Efficient Mesh Router Placement in Wireless Mesh Networks

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Abstract

The placement of mesh routers (MRs) in building a wireless mesh network (WMN) is the first step to ensure the desired network performance. Given a network domain, the fundamental issue in placing MRs is to find the minimal configuration of MRs so as to satisfy the network coverage, connectivity, and Internet traffic demand. In this paper, the problem is addressed under a constraint network model in which the traffic demand is non-uniformly distributed and the candidate positions for MRs are pre-decided. After formulating the MR placement problem, we first provide the theoretical analysis to validate the traffic demand and determine the optimal position of Internet gateway (IGW). To reduce complexity of determining the locations of MRs while satisfying the traffic constraint, we propose an effective heuristic algorithm to obtain an close-to-optimal solution. Finally, our simulation results verify our analytical model and show the effectiveness of our proposed algorithm.

1. Introduction

In recent years, the popularity of wireless mesh networks (WMNs) based on WiFi technology has continued to grow due to two main reasons. One is that WMNs offer the high bandwidth Internet access for mobile users at any time and anywhere by using the promising multi-radio multi-channel technology. The other primary reason is that the wireless backbone provided by WMNs achieves cost-efficient design and deployment as compared to traditional networks such WLANs due to reduction in dependency on wired connections [1]. On the other hand, this increasing popularity has also posed a challenging question: How to define a cost-efficient WMN connectivity that can provide desired level of Internet throughput to a large community.

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Prior research work has proposed several ways to improve the throughput of WMNs. However, all these assume that an existing WMN has already been constructed, i.e., each mesh router (MR) has been placed at given location and configured in advance. The efficient placement of MRs as the first important step to ensure satisfactory performance of a WMN is largely ignored. A bad deployment of MRs can not only result in significant interference that could otherwise be avoided but also create unfavorable hotspots that could sustain unexpected high traffic overload, while other MRs are rendered in a low resource utilization.

A practical MR placement scheme is a tradeoff between two key design constraints: cost and traffic demand. In particular, the placement problem ought to answer the following two crucial questions: (1) How many MRs should a WMN have? (2) Where these MRs ought to be placed? while taking into account a number of constraints.

In this paper, we investigate this problem and our main contributions can be summarized as follows:

- We analyze the complexity of the MR placement problem and seek for the optimal solution in a given local network. Then we propose a constraint network model for the local network and formulate the MR placement problem on it. To the best of our knowledge, this is the first work on the MR placement for a WMN.
- We provide the analytical results for validating the traffic demand and determining the optimal position of the IGW in the local network.
- We develop a heuristic algorithm that provides the trade-off between minimizing the number of MRs and guaranteeing the traffic demand, coverage and connectivity.

The rest of this paper is organized as follows: Section 2 considers the related work and Section 3 introduces the network model and the traffic model. Then, we address the approach for constructing a WMN in Section 4. In Section 5, we present simulation results and provide the underlying analyses. Finally, we discuss the future network in Section 6.

2. Related Work

In the previous research efforts, many approaches have been proposed to address how to maintain the desirable performance from the different aspects in WMNs. In [2][3], the authors target to reduce the interference by utilizing the appropriate channel assignment schemes. In [4][5], congested links are alleviated by using the load-balancing strategies, etc. In [10], the load-balancing among IGWs is studied by dynamically assigning MRs to different IGW domain. However, how to provide a cost-economic and capacity-effective WMN architecture is very much in its infancy.

In recent years, some researches have begun to study the positions of the IGWs. In [6], they have first considered the selection of IGW nodes in an existing WLAN and modeled the gateway selection problem as a linear program while considering some constraints, such as channel capacity, interference etc. In [8][9], the authors further considers the selection of IGWs for a large-scale multi-hop network. The network is divided into disjoint clusters and each cluster an organized as a tree structure rooted at the gateway nodes.

Although it is of practical importance to design a WMN with the minimum number of MRs, thereby reducing the cost of constructing a WMN while providing adequate throughput of the network domain, it is still not adequately addressed in the previous research. In this paper, we propose the approach to place MRs in the candidate positions that have been already decided in advance.

3. Network Model & Problem Formulation

The MR placement problem is *the determination of a minimum set of positions among the candidate positions in such a way that the MRs situated in these positions cover the given region, maintain the full connectivity toward the IGW and meet the traffic demand.* In this section, we describe the detailed network model and the problem.

Given a network domain with m candidate positions, we can assume that at least n positions have to be situated with MRs to satisfy the constraints. Such a process generates C_m^n configuration alternatives. To satisfy the Internet traffic demand, g MRs ($g \leq n$) among n MRs are required to be configured as IGWs. This generates C_n^g alternatives if each MR is a potential place for IGW. Therefore, the complexity of finding an optimal solution to the MR placement problem is $C_n^g \times C_m^n$, which increases exponentially as the number of MRs increases. The high complexity motivates us to find an effective approach for MR placement. Considering that the WMN is dominated by Internet traffic and each MR transmit or receive the Internet traffic through one of g IGWs [2][6], a large WMN can be partitioned into multiple disjointed local networks, each of which consists of one IGW and a group of MRs that are attached to the IGW by single hop or multi-hop path. The size of each local network is restricted by IGW's capacity. Therefore, rather than directly looking for the global optimal solution, we first deal with each of the local networks. Once we get these local placements, we can then incorporate them together and generate a global placement scheme. Note that the final placement scheme generated by this divide-and-conquer approach might not be an optimal solution. In this paper we first focus on the MR placement in a local network that has one IGW and plan to work on global optimization as our future research.

3.1. Constraint Network Model

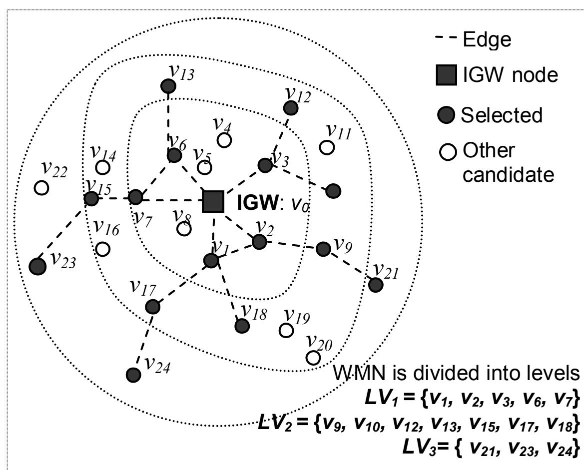

Figure 1. **WMN network model**

Figure 1 shows a candidate local network domain for placing the MRs and IGW in a two-dimensional disk of radius R. The candidate network can be represented by a graph $G = (V, E)$. The node set $V = \{v_0, v_1, v_2, \ldots, v_m\}$ is the set of all candidate positions (or nodes). For example, it has 25 candidate nodes in Figure 1. Since the locations of the candidate nodes are subjected to geographical constraint, we refer the network model as constraint network model. The position of a node $v_i \in V$ is represented with the coordinate (x_i, y_i) in a two dimensional Euclidean plane. We use node v_0 to denote the IGW that can be any one of the candidate positions. The number of nodes in G is $|V|$, i.e., m. E is the set of edges connecting the candidate nodes. Two candidate nodes are connected by an edge if and only if the Euclidean distance between them is not longer than the MR's transmission range, which is assumed to be identical for all the channels in this paper. The candidate coverage of a node $C(v_i)$, $v_i \in V$ is the transmission range of the MR located v_i.

The final WMN of the local network domain based on the candidate network can be accordingly represented by an induced subgraph $G' = (V', E')$ of G. The nodes in V' consists of selected MR nodes and the IGW node. In Figure 1, it has 16 nodes among the 25 candidate nodes are chose as the MR nodes. The number of nodes in G' is $|V'|$, i.e., $n \leq m$. The edges in E' correspond to edges of the graph G'. Note that one edge between two neighboring MRs represent the multiple wireless links between them.

$CH = \{1, 2, \ldots, h\}$ is the set of the orthogonal channels and the number of these channels is $|CH|$, i.e., h. For a specific channel $k \in CH$, its maximum transmission capacity is w_k bps. A MR can be equipped with multiple wireless radios (e.g., IEEE 802.11 a/b/g). Let $I(v_i)$ be the set of wireless radios on a node $v_i \in V'$. The number of these radios is $|I(v_i)|$. For any given node, $|I(v_i)| \leq h$. Multiple radios $I(v_i)$ at a node allow simultaneous transmission and reception. The transmission range of a radio by using its corresponding channel $k \in CH$ is r_k.

To provide the Internet connectivity for MRs, the IGW serves as an Internet gateway and connects to Internet by wired link. The throughput capacity of the wired link is assumed to be infinite compared to wireless links. To simplify the analysis, a WMN graph G' is divided into levels in terms of the number of minimal hops from a MR to the IGW as shown in Figure 1. It is symbolized by the set $\{LV_0, LV_1, \ldots, LV_q\}$, where LV_i is again the set of nodes at a minimal i hops to the IGW, and q is the furthest level. Thus, it has $\bigcup_{i=0}^{q} LV_i = V'$. The IGW node v_0 is the only element in the set LV_0 (i.e., $LV_0 = \{v_0\}$). LV_1 consists of the MR nodes directly connected to IGW as shown in Figure 1.

For a MR node $v_i \in V'$, we use $P_i = \{P_i^1, P_i^2, \ldots, P_i^j, \ldots\}$ to represents the set of all possible routing paths from to v_i the IGW. The number of these paths is $|P_i|$.

3.2. Heterogeneous Traffic Model

In design stage, the throughput of each MR is not easy to be effectively evaluated because the channel assignment and multi path routing have the significant impact on the MRs's real throughput in the running time; however, whether the channel assignment scheme or multi-path routing discovery information is hard to predict before the network is established. It is necessary to develop an analytical model to capture main traffic characteristics, which allows us to estimate throughput capacity for each MR in the stage of placing them.

We consider the traffic in a WMN dominated by the Internet traffic and neglect the peer-to-peer traffic. We address the MR placement problem in a *heterogeneous* traffic model in which the Internet traffic demand is non-uniformly distributed in the constraint network. We use a traffic density function μ to characterize the Internet traffic demand. For a given infinitesimal traffic area a, the traffic demand is $\mu(a)$ bps, which is the traffic needed to send either from this area to the Internet or from the Internet to this area. Thus, $\mu(a)$ varies according to the traffic density in a given area a. The total Internet traffic demand of the disk can be calculated as $T(D) = \int_0^{\pi R^2} \mu(a)da$ bps.

For a MR node $v_i \in V'$, we use $\tau(v_i)$ to denote the traffic demand it needs to transmit. This consists of two parts: $\tau(v_i) = \alpha(v_i) + \beta(v_i)$, where $\alpha(v_i)$ is the traffic demand originated from its own service area, and $\beta(v_i)$ is the traffic demand received from the other MRs for relaying. It can be seen that for the IGW it has $\tau(v_0) = T(D)$, since the IGW has to deal with all the traffic in the network domain. We also define the *Internet Throughput* $\varphi(v_i)$ for a given MR node v_i as the amount of traffic it can successfully send to and receive from the IGW.

3.3. Problem Formulation

Determination of MR nodes from a candidate network G has to minimize the required number of MR nodes in G' to meet the full coverage, full connectivity, and traffic demand of the given network domain. The minimum number of MRs is again to maximally reduce the investment cost imposed by MR hardware. Based on the above network and traffic models, we formulate the MR placement problem is to find the minimal sub-graph $G' = (V', E')$ as stated by following inequality (1) with additional constraints:

$$|V'| \le |V_j|, \quad V' \subseteq V, \; \forall\, V_j \subseteq V, \qquad (1)$$

$$\bigcup_{i=0}^{|V'|-1} C(v_i) \ge \pi R^2, \qquad v_i \in V', \qquad (2)$$

$$|P_i| > 0, \quad \forall v_i \in V', \qquad (3)$$

$$\varphi(v_i) \ge \tau(v_i), \qquad \forall\, v_i \in V'. \qquad (4)$$

Inequality (2) says the set of selected MR nodes provide full coverage of the given local domain and inequality (3) models the full connectivity of all nodes. Full connectivity implies that every node to be connected to the IGW by at least one path. In addition, inequality (4) says all MR nodes in G' have the Internet throughput capacity (i.e., $\varphi(v_i)$) to deliver all the traffic (i.e., $\tau(v_i)$) aggregated by itself or received from other nodes. Therefore, every MR in G' is able to successfully send and receive traffic to/from the IGW as the traffic demand.

4. IGW and MR Placement Scheme

In this section, we propose a scheme for placing MRs in the constraint network that could satisfy the objective and constraints as discussed in Section 3. The proposed scheme includes three steps: (*A*) validation of the traffic demand in the network, (*B*) placement of IGW, and (*C*) placement of MRs.

4.1. Model Validation of the Traffic Demand

THEOREM 1. The upper bound of the Internet throughput in the WMN with one IGW is $\sum_{k=1}^{h} w_k$.

Proof: In a WMN with one IGW, all Internet traffic has to pass through the IGW, including the Internet-oriented traffic aggregated from MRs, and the traffic from the Internet that is directed to any MR in the WMN. Therefore, the overall Internet throughput upper bound is limited by the IGW's throughput $\varphi(v_0)$. Otherwise, the IGW will be congested due to overloaded traffic in the IGW. At a given instant, there are at most h radios in the IGW that can simultaneously transmit/receive data to/from its neighboring MR nodes. The maximal throughput capacity of h channels is $\sum_{k=1}^{h} w_k$, which again is the maximal Internet throughput of the WMN with one IGW. ■

COROLLARY 1. The total traffic demand of the network domain, i.e., $T(D)$, can not exceed $\sum_{k=1}^{h} w_k$, otherwise Inequality (3) (i.e., $\varphi(v_i) \ge \tau(v_i)$) is violated at IGW node v_0 since $\varphi(v_0) \le \sum_{k=1}^{h} w_k$ but $\tau(v_0) > \sum_{k=1}^{h} w_k$.

Based on Theorem 1 and Corollary 1, we have the following remark.

REMARK 1. *MR placement can proceed only if the overall network traffic demand* $T(D) = \int_0^{\pi R^2} \mu(a)\, da$ *is less than the IGW's throughput capacity* $\sum_{k=1}^{h} w_k$. *Otherwise, the current network area ought to be split into two or more network local domains with multiple IGWs.*

4.2. Placement of IGW

In this subsection, we target to find the best location of the IGW in terms of traffic effort. We set the two dimensional coordinate for the network disk, with the center-point as the coordinate (0, 0) as shown in Figure 2. We first assume the IGW is located at the coordinate (x_0, y_0). Given an infinitesimal traffic area a, whose center is at the point (x, y), a can be measured by $dx \times dy$. The traffic density function $\mu(a)$ can be transformed as $\mu(x, y)$. Then, the traffic demand at a is $\mu(x, y)\ dxdy$. Assuming a shortest path routing protocol is adopted, the length of the route from a to the IGW can be approximated by the Euclidean distance between a and the IGW, which is $d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$. Thus, the traffic effort in area a can be defined as $\mu(x, y)\ dxdy \times d$. The total traffic effort of the disk is the sum of the traffic effort from all the areas in the disk to the IGW, yielding $\int_y \int_x \mu(x, y) \times d\ dxdy$. Therefore, to minimize the total traffic effort is to minimize:

$$\int_{-R}^{+R} \int_{-\sqrt{R^2-y^2}}^{+\sqrt{R^2-y^2}} u(x, y) \sqrt{(x-x_0)^2 + (y-y_0)^2}\, dxdy \qquad (5)$$

If the IGW is situated at any other positions, the total traveling path length increases. As a result, the number of MRs has to increase because the high traffic effort is needed to transmit the same amount of traffic. The delay of traffic may increase because of longer traffic traveling paths. Additionally the investment cost of MR hardware is accordingly increased.

Given $\mu(x, y)$, it is not hard to find the location (x_0, y_0) that achieves the minimum traffic effort in the heterogeneous traffic model. In case that none of the candidate nodes in V has the coordinate (x_0, y_0), we choose the candidate node closest to (x_0, y_0) in V and donate it as (x_0', y_0').

REMARK 2. T*he optimum position of IGW is located at the candidate node which has the shortest Euclidean* *distance with the point (i.e.,(x_0, y_0)) that achieves the minimum efforts in terms of routing length of the traffic as follows:*

$$Minimize \int_{-R}^{+R}\int_{-\sqrt{R^2-y^2}}^{+\sqrt{R^2-y^2}} \mu(x,y)\times\sqrt{(x-x_0)^2+(y-y_0)^2}\, dxdy \quad (5')$$

4.3. MR Placement

In this step, the MR placement scheme aims at finding out the minimum number of n-1 positions from m-1 candidate positions so that the MRs situated in these n-1 positions together with the IGW can cover the disk, every MR is connected to the IGW, and the traffic demand in the network is satisfied. It is noted that n is an unknown before finding the solution.

In this section, we first consider the problem in a simplified constraint network model with $m \to \infty$ and present the theoretical results for the MR placement in this environment. Then, we extend the results into the common constraint network model and propose a heuristic algorithm to approximate the optimal number of MRs.

4.3.1. Place the MRs when $m \to \infty$

$m \to \infty$ indicates that the number of the candidate positions in the network disk is infinite. In other words, MRs can be situated anywhere in the disk.

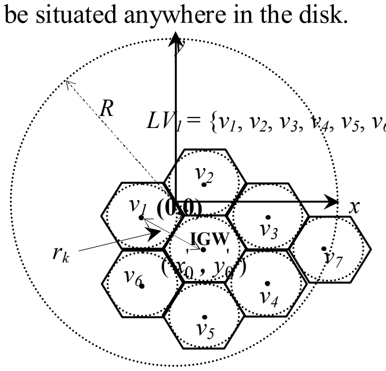

Figure 2. **MR placement when** $m \to \infty$

Thus, when every MR has the same radio coverage for all channels, the determination of the minimum number of MRs to satisfy the coverage and connectivity, traffic requirement of the network disk can be further modeled as a hexagonal tessellation packing problem [7] (chapter 2.3) as partly shown in Figure 2. We adopt the hexagonal tessellation among the only three types of regular tessellations [7], equilateral triangles, squares, and hexagons, because the hexagon is closest to MR's radio coverage and thus reduces the number of nodes in the network. The resulted tessellation has the following key properties:

- The final network, i.e., the tessellation, is formed by n regular hexagons. Each MR is placed in the center of each hexagon. We show the IGW and six LV_1 hexagons, v_1 - v_6, in Figure 2.
- Each regular hexagon is surrounded by at most six other regular hexagons to cover the disk without gaps.
- The distance between any two neighboring MR nodes is the transmission range r_k so that the network connectivity is guaranteed.

The hexagons fill the network plane with no overlaps and no gaps if we ignore the boundary of the disk. No overlap is necessary for minimizing the number of MRs in the disk; no gap is needed to satisfy the coverage requirement. Based on the above design and analysis, we can introduce the following remark:

REMARK 3. *If MRs can be placed anywhere in the network disk,* $\left\lceil \pi R^2/(\sqrt{3}r_k^2/2) \right\rceil$ *MRs is required when the boundary effect is neglected.*

4.3.2. Heuristic Algorithm for the constraint network

When m is a finite number, MRs can only be placed at those candidate positions. The optimal solution can be found by a brute-force search to select the minimum number of MR nodes (i.e., n) from the candidate node set V. In the worst case, the brute-force search totally results in $\sum_{j=1}^{n} C_m^j$ graph G' alternatives, and at each G' it involves the validation of the constraints (2)-(4). Although the brute force approach is simple, its complexity suffers from exponential growth. Thus, it is impractical when m and n increase.

We thus propose a heuristic algorithm that is able to find an approximate placement solution in the constraint network. Here we assume the candidate nodes and edges in the initial graph G have to meet the coverage and connectivity constraints if all candidate nodes are selected as the MR nodes. Otherwise, there is no MR placement solution unless more candidate nodes are added. Our heuristic algorithm includes two phases. In the first phase, the coverage is guaranteed by selecting a minimum set of candidate nodes for G', and in the second phase, we validate the full connectivity of G'. If the full connectivity is not met, then we enforce an add-and-merge scheme to create a full connectivity graph to the IGW.

***Phase I (Minimal Coverage Graph)**:* The first phase of our heuristic algorithm is to greedily find a minimum number of nodes that can cover the network disk. The corresponding induced graph is $G' = (V', E')$. Initially the disk is covered by the *m* candidate nodes, i.e., $G' = G$. The heuristic algorithm in phase *I* first sorts these candidate nodes in G' by their Euclidean distances to the IGW in the descending order. After that, the algorithm checks all the nodes, starting from the furthest node. If removing the node from G' does not introduce any uncovered area in the network disk, our algorithm then safely removes this node from the graph G'. The nodes left in the final graph G' generated by phase one are able to cover the network disk.

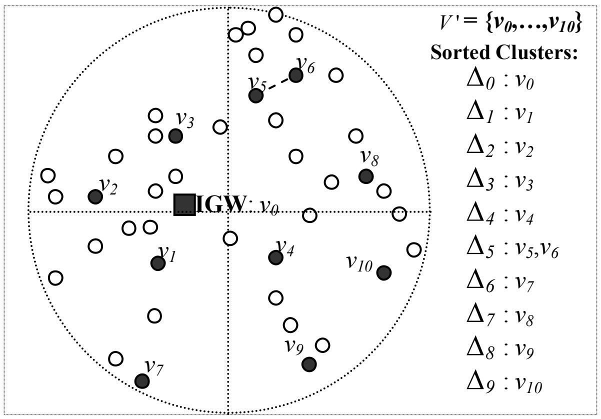

Figure 3. G' **of Full Coverage in Phase I**

For example, the network disk in Figure 3 has 43 candidate nodes in the network. The location of IGW is first determined by using the process of Section 4.2 and marked as a square in Figure 3. After removal of unnecessary nodes by phase *I*, it generates a graph G', including 10 MR nodes (marked as black points) as well as the IGW by which the network full coverage is granted. The unmarked points are the left candidate nodes. Although the nodes in G' provides full coverage, they may not be a connected graph. For example, v_1 and its nearest node v_0 (i.e., IGW) in Figure 3 are not within the transmission range of each other.

***Phase II (Add-and-Merge)**:* Phase *II* is an Add-and-Merge process to create a full connectivity graph by adding minimal nodes to G' generated by Phase *I*.

Before illustrating the algorithm, we first define *connected-cluster* used in the algorithm. A connected-cluster Δ_i is a subgraph of G' where every pair of nodes in Δ_i has at least one path between them. Any two clusters (i.e., Δ_i and Δ_j) are disconnected. Otherwise, they can be merged as one. For example, G' in Figure 3 has 10 connected-clusters since only nodes v_5 and v_6 are within the transmission range of each other. Nodes v_5 and v_6 belong to the same connected-cluster and every other connected-cluster only has one node. The union of all connected-clusters is the graph G'. It is easy to see that the graph G' generated by Phase *I* with more than one cluster does not have full connectivity.

The connected-cluster having the IGW node is donated by Δ_0. We define a power set Δ that contains all connected-clusters. The distance (i.e., $\Delta_i(d)$) between a connected-cluster Δ_i and the IGW is defined as the distance between the node closest to IGW in Δ_i and the IGW. In addition, the traffic demand of a connected-cluster Δ_i is the total traffic demands of all the nodes in the cluster, i.e., $\sum \alpha(v_{i'}) \quad v_{i'} \in \Delta_i$.

THEOREM 2. The Internet throughput upper bound for each MR is $(\sum_{k=1}^{h} w_k)/2$.

Proof: In order for a MR node $v_i \in V'$ to support a given throughput capacity (i.e., $\varphi(v_i)$), it has to provide the capacity for simultaneous traffic in two directions: entering the node $v_i \in V'$ and exiting the node $v_i \in V'$. At a given instant, let $\varphi_{in\max}(v_i)$ and $\varphi_{out\max}(v_i)$ represent the maximal throughput capacity on each direction. According to Theorem 1, it has $\varphi_{out\max}(v_i) + \varphi_{in\max}(v_i) \leq \sum_{k=1}^{h} w_k$. However, If $\varphi_{out\max}(v_i) > \varphi_{in\max}(v_i)$, the actual traffic capacity can only be $\varphi(v_i) = \varphi_{in\max}(v_i)$. On the contrary, if $\varphi_{out\max}(v_i) < \varphi_{in\max}(v_i)$, the actual traffic capacity is $\varphi(v_i) = \varphi_{out\max}(v_i)$. Therefore, the maximal $\varphi(v_i)$ is achieved if and only if $\varphi(v_i) = \varphi_{out\max}(v_i) = \varphi_{in\max}(v_i)$. So $\varphi(v_i) \leq (\sum_{k=1}^{h} w_k)/2$. ■

COROLLARY 2. Given any a connected cluster $\Delta_i \in \Delta - \{\Delta_0\}$, the traffic demand of Δ_i cannot exceed $(\sum_{k=1}^{h} w_k)/2$. Otherwise inequality (3) (i.e., $\varphi(v_i) \geq \tau(v_i)$) can not be guaranteed at the node v_j ($v_j \in \Delta_i$) nearest to IGW.

The basic idea of Add-and-Merge procedure is shown in Figure 4. First of all, as Line 1 of Figure 4 describes, we obtain the initial connected-clusters after traversing the minimal coverage graph G' resulted from phase I. If the traffic demand of $\Delta_i > (\sum_{k=1}^{h} w_k)/2$, it is equally split into two connected-clusters. Since the algorithm has to iteratively deal with the furthest cluster first and the closest one last in the following steps (i.e., Lines 4-10 of Figure 4), the clusters are sorted by their distance to IGW in the ascending order as Line 2 of Figure 4 does. Then, for any a connected-cluster Δ_i ($\Delta_i \in \Delta$-$\{\Delta_0\}$) that is not connected to the IGW, we add an additional candidate node in V-V' into cluster Δ_i one by one to reduce the distance between Δ_i and the IGW as shown by Line 5 of

Figure 4. Considering the efficiency, we select the node in $V\text{-}V'$ that has the direct connection with Δ_i and meanwhile can maximally reduce the distance from Δ_i to the IGW.

1: Generate connected-clusters of G': $\Delta = \{\Delta_0, \Delta_1, \ldots, \Delta_i \ldots\}$
2: Sort the clusters by their distances to the IGW in the ascending order.
3: while $|\Delta| > 1$ do
4:   for $i = |\Delta|$ to 1 do
5:     Find a neighboring node v_j of Δ_i in $V\text{-}V'$ that maximally reduces $\Delta_i(d)$, then ***add*** v_j to Δ_i (i.e., add v_j and related edges to G').
6:     for k =1 to the number of v_j 's neighboring nodes do
7:       if (k is in the other cluster) and (the sum of the traffic demand of the two clusters is less than $(\sum_{k=1}^{h} w_k)/2$)
8:         ***Merge*** the two clusters;
9:     endfor;
10:   endfor;
11: endwhile

Figure 4. **Procedure Add-and-Merge**

When a candidate node $v_j \in V\text{-}V'$ is added to a connected-cluster Δ_i, our algorithm then checks if v_j can connect Δ_i to other connected-clusters (Lines 7-9 of Figure 4). If yes, our algorithm merges Δ_i with these clusters as long as the merged traffic demand is less than node throughput upper bound, i.e., $(\sum_{k=1}^{h} w_k)/2$, which is stated in Corollary 2. Otherwise, the merge will not happen. The mergence is necessary to minimize the number of candidate nodes used for G'.

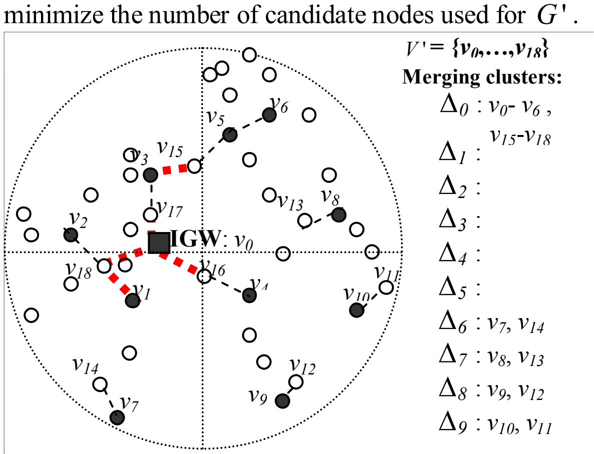

Figure 5. **Add-and-Merge Activities**

Following the minimal coverage graph G' shown in Figure 3, Figure 5 illustrates the connectivity graph after the first round of phase II (lines 4-9 in Figure 4). As shown in Figure 3, the initial coverage graph G' has nine connected-clusters, excluding Δ_0. As beginning of phase II (line 2), these nine clusters are sorted by the distance to the IGW. Δ_9 is the furthest cluster and Δ_0 is the closest. Phase II considers from Δ_9 to Δ_0 clusters for adding additional node. Each of the dash lines in Figure 5 represents that a candidate node has been added into a connected-cluster. For example v_{11} is introduced into Δ_9 and v_{15} into Δ_5. When a node is added, two connected-clusters may be able to merge as one connected-cluster. The square dot dash lines represents that two connected-clusters have been merged. For instance, when v_{15} is added into Δ_5, Δ_5 is then able to merge into Δ_3 because v_{15} and v_3 are neighboring with each other. After the first iteration, five of the ten initial connected-clusters: Δ_1, Δ_2, Δ_3, Δ_4, Δ_5 have been connected to the IGW.

The above Add-and-Merge process (Lines 3-11 of Figure 4) will continue until all connected-clusters are merged to Δ_0, which means all nodes are connected to the IGW. In the above example, the final G' generated by our algorithm is shown in Figure 6, in which 24 MR nodes have been selected to cover the network and provide full connectivity.

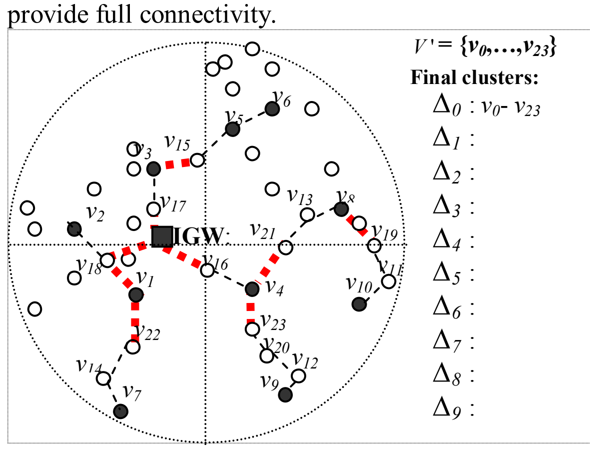

Figure 6. G' **of Full Coverage & Connectivity**

5. Algorithm Efficiency and Experimental Results

In this section, we analyze the complexity of the proposed heuristic algorithm and study the performance of our heuristic algorithm.

In the experiments, the transmission range and the network size are measured in terms of "unit", where the transmission range is scaled as one unit while the network size varies in the range of one to eight units. Considering that the 802.11a networks and 802.11b/g networks offer 12 and 3 non-overlapping channels respectively, we set the maximal number of orthogonal channels h at 12. The capacity of each channel w_k is normalized to 10M bps. The initial candidate positions are randomly generated and are ensured that they are enough to satisfy both coverage and connectivity constraints. The traffic density obeys random distribution in the network.

5.1. Algorithm Complexity

Table I. **Complexity Comparison**

Algorithm	Time Complexity		
	n=4	*n =67*	*n=501*
	m=22	*m=346*	*m=4186*
Brute Search	9180	5.03257E+72	$>>10^{307}$
Theoretical complexity of heuritic algorithm	O(487)	O(119839)	O(17523949)
Experimental complexity of heuritic algorithm	487	120386	17546290

The procedure of sorting the candidate nodes in G' by their physical distances to the IGW contributes to the mainly running time consumption in Phase I. This traversal procedure is bounded by the number of nodes in the network, i.e., m. Therefore, the time complexity of the sorting algorithm is O(m^2), in the worse case.

The time complexity of phase II is dominated by the merging procedure. Since the nodes in a connected-cluster are sorted in the order of the Euclidean distance to the IGW, merging two clusters actually uses the merge sort algorithm whose complexity is bounded by $O((|\Delta_i| + |\Delta_j|) * log((|\Delta_i| + |\Delta_j|))$. The merging procedure is called $|\Delta|$ times. If each cluster is assumed to have $n/|\Delta|$ number of nodes, the complexity of all merging activities is $|\Delta| * (2n/|\Delta|) * log(2n/|\Delta|)$. Therefore, the complexity of all merging activities is bounded by O($n$log(n)), where n is the number of MRs in the final connected-cluster.

Thus, combining phase I and II together, the time complexity of our heuristic algorithm is O(m^2)+ O($n$log(n)). Table I lists the detailed complexity given the specific m, n.

5.2. Scale to Larger Networks

We now study the impact of network size on our algorithm. In this experiment, we increase the disk radius from one to eight units. Larger networks are not under consideration here since it is infeasible to route traffic over a longer path and then multiple IGWs are required.

We compare the results calculated by our heuristic algorithm in the constraint network with the results calculated in the ideal case where a MR can be placed anywhere. In addition to the number of MRs in the final G', the number of MRs in G' that satisfies full coverage are also compared. Given a scenario, the heuristic algorithm runs for 10 times and we use the average sampled values as the final results. The standard deviation of the results is low and thus it is not present in the Figure.

The results in Figure 7 show that the number of selected MRs in each approach increases as the network size grows. It is because more MRs are needed to cover a larger network and maintain the IGW connectivity for the selected MRs. Figure 7 also shows the number MRs in the final G' is more than the number of MRs in the coverage-node set. The reason is that additional MRs are needed so that the IGW connectivity can be established for the MRs that only have the full coverage. We can also see from Figure 7 that our heuristic algorithm always results in more MRs than that generated by the ideal scenario (i.e., $m \to \infty$). Furthermore, the gap between them widens as the network area increases. Unlike the ideal scenario where the MR can be positioned anywhere in the network, the constraint network only has a fixed, non-optimized candidate to place MRs. Therefore, more MRs are likely to be used to satisfy the coverage and connectivity constraints. This result also indicates that increasing the candidate numbers of MRs will decrease the number of MRs in the final G', thereby decreasing the investment cost in terms of number of MRs.

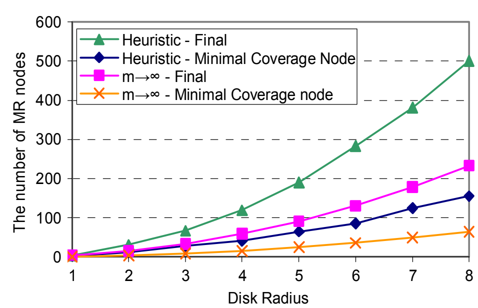

Figure 7. **Effect of network size**

5.3. Scale to Higher Traffic Density

We now examine the impact of traffic density. In the first round of experiments, the traffic demand $\mu(a)$ is randomly selected in the range of [0, xM bps] bits per square unit. Then, in the second round of experiments, the traffic demand is doubled (i.e., [0, 2xM bps] per square unit), and it increases proportionally with the round of experiments. The experiments are repeated six rounds. If the traffic demand increases, the traffic constraint could be violated and the MR placement may not be feasible with one IGW. In Figure 8, we can see such a missing point when the traffic demand is increased to the range of [0, 6*x*M bps] and the disk radius is eight units. The results are not present when the disk radius is smaller than four units. This is because the number of the MRs doesn't change in these cases even though the traffic density increases. Then, we can see from Figure 8 that when the disk radius is beyond four, the number of MRs increases with the increment of the radius. The degree of the increase with respect to the number of MRs goes up with the network size, which indicates that the traffic density has significant impact on the number of MRs in the larger size of networks. It also reflects the effectiveness of our heuristic algorithm. When the disk radius is small, the total traffic demand of the disk is relatively low. Even if we increase the traffic density, it does not cause significant difference and so the accumulated traffic demand of each connected-cluster is still below $(\sum_{k=1}^{h} w_k)/2$. As a result, the additional MRs are not necessary. In contrast, denser traffic leads to more increment of the accumulated traffic density when the disk radius turns larger. Correspondingly, more clusters can not be merged together any more because of the overload traffic demand. Therefore, more new nodes are introduced to share the burden.

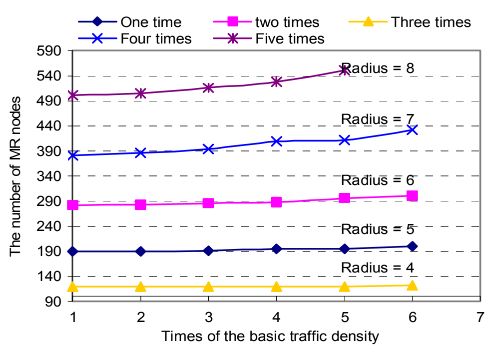

Figure 8. **Effect of traffic density**

6. Future work

There are several open issues in the MR placement. For example, interference model for the MR placement is also an interesting issue and deserves future research. Also, the impact of interconnecting several local networks to constitute a generic WMN and presence of other IGWs for alternate routes ought to be considered carefully. But, at least, this work is an initial attempt in designing WMN and could attract attention of researchers and service providers to this important area.

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