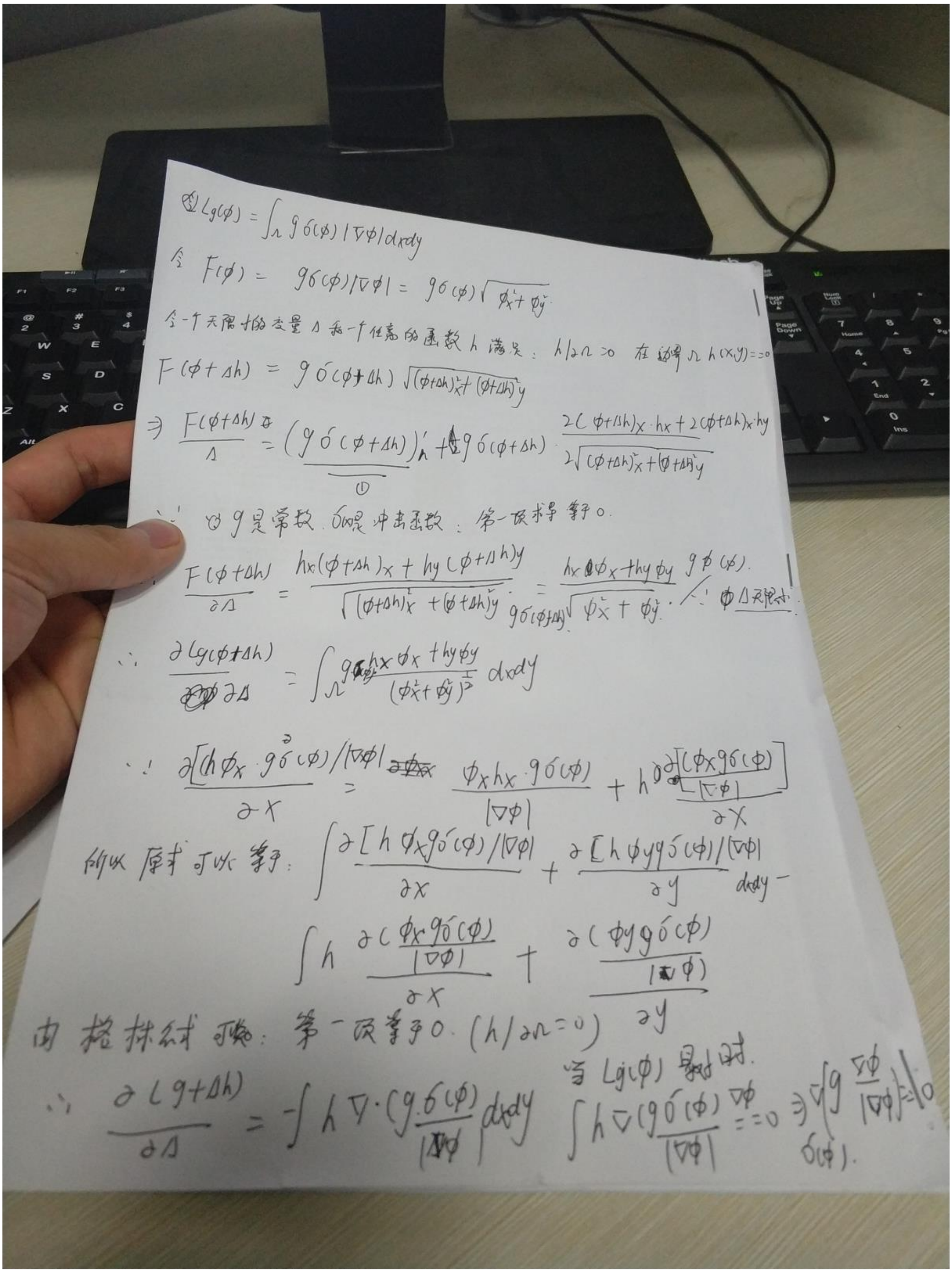
公式推导主要对第一项进行推导



$$L_g(\phi)=\int_\Omega g\delta(\phi)|\nabla\phi|\,dxdy$$

令 $F(\phi)=g\delta(\phi)|\nabla\phi|=g\delta(\phi)\sqrt{\phi_x^2+\phi_y^2}$

令一个无限小的变量 Δ 和一个任意的函数 h 满足：$h|_{\partial\Omega}=0$ 在边界 Ω $h(x,y)=0$

$$F(\phi+\Delta h)=g\delta(\phi+\Delta h)\sqrt{(\phi+\Delta h)_x^2+(\phi+\Delta h)_y^2}$$

$$\Rightarrow \frac{F(\phi+\Delta h)}{\Delta}=\underbrace{\left(g\delta(\phi+\Delta h)\right)'_\Delta}_{①}+g\delta(\phi+\Delta h)\cdot\frac{2(\phi+\Delta h)_x\cdot h_x+2(\phi+\Delta h)_y\cdot h_y}{2\sqrt{(\phi+\Delta h)_x^2+(\phi+\Delta h)_y^2}}$$

∵ g是常数 δ(φ)是冲击函数 ∴ 第一项求导等于0。

$$\frac{F(\phi+\Delta h)}{\partial\Delta}=\frac{h_x(\phi+\Delta h)_x+h_y(\phi+\Delta h)_y}{\sqrt{(\phi+\Delta h)_x^2+(\phi+\Delta h)_y^2}\,g\delta(\phi+\Delta h)}=\frac{h_x\phi_x+h_y\phi_y}{\sqrt{\phi_x^2+\phi_y^2}}\,g\delta(\phi)$$

∵ $\phi\Delta$ 无限小

$$\therefore \frac{\partial L_g(\phi+\Delta h)}{\partial\Delta}=\int_\Omega g\delta\frac{h_x\phi_x+h_y\phi_y}{(\phi_x^2+\phi_y^2)^{\frac{1}{2}}}\,dxdy$$

$$\because \frac{\partial[h\phi_x\cdot g\delta(\phi)/|\nabla\phi|]}{\partial x}=\frac{\phi_x h_x\cdot g\delta(\phi)}{|\nabla\phi|}+h\frac{\partial\left[\frac{\phi_x g\delta(\phi)}{|\nabla\phi|}\right]}{\partial x}$$

所以原式可以等于：

$$\int\frac{\partial[h\phi_x g\delta(\phi)/|\nabla\phi|]}{\partial x}+\frac{\partial[h\phi_y g\delta(\phi)/|\nabla\phi|]}{\partial y}\,dxdy-\int h\left(\frac{\partial\left(\frac{\phi_x g\delta(\phi)}{|\nabla\phi|}\right)}{\partial x}+\frac{\partial\left(\frac{\phi_y g\delta(\phi)}{|\nabla\phi|}\right)}{\partial y}\right)$$

由格林公式可知：第一项等于0. $(h/\partial\Omega=0)$

$$\therefore \frac{\partial L(g+\Delta h)}{\partial\Delta}=-\int h\,\nabla\cdot\left(g\delta(\phi)\frac{\nabla\phi}{|\nabla\phi|}\right)dxdy$$

当 $L_g(\phi)$ 最小时.

$$\int h\nabla\left(g\delta(\phi)\frac{\nabla\phi}{|\nabla\phi|}\right)==0\Rightarrow \nabla\left(g\frac{\nabla\phi}{|\nabla\phi|}\right)=0$$

$\delta(\phi)$.

$$\frac{\partial \phi}{\partial t} = \lambda \cdot \frac{\partial L_g(\phi)}{\partial t} + v \cdot \partial \frac{A_g(\phi)}{\partial t}$$

$$= \lambda \delta(\phi) \nabla \left(g \frac{\nabla \phi}{|\nabla \phi|} \right) + v g \delta(\phi) \quad \because \frac{\partial H'(\phi)}{\cancel{\partial t}} = \delta(\phi).$$

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