

HW 1

Exercises

Exercise 1 (Learning the mechanics.).

Use the method of least squares to fit a straight line to these six data points:

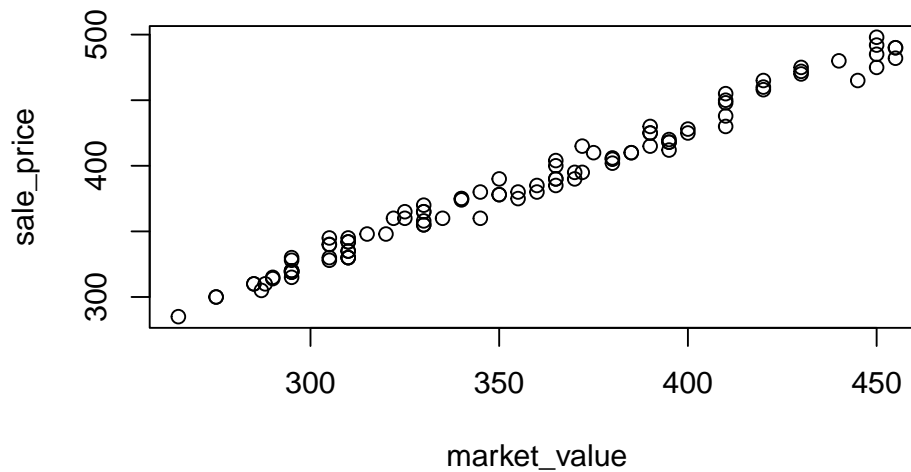
x	1	2	3	4	5	6
y	2	4	5	4	2	7

- (a) What are the least squares estimates of β_0 and β_1 ? Compute manually.
- (b) Plot the data points and graph the least squares line on the scatterplot.

Exercise 2 (Home sales price.).

Real estate investors, homebuyers, and homeowners often use the appraised (or market) value of a property as a basis for predicting sale price. Please look at the provided dataset [MARKET.csv](#). All the money are in 1000 dollars.

- (a) Propose a simple linear model to relate the appraised market value x to the sale price y .
- (b) A scatterplot of the data is shown below. Does it appear that a straight-line model will be an appropriate fit to the data?
- (c) A R simple linear regression printout is also shown below. Find the equation of the best-fitting line through the data on the printout.
- (d) Interpret the y -intercept of the least squares line. Does it have a practical meaning for this application? Explain.
- (e) Interpret the slope of the least squares line.
- (f) Over what range of x is the interpretation meaningful?
- (g) Use the least squares model to estimate the mean sale price of a property appraised at \$300,000.



```
Call:
lm(formula = sale_price ~ market_value)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-14.674  -5.480  -1.287   6.300  13.409
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  10.72069    5.01930   2.136  0.0352 *
market_value   1.05305    0.01399  75.256 <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 7.234 on 98 degrees of freedom
Multiple R-squared:  0.983, Adjusted R-squared:  0.9828
F-statistic: 5663 on 1 and 98 DF, p-value: < 2.2e-16
```

Exercise 3 (Heart rate).

A study shows that during a certain sport the mean heart rate y and the maximal oxygen uptake x might have relations. The dataset `SPORTHR.csv` shows y (expressed as a percentage of maximum heart rate) and x (VO2max).

player	VO2max	meanHR
1	140	68.2
2	150	71.1
3	160	74.4
4	170	76.5
5	180	78.8
6	185	80.1

player	VO2max	meanHR
7	190	82.4
8	200	84.6

- (a) Find the equation of the least squares line.
- (b) Give a practical interpretation (if possible) of the y -intercept of the line.
- (c) Give a practical interpretation (if possible) of the slope of the line.

Exercise 4 (Spilled liquid.).

A researcher studied the rate at which a spilled liquid will spread across a surface. The mass (in pounds) of the spill after a period of time ranging from 0 to 60 minutes is recorded and shown below (based on the dataset `SPILLS.csv`). Do the data indicate that the mass of the spill tends to diminish as time increases? If so, how much will the mass diminish each minute?

time_min	mass_lb
0	6.61
5	6.38
10	6.21
15	6.01
20	5.89
25	5.74
30	5.66
35	5.54
40	5.47
45	5.44
50	5.39
55	5.38
60	5.41

Exercise 5 (Sweetness of orange juice.).

To study the sweetness of orange juices, researchers collect some data on the sweetness index (y) and the amount of pectin (x) in the orange juice (in g/L). The dataset is `ORANGEJUICE.csv`.

sample	pectin	sweetness
1	100	6.72
2	120	6.41
3	140	6.58
4	160	6.11
5	180	6.33
6	200	5.98
7	220	6.21

sample	pectin	sweetness
8	240	5.87
9	260	6.03
10	280	5.72
11	300	5.95
12	320	5.61
13	340	5.84
14	360	5.53
15	380	5.77
16	400	5.46
17	420	5.69
18	440	5.38
19	460	5.62
20	480	5.31
21	500	5.55
22	520	5.24
23	540	5.49
24	560	5.21

- Find the values of SSE , s^2 , and s for this regression.
- Estimate σ^2 , the variance of the random error term in the model.
- Estimate σ , the standard deviation of the random error term in the model.
- Why is it difficult to give a practical interpretation to s^2 , the estimate of σ^2 ?
- Give a practical interpretation of the value of s .