

# HW 1

## Exercises

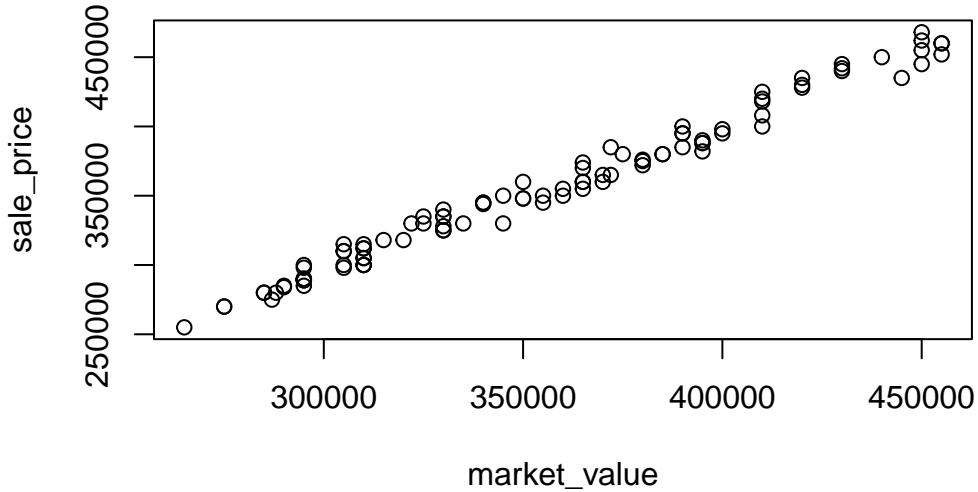
**Exercise 1** (Learning the mechanics.). Use the method of least squares to fit a straight line to these six data points:

x	1	2	3	4	5	6
y	2	4	5	4	2	7

- (a) What are the least squares estimates of  $\beta_0$  and  $\beta_1$ ?
- (b) Plot the data points and graph the least squares line on the scatterplot.

**Exercise 2** (Predicting home sales price.). Real estate investors, homebuyers, and homeowners often use the appraised (or market) value of a property as a basis for predicting sale price. Please look at the provided dataset [MARKET.csv](#). The first five and last five observations of the data set are listed in the accompanying table.

- (a) Propose a simple linear model to relate the appraised market value  $x$  to the sale price  $y$ .
- (b) A scatterplot of the data is shown below. Does it appear that a straight-line model will be an appropriate fit to the data?
- (c) A R simple linear regression printout is also shown below. Find the equation of the best-fitting line through the data on the printout.
- (d) Interpret the  $y$ -intercept of the least squares line. Does it have a practical meaning for this application? Explain.
- (e) Interpret the slope of the least squares line. Over what range of  $x$  is the interpretation meaningful?
- (f) Use the least squares model to estimate the mean sale price of a property appraised at \$300,000.



Call:

```
lm(formula = sale_price ~ market_value)
```

Residuals:

Min	1Q	Median	3Q	Max
-14674	-5480	-1287	6300	13408

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )		
(Intercept)	-1.928e+04	5.019e+03	-3.841	0.000217 ***		
market_value	1.053e+00	1.399e-02	75.256	< 2e-16 ***		
---						
Signif. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.'	0.1 ' '	1

Residual standard error: 7234 on 98 degrees of freedom

Multiple R-squared: 0.983, Adjusted R-squared: 0.9828

F-statistic: 5663 on 1 and 98 DF, p-value: < 2.2e-16

**Exercise 3.** A study shows that during a certain sport the mean heart rate  $y$  and the maximal oxygen uptake  $x$  might have relations. The dataset SPORTHR.csv shows  $y$  (expressed as a percentage of maximum heart rate) and  $x$  ( $V_{O_2\text{max}}$ ). The data are shown in the table.

```

player vo2max meanHR
 1    140    68.2
 2    150    71.1
 3    160    74.4
 4    170    76.5
 5    180    78.8
 6    185    80.1
 7    190    82.4
 8    200    84.6

```

- (a) Find the equation of the least squares line.
- (b) Give a practical interpretation (if possible) of the  $y$ -intercept of the line.
- (c) Give a practical interpretation (if possible) of the slope of the line.

**Exercise 4** (Spreading rate of spilled liquid.). A researcher studied the rate at which a spilled liquid will spread across a surface. The mass (in pounds) of the spill after a period of time ranging from 0 to 60 minutes is recorded and shown below (based on the dataset SPILLS.csv). Do the data indicate that the mass of the spill tends to diminish as time increases? If so, how much will the mass diminish each minute?

```

time_min mass_lb
 0    330.0
 5    325.3
10    319.8
15    315.2
20    309.7
25    305.1
30    300.3
35    295.4
40    290.2
45    284.7
50    279.9
55    274.8
60    269.5

```

**Exercise 5** (Sweetness of orange juice.). To study the sweetness of orange juices, researchers collect some data on the sweetness index ( $y$ ) and the amount of pectin ( $x$ ) in the orange juice (in  $g/L$ ). The dataset is ORANGEJUICE.csv.

```

sample pectin sweetness
 1    0.15      6.8
 2    0.20      7.1

```

3	0.25	7.4
4	0.30	7.9
5	0.35	8.2
6	0.40	8.6
7	0.50	9.1
8	0.60	9.5
9	0.70	10.1
10	0.80	10.4
12	0.90	10.9
13	1.00	11.3

- (a) Find the values of  $SSE$ ,  $s^2$ , and  $s$  for this regression.
- (b) Estimate  $\sigma^2$ , the variance of the random error term in the model.
- (c) Estimate  $\sigma$ , the standard deviation of the random error term in the model.
- (d) Explain why it is difficult to give a practical interpretation to  $s^2$ , the estimate of  $\sigma^2$ .
- (e) Give a practical interpretation of the value of  $s$ .