

REVIEW OF “ON TOPOLOGICAL APPROACH TO LOCAL THEORY OF SURFACES IN CALABI-YAU THREEFOLDS”

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In this paper the authors explore a series of enumerative invariants and study the inter-relations and dualities between them. In particular the authors focus on Donaldson-Thomas gauge theory on 3-folds and its reduction to 4-dimensional and 2-dimensional case which are relevant to the local theory of surfaces in Calabi-Yau 3-folds.

The paper consists of five sections. Section 1 is a quick introduction and Section 2 is a review of Gromov-Witten theory, Donaldson-Thomas theory and Seiberg-Witten theory.

In Section 3 the authors study the case that the 3-fold X is an elliptic fibration over a surface S . In this case the Donaldson-Thomas theory on X can be computed by the Vafa-Witten theory on S through the dimensional reduction process. Two examples are computed explicitly under this setting, and the Donaldson-Thomas theory is compared to the Gromov-Witten theory for both of them. The case of the total space of the canonical line bundle of a Fano surface is also discussed.

In Section 4 the authors first study the case that $X = \Sigma_\ell \times S$ where Σ_ℓ is a Riemann surface of genus ℓ and S is a Kähler surface. In this case the Donaldson-Thomas theory on X can be reduced to the 4-dimensional gauge theory on S , or to the $\mathcal{N} = 2$ sigma-model on Σ_ℓ . The moduli spaces of each theory are discussed and the correlation functions are computed. Then at the end of Section 4 the authors summarize many theorems and conjectures which describe the relations between various enumerative invariants on various spaces.

In Section 5 the authors study the case that X is the total space of the canonical line bundle over a smooth projective surface S which satisfies a specific condition. In this case, a modified Donaldson-Thomas theory of X is computed by the Seiberg-Witten theory on S and some invariants of “nested Hilbert schemes” on S . The formula is given explicitly.