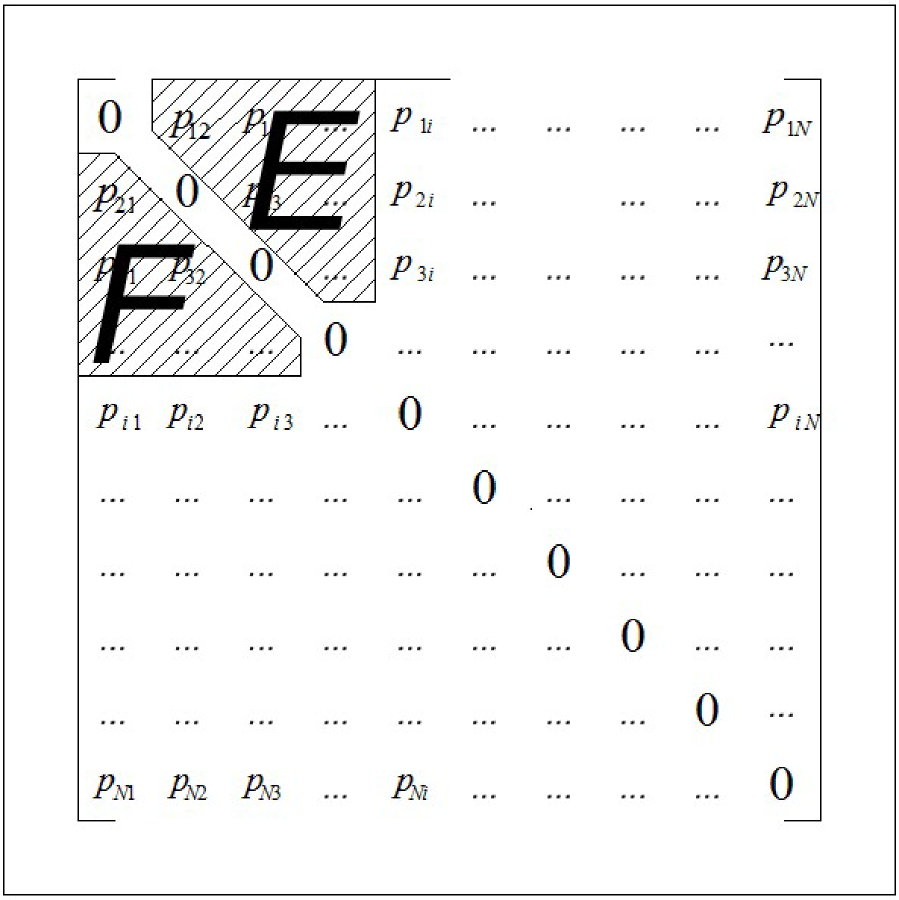

Figure 6: Shaded areas *E* and *F* used in deriving the probability of the lowest reversal floor being larger than *i*.

Applying the same principle to the probability of the lowest reversal floor being larger than *i*+1, the two shaded areas *G* and *H* (shown in Figure 7) can be used as shown below:

$$\Pr(L \geq i+1) = \left(1 - \sum\sum(G) - \sum\sum(H)\right)^{P} \qquad \text{……….(18)}$$

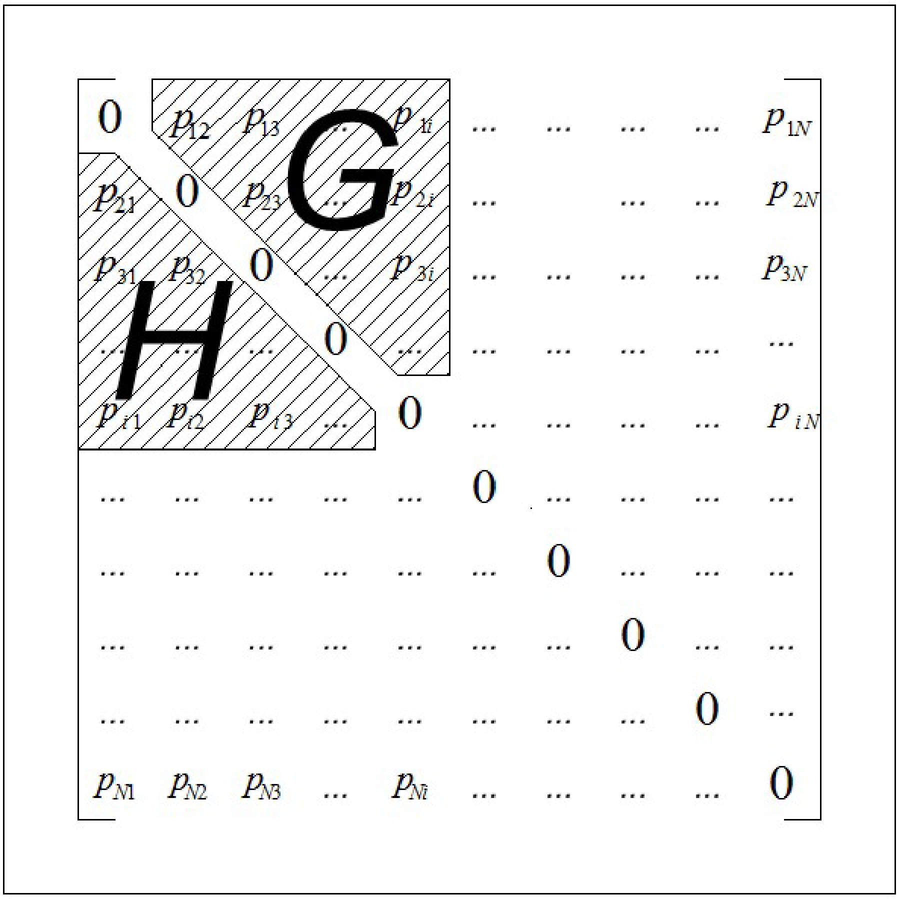

Figure 7: Shaded areas *G* and *H* used in deriving the probability of the lowest reversal floor being larger than *i*+1.

The probability of the lowest reversal floor being equal to *i* is the difference between the probability of the two events shown in (17) and (18) above:

$$\Pr(L = i) = \Pr(L \geq i) - \Pr(L \geq i + 1) \qquad \text{……….(19)}$$

It is now possible to find the formula for the expected values of the highest reversal floor, *H*, and the lowest reversal floor, *L*, as shown below as the weighted average of the floor from 1 to *N*, first for *H*:

$$E(H) = \sum_{i=1}^{N} \left(i \cdot \Pr(H = i) \right) \qquad \text{……….(20)}$$

...and for *L*:

$$E(L) = \sum_{i=1}^{N} \left(i \cdot \Pr(L = i) \right) \qquad \text{……….(21)}$$

4.2 Derivation of Formulae for the Number of Stops

A previous paper derived a formula for the expected number of stops in a round trip for the general case of mixed traffic conditions [6]. These formulae are reproduced here without proof.

The number of stops in a round trip has two main components: the stops that take place during the up part of the journey, denoted as S_U, and the stops that take place during the down part of the journey, denoted S_D.

However, it is necessary to remember that in some journeys, there is a common stop at the top reversal of the elevator during the round trip due to the fact that the last up stop floor (where the last up passenger(s) alight(s)) happens to be the same as the first down stop floor (where the first down passenger alight(s)). Such a stop is denoted as Upper Coincidental Stop (*UCS*), and must be subtracted from the total number of stops. The average number of *UCS* in a round trip is denoted as S_{UCS}.

A similar argument can be applied to the lower elevator reversals. In some journeys, there is also a common stop at the lower reversal of the elevator during the round trip due to the fact that the last down stop floor happens to be the same as the first up stop floor. Such a stop is denoted as a Lower Coincidental Stop (*LCS*) and must also be subtracted from the total number of stops.

The expected number of stops in the up direction, S_U and the expected number of stops in the down direction, S_D, can be evaluated as shown below in equations (22) and (23). The expected number of stops in the up direction is simply the sum of the probabilities of making a stop at all the floors from 1 to *N*, as shown below:

$$S_U = \sum_{i=1}^{N} P(S_{Ui}) = \sum_{i=1}^{N} \left(1 - P\left(\overline{S}_{Ui}\right)\right) \qquad (22)$$

...where the probability of stopping at a floor in the up direction can be calculated as shown below:

$$P\left(\overline{S}_{Ui}\right) = \left(1 - \sum_{k=1}^{i-1} p_{ki} - \sum_{k=i+1}^{N} p_{ik}\right)^{P} \qquad (23)$$

The expected number of stops in the down direction is simply the sum of the probabilities of making a stop at all the floors from 1 to *N*, as shown below:

$$S_D = \sum_{i=1}^{N} P(S_{Di}) = \sum_{i=1}^{N} \left(1 - P\left(\overline{S}_{Di}\right)\right) \qquad (24)$$

...where the probability of not stopping at a floor in the down direction can be calculated as shown below:

$$P\left(\overline{S}_{Di}\right) = \left(1 - \sum_{k=1}^{i-1} p_{ik} - \sum_{k=i+1}^{N} p_{ki}\right)^{P} \qquad (25)$$

The expected number of Upper Coincidental Stops (S_{UCS}) can be evaluated by summing the probability of an upper coincidental stop (*UCS)* taking place at each of the floors 1 to *N*. In fact the sum should run from 2 to *N* as the probability of floor 1 being an Upper Coincidental Stop is zero.

$$S_{UCS} = \sum_{i=1}^{N} P(UCS_i) = \sum_{i=1}^{N} P(UCJ_{ii}) \qquad (26)$$

...where the probability of an Upper Coincidental Stop (*UCS*) taking place at the i^{th} floor can be calculated as shown below:

$$P(UCS_i) = P(UCJ_{ii}) = \\ P\left(\overline{S}_{Ui+1,i+2...UN,DN....Di+2,i+1}\right) \\ - P\left(\overline{S}_{Ui,i+1,i+2...UN,DN....Di+2,i+1}\right) \\ - P\left(\overline{S}_{Ui+1,i+2...UN,DN....Di+2,i+1,i}\right) \\ + P\left(\overline{S}_{Ui,i+1,i+2...UN,DN....Di+2,i+1,i}\right) \tag{27}$$

Detailed equations for the probability of an up journey between two floors can be found in reference [6].

The expected number of Lower Coincidental Stops (S_{LCS}) can be evaluated by summing the probabilities of *a* Lower Coincidental Stop (*LCS*) taking place on each of the floors 1 to *N*. In fact the sum should run from 1 to *N-1* as the probability of floor *N* being a Lower Coincidental Stop is zero.

$$S_{LCS} = \sum_{i=1}^{N} P(LCS_i) = \sum_{i=1}^{N} P(LCJ_{ii}) \tag{28}$$

...where the probability of a lower coincidental stop taking place at the i^{th} floor can be evaluated by evaluating the probability of the Lower Connecting Journey *(LCJ)* taking place between the i^{th} floor and the i^{th} floor:

$$P(LCS_i) = P(LCJ_{ij}) = \\ P\left(\overline{S}_{Di-1,i-2...D1,U1....Ui-2,i-1}\right) \\ - P\left(\overline{S}_{Di,i-1,i-2...D1,U1....Ui-2,i-1}\right) \\ - P\left(\overline{S}_{Di-1,i-2...D1,U1....Ui-2,i-1,i}\right) \\ + P\left(\overline{S}_{Di,i-1,i-2...D1,U1....Ui-2,i-1,i}\right) \tag{29}$$

Detailed equations for the probability of a down journey between two floors can be found in reference [6].

Once the four components have been evaluated, the expected number of stops in a round trip (denoted as *S*) can be evaluated by adding the first two components and subtracting the last two components. It is worth not that the units of S_U, S_D, S_{UCS} and S_{LCS} are stops per round trip.

$$S = S_U + S_D - S_{UCS} - S_{LCS} \tag{30}$$

4.3 The Final Round Trip Time Equation

Having derived equations for the highest reversal floor, *H*, the lowest reversal floor, *L*, and the number of stops in a round trip, *S*, it is now possible to formulate an equation for the round trip time. The round trip time will be based on the classical format of the round trip time that is used for the case of incoming traffic condition and a single entrance, show below.

$$RTT = 2 \cdot H \cdot \frac{d_f}{v} + (S+1) \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} \right) + P \cdot (t_{pi} + t_{po}) \quad (31)$$

It is worth noting that S in the in the term (S+1) is the number of stops in the up direction, and that the term 1 in the term (S+1) is the number of down stops (i.e., only one stop in the down direction).

In order to convert equation (31) to the case of multiple entrances and general traffic conditions, H will be replaced by H-L and the number of stops (S+1) will be replaced by the terms in equation (30) (i.e., S_U+S_D-S_{UCS}-S_{LCS}), which gives the final equation shown below:

$$RTT = 2 \cdot (H - L) \cdot \frac{d_f}{v} + (S_U + S_D - S_{UCS} - S_{LCS}) \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} \right) + P \cdot (t_{pi} + t_{po}) \quad (32)$$

Equation (32) is the more general form of equation (31). When equation (32) is applied to the special case of incoming traffic only and a single entrance, S_D becomes equal to 1 (only one stop in the down direction), S_{UCS} and S_{LCS} become zero (i.e., no upper or lower coincidental stops) and the value of lowest reversal floor (L) becomes 0 (as the elevator must always start from the lowest floor which is the single entrance). Thus equation (32) reverts back to the special case represented by equation (31).

5. RE-DEFINING THE INTERVAL FOR THE CASE OF MULTIPLE ENTRANCES AND GENERAL TRAFFIC: THE VIRTUAL INTERVAL

In buildings with one single entrance under incoming traffic conditions, the definition of the concept of the interval is straightforward. The interval is defined as the time between successive arrivals (or departures) of the elevator at the single entrance (i.e., main lobby). However, in the case where there are multiple entrances or general traffic conditions, it is no longer possible to use such a definition of the interval, as each elevator may not stop at every entrance in every round trip. This is especially true in multiple entrance cases with no one single dominant entrance. It is thus necessary to amend the definition of the interval to account for the case of multiple entrances and general traffic conditions.

In order to do so, it is necessary to introduce two new definitions of the interval and two corresponding terms: The physical interval (to be denoted as int_p) and the virtual interval (to be denoted as int_v). The physical interval is the one that has been traditionally used in incoming traffic design.

The virtual interval is defined as the time between successive reversals of the elevators in the same of sense of reversal. It is important to define the term "sense" of reversal. A reversal of direction can either take place at the lower end of the journey or the upper end of the journey. A reversal of direction at the upper end of the journey will be from up to down and will be denoted as an up/down sense of reversal. A reversal of direction at the lower end of the journey will be from down to up and will denoted as a down/up sense of reversal. Thus the interval has to be re-defined as the time between successive reversals of the elevators in the same sense of reversal. It will be more intuitive to use the lower journey reversal (i.e., down/up sense of reversal) as the basis for the virtual interval, as this is usually the point where one round trip *finishes* and the next one *starts*.

The physical interval (denoted as int_p) is similar to the original concept of the interval, where it is the time between successive arrivals of elevators at the same floor travelling in the same direction. In general, the physical interval is larger than the virtual interval. Moreover, the physical interval will be specific to each entrance. Each entrance will have its own specific physical interval. So the fact that the physical interval for a certain floor is larger than the virtual interval for the whole group of elevators is a reflection of the fact that the elevators will not stop at that floor in every round trip.

An expected question will arise at this point: Why is it important to keep the concept of interval in the case of multiple entrances or in the case of general traffic conditions? The answer is related to the quality of service. The virtual interval can still be used as the indicator of the quality of service. The virtual interval is still calculated by dividing the round trip time by the number of elevators, L. On the other hand, the physical interval is an indication of the quality of service at a certain entrance floor.

Obviously, the two intervals become equal for the case of a single entrance and incoming traffic conditions, whereby the physical entrance becomes equal to the virtual interval.

6. NUMERICAL EXAMPLE

This section presents a numerical example to illustrate the methodology presented in this paper. The following conditions are true for this building:

1. Top speed attained in one floor journey.
2. Equal floor heights.
3. Unequal floor populations.
4. Multiple entrances.
5. General traffic conditions, where the traffic under which the design is carried out is a mixture of incoming traffic, out-going traffic and inter-floor traffic.

The following are the parameters for the building:

Total number of floors: 8 floors
Two entrances: *B* and *G*
Six occupant floors: 1, 2, 3, 4, 5 and 6
Traffic mix: 40%:40%:20% incoming, outgoing and inter-floor respectively.
The number of passengers boarding each elevator in every round trip, P, is 10
Equal floor heights: floor height, d_f, 4.5 m
Arrival rate, $AR\%$: 12.5% of the building population arriving in five minutes
Target interval, int_{tar}: 40 seconds
Rated speed, v: 1.6 m/s
Rated acceleration, a: 1 m/s^2
Rated jerk, j: 1 m/s^3
Door opening time, t_{do}: 2 s
Door closing time, t_{dc}: 3 s
Passenger boarding time, t_{pi}: 1.2 s
Passenger alighting time, t_{po}: 1.2 s

The percentage arrivals and percentage populations of the different floors in the building are shown in Table 11 below.

Table 11: Description of the arrival percentages and population percentages in the building.

Floor	Type of floor	Percentage arrival rate	Percentage population
B	Entrance/exit	30%	-
G	Entrance/exit	70%	-
1	Occupant	-	150
2	Occupant	-	150
3	Occupant	-	100
4	Occupant	-	100
5	Occupant	-	50
6	Occupant	-	50

The initial origin destination matrix is produced by multiplying the floor percentages and is shown below in Table 12.

Table 12: The initial origin-destination matrix resulting from the multiplication of the floor percentages.

	floor	**B**	**G**	**1**	**2**	**3**	**4**	**5**	**6**
	percentage	**0.3**	**0.7**	**0.25**	**0.25**	**0.166667**	**0.166667**	**0.083333**	**0.083333**
B	**0.3**	0	0	0.075	0.075	0.05	0.05	0.025	0.025
G	**0.7**	0	0	0.175	0.175	0.116667	0.116667	0.058333	0.058333
1	**0.25**	0.075	0.175	0.0625	0.0625	0.041667	0.041667	0.020833	0.020833
2	**0.25**	0.075	0.175	0.0625	0.0625	0.041667	0.041667	0.020833	0.020833
3	**0.166667**	0.05	0.116667	0.041667	0.041667	0.027778	0.027778	0.013889	0.013889
4	**0.166667**	0.05	0.116667	0.041667	0.041667	0.027778	0.027778	0.013889	0.013889
5	**0.083333**	0.025	0.058333	0.020833	0.020833	0.013889	0.013889	0.006944	0.006944
6	**0.083333**	0.025	0.058333	0.020833	0.020833	0.013889	0.013889	0.006944	0.006944

The value of *M* is calculated using equation (9) giving a value of 0.805556. Once the inter-entrance area has been zeroed, the incoming traffic and the outgoing traffic areas have been multiplied by 0.4, the inter-floor area multiplied by 0.2, its diagonal zeroed and the inter-floor values divided by *M*, the final origin-destination matrix is derived and is shown in Table 13.

Table 13: Final origin-destination matrix.

	floor	**B**	**G**	**1**	**2**	**3**	**4**	**5**	**6**
	percentage	**0.3**	**0.7**	**0.25**	**0.25**	**0.166667**	**0.166667**	**0.083333**	**0.083333**
B	**0.3**	0	0	0.03	0.03	0.02	0.02	0.01	0.01
G	**0.7**	0	0	0.07	0.07	0.046667	0.046667	0.023333	0.023333
1	**0.25**	0.03	0.07	0	0.015517	0.010345	0.010345	0.005172	0.005172
2	**0.25**	0.03	0.07	0.015517	0	0.010345	0.010345	0.005172	0.005172
3	**0.166667**	0.02	0.046667	0.010345	0.010345	0	0.006897	0.003448	0.003448
4	**0.166667**	0.02	0.046667	0.010345	0.010345	0.006897	0	0.003448	0.003448
5	**0.083333**	0.01	0.023333	0.005172	0.005172	0.003448	0.003448	0	0.001724
6	**0.083333**	0.01	0.023333	0.005172	0.005172	0.003448	0.003448	0.001724	0

Having arrived at the final origin-destination matrix, as shown in Table 13, it is now possible to find the value of the round trip time using equations in this paper.

Table 14: The values of the different parameters that constitute the round trip time equation using equations and the Monte Carlo Simulation (MCS) method.

Parameter	Value from the equations in this paper	Value from the Monte Carlo Simulation (MCS) method
S_U	5.4167	5.42
S_D	5.4167	5.4237
S_{UCS}	0.2775	0.2777
S_{LCS}	0.5703	0.566
S	9.9856	10
H	7.562	7.579
L	1.0643	1.05

Using the values of the parameters shown in Table 14 above and substituting them into equation (32) gives the final value of the round trip time. It has been verified versus the general round trip time equation from reference that was derived for the general case of top speed not attained in one floor journey and unequal floor heights, and against the Monte Carlo simulation method [15]. The match between the two equation methods is excellent. The match with the Monte Carlo simulation method is also very good with a difference of 0.036%.

Table 15: The resultant values of the round trip time using three methods.

Method	Value of the round trip time (s)
Calculation using equations in this paper (equation (32))	136.4403
Calculation using equations in reference [6] (the most general set of equations for all conditions)	136.4403
Monte Carlo Simulation method [15]	136.49

The required number of elevators, the actual interval and the handling capacity can be calculated as shown below in equation (33), (34) and (35) respectively.

$$E = \left\lceil \frac{RTT}{\text{int}_{tar}} \right\rceil = \left\lceil \frac{136.44}{40} \right\rceil = 4 \tag{33}$$

$$\text{int}_{act} = \frac{RTT}{E} = \frac{136.44}{4} = 34.11\text{s} \tag{34}$$

$$HC\% = \frac{300 \cdot E \cdot P}{RTT \cdot U} = \frac{300 \cdot 4 \cdot 10}{136.44 \cdot 600} = 14.66\% \tag{35}$$

The design is compliant as the actual interval is smaller than the target interval and the handling capacity, *HC%*, is larger than the arrival rate *AR%*. The design can be finalised using the *HARint* plane [7] and the *HARint* space [8].■

7. CONCLUSIONS

An integrated framework has been introduced that can be used to carry out an elevator traffic design under general traffic conditions (i.e., a mixture of incoming, outgoing and inter-floor traffic conditions). The first step comprises converting the traffic conditions (i.e., traffic mix) and the floor percentages (the floor percentage arrivals and the floor percentage populations) into an origin destination matrix. The origin destination matrix is a compact concise form for expressing the passenger movements within the building.

Once the origin-destination matrix has been developed, it is then used to evaluate the round trip time. This can be done using one of two methods: the Monte Carlo Simulation (MCS) method that relies on the origin-destination matrix to generate random passenger origin-destination pairs; or using formulae for evaluating the round trip for general traffic conditions. A novel set of equations have been derived in this paper for the case of top speed attained in one floor journey and equal floor heights under general traffic conditions.

The virtual interval is presented as an alternative to the physical interval that can be used as a critical user requirements for the case of multiple entrances and general traffic conditions.

A detailed numerical example has been presented for the design of the elevator traffic system for a building under general traffic conditions, equal floor heights and top speed attained in one floor journey. The origin-destination matrix is produced from the floor percentages and the specified traffic conditions. The round trip time is then derived using the novel equations derived in this paper. By dividing the value of the round trip time by the virtual interval and rounding up, the required number of elevators can be found.

Although not discussed in this paper, the *HARint* space methodology can be used to complement this integrated framework. The *HARint* space allows the designer to stipulate the passenger average waiting time and the passenger average travelling time and integrate them into the user requirements. It can then recommend the elevator car capacity; the elevator rated speed as well as making recommendations on building zoning/banking as appropriate.

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REFERENCES

[1] G. C. Barney, “Elevator Traffic Handbook: Theory and Practice”, Spon Press, London and New York, ISBN 0-415-27476-1, 2003.

[2] CIBSE, “CIBSE Guide D: Transportation systems in buildings”, published by the Chartered Institute of Building Services Engineers, Fourth Edition, 2010, pp 4-2, section 4.3.2.

[3] British Council for Offices, "BCO Guide to Specification 2009", British Council for Offices, 2009.

[4] Richard Peters, “Lift traffic analysis: Formulae for the general case”, Building Services Engineering Research & Technology, 11(2), 1990, pp 65-67.

[5] Henri Hakonen, Risto Lahdelma, “Calculation of elevator round-trip time for the collective control algorithm in general traffic situations”, Turku Centre for Computer Science, TUCS Technical Report No 671, March 2005.

[6] Lutfi Al-Sharif and Ahmad M Abu Alqumsan, “Stepwise derivation and verification of a universal elevator round trip time formula for general traffic conditions”, Building Services Engineering Research & Technology, published online before print, DOI: 10.1177/0143624414542111.
[7] Lutfi Al-Sharif, Ahmad M. Abu Alqumsan, Osama F. Abdel Aal, “Automated optimal design methodology of elevator systems using rules and graphical methods (the HARint plane)”, Building Services Engineering Research & Technology, August 2013 vol. 34 no. 3, pp 275-293, doi: 10.1177/0143624412441615.
[8] Lutfi Al-Sharif, Osama F. Abdel Aal, Ahmad M. Abu Alqumsan, Mohammad A. Abuzayyad, “The *HARint* Space: A Methodology for Compliant Elevator Traffic Designs”, Building Services Engineering Research and Technology, first published online June 20, 2014, doi: 0143624414539968.
[9] Rosa Basagoiti, Maite Beamurgia, Richard Peters, Stefan Kaczmarczyk, “Origin destination matrix estimation and prediction in vertical transportation”, 2nd Symposium on Lift and Escalator Technologies, University of Northampton, Northampton, U.K., 27th September 2012.
[10] Rosa Basagoiti, Maite Beamurgia, Richard Peters, Stefan Kaczmarczyk, “Passenger flow pattern learning based on trip counting in lift systems combined with on-line information”, 3rd Symposium on Lift and Escalator Technologies, University of Northampton, Northampton, U.K., 26th - 27th September 2013.
[11] Juha-Matti Kuusinen, Janne Sorsa, Marja-Liisa Siikonen, “The elevator trip origin-destination matrix estimation problem”, Transportation Science, articles in advance, pp 1-18, 2014, INFORMS
[12] Juha-Matti Kuusinen, Arnaud Malapert, “The effect of randomisation on constraint based estimation of elevator trip origin-destination matrices”, 4th Symposium on Lift and Escalator Technologies, University of Northampton, 25th to 26th September 2014
[13] Marja-Liisa Siikonen, “Procedure for controlling an elevator group where virtual passenger traffic is generated”, US patent number 6 345 697 B1, 12th February 2002.
[14] Nader Kameli, “Predictor elevator for traffic during peak conditions”, US patent number 5 276 295, 4th January 1994.
[15] Lutfi Al-Sharif, Hussam Dahyat, Laith Al-Kurdi, “ The use of Monte Carlo Simulation in the calculation of the elevator round trip time under up-peak conditions”, *Building Services Engineering Research and Technology, volume 33, issue 3 (2012) pp. 319–338, doi:10.1177/0143624411414837*.