计算方法实验报告

本文是计算方法的实验报告。

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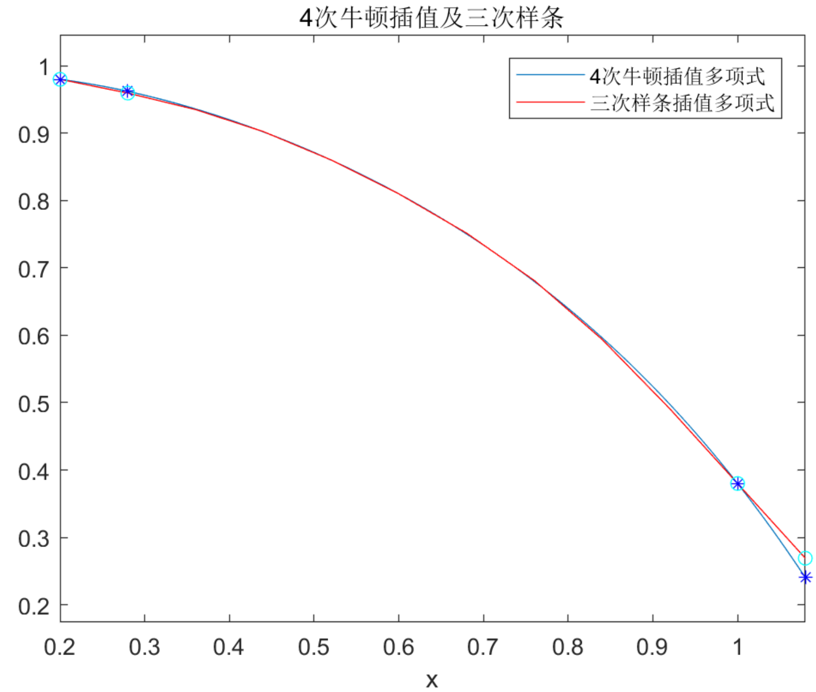
（1）4 次牛顿插值及三次样条函数（自然边界条件）插值

```
clear;clc;
x1=[0.2 0.4 0.6 0.8 1.0];
y1=[0.98 0.92 0.81 0.64 0.38];
n=length(y1);
c=y1(:);
for j=2:n
    for i=n:-1:j
        c(i)=(c(i)-c(i-1))/(x1(i)-x1(i-j+1));
    end
end
syms x df d;
df(1)=1;d(1)=y1(1);
for i=2:n
    df(i)=df(i-1)*(x-x1(i-1));
    d(i)=c(i)*df(i);
end
disp('4´ÎÅ£¶Ù²åÖµ¶àÏîÊ½');
P4=vpa(collect((sum(d))),5)
pp=csape(x1,y1, 'variational');
q=pp.coefs;
disp('Èý´ÎÑùÌõº¯Êý');
for i=1:4
S=q(i,:)*[(x-x1(i))^3;(x-x1(i))^2;(x-x1(i));1];
S=vpa(collect(S),5)
end
x2=0.2:0.08:1.08;
dot=[1 2 11 10];
figure
ezplot(P4,[0.2,1.08]);
hold on
y2=fnval(pp,x2);
x=x2(dot);
y3=eval(P4);
y4=fnval(pp,x2(dot));
plot(x2,y2,'r',x2(dot),y3,'b*',x2(dot),y4,'co');
legend('4´ÎÅ£¶Ù²åÖµ¶àÏîÊ½', 'Èý´ÎÑùÌõ²åÖµ¶àÏîÊ½');
title('4´ÎÅ£¶Ù²åÖµ¼°Èý´ÎÑùÌõ');
```



A. 4 次牛顿插值多项式

P4 = − 0.52083*x^4 + 0.83333*x^3 − 1.1042*x^2 + 0.19167*x + 0.98

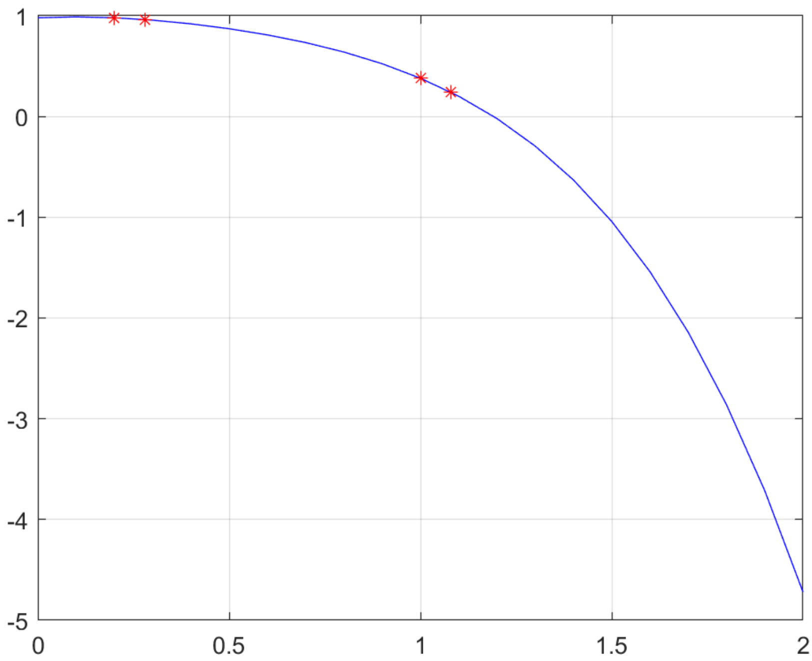
B. 三次样条函数

x∈[0.2,0.4]时，S = − 1.3393*x^3 + 0.80357*x^2 − 0.40714*x + 1.04
x∈[0.4,0.6]时，S = 0.44643*x^3 − 1.3393*x^2 + 0.45*x + 0.92571
x∈[0.6,0.8]时，S = − 1.6964*x^3 + 2.5179*x^2 − 1.8643*x + 1.3886
x∈[0.8,1.0]时，S =2.5893*x^3 − 7.7679*x^2 + 6.3643*x − 0.80571

（2）

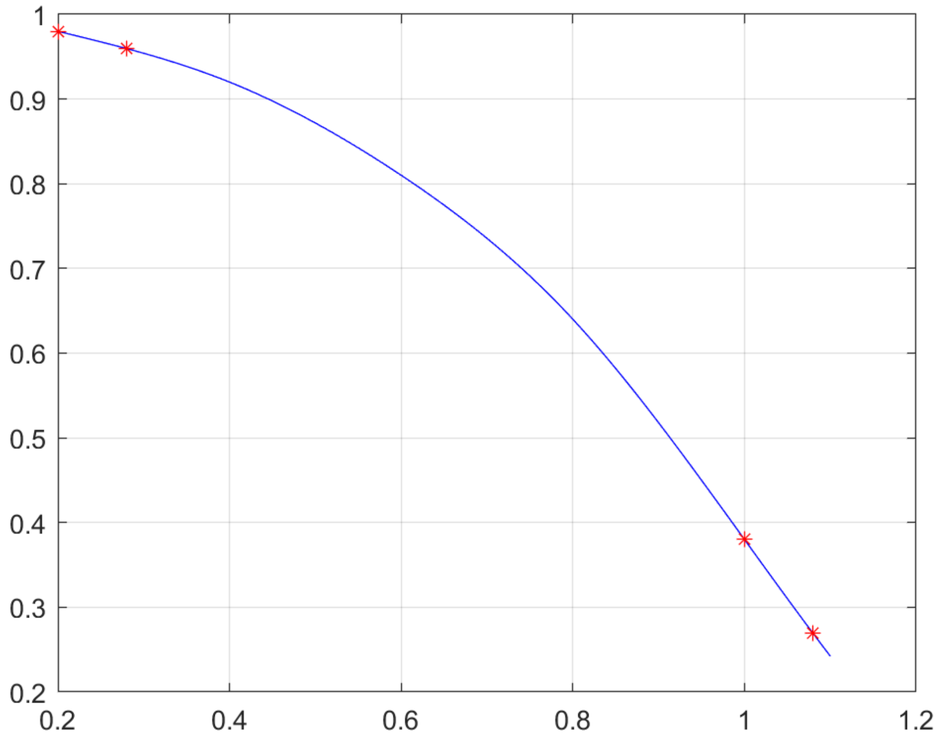
A 牛顿插值作图

```
x = 0:0.1:2;
P4 = - 0.52083.*x.^4 +
0.83333.*x.^3 -
1.1042.*x.^2 +
0.19167.*x + 0.98;
plot(x, P4, 'b');
x2 = [0.2 0.28 1 1.08] ;
y1 = - 0.52083.*x2.^4 +
0.83333.*x2.^3 -
1.1042.*x2.^2 +
0.19167.*x2 + 0.98;
hold on
plot(x2, y1, 'r*');
hold on
grid on
```



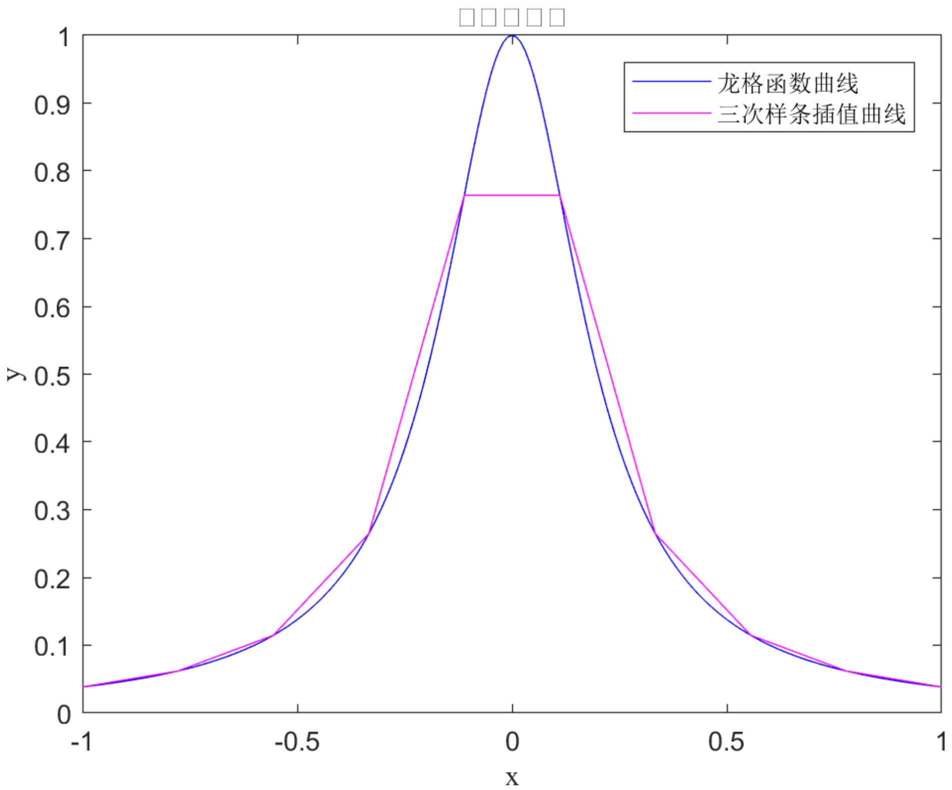
B 三次样条插值作图

```
x = 0.2:0.01:0.4;
S = - 1.3393.*x.^3 + 0.80357.*x.^2 - 0.40714.*x + 1.04;
plot(x, S, 'b');
hold on
x = 0.4:0.01:0.6;
S = 0.44643.*x.^3 - 1.3393.*x.^2 + 0.45.*x + 0.92571;
plot(x, S, 'b');
hold on
x = 0.6:0.01:0.8;
S = - 1.6964.*x.^3 + 2.5179.*x.^2 - 1.8643.*x + 1.3886;
plot(x, S, 'b');
hold on
x = 0.8:0.01:1.1;
S = 2.5893.*x.^3 - 7.7679.*x.^2 + 6.3643.*x - 0.80571;
plot(x, S, 'b');
hold on
x2 = [0.2 0.28];
y2 = - 1.3393.*x2.^3 + 0.80357.*x2.^2 - 0.40714.*x2 +
1.04;
plot(x2, y2, 'r*');
hold on
x2 = [1 1.08];
y2 = 2.5893.*x2.^3 - 7.7679.*x2.^2 + 6.3643.*x2 - 0.80571;
plot(x2, y2, 'r*');
hold on
grid on
```



（1）10 个节点时

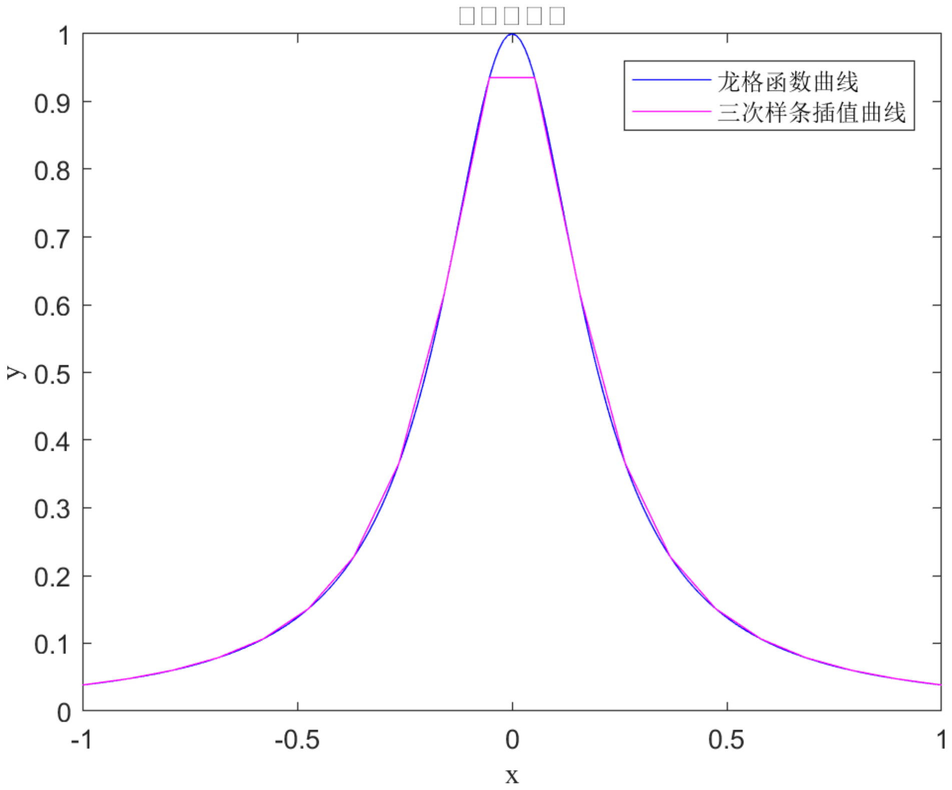
```
clear;
x0 =[-1:0.01:1];
y0 = 1./(1+25*x0.^2);
x1 = linspace(-1, 1, 20);
y1 = interp1(x0,y0,x1,'pchip');
plot(x0,y0,'b');
hold on
plot(x1,y1,'m');
title('Èý´Î¶àÏîÊ½','FontName','Times New
Roman','FontSize',11);
legend('Áú¸ñº¯ÊýÇúÏß', 'Èý´ÎÑùÌõ²åÖµÇúÏß');
axis([-1,1,0,1]);
xlabel('x','FontName',' Times New Roman ','FontSize',11);
ylabel('y','FontName',' Times New Roman ','FontSize',11);
```



（2）20 个节点时

```
clear;
x0 =[-1:0.01:1];
y0 = 1./(1+25*x0.^2);
x1 = linspace(-1, 1, 20);
y1 = interp1(x0,y0,x1,'pchip');
plot(x0,y0,'b');
hold on
plot(x1,y1,'m');
title('Èý´Î¶àÏîÊ½','FontName','Times New
Roman','FontSize',11);
```

```
legend('Áú¸ñº¯ÊýÇúÏß', 'Èý´ÎÑùÌõ²åÖµÇúÏß');
axis([-1,1,0,1]);
xlabel('x','FontName',' Times New Roman ','FontSize',11);
ylabel('y','FontName',' Times New Roman ','FontSize',11);
```



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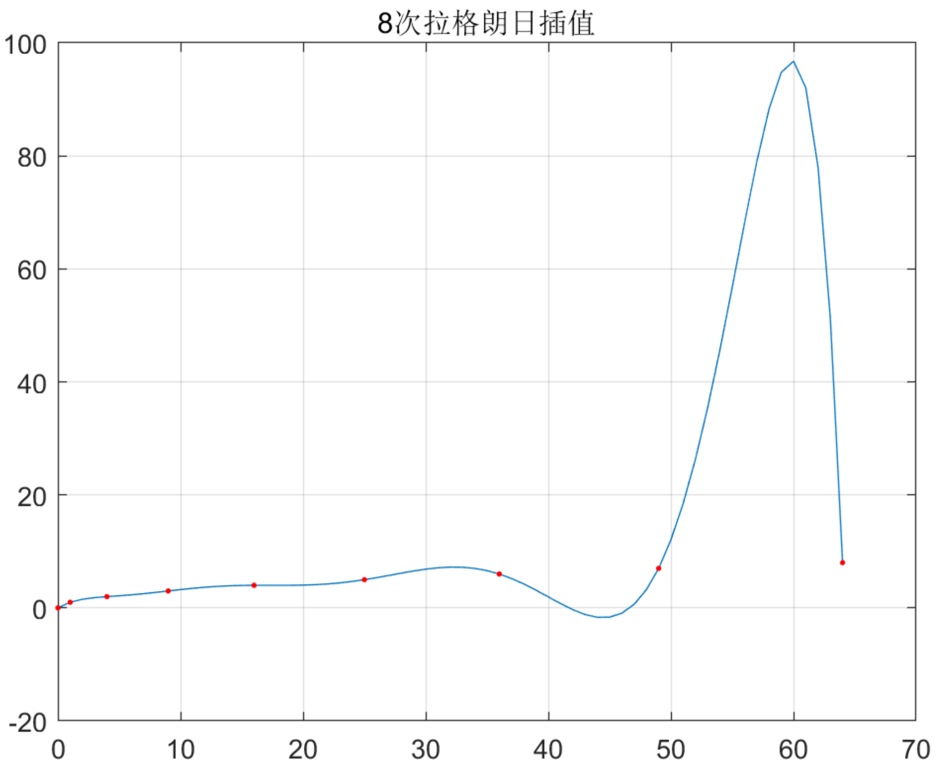
(1) 8 次拉格朗日插值:

```
clear;
x1=[0 1 4 9 16 25 36 49 64];
y1=[0 1 2 3 4 5 6 7 8];
n=length(y1);
a=ones(n,2);
a(:,2)=-x1';
c=1;
for i=1:n
    c=conv(c,a(i,:));
end
q=zeros(n,n);
r=zeros(n,n+1);
for i=1:n
    [q(i,:),r(i,:)]=deconv(c,a(i,:));
end
Dw=zeros(1,n);
for i=1:n
Dw(i)=y1(i)/polyval(q(i,:),x1(i));
end
p=Dw*q;
syms x L8;
```

```
for i=1:n
L8(i)=p(n-i+1)*x^(i-1);
end
disp('8´ÎÀ-¸ñÀÊÈÕ²åÖµ');
L8=vpa(collect((sum(L8))),5)
xi=0:64;
yi=polyval(p,xi);
figure
plot(xi,yi,x1,y1,'r.');
hold on
grid on
title('8´ÎÀ-¸ñÀÊÈÕ²åÖµ');
```



8 次拉格朗日插值:

L8 = - 3.2806e-10*x^8 + 6.7127e-8*x^7 - 5.4292e-6*x^6 + 0.00022297*x^5 - 0.0049807*x^4 + 0.060429*x^3 - 0.38141*x^2 + 1.3257*x

（2）三次样条插值

定义函数:

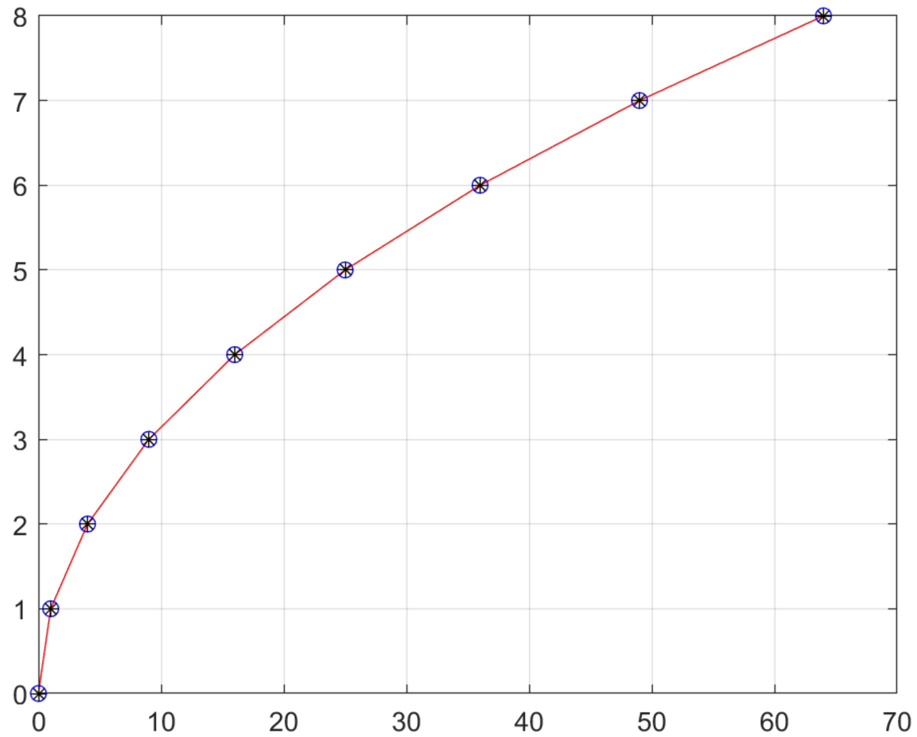
```
function yy = Interpolation_Spline0(x, y, xx)

n = length(x);
a = y(1 : end - 1);
b = zeros(n - 1, 1);
d = zeros(n - 1, 1);
dx = diff(x);
dy = diff(y);
```

```
A = zeros(n);
B = zeros(n, 1);
A(1, 1) = 1;
A(n, n) = 1;
for i = 2 : n - 1
    A(i, i - 1) = dx(i - 1);
    A(i, i) = 2*(dx(i - 1) + dx(i));
    A(i, i + 1) = dx(i);
    B(i) = 3*(dy(i) / dx(i) - dy(i - 1) / dx(i - 1));
end
c = A \ B;
for i = 1 : n - 1
    d(i) = (c(i + 1) - c(i)) / (3 * dx(i));
    b(i) = dy(i) / dx(i) - dx(i)*(2*c(i) + c(i + 1)) / 3;
end
[mm, nn] = size(xx);
yy = zeros(mm, nn);
for i = 1 : mm*nn
    for ii = 1 : n - 1
        if xx(i) >= x(ii) && xx(i) < x(ii + 1)
            j = ii;
            break;
        elseif xx(i) == x(n)
            j = n - 1;
        end
    end
    yy(i) = a(j) + b(j)*(xx(i) - x(j)) + c(j)*(xx(i) -
x(j))^2 + d(j)*(xx(i) - x(j))^3;
end
```

插值实现：

```
clear; clc;
x=[0 1 4 9 16 25 36 49 64];
y=[0 1 2 3 4 5 6 7 8];
xx =[0 1 4 9 16 25 36 49 64];
yy= Interpolation_Spline0(x, y, xx);
yyy = spline(x, y, xx);
plot(x, y, '-r', xx, yy, 'ob', xx, yyy, '*k');
grid on
```



经过观察，可知 y = x^0.5 **三次样条**插值在[0,64]上更精确。而经过实验验证，此题使用**牛顿插值与拉格朗日插值均会在 x 较大时出现大波动**。而显然，在 x 较小时拉格朗日插值较为精确。