

# HW 1

## Exercise 1 (Learning the mechanics.).

Use the method of least squares to fit a straight line to these six data points:

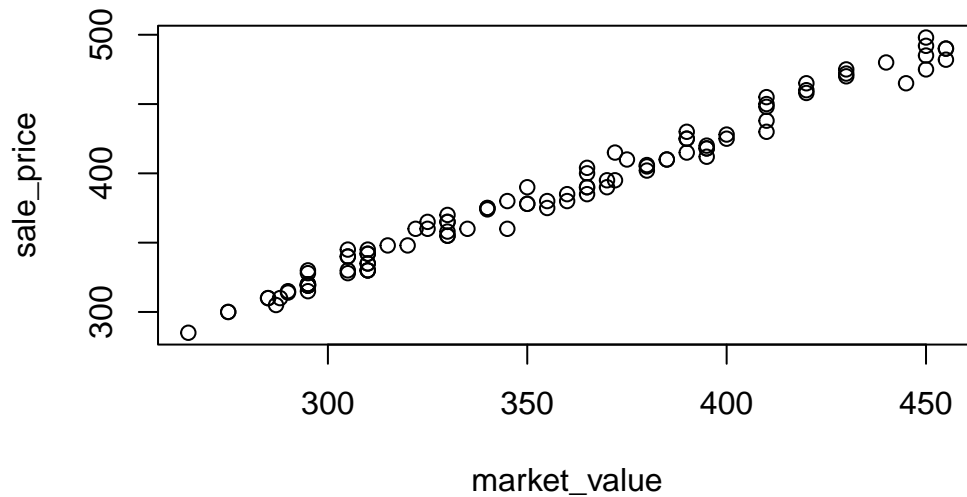
|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 2 | 4 | 5 | 4 | 2 | 7 |

- (a) What are the least squares estimates of  $\beta_0$  and  $\beta_1$ ? Compute manually.
- (b) Plot the data points and graph the least squares line on the scatterplot.

## Exercise 2 (Predicting home sales price.).

Real estate investors, homebuyers, and homeowners often use the appraised (or market) value of a property as a basis for predicting sale price. Please look at the provided dataset [MARKET.csv](#). All the money are in 1000 dollars.

- (a) Propose a simple linear model to relate the appraised market value  $x$  to the sale price  $y$ .
- (b) A scatterplot of the data is shown below. Does it appear that a straight-line model will be an appropriate fit to the data?
- (c) A R simple linear regression printout is also shown below. Find the equation of the best-fitting line through the data on the printout.
- (d) Interpret the  $y$ -intercept of the least squares line. Does it have a practical meaning for this application? Explain.
- (e) Interpret the slope of the least squares line.
- (f) Over what range of  $x$  is the interpretation meaningful?
- (g) Use the least squares model to estimate the mean sale price of a property appraised at \$300,000.



Call:

```
lm(formula = sale_price ~ market_value)
```

Residuals:

| Min     | 1Q     | Median | 3Q    | Max    |
|---------|--------|--------|-------|--------|
| -14.674 | -5.480 | -1.287 | 6.300 | 13.409 |

Coefficients:

|              | Estimate | Std. Error | t value | Pr(> t )   |
|--------------|----------|------------|---------|------------|
| (Intercept)  | 10.72069 | 5.01930    | 2.136   | 0.0352 *   |
| market_value | 1.05305  | 0.01399    | 75.256  | <2e-16 *** |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.234 on 98 degrees of freedom

Multiple R-squared: 0.983, Adjusted R-squared: 0.9828

F-statistic: 5663 on 1 and 98 DF, p-value: < 2.2e-16

### Exercise 3 (Heart rate).

A study shows that during a certain sport the mean heart rate  $y$  and the maximal oxygen uptake  $x$  might have relations. The dataset `SPORTH.R.csv` shows  $y$  (expressed as a percentage of maximum heart rate) and  $x$  ( $VO_{2max}$ ). The data are shown in the table.

| player | $VO_{2max}$ | meanHR |
|--------|-------------|--------|
| 1      | 140         | 68.2   |
| 2      | 150         | 71.1   |
| 3      | 160         | 74.4   |

|   |     |      |
|---|-----|------|
| 4 | 170 | 76.5 |
| 5 | 180 | 78.8 |
| 6 | 185 | 80.1 |
| 7 | 190 | 82.4 |
| 8 | 200 | 84.6 |

- Find the equation of the least squares line.
- Give a practical interpretation (if possible) of the  $y$ -intercept of the line.
- Give a practical interpretation (if possible) of the slope of the line.

**Exercise 4** (Spreading rate of spilled liquid.).

A researcher studied the rate at which a spilled liquid will spread across a surface. The mass (in pounds) of the spill after a period of time ranging from 0 to 60 minutes is recorded and shown below (based on the dataset `SPILLS.csv`). Do the data indicate that the mass of the spill tends to diminish as time increases? If so, how much will the mass diminish each minute?

| time_min | mass_lb |
|----------|---------|
| 0        | 6.61    |
| 5        | 6.38    |
| 10       | 6.21    |
| 15       | 6.01    |
| 20       | 5.89    |
| 25       | 5.74    |
| 30       | 5.66    |
| 35       | 5.54    |
| 40       | 5.47    |
| 45       | 5.44    |
| 50       | 5.39    |
| 55       | 5.38    |
| 60       | 5.41    |

**Exercise 5** (Sweetness of orange juice.).

To study the sweetness of orange juices, researchers collect some data on the sweetness index ( $y$ ) and the amount of pectin ( $x$ ) in the orange juice (in  $g/L$ ). The dataset is `ORANGEJUICE.csv`.

| sample | pectin | sweetness |
|--------|--------|-----------|
| 1      | 100    | 6.72      |
| 2      | 120    | 6.41      |
| 3      | 140    | 6.58      |
| 4      | 160    | 6.11      |

|    |     |      |
|----|-----|------|
| 5  | 180 | 6.33 |
| 6  | 200 | 5.98 |
| 7  | 220 | 6.21 |
| 8  | 240 | 5.87 |
| 9  | 260 | 6.03 |
| 10 | 280 | 5.72 |
| 11 | 300 | 5.95 |
| 12 | 320 | 5.61 |
| 13 | 340 | 5.84 |
| 14 | 360 | 5.53 |
| 15 | 380 | 5.77 |
| 16 | 400 | 5.46 |
| 17 | 420 | 5.69 |
| 18 | 440 | 5.38 |
| 19 | 460 | 5.62 |
| 20 | 480 | 5.31 |
| 21 | 500 | 5.55 |
| 22 | 520 | 5.24 |
| 23 | 540 | 5.49 |
| 24 | 560 | 5.21 |

- Find the values of  $SSE$ ,  $s^2$ , and  $s$  for this regression.
- Estimate  $\sigma^2$ , the variance of the random error term in the model.
- Estimate  $\sigma$ , the standard deviation of the random error term in the model.
- Explain why it is difficult to give a practical interpretation to  $s^2$ , the estimate of  $\sigma^2$ .
- Give a practical interpretation of the value of  $s$ .