计算方法实验报告

本文是计算方法第三次实验报告。
一、实验原理
数值积分与数值微分等。
二、实验过程
（一）环境：Matlab
（二）实验题目与核心代码
1.用不同的数值积分方法计算积分
（1）复合梯形
自定义一个使用复合梯形公式计算的函数，一共有 4 个参数：待积分函数、区间始末位置、分点数。
feval 用来计算指定函数在某点的函数值。

```
function t = juxing(fun,a,b,n)
h=(b-a)/n;
fa=feval(fun,a);
fb=feval(fun,b);
f=feval(fun,a+h:h:b-h+0.001*h);
t=h*(0.5*(fa+fb)+sum(f));
end
```

主函数：

```
syms x
f=inline('sqrt(x).*log(x);');
juxing(f,eps,1,10)
```

实验结果：

```
ans =

   -0.4171
```

将 10 改为 100，主函数：

```
syms x
f=inline('sqrt(x).*log(x);');
juxing(f,eps,1,100)
```

实验结果：

```
ans =

   -0.4431
```

(2) 复合辛普森

自定义一个使用复合辛普森公式计算的函数，一共有 4 个参数：待积分函数、区间始末位置、分点数。

```
function t = comsimpson(fun,a,b,n)
h=(b-a)/n;
fa=feval(fun,a);
fb=feval(fun,b);
f1=feval(fun,a+h:h:b-h+0.001*h);
f2=feval(fun,a+h/2:h:b-h+0.001*h);
t=h/6*(fa+fb+2*sum(f1)+4*sum(f2));
```

主函数：

```
syms x
f=inline('sqrt(x).*log(x);');
format long;
comsimpson(f,eps,1,10)
```

结果：

```
ans =

  -0.435297890074689
```

将 10 改为 100，结果为

```
ans =

  -0.444161178415673
```

（3）龙贝格

自定义函数：RT是龙贝格积分表；R是数值积分值；wucha是误差估计；h是最小步长；fun是被积函数；a b是积分下、上限；m是龙贝格积分表中行最大数目，wucha是两次相邻迭代值的绝对误差限。

```
function [RT,R,wucha,h] = Romberg(fun,a,b,wucha,m)
n=1;
h=b-a;
wucha=1;
x=a;
k=0;
RT=zeros(4,4);
RT(1,1)=h*(feval(fun,a)+feval(fun,b))/2;
while((wucha>wucha)&&(k<m)||(k<4))
    k=k+1;h=h/2;s=0;
    for j=1:n
        x=a+h*(2*j-1);s=s+feval(fun,x);
    end
    RT(k+1,1)=RT(k,1)/2+h*s;n=2*n;
    for i=1:k
        RT(k+1,i+1)=((4^i)*RT(k+1,i)-RT(k,i))/(4^i-1);
    end
    wucha=abs(RT(k+1,k)-RT(k+1,k+1));
end
R=RT(k+1,k+1);
```

命令行中输入：

```
>> fun=inline('sqrt(x).*log(x)');
>> [RT,R,wucha,h]=Romberg(fun,eps,1,1e-5,13)

RT =

  -0.000000268546145       0         0         0         0         0
  -0.245064670140209  -0.326752804004897         0         0
0         0
  -0.358104125949240  -0.395783944552250  -
0.400386020588741         0         0         0
  -0.408090073087781  -0.424752055467295  -
0.426683262861631  -0.427100679405645         0         0
  -0.429474601629505  -0.436602777810080  -
0.437392825966266  -0.437562819031419  -0.437603847029951
0
  -0.438389494461832  -0.441361125405941  -
0.441678348578999  -0.441746372747455  -0.441762778840459
-0.441766844267449
RT =

   -0.0000         0         0         0         0         0
   -0.2451   -0.3268         0         0         0         0
   -0.3581   -0.3958   -0.4004         0         0         0
   -0.4081   -0.4248   -0.4267   -0.4271         0         0
   -0.4295   -0.4366   -0.4374   -0.4376   -0.4376         0
   -0.4384   -0.4414   -0.4417   -0.4417   -0.4418   -0.4418


R =

  -0.441766844267449

wucha =

     4.065426989774412e-06


h =

   0.031250000000000
```

（4）自适应辛普森

```
>>  f=inline('sqrt(x).*log(x)');
>>  q=quad(f,0,1,1e-4)

q =

   -0.4440
```

2.计算二重积分

（1）复合辛普森

复合Simpson多元求积公式；fun被积函数；x0—被积函数自变量；[a,b]积分区间；n为区间份数

```
function q=combinesimpson(fun,x0,a,b,n)
x=linspace(a,b,n+1);
q=0;
for k=1:n

q=q+subs(fun,x0,x(k))+4*subs(fun,x0,(x(k)+x(k+1))/2)+subs
(fun,x0,x(k+1));
end
q=q*(b-a)/n/6;
```

主函数：

```
clear
syms x y;
F=exp(-x.*y);
s=combinesimpson(combinesimpson(F,'x',0,1,4),'y',0,1,4)
```

结果：

```
s =



exp(-1)/576 + exp(-1/2)/144 + exp(-1/4)/72 + exp(-
3/4)/144 + exp(-1/8)/36 + exp(-3/8)/36 + exp(-5/8)/72 +
exp(-7/8)/72 + (5*exp(-1/16))/144 + exp(-3/16)/24 + exp(-
5/16)/36 + exp(-7/16)/36 + exp(-9/16)/144 + exp(-1/32)/36
+ exp(-3/32)/18 + exp(-5/32)/36 + exp(-7/32)/36 + exp(-
9/32)/36 + exp(-15/32)/36 + exp(-21/32)/36 + exp(-
1/64)/36 + exp(-3/64)/18 + exp(-5/64)/18 + exp(-7/64)/18
+ exp(-9/64)/36 + exp(-15/64)/18 + exp(-21/64)/18 + exp(-
25/64)/36 + exp(-35/64)/18 + exp(-49/64)/36 + 47/576

>> double(s)

ans =

   0.796599967946203
```

（2）高斯求积公式

定义函数：

Gauss 求积公式；fun 被积函数；x0—被积函数自变量；[a,b]积分区间；n 为节点个数

```
function q=gauss(fun,x0,a,b,n)
syms t;
fun=subs(fun,x0,(b-a)/2*t+(a+b)/2);
[x,A]=gausspoints(n);
q=(b-a)/2*sum(A.*subs(fun,t,x));
```

主函数：

```
syms x y;
F=exp(-x.*y);
s=gauss(gauss(F,x,0,1,4),y,0,1,4)
```

结果：

```
s =

(18404408714424662147502004272695*exp(-
404656362295822206307837631302001/64903710731685345356631
2041152512))/162259276829213363391578010288128 +
(18404408714424662147502004272695*exp(-
14871495419127870058979628190065/649037107316853453566312
041152512))/162259276829213363391578010288128 +
(39267716369383881636165367255225*exp(-
281019143215446796775403056299801/32451855365842672678315
6020576256))/1298074214633706907132624082305024 +
(39267716369383881636165367255225*exp(-
1564432874192569572515014803225/3245185536584267267831560
20576256))/1298074214633706907132624082305024 +
(8625972973352844496136100010969*exp(-
582689028487106428157024779144201/12980742146337069071326
24082305024))/81129638414606681695789005144064 +
(8625972973352844496136100010969*exp(-
141368402345345991973144932682761/12980742146337069071326
24082305024))/81129638414606681695789005144064 +
(8625972973352844496136100010969*exp(-
287008391900627243501227185239031/12980742146337069071326
24082305024))/40564819207303340847894502572032 +
(39267716369383881636165367255225*exp(-
20967488784393680217618974736615/324518553658426726783156
020576256))/649037107316853453566312041152512 +
(18404408714424662147502004272695*exp(-
199316901703858747678206430770831/64903710731685345356631
2041152512))/162259276829213363391578010288128 +
(18404408714424662147502004272695*exp(-
30192347898044629521288350889615/649037107316853453566312
041152512))/162259276829213363391578010288128

>> double(s)
ans =

    0.7966
```

（3）积分区域为圆形（复合辛普森）

```
syms x y;
f=exp(-x.*y);
s=combinesimpson(combinesimpson(f,y,0,sqrt(1-
x^2),4),x,0,1,4)
```

结果：

```
s =

(3^(1/2)*(exp(-3^(1/2)/4) + 2*exp(-3^(1/2)/8) + 2*exp(-
3^(1/2)/16) + 2*exp(-(3*3^(1/2))/16) + 4*exp(-3^(1/2)/32)
+ 4*exp(-(3*3^(1/2))/32) + 4*exp(-(5*3^(1/2))/32) +
4*exp(-(7*3^(1/2))/32) + 1))/576 + (7^(1/2)*(exp(-
(3*7^(1/2))/16) + 2*exp(-(3*7^(1/2))/32) + 2*exp(-
(3*7^(1/2))/64) + 2*exp(-(9*7^(1/2))/64) + 4*exp(-
(3*7^(1/2))/128) + 4*exp(-(9*7^(1/2))/128) + 4*exp(-
(15*7^(1/2))/128) + 4*exp(-(21*7^(1/2))/128) + 1))/1152 +
(15^(1/2)*(exp(-15^(1/2)/16) + 2*exp(-15^(1/2)/32) +
2*exp(-15^(1/2)/64) + 2*exp(-(3*15^(1/2))/64) + 4*exp(-
15^(1/2)/128) + 4*exp(-(3*15^(1/2))/128) + 4*exp(-
(5*15^(1/2))/128) + 4*exp(-(7*15^(1/2))/128) + 1))/1152 +
(15^(1/2)*(exp(-(7*15^(1/2))/64) + 2*exp(-
(7*15^(1/2))/128) + 2*exp(-(7*15^(1/2))/256) + 2*exp(-
(21*15^(1/2))/256) + 4*exp(-(7*15^(1/2))/512) + 4*exp(-
(21*15^(1/2))/512) + 4*exp(-(35*15^(1/2))/512) + 4*exp(-
(49*15^(1/2))/512) + 1))/1152 + (39^(1/2)*(exp(-
(5*39^(1/2))/64) + 2*exp(-(5*39^(1/2))/128) + 2*exp(-
(5*39^(1/2))/256) + 2*exp(-(15*39^(1/2))/256) + 4*exp(-
(5*39^(1/2))/512) + 4*exp(-(15*39^(1/2))/512) + 4*exp(-
(25*39^(1/2))/512) + 4*exp(-(35*39^(1/2))/512) + 1))/1152
+ (55^(1/2)*(exp(-(3*55^(1/2))/64) + 2*exp(-
(3*55^(1/2))/128) + 2*exp(-(3*55^(1/2))/256) + 2*exp(-
(9*55^(1/2))/256) + 4*exp(-(3*55^(1/2))/512) + 4*exp(-
(9*55^(1/2))/512) + 4*exp(-(15*55^(1/2))/512) + 4*exp(-
(21*55^(1/2))/512) + 1))/1152 + (63^(1/2)*(exp(-
63^(1/2)/64) + 2*exp(-63^(1/2)/128) + 2*exp(-63^(1/2)/256)
+ 2*exp(-(3*63^(1/2))/256) + 4*exp(-63^(1/2)/512) +
4*exp(-(3*63^(1/2))/512) + 4*exp(-(5*63^(1/2))/512) +
4*exp(-(7*63^(1/2))/512) + 1))/1152 + 1/24


>> double(s)

ans =

    0.6701
```