Competition, Signaling, and Status Externalities in Ph.D. Admissions*

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Abstract

Rising competition in imperfect markets pushes agents to invest in costly signals that differentiate themselves. Such investments can mitigate unraveling and improve matching efficiency, but also generate rat races that reallocate resources towards relative standing. I develop an empirical framework to quantify how competition affects signal adoption in matching markets and its welfare consequences, applying it to the role of pre-Ph.D. experiences—master's and predoctoral programs—in Ph.D. admissions. These experiences help programs screen applicants and provide research training. Yet when capacity is limited and grade inflation reduces informativeness, students pursue additional research experience to stand out. Using LinkedIn data on Economics and Business Ph.D.s, I find that pre-Ph.D. experience improves admission outcomes, with 54% of the gain attributable to signaling and 46% to training. While signaling restores about half of the matching efficiency lost under pooling, its opportunity costs exceed benefits, yielding a 15% net welfare loss. Benefits are concentrated among economics majors from top colleges; other groups are worse off. Grade inflation explains roughly one-quarter of the rise in pre-Ph.D. experience.

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1 Introduction

Across many markets, rising competition has led agents to invest in increasingly costly signals to improve their relative standing. In education, college admissions have become increasingly competitive: where strong grades, standardized test scores, and a few advanced courses once sufficed, students now devote considerable resources to supplementary coursework, research experience, and test preparation (Ramey and Ramey, 2010). In the labor market, undergraduates who once relied on coursework and grades to secure positions in elite finance or consulting firms now pursue multiple internships, including unpaid ones, to remain competitive for full-time offers (Wolfgram and Ahrens, 2022). In professional settings, junior associates in law and consulting firms routinely work exceptionally long hours to signal commitment and ability in the contest for promotion (Landers et al., 1996).[1] Such patterns reveal a common feature of imperfect markets: as opportunities become more competitive and information less transparent, individuals escalate costly efforts to stand out.

While theory provides a rich understanding of how competition and signaling shape markets, it offers ambiguous welfare implications. Costly signals can improve sorting efficiency by revealing hidden ability or motivation, helping allocate scarce opportunities to the most qualified agents(Cole et al., 1995, 2001; Hoppe et al., 2009). Yet when rewards depend on relative ranking, signaling generates status externalities: each individual's effort lowers others' standing and can trigger rat race in which everyone invests more without improving aggregate outcomes (Akerlof, 1976; Frank, 2005; Hopkins, 2023). The welfare effect of signaling therefore hinges on how much informational value it adds relative to the social cost of intensified competition. Moreover, how the degree of competition itself shapes signaling equilibria—whether fiercer competition mainly amplifies efficiency or merely magnifies waste—remains an open question.

This paper develops an empirical framework to quantify how signaling affects welfare in competitive matching markets and how intensified competition reshapes those effects. The framework unifies signaling and matching under asymmetric information, allowing both the informational gains from improved sorting and the welfare losses from excessive competition to be identified within the same structure. I apply this framework to the market for Ph.D. admissions, where applicants increasingly use master's and predoctoral research programs as costly signals of research ability. These programs provide an ideal setting to examine the dual role of signaling and training in a high-stakes matching tournament, where students'

[1]Similar phenomenon arises in other markets. In China, a skewed sex ratio intensifies competition in the marriage market, prompting families to save excessively, purchase housing, or pay large dowries to improve their relative standing (Wei and Zhang, 2011). Local governments likewise compete for fiscal or political rewards by overstating economic performance, turning data reporting into a tournament (Wallace, 2016)

choices and program admissions jointly determine equilibrium outcomes and welfare.

To capture how competition has intensified in the academic environment, I focus on grade inflation as an empirical measure of declining signal informativeness. Over recent decades, both the average undergraduate GPA and the share of A grades in U.S. colleges have risen markedly (Rojstaczer and Healy, 2012).[2] Prior studies attribute this pattern primarily to intensified labor-market competition, as universities strategically relax grading standards to improve their graduates' job placement outcomes (Popov and Bernhardt, 2013). While such practices may benefit institutions, they reduce the informativeness of grades, pooling students at the top and making high-ability applicants harder to distinguish (Boleslavsky and Cotton, 2015). As grades become less informative, students face stronger incentives to acquire alternative credentials—such as pre-Ph.D. training or research experience—to stand out in admissions. In this sense, grade inflation provides an externally generated source of competitive pressure that amplifies signaling incentives in the Ph.D. admission market.[3]

I begin my analysis by documenting empirical patterns that highlight the role of master's programs and predoctoral experience in Ph.D. admissions. Using a new panel of Economics and Business Ph.D. students constructed from LinkedIn profiles covering cohorts from 2000 to 2017, I find systematic differences in pre-Ph.D. experience across student backgrounds and a strong positive association between pre-Ph.D. experience and admission to top Ph.D. programs. These patterns suggest that pre-Ph.D. experiences function both as selective training opportunities and as costly signals that help applicants stand out when traditional indicators, such as grades, convey limited information. Moreover, the substantial return to entering top Ph.D. programs implies that students face strong incentives to acquire such credentials, even when the private competition may not translate into social efficiency. In this environment, grade inflation further amplifies these incentives: as undergraduate grades become less informative, students increasingly rely on pre-Ph.D. credentials to separate themselves, reinforcing the cycle of competition and signaling that this paper seeks to quantify.

Motivated by the empirical facts, I develop a structural model that integrates signaling and training in a matching market for Ph.D. admissions. A continuum of students differ in privately known research ability and publicly observed background characteristics. Before

[2] Grade inflation is not unique to the United States; similar upward trends have been documented in the United Kingdom, Canada, and East Asian countries (Johnes and Soo, 2017; Park and Cho, 2023; Ahn et al., 2024). I focus on the U.S. because even before inflation, admissions committees already faced challenges comparing applicants across international institutions.

[3] In the analysis, I treat grade inflation as exogenous to the Ph.D. admission market. Universities primarily inflate grades to improve students' labor-market outcomes, while only a small share of graduates pursue doctoral studies. Hence, the inflation of grades can be viewed as an external shock to the academic segment, affecting Ph.D. applicants through reduced signal informativeness rather than through endogenous response within the Ph.D. market.

applying to Ph.D. programs, each student decides whether to pursue pre-Ph.D. experience. These programs serve a dual role. First, they provide training, increasing a student's effective research ability and improving eventual job-market outcomes after the Ph.D. Second, they generate signals of underlying ability: because higher-ability students benefit more from the training provided by top Ph.D. programs, they have stronger incentives to invest in pre-Ph.D. experience to distinguish themselves and secure admission.

Ph.D. programs seek to admit students with high ability. They cannot directly observe ability but form rational posterior beliefs based on each applicant's background and pre-Ph.D. experiences decision. Admissions decisions are made under capacity constraints, and a stable matching forms between programs and students. After the match, each student's true research ability and training translate into realized job-market performance.

Ph.D. programs interpret pre-Ph.D. experience as evidence of higher research ability, recognizing that students who expect to perform well in research are more willing to incur its cost. Anticipating this inference, higher-ability students choose pre-Ph.D. experience to strengthen their applications and improve their chances of admission. These mutually consistent expectations sustain a self-fulfilling signaling equilibrium in which pre-Ph.D. experience both reflects and reveals underlying ability.

Relative to a market with training and status externalities alone (Krishna et al., 2018; Kim et al., 2024), the presence of this signaling amplifies competition. Because programs can now better identify high-ability students, matching efficiency improves and the expected payoff to entering a top Ph.D. program rises. The increase in expected returns widens the incentive gap between student types: high-ability students become even more willing to bear the cost of signaling, while others must invest more merely to remain competitive. As a result, overall effort and pre-Ph.D. training participation rise beyond what would be optimal in the absence of signaling. The equilibrium therefore combines training-driven productivity gains with signaling-induced escalation of effort, and welfare depends on the balance between these opposing forces—an evaluation quantified empirically relative to a benchmark without signaling.

The model characterizes how changes in the competitive environment shape equilibrium pre-Ph.D. training participation. When grade inflation erodes the informativeness of undergraduate records, programs learn less about applicants' ability from observed backgrounds alone. The resulting compression of posterior beliefs increases the value of costly signals, leading more students to pursue pre-Ph.D. experiences to separate themselves from peers. Moreover, higher returns to top Ph.D. programs strengthen incentives to invest in pre-Ph.D. training, whereas tighter quotas induce effort only up to the point where admission prospects become too low.

To separately identify the various mechanisms that may shape the incentives to participate in pre-Ph.D. experience, the paper leverages two kinds of outcomes and one source of quasi-experimental variation. First, it disentangles signaling from training by combining short-run admissions with long-run job placements: signaling mainly reshuffles students across programs (shows up in admissions) without raising unconditional job outcomes, whereas training raises unconditional outcomes; aligning both moments pins down each channel's contribution. Second, to isolate Ph.D. program training from selection, it exploits exogenous fluctuations in program quotas across cohorts (capacity shocks unrelated to the applicant pool), comparing outcomes of marginal admits across years to identify program-specific training effects. Third, the paper ties grade-inflation–driven signal compression to behavior by showing that faster undergraduate GPA growth predicts higher pre-Ph.D. participation, providing reduced-form evidence that weaker exogenous signals induce costlier endogenous signaling. Together—admissions vs. jobs to split signaling vs. training, cohort-level quota shifts to get program training, and GPA trends to predoc uptake for the grade-inflation channel—the identification strategy cleanly attributes the rise in competition to specific mechanisms rather than generic selection. The empirical estimates suggest that pre-Ph.D. experiences—master's or predoctoral research programs—enhance admission prospects by 54% through signaling and 46% through genuine training.

To assess the welfare implications of signaling, I compare the estimated equilibrium with several counterfactual experiments: a full-information equilibrium in which programs perfectly observe ability, a no-information (pooling) equilibrium where admissions offices do not account for pre-Ph.D. experience, and a training-only equilibrium where pre-Ph.D. experience enhance ability but convey no information. The comparison shows that signaling accounts for roughly two-thirds of total pre-Ph.D. experience participation, with the remaining one-third driven by training incentives. In welfare terms, signaling improves allocation by restoring about 48% of the matching efficiency lost under the no-pre-Ph.D. experience equilibrium, but its opportunity costs exceed these benefits, resulting in a net welfare loss of roughly 15% relative to the pooling equilibrium.

The welfare effects of signaling are uneven across student groups. Students from top undergraduate institutions, particularly those majoring in economics, face the lowest cost of pursuing pre-Ph.D. experiences. As a result, they participate in pre-Ph.D. experiences at much higher rates and gain disproportionately from the signaling channel. In contrast, students from less selective or non-economics backgrounds incur higher costs and receive little benefit, as the equilibrium rise in competition offsets potential training gains.

Finally, I evaluate how the information environment shapes competition by comparing the estimated equilibrium to a counterfactual without grade inflation. The results show that grade inflation accounts for roughly 24% of total pre-Ph.D. experience participation. By reducing the informativeness of undergraduate grades, grade inflation amplifies the reliance on alternative credentials and strengthens incentives to signal through pre-Ph.D. experience.

1.1 Related Literature

This paper contributes to the literature on status externalities and signaling in matching markets. Theoretical models date back to Cole et al. (1995) and Pesendorfer (1995), who show how individuals exert costly effort to improve their relative standing. Subsequent work has examined the inefficiencies such behavior can generate (Rege, 2008; Hoppe et al., 2009; Hopkins and Kornienko, 2010; Hopkins, 2012; Coles et al., 2013).[4] Empirical studies have documented status externalities in education and consumption (Landers et al., 1996; Ramey and Ramey, 2010; Wei and Zhang, 2011; Johnsen et al., 2023), while most rely on reduced-form evidence without structural modeling of welfare implications. The most closely related papers are Krishna et al. (2018, 2025) and Kim et al. (2024), who build structural models of college admissions showing how retaking exams and investing in private tutoring induce rat races. A key distinction is that in their settings, these behaviors provide only training effects: conditional on exam performance, such investments do not further affect admissions. By contrast, in my model, pre-Ph.D. experience yields both training and signaling benefits, as higher-ability students are more willing to enroll. This dual role intensifies competition and changes welfare outcomes.

In addition to the status externalities literature, this paper is related to research on signaling. The theoretical foundations of signaling date back to the classic model of Spence (1973), and have since generated a large body of work exploring how costly signals can improve market outcomes. While theoretical models are abundant, empirical studies quantifying signaling effects are relatively scarce.[5] Among the most closely related empirical structural models are Fang (2006); Backus et al. (2019), and Kawai et al. (2020). A key difference in my model is that receivers, Ph.D. programs, have fixed quotas, rather than engage in perfect competition and thus payoffs depend purely on beliefs about individuals' types. This constraint transforms signaling into a matching tournament: when slots are limited, signaling increases competition and may reduce aggregate welfare. Unlike previous work that highlights the efficiency gains from signaling, this paper simultaneously quantifies

[4]Related theory on status externalities driven purely by training effects dates back to Cole et al. (2001) and subsequent work. For a review, see Nöldeke and Samuelson (2024).

[5]In labor markets, there is a large literature documenting signaling, for example through educational credentials and certification, but most studies rely on reduced-form strategies to test the existence of signaling effects rather than fully modeling strategic behavior; see Altonji and Pierret (2001); Arcidiacono et al. (2010); Barrera-Osorio and Bayona-Rodríguez (2019) among others.

both the benefits and rat race costs produced by signaling under capacity constraints.

A growing literature studies how competition among colleges shapes grading standards and the informativeness of academic signals. Johnson (2006) and Bar et al. (2009) document widespread grade inflation, while Chan et al. (2007) and Popov and Bernhardt (2013) model how universities strategically inflate grades to improve graduates' placement outcomes. Subsequent work (Boleslavsky and Cotton, 2015; Ahn et al., 2024; Denning et al., 2022; Bowden et al., Forthcoming) analyze how equilibrium grading policies respond to market incentives and how such leniency affects student sorting and effort. My paper adds to this literature by emphasizing a downstream consequence of grade inflation: when grades become less informative, students adopt costly alternative signals, such as pre-Ph.D. experiences or internships, to differentiate themselves in the competition for limited elite Ph.D. slots.

2 Background and Data

2.1 Ph.D. Graduates in Economics and Business

The primary data source is the LinkedIn People's Public Profile, collected and provided by Bright Data. LinkedIn is the world's largest professional networking website, with more than 1.1 billion users as of 2025.[6] Each user's profile is formatted as a résumé through self-report, and typically contains detailed records of education and work history. For each degree, profiles report the institution, major field of study, type of degree earned, and the duration of enrollment. For employment, profiles record the institution or firm, job title, and start and end dates.

The dataset was collected by Bright Data, a global technology company specializing in web data collection and proxy services. Bright Data maintains large-scale archives of publicly available LinkedIn profiles. Through a pro bono initiative supporting academic research, I collaborated with Bright Data to extract all individuals who obtained a Ph.D. in Economics or Business from a U.S. News top 300 universities with starting years between 2000 and 2017. This yields a sample of graduates from 197 universities. To ensure completeness of academic and career records, I retain only profiles that report both (i) undergraduate degree information and (ii) the first employer following the Ph.D. degree. This filtering procedure produces a final dataset of 15,422 Ph.D. recipients.

I complement the LinkedIn profiles with several external sources to enrich information on students and institutions. Undergraduate and Ph.D. institutions are linked to the NCES Integrated Postsecondary Education Data System (IPEDS), which provides measures of selec-

[6]LinkedIn Statistics and Facts for 2025.

tiveness, location, and public/private status. Program quality is further characterized using multiple ranking systems: QS and U.S. News for undergraduate universities,[7] and Tilburg Business Economics publication scores, IDEAS citation counts, and U.S. News rankings for Economics Ph.D. programs.

Geographic and cost-of-living conditions are captured using Zillow data on average city-level rents for each Ph.D. program location. To measure grading standards at the undergraduate level, I incorporate institution-specific average GPA trends from 2000 to 2013 compiled by Rojstaczer and Healy (2012).[8] Finally, I infer gender from first names using the `genderize.io` API.

Despite the popularity of LinkedIn as the largest professional networking platform today, there may be important selection into the matched sample of Ph.D.s. To evaluate how severe this selection problem is, I construct a benchmark dataset from the *ProQuest Dissertations and Theses* database. I collect all doctoral dissertations in Economics and Business from the same set of U.S. universities between 2005 and 2022.[9] Each record provides the student's full name, doctoral institution, and year of completion. In total, this yields 35,928 dissertations, with roughly 1,700 to 2,100 new Ph.D.s awarded each year.

Figure 1 compares the coverage of Ph.D.s between the LinkedIn dataset and ProQuest by graduation year. In the early 2000s, LinkedIn captures about one-third of the population of new Ph.D.s. Coverage improves steadily over time, reaching about one-half of graduates by the late 2010s. This trend reflects both the increasing adoption of LinkedIn among professionals and improvements in data collection. The extent of this bias is mitigated in later cohorts. In future work I will seek to complement LinkedIn coverage with academic CVs of professors to improve representation of the academic career track.

2.2 Pre-Ph.D. Experience and Job Outcomes

A central component of the analysis is the definition of pre-Ph.D. training experience. I classify pre-Ph.D. training credentials into two categories: master's programs and research work experience. For master's programs, I retain only those degrees that begin after the completion of the student's first bachelor's degree and end prior to Ph.D. enrollment, with a reported major in economics or business. To focus on programs that credibly serve as

[7]Because students in the sample come from diverse majors, approximately 50% from economics or business disciplines, we use the overall university ranking rather than field-specific rankings to maintain comparability across majors and countries.

[8]Data available at `www.gradeinflation.com`.

[9]The LinkedIn dataset records both program starting and finishing years, whereas the ProQuest dissertation data contain only the finishing year of the Ph.D. program. To ensure consistency across data sources, we align the comparison sample using finishing years between 2005 and 2022.

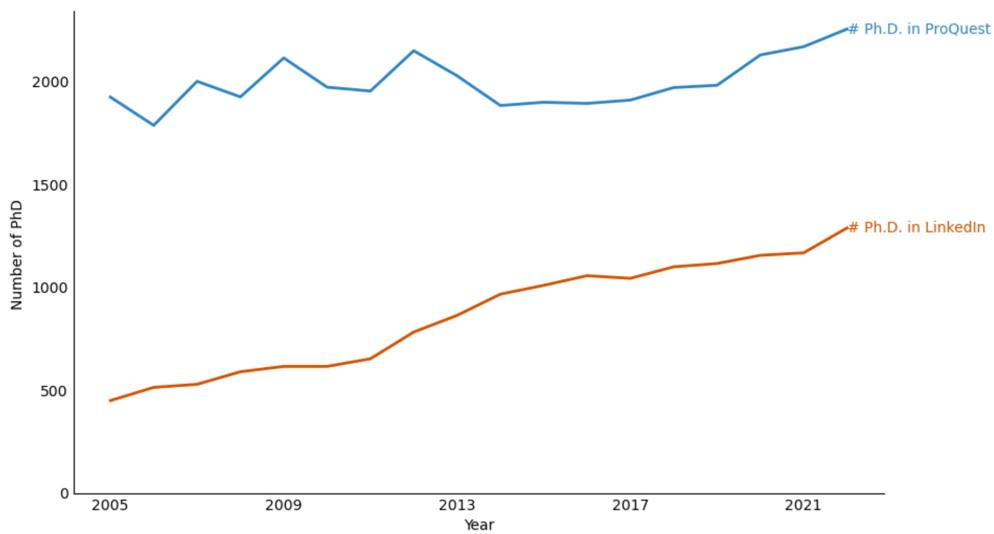

Figure 1: Coverage of Economics and Business Ph.D.s: LinkedIn vs. ProQuest, 2005–2022

pipelines to doctoral study, I restrict attention to master's degrees offered by top-50 universities in the United States or by universities ranked among the top three in their country according to the QS ranking. In addition, I require that at least ten students from a given program subsequently entered a Ph.D. program during the sample period.

For research work experience, I restrict attention to positions that occur in the interval between the bachelor's degree and Ph.D. enrollment. I classify an experience as predoctoral research only if the reported job title indicates a research-oriented role, excluding teaching, volunteer, or administrative positions. To ensure that these positions are academically relevant, I require that at least five individuals from the same institution or employer subsequently entered a Ph.D. program during sample period, and I exclude positions in finance institutions or management consulting.

Figure 2 plots the share of Ph.D. students with predoctoral experience by cohort. The prevalence of master's degrees rose from roughly 20 percent among students entering Ph.D. programs in 2000 to about 30 percent in 2017. Over the same period, the prevalence of predoctoral research experience increased even more dramatically, from around 5 percent to 20 percent. This trend illustrates the growing importance of predoctoral pathways in the Ph.D. admission process.

The second key outcome variable is the student's job market placement following Ph.D. completion. I define an "elite" outcome to include both academic and non-academic placements at the top tier of the profession. On the academic side, this category includes tenure-track appointments at the top 70 universities in the United States, as well as faculty positions at one of the top three universities within each country outside the United States. On the

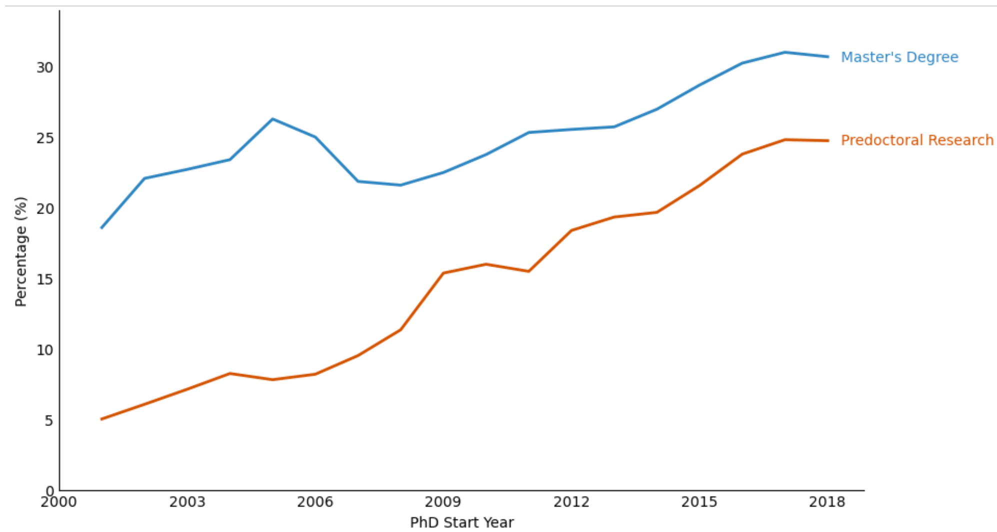

Figure 2: Share of Ph.D. Students with Master's Degrees or Predoctoral Research Experience, 2000–2017

industry and policy side, elite outcomes include positions at the four leading technology companies, the four leading economic consulting firms, and major investment banks or hedge funds. I also classify professional appointments at major government agencies such as the Federal Trade Commission, the Department of Justice, and the Consumer Financial Protection Bureau as elite outcomes, as well as placements at leading international organizations including the International Monetary Fund and the World Bank.

Tables 1 and 2 report descriptive statistics for Ph.D. institutions and students in the LinkedIn sample. Institutions vary widely in size and quality: the average program graduates about six students per year, though the standard deviation is nearly as large as the mean. Research productivity and reputation also differ substantially across programs, as reflected in publication scores, citation measures, and rankings. Roughly two-thirds of Ph.D. programs are housed in public universities, and about 14 percent are located in rural areas.

Student characteristics mirror the heterogeneity across programs. About 61 percent of students are male, and half majored in economics or business as undergraduates. Undergraduate backgrounds span a wide range of institutions, with average QS rankings in the mid-300s and U.S. News rankings near 100. Nearly 30% of the students completed some form of pre-Ph.D. experiences, most often a U.S. master's degree, though university and non-university research positions are also represented.

Table 1: Summary Statistics of Ph.D. Institutions

Characteristic	Mean	s. e.
# of Students per Year	5.78	(5.18)
Econ Top 5 Publication	49.3	(123.8)
Fin Top 3 Publication	34.2	(57.7)
Accounting Top 3 Publication	18.5	(29.7)
Management Top 5 Publication	43.2	(64.4)
Tilburg BE Score	4901	(8448)
IDEAS Citation Score	64.6	(53.7)
U.S. News Ranking	93.7	(88.0)
Average Rent in City	2059	(1144)
Public	0.63	(0.48)
Rural	0.14	(0.34)
Region:		
Northeast	51	
West	41	
Midwest	40	
South	63	

Table 2: Student Summary Statistics

Panel A. Means and Standard Deviations		
Male	0.61	(0.49)
UG Major=Econ/Busi	0.50	(0.50)
QS University Ranking	345.07	(304.07)
US News University Ranking	95.10	(84.21)
US News College Ranking	106.78	(85.83)
UG Selectiveness	0.55	(0.31)
Job Market Outcome	0.30	(0.46)
Panel B. Category Counts		
US Master	1545	
Non US Master	1345	
US Univ. Predoc	272	
US Non Univ. Predoc	633	
Non US Predoc	172	
Other Degree/Work	5465	
No Predoc	6936	

Notes: Job Market Outcome is an indicator for whether the student obtained a position at a highly desirable placement: a tenure-track job in a U.S. R1 university or one of the QS top three universities in another country, a position at one of the top four technology firms or top four economic consulting firms, or a professional appointment at leading government agencies and international organizations.

3 Evidence of Pre-Ph.D. Experience as Signals

In this section, I present reduced-form evidence on the role of pre-Ph.D. experience in Ph.D. admissions and subsequent career outcomes. Students with pre-Ph.D. experience are more likely to be admitted to top-ranked Ph.D. programs. They are also more likely to obtain elite job placements, although this association is largely explained by their enrollment in a top Ph.D. program. Finally, students from undergraduate institutions with faster increases in average GPA—a measure of grade inflation—are more likely to pursue pre-Ph.D. experiences, consistent with the interpretation that pre-Ph.D. experience becomes more valuable when undergraduate signals are less informative.

3.1 Pre-Ph.D. Experience and Admission to Top Ph.D. Programs

I begin by examining the relationship between pre-Ph.D. experiences and the likelihood of admission to a top Ph.D. program. Specifically, I estimate the following linear probability model:

$$\text{Top-Ph.D.}_{ikct} \;=\; \sum_k \alpha_k \mathbf{1}\{\text{Pre-Ph.D.}_i = k\} \;+\; \beta B_i \;+\; \delta_t \;+\; \delta_c \;+\; \varepsilon_{ikct}, \tag{1}$$

where Top-Ph.D._{ikct} is an indicator equal to one if student i enrolled in a top-30 Ph.D. program, and zero otherwise. The main variable of interest are indicators for different types of predoctoral experiences, $\mathbf{1}\{\text{pre-Ph.D.}_i = k\}$. The vector B_i includes student-level controls: whether the student attended a top U.S. undergraduate institution, a top non-U.S. undergraduate institution, whether the undergraduate major was economics, and gender. All specifications include Ph.D. start-year fixed effects δ_t, and non-U.S. subsamples also include country fixed effects δ_c.

Table 3 shows that all types of pre-Ph.D. training experience are positively associated with admission to top-30 Ph.D. programs. The magnitudes are sizable: U.S. master's degrees are linked to a 5 percentage point higher probability of top admission for U.S. students, compared to 13 percentage points for non-U.S. students, and non-U.S. master's degrees are associated with even larger margins (12 vs. 20 percentage points). Research-based predoctoral experience show the strongest associations overall, with U.S. non-university research positions linked to a 27 percentage point increase for U.S. students, while U.S. university research is associated with gains of 18–23 percentage points across groups. The larger coefficients for non-U.S. master's students suggest that admissions committees rely more on pre-Ph.D. training signals when evaluating foreign applicants, while within the U.S., research positions appear to be particularly powerful signals relative to master's programs.

Table 3: Pre-Ph.D. Training and Top Ph.D. Enrollment

	Full	U.S.	Non-U.S.
U.S. Master	0.089***	0.052**	0.132***
	(0.014)	(0.018)	(0.019)
Non U.S. Master	0.186***	0.119***	0.196***
	(0.014)	(0.039)	(0.017)
U.S. University Predoc	0.216***	0.175***	0.228***
	(0.026)	(0.035)	(0.048)
U.S. non University Predoc	0.239***	0.275***	0.146***
	(0.018)	(0.022)	(0.046)
Non U.S. Predoc	0.126***		0.118***
	(0.0030)		(0.030)
R^2	0.222	0.276	0.167
N	15422	7919	7503

Notes: Coefficients shown with significance stars; variance of the robust (HC3) estimator shown in parentheses beneath each coefficient. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

3.2 Pre-Ph.D. Training Experience and Job Outcomes

I next study the relationship between pre-Ph.D. training experience and elite job market outcomes. The estimating equation is given by:

$$\text{Elite-Job}_{ikct} \ = \ \sum_k \alpha_k \mathbf{1}\{\text{pre-Ph.D.}_{\cdot i} = k\} \ + \ \gamma \mathbf{1}\{\text{Top-Ph.D.}_{\cdot i} = 1\} \ + \ \beta B_i \ + \ \delta_t \ + \ \delta_c \ + \ \varepsilon_{ikct}, \tag{2}$$

where Elite-Job$_{ikct}$ is an indicator equal to one if student i obtains an elite placement as defined in Section 2.2. The key regressors are indicators for different types of pre-Ph.D. training $\mathbf{1}\{\text{pre-Ph.D.}_{\cdot i} = k\}$, and an indicator for whether the student enrolled in a top-30 Ph.D. program. As before, B_i includes undergraduate controls (top U.S. UG, top non-U.S. UG, UG major in economics, gender), δ_t are start-year fixed effects, and δ_c are country fixed effects in the non-U.S. subsample.

Table 4 reports the results. Without conditioning on Ph.D. program, pre-Ph.D. training experience is strongly associated with a higher probability of elite placement. For example, U.S. non-university research positions are linked to an increase of about 12 percentage points among U.S. students, and non-U.S. master's degrees and research positions are also associated with gains in the range of 9–12 percentage points. These magnitudes closely mirror the patterns found in Table 3 for top-30 Ph.D. admissions.

Once I control for whether a student enrolled in a top-30 Ph.D. program, however, the coefficients on pre-Ph.D. training experience decline substantially and, in many cases, lose statistical significance. For instance, the coefficient for U.S. non-university research positions among U.S. students falls by more than half, from 0.117 to 0.051. Similarly, the coefficients for master's programs are small and statistically insignificant once Ph.D. program quality is taken into account. By contrast, the top-30 Ph.D. indicator itself is strongly associated with elite placement, with magnitudes around 0.23–0.24 across subsamples.

These results suggest that much of the observed association between pre-Ph.D. training experience and job outcomes operates through sorting into stronger Ph.D. programs, rather than through direct gains from predoc training. Conditional on Ph.D. program attended, pre-Ph.D. training experience has little residual predictive power for elite placement. This pattern highlights the importance of moving beyond reduced-form analysis: if students are of similar quality after pre-Ph.D. training, we would not expect pre-Ph.D. training to independently predict job outcomes once Ph.D. program quality is accounted for. A structural model is therefore helpful to separate the contributions of sorting into Ph.D. programs from the training effects of pre-Ph.D. training.

Table 4: Pre-Ph.D. Training and Job Outcome

	U.S.	U.S.	Non-U.S.	Non-U.S.
Top-30 Ph.D.	–	0.230***	–	0.241***
		(0.016)		(0.015)
US Master's	0.022**	0.008	0.075***	0.017
	(0.010)	(0.008)	(0.018)	(0.017)
Non-US Master's	0.028	-0.014	0.092***	0.049***
	(0.039)	(0.039)	(0.015)	(0.015)
US Univ. Research	0.054**	-0.009	0.082*	0.040
	(0.024)	(0.026)	(0.045)	(0.043)
US Non-Univ. Research	0.117***	0.051**	0.102**	0.085**
	(0.026)	(0.026)	(0.039)	(0.038)
Non-US Research	–	–	0.119***	0.102***
			(0.033)	(0.032)
R^2	0.108	0.108	0.098	0.098
N	7919	7919	7503	7503

Notes: Coefficients shown with significance stars; variance of the robust (HC3) estimator shown in parentheses beneath each coefficient. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

3.3 Grade Inflation and Pre-Ph.D. Experience Adoption

Finally, I examine how grade inflation at undergraduate institutions relates to students' incentives to pursue a pre-Ph.D. training. The estimating equation is:

$$\mathbb{1}\{\text{Pre-Ph.D.}\}_{iut} = \alpha \cdot \Delta\overline{\text{GPA}}_u + \beta B_i + \delta_t + \varepsilon_{iut}, \tag{3}$$

where $\mathbb{1}\{\text{Pre-Ph.D.}\}_{iut}$ is an indicator equal to one if student i from university u pursued a master's or predoctoral research position before Ph.D. enrollment. The independent variable of interest, $\Delta\overline{\text{GPA}}_u$, measures the average annual change in GPA at undergraduate institution u between 2000 and 2013. This measure captures the extent to which an institution has experienced grade inflation. A larger value of $\Delta\overline{\text{GPA}}_u$ reflects a reputation for more lenient grading, making undergraduate GPAs less informative for admissions committees. As a result, students from such institutions may have stronger incentives to acquire additional signals through pre-Ph.D. training experience.[10] In addition to student controls (gender, top U.S. or non-U.S. UG, UG major in economics), I control for the selectiveness and ranking of the undergraduate institution and include year fixed effects.

Table 5 reports the results. The estimates indicate that a 0.01 increase in average GPA per year at a university is associated with a 0.026 percentage point increase in the share of its graduates pursuing a pre-Ph.D. training, equivalent to about a 10 percent rise relative to the mean. This pattern is consistent with the interpretation that when undergraduate transcripts become noisier signals of ability, students are more likely to invest in pre-Ph.D. training credentials to distinguish themselves in the Ph.D. admissions process.

Taken together, the reduced-form patterns provide a coherent picture of how pre-Ph.D. experience operates in the Ph.D. admission market. Pre-Ph.D. training participation varies systematically across student backgrounds: U.S. research positions appear more selective than master's programs, and non-U.S. students are disproportionately represented among pre-Ph.D. experience participants. Students with pre-Ph.D. experience are more likely to enroll in top Ph.D. programs, consistent with the interpretation that pre-Ph.D. experience serves as a costly signal of research ability. At the same time, the strong association between attending a top Ph.D. program and subsequent elite placement indicates that the return to securing admission is substantial. This suggests that students have strong incentives to invest in pre-Ph.D. experience credentials to compete for limited slots, potentially leading to excessive competition in equilibrium.

[10] Although the GPA series begins in 2000, the absence of a nationwide shock to grading standards implies that institutional reputations for leniency can be thought of as persistent and relevant even for students graduating at the start of our period. Since LinkedIn coverage is more comprehensive for later cohorts, this limitation is unlikely to materially affect the analysis.

Table 5: UG Average GPA Change and Pre-Ph.D. Experience Adoption

	Any Master's/Predoc	
	Coefficient	s.e.
$\Delta\overline{\text{GPA}}$	2.65**	(1.34)
Controls:		
Gender	Yes	
Year FE	Yes	
Selectiveness	Yes	
UG Rankings	Yes	
Observations	7321	
R^2	0.075	

Notes: $\Delta\overline{\text{GPA}}$ measures the annualized change in institutional average GPA between 2000-2013. Robust standard errors are reported in parentheses. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

These descriptive findings motivate the structural model developed in the next section, which formalizes how signaling and training channels jointly determine pre-Ph.D. experience choice, Ph.D. admissions, and eventual job outcomes, and quantifies the welfare implications.

4 Model

We develop a model of the market for pre-Ph.D. experience, hereafter predoc, and Ph.D. program admissions. In this market, a continuum of students make forward-looking decisions about whether to pursue a predoc, anticipating how this choice will affect their likelihood of admission to a Ph.D. program and, ultimately, their job market outcomes. Ph.D. programs evaluate students based on noisy signals of their underlying ability and admit qualified students up to their constrained capacity.

4.1 Model Setup

4.1.1 Players

The economy comprises a continuum of students, indexed by $i \in [0, 1]$, a finite number of Ph.D. Program, indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$, a finite number of predoc program, indexed by $k \in \mathcal{K} = \{0, 1, 2, .., K\}$ where 0 denotes applying directly without a predoc. Students are endowed with their academic background (B_i). They also differ in research ability ψ_i, which is known only to the students themselves. For the Ph.D. program, each Ph.D. program is endowed with characteristics (Z_j), as well as exogeneous quality denoted by h_j.[11] Each program has a fixed capacity κ_j where $\kappa_j > 0$ and $\sum_{j\in J} \kappa_j = 1$. These programs do not observe students' true ability ψ_i, but instead receive signals based on students' observable characteristics and educational histories. Before applying to Ph.D. programs, students may choose to pursue a master or predoc from $\mathcal{K}$. Each predoc differs in its cost $C_k(B_i)$ and training effect $S_k(\psi_i)$. Student's research ability after adopting predoc is $\tilde{\psi}_{ik} = S_k(\psi_i)$, where $S_0(\psi_i) = \psi_i$ for all ψ_i.[12]

After matching to a Ph.D. program, the student undergoes training, then enters the job market and realizes a job outcome. We model the labor market outcome as a function of the student's ability and the training of their Ph.D. program.

4.1.2 Job market outcome

After completing a Ph.D. program, a student's job market outcome is given by:

$$Y_{ijk} = T(\tilde{\psi}_{ik}, h_j) + \nu_{ijk}$$

where $T(\tilde{\psi}, h)$ denotes how a student with research ability $\tilde{\psi}$ can benefit from a Ph.D. program h. This indicates that if students are direct admitted to the Ph.D. program, their private research ability is the same as their endowment.

To make the positive assortative matching outcome socially desirable, that is, a social planner prefers to match students with high research ability with Ph.D. programs with high training intensity, following assumptions are imposed:

Assumption 1 *Ph.D. program Training $T(\tilde{\psi}, h)$ is strictly increasing in both $\tilde{\psi}$ and h, in addition, it is supermodular.*

Assumption 2 *Predoc program training $S_k(\psi)$ is weakly increasing in ψ.*

The first assumption ensures that allocating better Ph.D. programs to more capable students is socially desirable: students with higher research ability accumulate more human capital in top Ph.D. programs than their peers, so a social planner aiming to maximize aggregate human capital would assign them to top programs. The second assumption guarantees that predoc training preserves students' relative ranking. These two assumptions are mild, as

[11] We assume that the quality of Ph.D. programs are exogenous and constant over the sample period.

[12] In the model, we allow the expected payoff from taking a predoc to depend on the student's private type ψ_i, but we assume that the cost function $C_k(B_i)$ does not vary with ψ_i. This restriction is mainly for identification: from the model's perspective, increasing difference between high- and low-type students in either expected gain or cost from taking a predoc can generate signaling behavior. Empirically, however, only the total signaling effect can be identified, not its decomposition into payoff and cost components.

long as we believe that matching between stronger students and higher-quality Ph.D. (or predoc) programs is efficient.

Assumption 3 *The noise term ν is mean independent with students' research ability and Ph.D. program's training intensity.*

The third assumption is not strong, we just assume the programs do not have additional information about the students' job market outcome prediction other than research ability and the training they receive. If the assumption does not hold, we can simply demean ν and add $E\mathbb{E}(\nu|\psi, h)$ in to the training function term.

4.1.3 Ph.D. program admission

The students and Ph.D. program enters a matching market at the application-admission stage to get matched. Following Agarwal (2015) and other literature, we abstracted from the application admission details and assume they form a stable and individual rational matching outcome. A matching between students and schools is considered stable if there is no student and school who would both rather be paired with each other than with the assignments they receive in the proposed matching, i.e., no blocking pairs. A matching is individually rational if everyone prefer the matching pairs compared to being unmatched. For detailed definition, refers to Agarwal (2015); Azevedo and Leshno (2016).

Students' preference over Ph.D. program based on the training intensity and other program attributes. The utility of student i for program j is:[13]

$$U(\tilde{\psi}_{ik}, h_j, Z_j) = \mathbb{E}[y_{ijk}|\tilde{\psi}_{ik}, h_j] + \beta_i Z_j + \zeta_j = T(S_k(\psi_i), h_j) + \beta_i Z_j + \zeta_j \tag{4}$$

where Z_j includes characteristics such as program location, private or public and average local rent, β_i captures the student's taste for these attributes, and ξ_j is Ph.D. program's fixed latent attributes.

Ph.D. program prefers students with high research ability. While research ability is unobservable to the program, they infer that from students' background B_i and their predoc choice k_i. The program's preference over student is the posterior of students' quality and a private shock that is only observable to the program:

$$q(B_i, k_i) = \mathbb{E}[\tilde{\psi}_{ik}|B_i, k_i] + \eta_i \tag{5}$$

The private taste shock can be explained as the information that is only observable to

[13]Following Agarwal (2015), we exclude individual-level preference shocks ε_{ijt} because their variance cannot be separately identified from program-level unobservables ξ_j in our setting.

the Ph.D. program. For instance, reference letter, how the student's background can be transformed to future research.

Discussion: We model Ph.D. program's preference directly over expectation on students' private type instead of over their expectation on students' job market outcome. This model specification is not restrictive, because these two specifications would generates same equilibrium prediction. For a given Ph.D. program, h_j is fixed. The ranking over students' expected job market outcome is identical to the ranking over posterior mean on students' belief given $T(\tilde{\psi}_{ik}, h_j)$ strictly increasing.

4.1.4 Students

Students decide whether and which predoc or master to take before they apply to a Ph.D. program. If the student take the predoc or master, he receives the training and increase research ability while pay the cost of the master or predoc. If a student believe his possibility of being matched with Ph.D. program follows distribution $\mu(i)$, his utility function would be:

$$u_{ik} = \mathbb{E}_\mu[U(S_k(\psi_i), j_{\mu(i)}, Z_{\mu(i)}|B_i, k] - C_k(B_i) - \varepsilon_{ik}$$

Discussion: In the model we do not model the application and admission of predoc programs. Instead we specify the model as the student are always free to choose whether to do a predoc and can choose any predoc available in the market, while different predoc have different cost. This simplification is not as strong as its first glance. The intuition is as follows: if we write down a full model to include students' application cost and admission. Competition in predoc may generate additional signaling information to the Ph.D. program. In this simplified model, the additional signaling information, or screening power will go to cost differentiation. That is to say, if a predoc program is selective in real world, our model explain it as having sharper cost differentiation between students. Taking the predoc means the students have low cost, thus he has strong research ability. Since we are not focusing on the cost *per se*, but the signaling effect generated by the cost, our model will generate the same prediction as the original model.

Another simplification in this model is that we do not model the signal realization after they finish their predoc. In the market, student works on master classes or as research assistant for professors. They usually expect good grades, publications, or strong recommendation that will help them in their next round Ph.D. application. These outcomes in addition helps the Ph.D. program separate students within the same predoc program.

4.2 Characterization of Equilibrium

We characterize the equilibrium of the model through sequential rationality. In particular, we begin with the student-Ph.D. program matching stage, where students have already made predoc choices and programs have drawn their idiosyncratic preference shocks.[14] Then we compute the student's expected payoff for him doing different predocs. Finally the students choose the predoc that maximizes his utility.

4.2.1 Student-Program Matching

At the matching stage, the distribution of student characteristics (B_i, k_i) and private preference shocks η_i is realized. Given these, each Ph.D. program forms a posterior belief about student i's ability, denoted by $\mathbb{E}[\tilde{\psi}_i \mid B_i, k_i]$. Programs rank students based on this posterior plus the program-specific shock η_i, and admit students up to capacity. Students form their preference based on (h_j, Z_j) and accept the offer from the most preferred program that admits them.

From Azevedo and Leshno (2016), under the assumption of strict preferences and a continuum of students, the stable matching is unique and can be fully characterized by a set of program-specific cutoffs $\{\delta_j\}_{j\in\mathcal{J}}$. Each cutoff δ_j represents the lowest posterior value among students admitted to program j:

$$\delta_j = \min\left\{\mathbb{E}[\tilde{\psi}_i \mid B_i, k_i] + \eta_i : \mu(i) = j\right\}.$$

The distribution of posterior mean q_i follows $G(\cdot)$. Since the predoc choice is chosen by the student, the distribution of k and B are correlated. Specifically,

$$\begin{aligned} G(q) &= Pr[\mathbb{E}(S_k(\psi)|B,k) + \eta \leq q] \\ &= \int\int\sum_k Pr(k|B_i)\mathbb{1}\left\{\mathbb{E}[S_k(\psi) \mid B,k] + \eta \leq q\right\}\, dF_B(B)\, dF_\eta(\eta). \end{aligned} \tag{6}$$

Since we have one side homogeneous preference, the matching outcome follows a serial dictatorship. The unique equilibrium outcome can be implemented as follows, let the student with highest q_i in the pool pick his most preferred available program, then remove the student from the pool and iterate until all the students are matched with the program.[15]

[14] The job outcome realization stage is mechanical and does not involve additional strategic behavior.

[15] Formally, the serial-dictatorship implementation applies to a finite-agent matching market, whereas our model assumes a continuum of students. This distinction raises technical issues of measurability in defining individual selection under a continuum, but it does not affect the equilibrium logic. We maintain the continuum assumption to preserve the general-equilibrium property that a single student's deviation does not affect aggregate beliefs, while using the finite-agent matching algorithm for numerical implementation.

In equilibrium, Ph.D. program's posterior is formed on student's choice on whether to take predoc. The effect of taking a predoc, similar to education in labor market, can be decomposed into training effect and signaling effect. The posterior belief is:

$$\begin{aligned} q_i &= \mathbb{E}[S_k(\psi_i) \mid B_i, k_i] + \eta_i \\ &= \underbrace{\mathbb{E}[\psi_i \mid B_i, k_i] - E[\psi_i|B_i, 0]}_{\text{Signaling}} + \underbrace{\mathbb{E}[S_k(\psi_i) - \psi_i \mid B_i, k_i]}_{\text{Training}} + \mathbb{E}[\psi_i|B_i, 0] + \eta_i. \end{aligned}$$

The training effect is expected gain in ability from the predoc program itself, conditional on observables: $\mathbb{E}[S_k(\psi_i) - \psi_i \mid B_i, k_i]$, which captures how the predoc improves the research ability of a given student, holding their type fixed. The signaling effect is the difference in expected ability between students who choose predoc k versus those who do not, holding fixed their observable background: $\mathbb{E}[\psi_i \mid B_i, k_i] - \mathbb{E}[\psi_i \mid B_i, k = 0]$. This term reflects how students who opt into predoc k are selected in terms of their unobservable ability relative to those who apply directly without predoc.[16] The training effect is exogenous and the signaling effect is endogenous and determined by the population who choose to do the predoc.

Remark: We assume the program have identical preference among the students. This simplification is both analytically convenient and empirically plausible. While stable matching exists and is generally unique even when both sides have heterogeneous preferences (Azevedo and Leshno, 2016), assuming identical preferences allows us to decompose the posterior q_i into training and signaling effects in a clean and interpretable way. Introducing heterogeneity on the program side would increase computational burden and obscure the core economic mechanisms by which predocs influence outcomes. Moreover, in practice, Ph.D. programs typically evaluate students using similar materials—such as transcripts, recommendation letters, and CVs—and are less likely to have strong, systematic preferences over student-specific attributes like location or gender. As such, our assumption of uniform program preferences is a reasonable abstraction that does not materially reduce the generality or relevance of the model.

4.2.2 Student's Expected payoff

To evaluate their expected payoff from entering the matching process, each student must form beliefs about which Ph.D. programs they are likely to match with. While students

[16] We abstract from "muddled" signaling and assume that all signals are fully credible. If students possess multi-dimensional private information—such as both research ability and "gaming" ability that helps them navigate admissions—then signaling may convey information along multiple dimensions. As shown in Frankel and Kartik (2019), such multi-dimensional signaling can lead to equilibria in which signals partially lose their value, as admissions committees can no longer separate true ability from strategic manipulation.

know their own characteristics B_i, predoc choice k_i, and research ability ψ_i, they do not observe their own private preference shock η_i, and therefore do not know their exact ranking within the applicant pool. However, they hold correct beliefs about the posterior distribution $G(q)$.[17]

Since the student does not observe q_i, they compute the probability of being admitted to each program as the probability that q_i exceeds the program's cutoff and that all more-preferred admissible programs reject them.

Each student has an individual preference ranking over programs, denoted $j_i(1), j_i(2), \ldots,$ from most to least preferred. Given the inferred cutoff vector $\{\delta_j\}$, the student compares this sequence of cutoffs against their own (unknown) score q_i. If a less preferred program has a higher cutoff than a more preferred one (i.e., $\delta_{j_i(m+1)} \geq \delta_{j_i(m)}$), then the student will never match to program $j_i(m+1)$, and it can be eliminated from their consideration. After eliminating such dominated options, the student faces a strict sequence of admissible programs with strictly decreasing cutoffs.

Thus, the student is matched to the first program in their preference list for which $q_i \geq \delta_j$. Thus, their expected utility is:

$$\begin{aligned}\mathbb{E}[U_i|B_i, k_i, \psi_i] &= \sum_{m=1}^{M_i} \Pr\left(q_i \in [\delta_{j_i(m)}, \delta_{j_i(m+1)}]|B_i, k_i\right) \cdot U_i(\tilde{\psi}_{ik}, h_{j_i(m)}, Z_{j_i(m)}) \\ &= \sum_{m=1}^{M_i} \left[F_\eta\left(\delta_{j_i(m+1)} - \mathbb{E}[\tilde{\psi}_{ik} \mid B_i, k_i]\right) - F_\eta\left(\delta_{j_i(m)} - \mathbb{E}[\tilde{\psi}_{ik} \mid B_i, k_i]\right)\right] U_i(\tilde{\psi}_{ik}, h_{j_i(m)}, Z_{j_i(m)})\end{aligned} \tag{7}$$

where M_i is the number of admissible programs remaining after the elimination step and $F_\eta(\delta_{j_i(0)} - c) = 1$ for all c. This formulation captures the student's expected benefit from entering the matching process given their beliefs about other student's choice.

4.2.3 Student Predoc Choice

Given these beliefs about their expected utility from Ph.D. matching, students decide whether and where to pursue a predoc. This decision is forward-looking and based on how the choice of a particular predoc affects the student's posterior from Ph.D. program's perspective. A predoc can shift the distribution of a student's posterior mean in the two channels we discussed above, training and signaling effect. The training effect arises from the direct improvement in a student's research ability due to their participation in a predoc program.

[17] Students are assumed to know the population distribution $G(q)$ but not their individual realization q_i. Because the economy contains a continuum of students, a single student's deviation does not affect the aggregate distribution $G(q)$. This assumption simplifies the analysis by allowing us to focus on representative behavior in equilibrium rather than tracking strategic interactions among finitely many students. In equilibrium, each student competes not against specific peers but against the overall population distribution.

It is determined exogenously by the training function $S_k(\psi_i)$ and is captured by the expected gain in ability, $\mathbb{E}[S_k(\psi_i) - \psi_i \mid B_i, k]$. In contrast, the signaling effect stems from how the student's choice of predoc conveys information about their underlying ability to Ph.D. programs. Because the distribution of ability among students who choose each predoc is endogenous and shaped in equilibrium, the Ph.D. program interprets a student's predoc choice as informative. The value of this signaling channel is measured by the difference in expected ability between students with the same background who choose to do the predoc versus those who do not, i.e., $\mathbb{E}[\psi_i \mid B_i, k] - \mathbb{E}[\psi_i \mid B_i, 0]$.

Combining these considerations, the student solves the following problem:

$$\max_{k \in \mathcal{K}} \mathbb{E}[U_i \mid \psi_i, B_i, k] - C_k(B_i),$$

where $\mathbb{E}[U_i \mid \psi_i, B_i, k]$ reflects the expected matching utility as shaped by both the training and signaling value of predoc k, and $C_k(B_i)$ is the cost of participating in it. This formulation captures the strategic nature of the predoc decision: students weigh the investment cost of predoc participation against the anticipated return in the Ph.D. matching market.

Discussion: We assume a large market in which each student takes the distribution of posterior beliefs and program cutoffs as given. That is, while students understand that predoc choices are informative, they treat the population-level inference from these choices as unaffected by their individual decision. This price-taking assumption simplifies the analysis by avoiding the need to solve a full Nash equilibrium among students. Instead, students respond optimally to an equilibrium environment, without internalizing their negligible influence on aggregate outcomes. This approach is standard in the large matching market literature and is particularly useful here, as it allows us to isolate and interpret the signaling and training effects of predoc decisions in a tractable way.

4.3 Equilibrium

We focus on weak perfect Bayesian equilibrium. Because students can choose different predoc options and thereby enter the application pool in different periods, we restrict attention to a stationary equilibrium in which the distribution of applicants and admission thresholds are time-invariant.

Definition 1 *A stationary equilibrium consists of individual optimal decision $k(B, \psi)$, Ph.D. program admission criteria δ_j and applicant distribution $Pr(k, B)$ such that:*

- *Given admission criteria δ_j, $k(B, \psi)$ is the optimal predoc choice for every (B, ψ);*

- *Given an predoc choice $k(B, \psi)$ and student distribution $Pr(B, \psi)$, Ph.D. program forms posterior belief $E(\tilde{\psi}|B, k)$. The matching generates Ph.D. program admission criteria δ_j;*
- *Predoc choice decision $k(B, \psi)$ generates distribution $Pr(k, B)$.*

4.3.1 Existence of Equilibrium

We next prove the existence of an equilibrium in a simplified version of the model. In this restricted environment, there are only two Ph.D. programs, a single predoc option, and no idiosyncratic taste shocks over predoc choice (i.e., ε_{ik} are degenerate).

Assumption 4 *In the two programs $j \in \{1, 2\}$ with quota $\kappa_1 + \kappa_2 = 1$, one predoc $k \in \{0, 1\}$ environment, the following assumptions hold: (i) The idiosyncratic predoc preference shocks ε_{ik} are degenerate (i.e., constant), program taste shock follows uniform distribution $\eta \sim U(-\bar{\eta}, \bar{\eta})$;(ii) The prior distribution $F(\psi \mid B)$ has an increasing hazard rate. (iii)The training effect of Ph.D. program 2 is normalized to 0: $T(\psi, h_2) = 0$.*

With these assumptions, we can show the equilibrium exist and is summarized in following proposition:

Proposition 1 (Existence) *Under Assumption 4, there exists a stationary equilibrium.*

In the original model, the positive assortative matching between students and programs yields a unique allocation and thus uniquely determined admission cutoffs given any distribution of applicants. Second, conditional on these cutoffs, the predoc choice problem reduces to a signaling game with finitely many signals and types. By previous literature (Cho and Kreps, 1987), such signaling games always has equilibrium. Together, these observations imply that the existence of equilibrium is of less concern. The computational method to compute equilibrium can be found in Appendix B.1.

5 Comparative Statics

In this section, we first compare environments with no predoc, predocs that provide training only, and predocs that combine training with signaling. We then analyze how program-side factors, such as selectivity and returns, and the informativeness of exogenous signals, like grades, affect students' incentives to pursue predoc experience.

To make intuition clear, we maintain assumption 4. In addition, we assume programs only have one dimensional characteristic h, so all students strictly prefer admission into the higher-quality program. Also, each student is characterized by a one-dimensional observable characteristic b and private type ψ. Admissions committees evaluate candidates based on the posterior expectation of ψ, conditional on observed characteristics and predoc enrollment status, combined with an unobservable noise term η:

$$q = \mathbb{E}\big[\psi \mid b, k\big] + \eta,$$

where $k \in \{0, 1\}$ indicates predoc participation.[18]

5.1 Effect of Predoc

In this part, We compare how introducing a predoc program—either with uniform training benefits or with benefits that depend on student ability—alters equilibrium admissions and sorting. This highlights how different forms of predoc design can generate distinct selection and signaling effects.

5.1.1 No Predoc Equilibrium

In the baseline scenario, no predoc program is available. As shown in Figure 3, admissions decisions rely exclusively on observable characteristics b, since no action or signal can reveal private information about ψ. Because the admissions committee has no private information and η is mean-zero, candidates are accepted if b exceeds a threshold $b^*_\varnothing$. This generates a flat cutoff line in the (b, ψ) space: all applicants with $b \geq b^*_\varnothing$ are admitted regardless of their private type. In this regime, no selection on unobservables arises.

5.1.2 Training Effect Equilibrium

Introducing a predoc that provides a pure training effect leads to a different pattern. In this scenario, participation yields a fixed benefit $S_1(\psi) = \psi + s$, and $T(\psi, h)$ gives fixed add up for all ψ. All students receive the same incremental benefit if they enroll. Admissions committees therefore set different thresholds for candidates depending on whether they have completed the predoc. Let b^*_1 denote the cutoff for predoc participants and b^*_0 the cutoff for non-participants. As illustrated in Figure 4, we have $b^*_0 > b^*_\varnothing \geq b^*_1$. The second inequality is strict if and only if ε is not degenerate.[19] This ordering generates a rat race effect: marginal

[18]The restriction to one-dimensional program quality and student heterogeneity is for expositional clarity. The main intuition extends to richer multidimensional environments where programs differ along multiple attributes and students have heterogeneous valuations. For numerical simulations and robustness checks under less restrictive specifications, see Appendix B.2.

[19]The presence of noise terms η and ε introduces uncertainty in admissions and cost realizations. Because students cannot perfectly predict whether predoc participation guarantees admission to a top program,

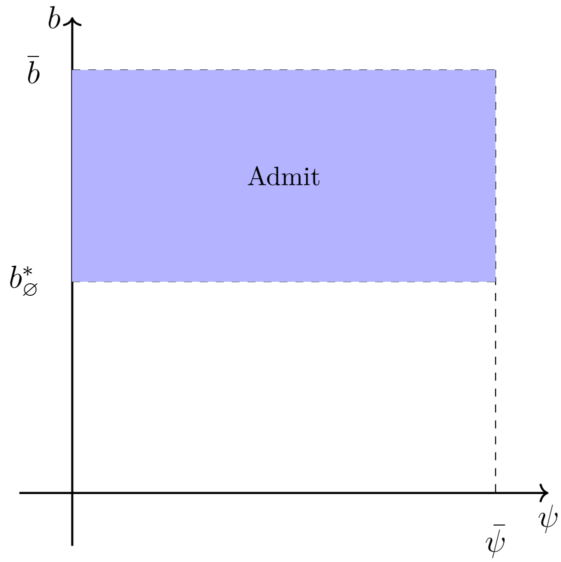

Figure 3: Top Ph.D. program admission cutoffs under no-predoc equilibrium.

Notes: The figure illustrates the admission cutoff in the absence of predoc programs. The horizontal axis represents students' private research ability ψ, and the vertical axis shows observable credentials b. The dashed line at $b^*_\varnothing$ indicates the admission threshold: applicants with $b \geq b^*_\varnothing$ are admitted. The shaded blue area marks admitted students, while those below the line are rejected. The flat cutoff reflects that admission decisions depend only on b, not on unobserved ability.

applicants who would otherwise be below $b^*_\varnothing$ have incentives to incur the (constant) cost of predoc enrollment to improve admission prospects.[20] This increase the admission criteria, while the program treat the two group of student differently, the marginal students are the same quality, $E(\psi|b^*_0) = E(\psi + s|b^*_1)$. This training effect equilibrium is consistent with the main mechanism in Krishna et al. (2018) and Kim et al. (2024).

5.1.3 Signaling Equilibrium

In the final scenario, predoc participation provides both training and signaling. The training effect still applies to all students. In addition, the function $T(\psi, h)$ is supermodular, so higher-type students derive a larger benefit from admission into the top program. This property creates an endogenous selection effect: students with higher ψ are more willing to enroll because their expected return from predoc participation is greater.

those near the indifference cutoff (the "red region") may optimally choose not to take a predoc when their idiosyncratic cost shock is high.

[20] If there were no uncertainty (i.e., η and ε were degenerate), training would not improve matching efficiency. In that case, the same cohort of students would be admitted in both the no-predoc and training equilibria, and predoc participation would only shift their human capital without affecting the allocation across programs.

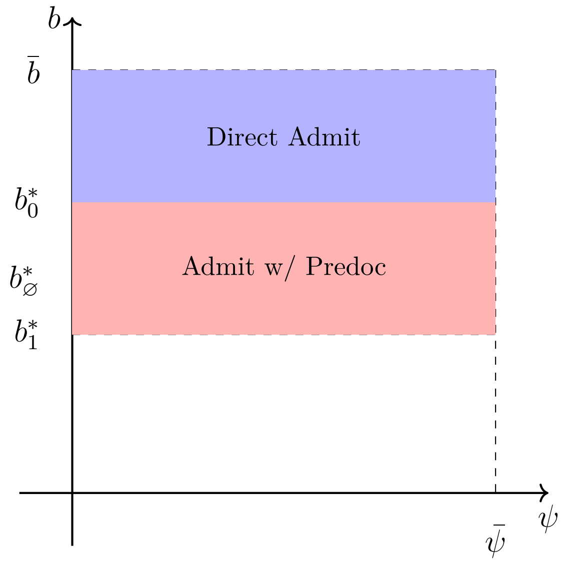

Figure 4: Top Ph.D. program admission cutoffs under training equilibrium.

Notes: The figure plots the admission thresholds when a predoc provides a fixed training gain. The vertical axis shows observable credentials b, and the horizontal axis represents latent ability ψ. Dashed lines at b_1^*, $b_\varnothing^*$, and b_0^* mark cutoff levels for predoc participants, the baseline case, and non-participants, respectively. The blue shaded area indicates applicants directly admitted without a predoc, while the red region shows those admitted after completing a predoc. The gap between b_0^* and b_1^* illustrates how the predoc lowers the admission threshold for participants by providing an equivalent training crenditials.

In this situation, the marginal student is indifferent between doing a predoc and increase the chance of admission or doing nothing:

$$U(\psi^*(b), k = 1) = Pr(h_1|b, 1)T(S(\psi^*(b)), h_1) - c = Pr(h_1|b, 0)T(\psi^*(b), h_1) = U(\psi^*(b), k = 0)$$

And the school will update their belief based on the cutoff $\psi^*(b)$ and admit students accordingly.

$$E(\psi|b, k = 1) = E(\psi|b, \psi > \psi^*(b))$$

So in equilibrium $E(\psi|b, \psi > \psi^*(b))$ always equal to the admission criteria if η is degenerate.

The equilibrium admission outcome is as shown in Figure 5, predoc enrollment becomes informative about a student's private type. Admissions committees update their posterior beliefs conditional on observing $k = 1$. Unlike the flat thresholds in the training-only case, the effective admissions frontier becomes tilt in ψ, reflecting the higher likelihood of predoc participation among high-type students. Compared to the previous regimes, this generates richer sorting: predoc enrollment now functions both as a skill investment and a credible signal of ability.

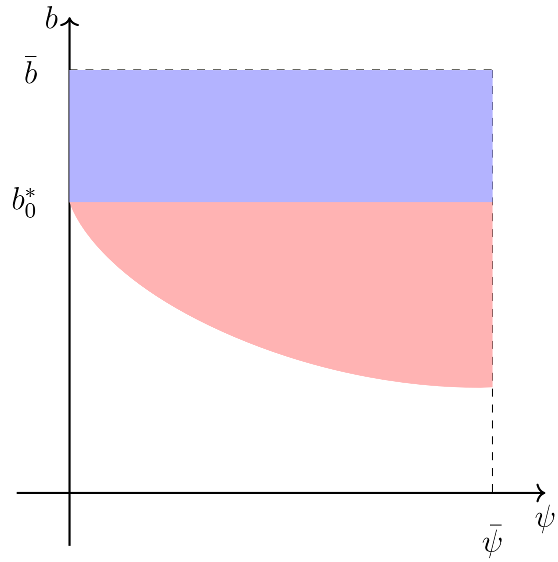

Figure 5: Top Ph.D. program admission cutoffs under signaling equilibrium.

Notes: The figure depicts the admission frontier when predoc participation conveys both training and information about private ability. The horizontal axis represents latent ability ψ, and the vertical axis shows observable credentials b. The blue region indicates students directly admitted without predoc, while the red curved area represents those admitted after completing a predoc. The admission boundary now slopes upward in ψ, reflecting that higher-ability students are more likely to undertake a predoc and thus receive favorable posterior evaluations. Predoc participation therefore acts as both a skill investment and a credible signal of high ability.

It worth mentioning that the high type do not signal. This pattern is closely related to the countersignaling behavior formalized in Feltovich et al. (2002). In both model, agents with the highest underlying quality refrain from engaging in costly signaling precisely because their existing reputation sufficiently distinguishes them from lower types. In both frameworks, the reward from signaling is not supermodular: once the posterior belief about the agent's type exceeds a threshold, the marginal benefit of additional signaling effort becomes flat. Consequently, agents whose observable characteristic b is already high enough to generate a high posterior admission probability find it optimal not to undertake the costly action.[21] In our setting, this implies that the most qualified applicants may choose not to participate in the predoc program, while intermediate types use it to credibly signal their ability. This resemblance underscores how the informational role of abstaining from a signal can arise endogenously when the payoff function exhibits this non-supermodular structure.

In summary, the three cases illustrate a progression in the role of predoc programs.

[21] In the model, all the high type students signal can also be a weak Perfect Bayesian Equilibrium. In this case, not doing predoc becomes off-path and a belief that the students not doing predoc are the lowest type can support the equilibrium. We rule out the weak PBE by assuming there is prefect competition for the best students so the programs are willing to admit students without predoc.

Without predoc, admissions depend solely on observable characteristics, and no selection on unobservable arises. When the predoc provides only training, it uniformly shifts admissions cutoffs and induces participation among marginal applicants without conveying information about private types. In contrast, when predoc participation yields supermodular benefits in $T(\psi, h)$, it functions both as an investment in skills and a credible signal of higher private ability, reshaping admissions thresholds and equilibrium sorting patterns.

5.1.4 Welfare

Beyond admissions thresholds and sorting patterns, an important question concerns the welfare implications of introducing a predoc program with signaling features. As noted in Proposition 3 in Hopkins (2023), the signaling equilibrium can be welfare superior to the no-predoc benchmark if and only if both the variance of program quality h and the variance of student private type ψ are sufficiently large. In such environments, random matching without any informative signals is more likely to allocate high-ability students to less-preferred programs, reducing aggregate welfare. However, in settings where heterogeneity is limited, or where signaling primarily induces costly rat race behavior without improving matching accuracy, the welfare gains can be negligible or even negative. In our context, this ambiguity implies that the introduction of a predoc as a signaling device does not unambiguously benefit students and may exacerbate competitive pressures without substantially improving the alignment between student quality and program quality.

5.2 Competition and Predoc Choice

In this subsection, we examine how different dimensions of competition affect students' predoc decisions. We organize the discussion into two parts. First, we explore how changes in students' observable characteristics, which is grade inflation in undergraduate programs that makes exogenous signals noisier and compresses the distribution of apparent ability, influence the equilibrium. Second, we consider how increased selectivity or higher rewards from admission to elite Ph.D. programs intensify students' investment in predoc.We show the theoretical proposition in the main text and the numerical simulation under less restrictive assumptions can be found in appendix B.2.

5.2.1 Grade Inflation on Predoc Choice

In this subsection, we investigate how the informativeness of students' observable signals affects their predoc decisions. Specifically, we consider the comparative statics with respect to the precision of exogenous information, such as undergraduate grades, when students compete for admission to selective Ph.D. programs.

Formally, let $F(\psi \mid B)$ denote the baseline conditional distribution of student ability and $\tilde{F}(\psi \mid B)$ an alternative distribution. Let $\psi^*(B)$ and $\tilde{\psi}^*(B)$ denote the associated predoc cutoff thresholds under the two distributions. The following proposition holds:

Proposition 2 *Under assumption 4, if $F(\psi \mid B)$ second-order stochastically dominates $\tilde{F}(\psi \mid B)$, then $\tilde{\psi}^*(B) > \psi^*(B)$. That is, when exogenous information becomes less precise, students are more likely to choose predoc experience.*

This result illustrates how grade inflation and other forms of signal compression shape students' strategic investments. Intuitively, when grades become more homogeneous and thus less informative about true ability, students perceive that it is harder to credibly distinguish themselves from their peers. As a consequence, they are more inclined to use predoc experience as a costly signal to separate from the mass of indistinguishable applicants.

This insight contributes to a growing literature on the effects of grade inflation and signal compression. Previous literature have documented the pervasiveness of grade inflation and analyze how it distorts incentives, increases effort on alternative signals, or exacerbates inequality.[22] Our analysis complements these findings by highlighting a novel channel through which noisier grades induce intensified competition for predoc positions, with potential welfare consequences. In equilibrium, more students invest in costly predoc training to offset the information loss from inflated undergraduate evaluations.

This mechanism also offers a potential explanation for broader trends in educational and labor markets, such as the increased prevalence of internships and pre-professional experiences. As exogenous signals about student quality become less reliable, both academic institutions and employers place greater weight on additional dimensions of differentiation, which raises the overall cost of competition. Understanding these dynamics is important for evaluating policies aimed at improving transparency and restoring the informativeness of educational credentials.

5.2.2 Ph.D. Program Selectivity and Returns on Predoc Choice

In this part, we analyze how changes in the characteristics of Ph.D. programs—specifically, their selectivity and the returns to admission—affect students' incentives to invest in predoc experience. We formalize these comparative statics in the following proposition:

[22] For instense, see Chan et al. (2007), Popov and Bernhardt (2013), Boleslavsky and Cotton (2015), and Ahn et al. (2024)

Proposition 3 *Under assumption 4*

1. *If the return to attending the top Ph.D. program increases, more students will choose to undertake a predoc, i.e.,*
$$\frac{\partial \psi}{\partial h_1} < 0.$$
2. *There exists a threshold κ^* such that, when $\kappa < \kappa^*$, as top Ph.D. program size decrease, more student take predoc, i.e.,*
$$\frac{\partial \psi}{\partial \kappa} < 0$$
when $\kappa > \kappa^*$, as top Ph.D. program size decrease, fewer student take predoc, i.e.,
$$\frac{\partial \psi}{\partial \kappa} > 0$$

This result highlights a subtle distinction between two sources of intensified competition in the market. When the return to attending a top program increases, students have stronger incentives to differentiate themselves through predoc experience, leading to higher participation. In contrast, when the quota of top programs contracts—making them more selective—students initially increase effort to compete among peers, but as capacity shrinks further, they anticipate a low probability of admission regardless of their predoc choice, which ultimately discourages investment in preparation.[23]

Thus, while both changes heighten competition by raising the value of scarce positions, they differ in how they shape equilibrium incentives—greater rewards expand signaling effort, whereas tighter quotas suppress it once the contest becomes excessively selective.

6 Identification and Estimation

In this section, we describe how the model is identified and estimated under a binary private research type $\psi \in \{\psi_l, \psi_h\}$ with distribution $\Pr(\psi_h \mid B, k, t)$.

6.1 Estimation

This section describes how the model parameters are estimated. We first specify the parameterization of the model and then outline the two-stage estimation procedure.

[23] A similar non-monotonic pattern arises in tournament models of effort provision: when the number of prizes or winning probability changes, participants first increase effort to compete and later reduce it once success becomes too unlikely(Lazear and Rosen, 1981; Moldovanu and Sela, 2001).

In the first stage, we jointly estimate the type distribution, training, and matching parameters using a *Simulated Method of Moments (SMM)* approach. This stage takes students' predoc choice k and the equilibrium belief $\Pr(\psi \mid B, k, t)$ as given.[24] The estimation targets moments that capture student–program sorting patterns, within-program heterogeneity, and job-outcome differences across programs.

In the second stage, conditional on the first-stage estimates, we estimate students' utility parameters by *maximum likelihood (MLE)*. It uses the structural parameters from the first stage to compute each student's expected admission probability and expected utility under different predoc choices. We also compute each student's posterior probability of being a high type using the estimated parameters. With these results, we estimate the cost parameters for different predoc types using a demand estimation.

6.1.1 Parameterization

Predoc and Ph.D. program training are assumed to be time-invariant, while predoc costs may vary across cohorts. The model parameters are grouped into four sets: type distribution, training, matching, and predoc choice. The type distribution is characterized by $\boldsymbol{\theta_\psi} = \{\psi_L, \psi_H, \gamma\}$, where each student's research ability $\psi \in \{\psi_L, \psi_H\}$ and the probability of being high-type depends on background characteristics, predoc choice, and cohort through a probit specification $\Pr(\psi \mid B_i, k, t; \gamma) = \Phi(\gamma_B B_i + \gamma_k + \gamma_t)$. Training parameters $\boldsymbol{\theta_{tr}} = \{\tau, w, s, \sigma_\nu\}$ govern how ability translates into research output: the training production function takes the form $T(\tilde{\psi}_i, h_j; \tau, w) = \tilde{\psi}_i + \tau_1 h_j + \tau_2 \tilde{\psi}_i h_j$, where $\tilde{\psi}_i = S_k(\psi_i; s) = \psi_i + s_k$ represents post-predoc ability and $\nu_{ijk} \sim N(0, \sigma_\nu)$ is an idiosyncratic shock. Program quality h_j is summarized by a weighted index of observable attributes $h_j = w^\top h_{jl}$, including publications in top field journals, Tilburg and IDEAS research scores, and *U.S. News* rankings.

The matching parameters $\boldsymbol{\theta_m} = \{\Pi, \sigma_e, \sigma_\zeta, \sigma_\eta\}$ capture how students and programs are paired in equilibrium. A student's preference over program characteristics is $\beta_i = \Pi B_i + e_i$ with $e_i \sim N(0, \sigma_e)$, while program- and student-specific shocks are $\zeta_j \sim N(0, \sigma_\zeta)$ and $\eta_i \sim N(0, \sigma_\eta)$. Finally, the utility parameters $\boldsymbol{\theta_u} = \{\alpha, \lambda\}$ govern predoc choice. A student of type ψ choosing predoc k obtains utility

$$u(\psi, k; \alpha, \lambda) = \alpha E[U \mid \psi_i, B, k, t] - (\lambda_k^0 + \lambda_k^B B + \lambda_k^t) + \varepsilon_{ik},$$

[24] In signaling models with finitely many signaling tools, equilibria are generally not unique. Multiple equilibria can arise from coordination: when agents collectively believe that one signaling tool conveys a stronger signal of ability, they crowded to that signal, and the market self-fulfills this belief. Our framework follows Fu (2014) and previous model, in which such beliefs are treated as exogenous and can be inferred directly from observed data. This ensures that equilibrium multiplicity does not affect the estimation procedure. However, as we later discuss in the counterfactual analysis, these self-fulfilling beliefs matter for welfare and comparative statics in how grade inflation influences predoc adoption.

6.1.2 Matching and Job Outcome Estimation

In the first stage, we jointly estimate the parameters governing types, training, and matching, $\boldsymbol{\theta} = \{\boldsymbol{\theta_\psi}, \boldsymbol{\theta_{tr}}, \boldsymbol{\theta_m}\}$, using a *Simulated Method of Moments (SMM).*

Given a parameter $\boldsymbol{\theta}$, we first compute each student's expected outcome from Ph.D. training, $T(\tilde{\psi}, h; \theta_{tr})$, and then simulate a matching equilibrium using a serial dictatorship procedure. Specifically, for each simulated market, we draw η_i, ζ_j, and e_i from their corresponding distributions, and use them to construct each student's posterior research ability q_i following equation 5. Conditional on q_i, we compute the student's expected utility from attending program j, $U(\tilde{\psi}, h_j, Z_j; \theta_m)$ following equation 4.

Students are then ranked according to their posterior q_i. For each observed student i, we generate R simulated counterparts with the same observable characteristics (B_i, k_i, β_i), drawing their private type ψ from $\Pr(\psi_h \mid B_i, k_i, t; \theta_\psi)$. Program capacities are proportionally scaled to match the simulated market size. The matching proceeds iteratively: starting from the top of the ranking, each student selects the most preferred Ph.D. program based on $U(\tilde{\psi}, h_j, Z_j; \theta_m)$. Once a student is matched, both the student and the occupied seat in that program are removed from the market. This process continues until all program quotas are filled and the market clears.

The resulting simulated matching outcome and predicted job outcomes are used to construct model-implied moments, which are compared to the empirical moments observed in the data. Parameter estimates minimize the weighted quadratic distance between simulated and empirical moments:

$$\hat{\theta} = \arg\min_{\theta} \big[\hat{m}(\theta) - m^{\text{data}}\big]^\top \hat{W} \big[\hat{m}(\theta) - m^{\text{data}}\big],$$

where $\hat{W}$ is a positive-definite weighting matrix and $\hat{m}(\theta)$ denotes the simulated moments given parameter vector θ. The detailed moments set and optimization method and bootstrap variance can be found in appendix C.

6.1.3 Predoc Choice Estimation

To estimate the choice of predoc, we first compute the expected utility gain for each student choosing different predoc programs, $E\big[U(S_k(\psi_i), h_{\mu(B_i,k)}, Z_{\mu(B_i,k)}) \mid \psi_i, B_i, k\big]$. This expected gain depends on the probability of admission into each Ph.D. program, which is obtained from the first-stage estimation. A key empirical challenge is that not all combinations of student background and predoc type are observed in the data. For example, some background groups appear in only a subset of predoc categories, leaving us without direct evidence on how those students would have performed had they chosen other types of predoc.

To address this missing-cell problem, we specify *off-path beliefs* about students' potential matching outcomes under counterfactual predoc choices. For each background B, we assign the predicted probability of admission to different Ph.D. programs using the average matching outcomes of students with the same background observed in other years. If no such observation exists across all sample years—that is, if no student with background B ever took a particular type of predoc—we impute the belief using the overall average matching probabilities from students with other backgrounds. This procedure preserves the observed cross-sectional heterogeneity while allowing us to evaluate how students with any observable background would have fared had they selected each available predoc option.

Given these computed expectations, we estimate the parameters governing students' predoc choices by maximum likelihood. For a student with private type ψ, the probability of choosing predoc k is specified as $\Pr(k \mid \psi, B, t; \theta_u)$. Since a student's type is unobserved, we integrate over the estimated type distribution obtained from the first-stage estimation. For each student i, the predicted probability of choosing predoc k is therefore given by

$$\Pr(k \mid B_i, t) = \hat{\Pr}(\psi_H \mid B_i, k, t; \hat{\theta}_\psi) \Pr(k \mid \psi_H, B_i, t) + \big(1 - \hat{\Pr}(\psi_H \mid B_i, k, t; \hat{\theta}_\psi)\big) \Pr(k \mid \psi_L, B_i, t),$$

where $\hat{\Pr}(\psi_H \mid B_i, k, t; \hat{\theta}_\psi)$ denotes the estimated probability being a high-type individual implied in the first-stage.

6.2 Identification

In this subsection we provide parametric identification justification for the first stage estimation. For detailed identification discussion see appendix A.4. the first stage is identified jointly from two sets of moments: (i) outcome moments that relate students' realized outcomes to their observed assignments, and (ii) matching moments that describe how students and programs are paired in equilibrium. We are to jointly identify type parameter $\boldsymbol{\theta_\psi}$, training parameter $\boldsymbol{\theta_{tr}}$ and matching parameters $\boldsymbol{\theta_m}$. We define the implied matching function $F_j(\cdot; \boldsymbol{\theta_m})$is as follows:

$$Pr(\psi_H|B,k,j,t) = F_j(Pr(\psi_H|B,k,t); \boldsymbol{\theta_m}) = F_j(\Phi(\gamma_B B + \gamma_k k + \gamma_t); \boldsymbol{\theta_m})$$

The key idea is that the two blocks depend on these primitives through different sources of variation. Outcome data pin down how training and ability affect performance given who matches where, while matching data pin down how ability and training affect preference of matching. By combining both sources, we obtain joint identification of all structural parameters.

Outcome Moments. Conditional on the matching function $F_j(\cdot;\boldsymbol{\theta_m})$ and $\Delta\psi$, the expected outcome can be written as

$$E(y \mid B,t) = \psi_L + \sum_k s_k \Pr(k \mid B,t) + (\tau_1 + \tau_2\psi_L)\sum_j h_j \Pr(j \mid B,t) \\ + \sum_k \tau_2 s_k \sum_j h_j \Pr(j,k \mid B,t) + \Delta\psi \sum_{j,k}(h_j+1)\, F_j\big(\Phi(\gamma_B B + \gamma_k k + \gamma_t);\boldsymbol{\theta_m}\big) \Pr(j,k \mid B,t).$$

Identifying $(\psi_L, \{s_k\}, \tau_1, \tau_2)$. The first four terms are linear in $(\psi_L, \{s_k\}, \tau_1, \tau_2)$. Because the observables

$$\Big\{ \Pr(k \mid B,t),\ \sum_j h_j \Pr(j \mid B,t),\ \big\{\sum_j h_j \Pr(j,k \mid B,t)\big\}_k \Big\}$$

vary independently across (B,k,t), and because the nonlinear component generated by $F_j(\cdot;\boldsymbol{\theta_m})$ is not in their linear span, these coefficients are point-identified. Intuitively, predoc share variation pins down $\{s_k\}$, Ph.D. program quota variation accross years pins down $(\tau_1+\tau_2\psi_L)$, and the interaction of predoc with program quality identifies τ_2, hence τ_1 separately once ψ_L is known.

Identifying γ *given* $\Delta\psi$ *and* F_j. Define the residual:

$$R(B,k,t) \;=\; \Delta\psi \sum_{j,k}(h_j+1)\, F_j\big(\Phi(\gamma_B B + \gamma_k k + \gamma_t);\boldsymbol{\theta_m}\big) \Pr(j,k \mid B,t).$$

Treating $\Delta\psi$ and $\{F_j(\cdot;\boldsymbol{\theta_m})\}_j$ as given, the right-hand side is a known transformation of the single index $x(B,k,t) \equiv \gamma_B B + \gamma_k k + \gamma_t$, passed through $g_{B,t}(x) \equiv \sum_{j,k}(h_j + 1)\, F_j\big(\Phi(x);\boldsymbol{\theta_m}\big) \Pr(j,k \mid B,t)$. Hence, cross-cell variation in (B,k,t) that moves $x(B,k,t)$ identifies γ up to standard single-index normalizations, provided: (i) $g_{B,t}(\cdot)$ is non-constant and (weakly) monotone in x on the support of (B,k,t), (ii) (B,k,t) has sufficient support so that $x(B,k,t)$ varies, and (iii) (B,k,t) enter outcomes only through $x(B,k,t)$ inside $F_j(\Phi(\cdot))$ (exclusion). Impose a standard normalization: $\gamma_{k=0}=0, \gamma_{t=0}=0$ and recover $(\gamma_B, \gamma_k, \gamma_t)$ from the variation in $R(B,k,t)$ across cells.

Matching Moments. Conditional on $(\psi_L, \{s_k\}, \tau_1, \tau_2, \gamma)$—which determine each student's effective training and the posterior probability of being high type

$$p_H(B,k,t) = \Phi(\gamma_B B + \gamma_k k + \gamma_t),$$

the observed assignment probabilities $\sigma(B, k, h, Z, t)$ identify the remaining parameters governing the two-sided preferences $(\beta, \Delta\psi, \sigma_\eta, \sigma_\zeta)$. The intuition parallels the logic of revealed preference on both sides of the market.

Student side. Holding (B, k, t) fixed, variation across programs (h, Z) identifies the student preference parameters β and the relative scale of idiosyncratic shocks σ_η. Because program attributes Z_j enter the utility function linearly,

$$u(\psi, h_j, Z_j) = \tau_1 h_j + \tau_2(\psi + s_k)h_j + \beta^\top Z_j + \eta_j,$$

substitution patterns across programs with different Z_j but similar h_j reveal β up to a normalization of the utility scale σ_η. The scale can be fixed by setting $\sigma_\eta = 1$ without loss of generality.

Program side. Holding program characteristics (h, Z) fixed, variation in (B, k, t) shifts the belief index $p_H(B, k, t)$ and thus the expected ability composition of applicants. These shifts identify how programs value expected ability—captured by $\Delta\psi$—as well as the dispersion σ_ζ of the unobserved evaluation component ζ_j in the latent admission index

$$q(B, k, t) = \psi_L + \Delta\psi\, p_H(B, k, t) + \zeta_j.$$

Given $(\beta, \Delta\psi)$, γ, and observed capacities, and assuming a unique stable assignment (as in the serial dictatorship mechanism), we can invert the observed assignment probabilities $\sigma(B, k, h, Z, t)$ to recover each program's unique acceptance probability $F_j(\cdot; \boldsymbol{\theta_m})$. The shape of F_j is determined by the slope of observed admission rates across $p_H(B, k, t)$, while its level is pinned down by capacity constraints.

Identification of $\Delta\psi$ conditional on γ. Since $p_H(B, k, t)$ is already identified from (B, k, t) through γ, cross-program variation in the acceptance margin with respect to p_H traces out $\Delta\psi$. Intuitively, as the posterior belief about ability increases, programs admit more students from high-p_H groups; the steepness of this response identifies $\Delta\psi$, while the curvature of the acceptance schedule identifies σ_ζ.[25]

Joint Identification. The outcome and matching moments share several primitives—$(\gamma, \Delta\psi, \tau_1, \tau_2, \{s_k\})$—and are tied together through equilibrium matching. The same beliefs $p_H(B, k, t)$ and program evaluations $q(B, k, t)$ must simultaneously rationalize both the observed sorting of students into programs and the resulting distribution of job outcomes. This

[25]Dispersion moments, the cross-program dispersion of assigned h_j, strengthen the identification. Under the two-type mixture, these variances scale with $(\Delta\psi)^2 F_j(p_H; \boldsymbol{\theta_m})[1-F_j(p_H; (\boldsymbol{\theta_m}))]$, introducing curvature in the data that is distinct from the mean relationships and thereby provide additional identification restriction.

cross-equation restriction rules out observational equivalence.

Under standard exclusion conditions, the mapping from the full parameter vector

$$(\psi_L, \{s_k\}, \tau_1, \tau_2, \gamma, \beta, \Delta\psi, \{F_j\})$$

to the joint distribution of outcomes and matches is injective. Intuitively, if one were to perturb the training parameters (τ_1, τ_2) or ability mapping γ, the expected outcome profile $E[y|B, k, t]$ would shift. To offset that change, the equilibrium admission rules $F_j(\cdot)$ would have to adjust to restore fit, but such an adjustment would also distort the observed matching pattern. Therefore, no alternative parameter configuration can satisfy both the outcome and matching moments simultaneously.

7 Estimation Results

The specification we use to estimate the model is that students' unobserved research ability follows a binary distribution, and the probability of being a high-type is modeled as a probit function of student background, predoc type, and cohort year. This specification allows heterogeneity across cohorts in both the composition of the applicant pool and the informational content of predoc participation.

Predoc training is modeled as an additive improvement to the student's underlying research ability, while the training provided by Ph.D. programs depends on both student ability and program quality. Specifically, Ph.D. training exhibits complementarity between a student's ability and the program's research intensity, such that higher-ability students benefit more from attending top programs. This supermodular structure captures the empirical sorting observed in the data, where stronger students tend to match with higher-ranked departments. Predoc costs are allowed to vary flexibly with student background and cohort. For structural parameter estimates, see appendix D.

7.1 Predoc Effects

We next examine how predoc participation affects students' relative ranking in the Ph.D. admissions market. In the model, a student's position in the match depends on the posterior mean of her research ability as perceived by Ph.D. programs. Predoc participation can alter this posterior through two distinct channels.

$$\Delta q = q(B_i, k) - q(B_i, 0) = \underbrace{[\mathbb{E}(\psi|B_i, k) - \mathbb{E}(\psi|B_i, 0)]}_{\text{signaling}} + \underbrace{s_k}_{\text{training}}$$

First, students who choose predocs differ systematically in latent ability from those who apply directly, generating a signaling effect. Second, predoc programs may enhance students' actual research ability, leading to a training effect. The overall change in relative ranking thus combines these two forces.

The estimated results are summarized in Table 6. On average, predoc participation raises a student's relative ranking by 0.93 standard deviations of the posterior mean. Approximately 54 percent of this gain is attributable to signaling and 46 percent to training. Both effects are quantitatively meaningful and together imply that predoc experience significantly reshapes the distribution of students across Ph.D. programs.There is substantial heterogeneity across predoc categories. The total gain in ranking is largest for non-U.S. master's programs, which increase students' posterior mean by about 1.16 standard deviations, and smallest for non-U.S. research positions, which increase it by about 0.56 standard deviations.[26] University-affiliated research predocs and U.S. master's programs deliver the largest signaling effects, reflecting their selectivity and the weight admissions committees place on them as indicators of potential research success. In contrast, non-university research and non-U.S. master's programs yield larger training effects, consistent with their more intensive research exposure and lower screening barriers.[27]

Table 6: Predoc Effects on Relative Ranking

$q(k) - q(0)\ /\ (\sigma_q)$	Total	Signaling	Training
U.S. Non-Univ. Research	0.944	0.309	0.635
U.S. Univ. Research	0.985	0.671	0.314
U.S. Master's	0.772	0.506	0.266
Non-U.S. Master's	1.156	0.607	0.549
Non-U.S. Research	0.560	0.051	0.509

Notes: The table reports the estimated change in students' posterior-mean ranking relative to no predoc.

7.2 Expected Utility and Cost of Predoc Participation

To evaluate students' welfare implications, we next compute the expected gains from undertaking each type of predoc. The expected utility for a student depends on the probability of

[26]The rising share of international students during the sample period may partly inflate the measured gains from non-U.S. master's programs, as these students disproportionately pursue such programs and are more likely to accept positions at elite universities in their home countries.

[27]Because training effects are inferred from average changes in job-market outcomes over time, expansion in industry placements may lead to overestimation of the training component, especially for students who come from non-university research roles and later return to same industry.

admission to different Ph.D. programs conditional on the predoc choice and the associated training and signaling effects.

With these constructed probabilities, we can compute each student's expected utility from taking predoc k as the expected value of admission outcomes net of costs, holding their latent research type fixed. The difference in expected utility between doing and not doing a predoc, $E(U|\psi, k) - E(U|\psi, 0)$, provides a measure of the ex-ante return to predoc participation. Table 7 reports the estimated expected-utility gains for high-type and low-type students, expressed in standard deviations of the expected-utility index.

Table 7: Expected Utility Gains from Predoc Participation

$E(U\|\psi, k) - E(U\|\psi, 0)$ (σ_{E_U})	ψ_H	ψ_L
U.S. Non-Univ. Research	1.565	1.122
U.S. Univ. Research	1.582	1.068
U.S. Master's	1.243	0.784
Non-U.S. Master's	1.592	1.134
Non-U.S. Research	1.024	0.623

Notes: The table reports the estimated increase in students' expected utility from taking a predoc relative to direct admission, expressed in standard deviations of expected utility.

The results indicate that high-type students consistently gain more from predoc participation than low-type students. This monotonic pattern implies that predoc participation amplifies ability differences and reinforces the signaling channel in equilibrium: high-ability students find predocs more rewarding and are therefore more likely to select into them, validating programs' beliefs that predoc participation signals higher underlying quality. Across program categories, the largest expected gains occur for U.S. research roles and non-U.S. master's programs, both of which combine substantive training opportunities with high returns in admission prospects.

7.3 Cost of Predoc Participation

With the expected utilities computed above, we infer the implied cost of taking each type of predoc from students' observed choices. For comparability across programs, we report all cost estimates in standard deviations of expected utility. Table 13 summarizes the results by student background and predoc category.

The results show that graduates from top U.S. universities face the lowest cost of undertaking predocs across all categories, while students from non-U.S. universities generally face higher costs. Among program types, U.S. master's programs have relatively similar costs

Table 8: Average Cost of Predoc Participation by Student Background

Avg. Cost (σ_{E_U})	Top U.S. UG	Top Non-U.S. UG	Other U.S. UG	Other Non-U.S. UG
U.S. Non-Univ. Research	2.363	3.464	3.341	3.656
U.S. Univ. Research	3.523	3.822	4.159	3.906
U.S. Master's	2.271	2.175	2.560	2.229
Non-U.S. Master's	3.861	1.834	4.019	2.151
Non-U.S. Research	–	3.375	–	3.204

Notes: The table reports the average cost of predoc participation, expressed in standard deviations of expected utility.

across student groups, suggesting that tuition levels are broadly uniform and that participation decisions are primarily demand driven. By contrast, non-U.S. master's programs appear to be the most cost-efficient option for international students, offering an accessible signal of ability at relatively low expected cost.

The comparison between U.S. university and non-university research roles highlights the selectiveness of these programs. Non-university research positions tend to screen students more heavily on observable characteristics such as prior coursework or recommendations, while university-based research roles select more on unobservables through faculty referrals and internal hiring. The higher inferred cost for the latter reflects limited capacity and tighter screening on research potential.

Remarks: our model does not separately identify the supply-side elasticity of predoc availability. Because the data capture only the realized equilibrium allocation, we cannot fully disentangle whether variation in participation arises from students' demand or from program supply constraints. As a result, the estimated costs likely overstate the true resource cost of research positions, which are partially supply driven. That's why research roles are typically paid positions, but have higher cost than master's programs which collects tuition, counter the intuition that master's program are of higher monetary cost.

Meanwhile, the problem is less severe with following institutional facts: whereas master's students pay tuition but can also benefit from transferable skills that improve industry employability even if they do not pursue a Ph.D. Research predocs, by contrast, are highly specific to the Ph.D. track and provide limited outside-option value, implying a high opportunity cost of access rather than direct monetary cost.

8 Counterfactual Experiments

This section uses the estimated model to evaluate how information frictions and grading environments shape equilibrium outcomes in the Ph.D. admission market. In the first set of experiments, we simulate four environments that differ in how admission committees interpret and use predoc information. The first is the observed *status quo* equilibrium. The second is a *predoc-free* or *pooling* equilibrium, in which admission offices commit to evaluating applicants solely on undergraduate background and ignore predoc experience altogether.[28] The third is a *full-information* equilibrium, which eliminates information asymmetry by allowing programs to observe students' true research ability directly. Finally, we consider a *training-only* equilibrium in which predoc participation enhances research ability but carries no informational content about unobserved ability. In the second set of experiments, we focus on the role of grade inflation at the undergraduate level. We construct a counterfactual environment in which college grades remain as informative about ability as they were prior to the grade inflation.

8.1 Does Predoc Benefit the Market?

The counterfactual exercises are not intended as policy forecasts but as quantitative results of how information frictions shape the Ph.D. admission market. Rather than predicting what would happen following a specific reform, we use these simulations to assess how much the market suffers from asymmetric information and whether the costs induced by status externality offset the information gains from signaling. This approach provides a quantitative resolution to the long-standing "rat race" debate surrounding predoctoral research experience. Moreover, distinguishing the training channel from the signaling channel helps clarify how each mechanism operates in the matching tournament game: both can generate excessive competition, but their welfare implications differ. Signaling primarily improves matching efficiency by helping programs identify high-ability students, while training increases the total output of human capital in the market.[29]

Figure 6 plots the equilibrium share of students undertaking predocs under four informational regimes over time. Even in the predoc-free (pooling) and full-information equilibria, a nontrivial fraction of students still choose to pursue predoc experience. This persistence re-

[28] This counterfactual analysis connects to recent work examining how changes in admission rules alter students' strategic behavior and welfare. Borghesan (2023) develop a structural equilibrium model showing that removing standardized tests reshapes both application choices and college completion by endogenizing students' effort and application behavior. Related empirical and theoretical studies—including Cotton et al. (2022), Leeds et al. (2017), Akhtari et al. (2024), Grau (2018), Bond et al. (2018), and Goodman et al. (2020)—document similar behavioral adjustments when admissions criteria change.

[29] Both the equilibrium predoc share and welfare are computed conditional on students who eventually enroll in a Ph.D. program. This implies that the model effectively conditions on successful transitions, excluding predoc participants who do not proceed to doctoral study. As a result, the estimated aggregate cost and observed predoc participation share are lower bounds relative to the true population-level values. Incorporating data on all predoc participants, including those who exit before Ph.D. enrollment, would correct for this sample-selection truncation and yield higher implied costs and overall predoc prevalence.

flects the intrinsic training value of predocs: even when admissions committees do not reward predoc participation, students may find the research training beneficial for their long-term career outcomes. The slightly higher adoption in the full-information equilibrium relative to the pooling case arises because, under full information, high-type students are matched with better Ph.D. programs. Given the supermodularity between ability and training, these students derive larger returns from predoc participation, leading to higher equilibrium take-up.

We next compare the training-only equilibrium to the pooling equilibrium. Predoc adoption is significantly higher when admissions recognize only the training effect. This pattern indicates a pure "rat race" channel: even though training has no signaling value in this equilibrium, students compete for better placements by accumulating additional skills. However, because the admission rule does not incorporate information revelation, this extra investment generates limited matching efficiency gains.

Conceptually, the training-only equilibrium resembles the effort environment of college entrance examinations in other countries. There, students can invest in tutoring to raise their human capital and test scores, but conditional on those scores, colleges are not permitted to make admissions decisions based on whether applicants engaged in tutoring. In both cases, investment is privately beneficial yet generates limited additional information for the selection process, creating competitive pressure without commensurate gains in efficiency.

Finally, comparing the status quo (signaling) equilibrium with the training-only equilibrium shows that allowing predocs to convey information sharply increases predoc participation. At first glance, this result appears counterintuitive given that our structural decomposition attributes roughly equal contributions of signaling and training to overall ranking improvements. The explanation lies in two reinforcing forces. First, signaling improves matching efficiency and allocates more high-type students to top programs, raising their incentive to undertake predocs relative to low-type students. Second, the rat-race effect is nonlinear: once a signal meaningfully shifts relative rankings, the incentive to adopt it expands rapidly, inducing a disproportionate rise in predoc participation.

Welfare Comparison. Figure 7 reports the welfare comparison across equilibria. We normalize welfare in the pooling equilibrium to zero and use the welfare gap between the full-information and pooling equilibria as the unit measure. This normalization allows welfare in other equilibria to be interpreted as a percentage of the information gain achieved under full information.

Although Ph.D. programs allocate a fixed number of seats, the market is not a pure zero-sum game. Because of the supermodularity between student ability and program quality in the training function, positive assortative matching is socially desirable: matching high-ability students to better programs increases total output. Full information therefore

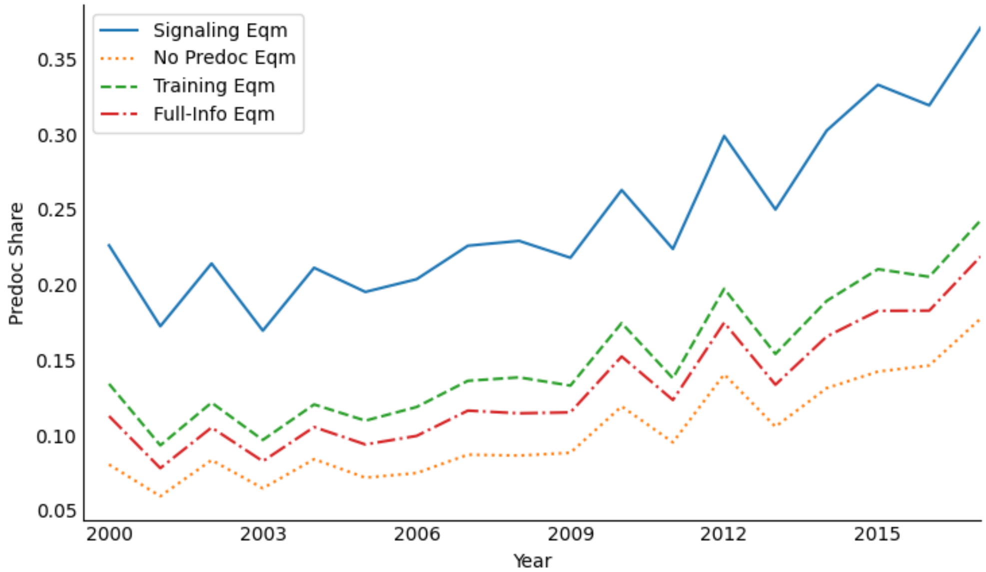

Figure 6: Predoc Adoption under Alternative Informational Equilibria

Notes: The figure plots the equilibrium share of students taking predocs across four informational regimes—status quo (signaling), training-only, full-information, and pooling (no predoc). Shares are computed by year using the estimated model and reflect equilibrium behavior conditional on observed program capacity and costs.

represents the efficient benchmark that maximizes social welfare.

In the benchmark (signaling) equilibrium, we find that signaling restores 48.3 percent of the matching efficiency lost under asymmetric information, meaning that predocs substantially alleviate the information problem. This efficiency gain explains why the profession broadly supports the rise of predoc programs. However, on the cost side, the welfare loss from status externality exceeds the information benefit: the total cost amounts to 63.2 percent of the matching gain, leading to a net welfare loss of 14.9 percent relative to the full-information benchmark.

When we compare the training-only equilibrium, we find that it restores only 17.5 percent of matching efficiency while generating costs equivalent to 28.7 percent of the matching efficiency. Because admissions in this case do not reward signaling, fewer high-type students self-select into predocs, limiting the program's ability to distinguish students. The smaller information gain and lower selectivity together yield an even larger welfare shortfall relative to the signaling equilibrium.

Heterogeneous Welfare Effects. We next examine how the welfare effects of predoc participation vary across student backgrounds. Figure 8 decomposes welfare differences between the signaling (status quo) and pooling equilibria by student group. For each group, the left panel shows the gain in matching efficiency, and the right panel shows the cost

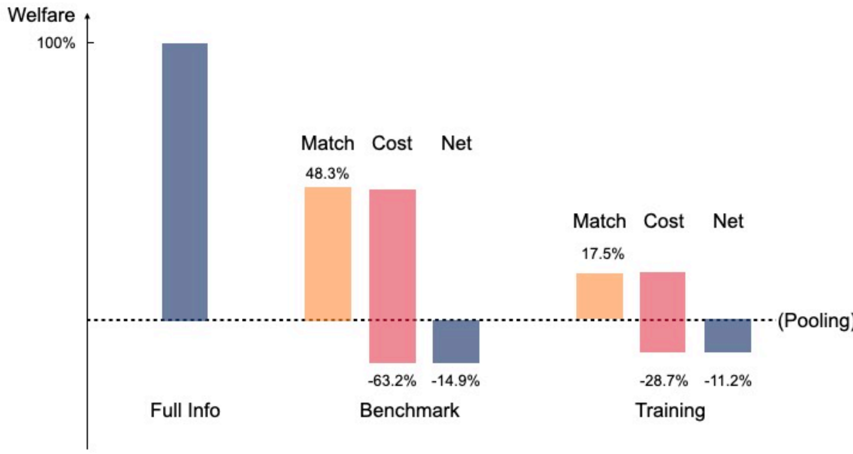

Figure 7: Welfare Comparison Across Informational Equilibria

Notes: Welfare is normalized relative to the pooling equilibrium. Bars decompose welfare into matching efficiency and status-externality costs, expressed as shares of the full-information welfare gap. "Benchmark" refers to the estimated signaling equilibrium.

incurred through predoc participation. If the matching gain (left bar) exceeds the cost (right bar), the group benefits on net; otherwise, the group is worse off.

From the figure, students from top undergraduate institutions and those majoring in economics obtain the largest matching efficiency gains. These students are also the most likely to undertake predocs and bear the highest direct costs. Nevertheless, as estimated before, they face the lowest average cost of participation, resulting in positive net welfare from predoc adoption. In contrast, students from non-top or non-economics backgrounds also experience some improvement in matching efficiency, but their costs exceed the benefits, yielding negative net welfare effects. In other words, predoc programs disproportionately benefit already advantaged students, while imposing net losses on others. We find no systematic qualitative difference between male and female students in this decomposition, suggesting that gender heterogeneity does not play a first-order role in welfare outcomes once background characteristics are controlled for.[30]

[30]It is worth noting that all student groups experience some increase in matching efficiency relative to the pooling equilibrium, even though the number of Ph.D. program slots is fixed. This occurs because we group students by observable characteristics, within which there remains unobserved heterogeneity in ability. Under the signaling equilibrium, positive assortative matching allows high-type students within each group to be more accurately allocated to higher-quality programs. The supermodularity of the training function implies that this improved sorting raises total output, making it possible for all groups to register positive efficiency gains despite the fixed number of available seats.

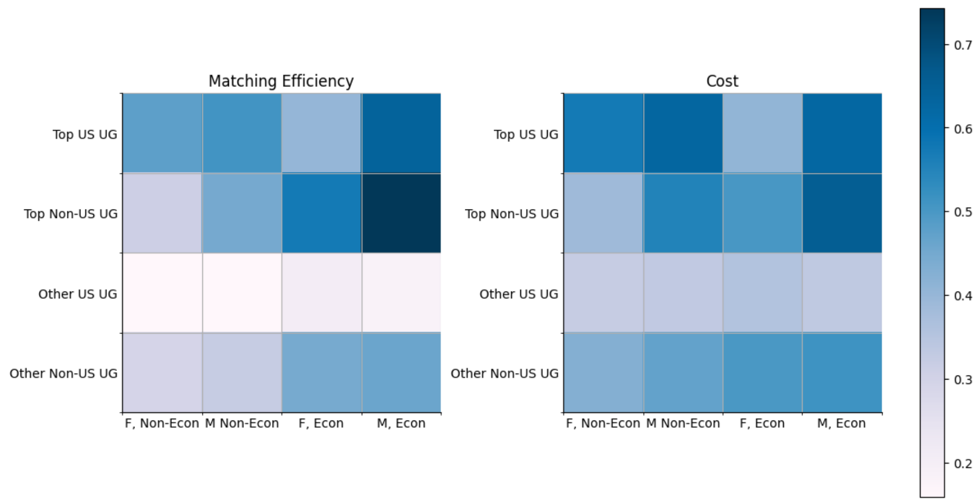

Figure 8: Heterogeneous Welfare Effects of Predoc Participation by Student Background

Notes: The figure compares welfare components between the signaling equilibrium and pooling equilibrium across observable student groups. Left bars show welfare gains from improved matching efficiency; right bars show welfare costs associated with predoc participation. Groups for which the left bar is darker (larger) experience positive net welfare.

8.2 Grade Inflation Counterfactuals

In Section 5 we showed that grade inflation reduces the informativeness of student profiles, weakening the signal value of undergraduate performance and lowering admission committees' posterior beliefs about applicants. Students respond by pursuing predocs as alternative signals of ability. To quantify this mechanism, we simulate a counterfactual market in which undergraduate grades remain as informative as before the onset of grade inflation. Specifically, we hold the average GPA distribution fixed while replacing each student's inflated undergraduate reputation with the counterfactual value implied by a no-inflation mapping, allowing predoc costs to evolve over time as in the data. This experiment isolates the effect of grade inflation from other factors that contribute to changing predoc participation, such as shifts in costs or program capacity.

Figure 9 reports the equilibrium share of students undertaking predocs under the baseline (status quo) equilibrium and the no-grade-inflation counterfactual. The results show that grade inflation meaningfully increases students' incentive to take predocs. In the model, the posterior mean of research ability is a function of the informativeness of undergraduate signals. When grades become inflated, the posterior mean distribution compresses, making it harder for programs to distinguish among applicants. This reduced informativeness lowers students' perceived relative ranking and induces more of them to seek costly predoc experience as an alternative signal. Under the counterfactual with no grade inflation, the posterior distribution is more dispersed—programs can infer ability more accurately from undergraduate records—and predoc participation declines accordingly. Quantitatively, grade inflation accounts for approximately 24.5% of the total predoc participation observed in the market.

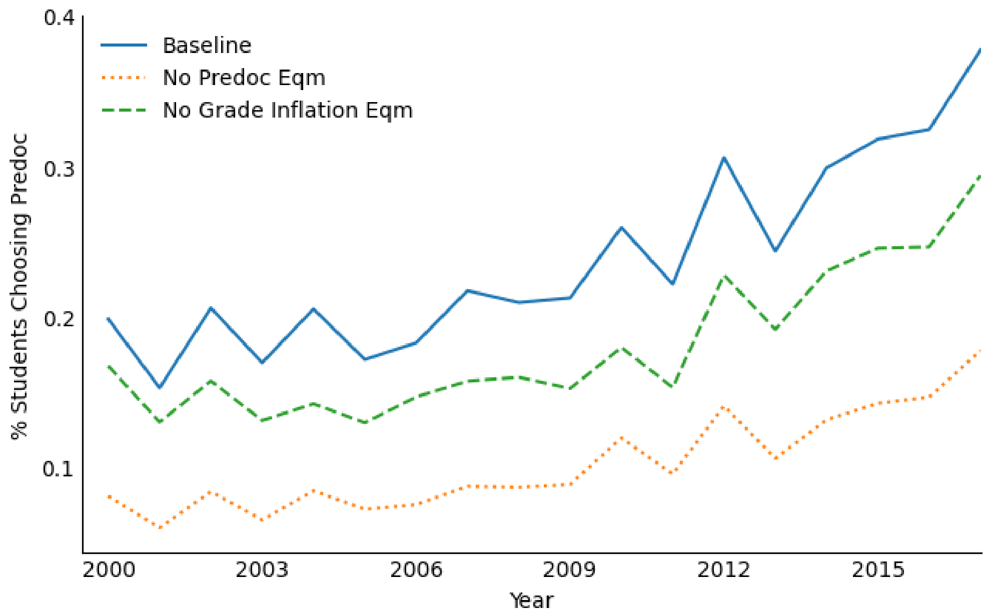

Figure 9: Predoc Adoption with and without Grade Inflation

Notes: The figure plots the equilibrium share of students choosing predocs under the baseline signaling equilibrium, a no-predoc (pooling) benchmark, and a counterfactual scenario without grade inflation. Shares are calculated by year using estimated model parameters and equilibrium matching conditions.

Remarks: Because the admission market is modeled as a signaling game, multiple equilibria exist. Students' beliefs about which predoc serves as an effective signal can become self-fulfilling, leading to coordination on different signaling conventions. Our counterfactual focuses on the equilibrium that is locally similar to the observed one—posterior belief differing only in the informativeness of grades—so that changes in predoc adoption can be attributed specifically to grade inflation. Other equilibria could in principle emerge under different coordination beliefs, potentially generating distinct welfare outcomes.

9 Conclusion

This paper studies how signaling and competition interact in a matching tournament and asks whether the information gain from signaling tools can offset the rat-race cost they generate. I develop a signaling and matching model in which agents compete for limited desirable slots and may overuse costly signals as competition intensifies. In equilibrium, signals that initially improve information can become diluted, and participants may ultimately be worse off despite better screening. I apply this framework to the Ph.D. admission market, where pre-Ph.D. experience has emerged as both a signal of ability and a source of training. The model provides a unified way to evaluate how signaling improves sorting efficiency while simultaneously generating excessive competition and welfare losses.

I empirically implement the model using a new dataset constructed from LinkedIn profiles of Economics and Business Ph.D. students. A key challenge is to separate Ph.D. program training effects, pre-Ph.D. experience training effects, and pre-Ph.D. experience signaling effects. To identify the Ph.D. program training effect, I exploit variation in program quotas across years. To disentangle signaling from training, I leverage differences between short-term admission outcomes and long-term job-market outcomes: signaling reallocates students across programs without changing unconditional outcomes, whereas training raises unconditional outcomes. The estimates show that pre-Ph.D. experience raises students' relative ranking with roughly 54% attributable to signaling and 46% to training. Signaling restores part of the matching efficiency lost under asymmetric information but imposes costs that exceed its benefits, resulting in a net welfare loss equivalent to 15% of matching efficiency. Counterfactual simulations further show that grade inflation erodes the informativeness of undergraduate records, leading to about 24% more pre-Ph.D. experience participation in equilibrium.

The analysis offers several implications. First, pre-Ph.D. experience acts as a partial solution to asymmetric information in admissions, but their private signaling value may outweigh their social benefit once the associated costs of competition are accounted for. Second, colleges inflate grades to help their students succeed in the job market, yet push students into a further rat race by weakening existing signals and encouraging costly new ones. The intuition extends beyond graduate education: the same forces underlie the expansion of Advanced Placement classes and extracurricular activities in college admissions, unpaid internships in labor markets, and long-hour tournaments in the law and finance industries. More broadly, this framework provides a foundation for studying the efficiency and welfare consequences of competitive signaling in other high-skill markets where information asymmetry and status externality jointly shape human-capital investment.

While the analysis provides new quantitative evidence on the interaction between signaling and competition, several limitations point to natural extensions. One important omission is the supply side of the pre-Ph.D. experience market. The current framework focuses on students' decisions but abstracts from how universities, research centers, and faculty labs determine how many and what types of pre-Ph.D. experience positions to offer. Exploring how the supply of pre-Ph.D. experience responds to market competition would enrich our understanding of the equilibrium dynamics of signaling and could reveal whether institutions themselves amplify or mitigate the rat race.

Another promising direction concerns students' information about their own research ability. In the present framework, students are assumed to know their type before choosing whether to undertake a pre-Ph.D. experience. In reality, pre-Ph.D. experience experience may also serve as a period of self-assessment, during which students learn about their research fit and adjust their career plans accordingly. This learning channel is partially captured by the estimated training effect, as students who discover a strong fit are more likely to proceed to Ph.D. programs, increasing the observed average quality among pre-Ph.D. experience participants. Future work using richer data that follow students who enter pre-Ph.D. experience but do not continue to Ph.D. programs could more precisely separate learning about fit from human-capital accumulation.

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A Proof:

A.1 Proof for Proposition 1

Proof. We prove the existence of equilibrium in the following steps: we first show in equilibrium follows a cutoff rule, and the equilibrium can be summarized into a sequence of cutoffs. Then we need to prove the existence of cutoffs. We do this by applying Tarski fixed point theorem.

Lemma 1 *(*Monotonicity of Predoc Choice*) Consider any equilibrium. Fix B. Let two students have types $\psi' > \psi$. If the student with type ψ chooses to do the predoc, then the student with type ψ' must also choose to do the predoc.*

This lemma implies that in equilibrium, the predoc choice follows a cutoff rule: for each background B, there exists a threshold $\psi^*(B)$ such that a student with type ψ chooses the predoc if and only if $\psi \geq \psi^*(B)$. For the marginal type $\psi^*(B)$, the student must be indifferent between doing the predoc and not doing it. The indifference condition is:

$$\Pr\big(q > \delta_1 \mid B, k=1\big)\, T\big(\psi^*(B), h_1\big) - c(B, k=1) \quad = \quad \Pr\big(q > \delta_1 \mid B, k=0\big)\, T\big(\psi^*(B), h_1\big).$$

Equivalently, we can write:

$$\Big[1 - F_\eta\big(G^{-1}(\kappa_2) - E\big(S(\psi^*(B)) \mid B, 1\big)\big)\Big] - \Big[1 - F_\eta\big(G^{-1}(\kappa_2) - E\big(S(\psi^*(B)) \mid B, 0\big)\big)\Big] = \frac{c(B,1)}{T\big(\psi^*(B), h_1\big)}.$$

Define:

$$D\big(\psi^*(B)\big) = \Big[1 - F_\eta\big(G^{-1}(\kappa_2) - E\big(\psi^*(B) \mid B, k=1\big)\big)\Big] - \Big[1 - F_\eta\big(G^{-1}(\kappa_2) - E\big(S(\psi^*(B)) \mid B, k=0\big)\big)\Big].$$

Then the indifference condition can be equivalently expressed as:

$$\psi^*(B) \quad = \quad T^{-1}\left(\frac{c(B,1)}{D\big(\psi^*(B)\big)}\right).$$

Therefore, existence of an equilibrium is equivalent to existence of a fixed point of this equation. To apply Tarski's fixed point theorem, we must show that the right-hand side is monotone decreasing in $\psi^*(B)$. This is established by the following two lemmas.

Lemma 2 *(*Monotonicity of G*) If $\tilde{\psi}^*(B) > \psi^*(B)$, then the posterior mean distribution $G\big(\cdot \mid \tilde{\psi}^*(B)\big)$ first-order stochastically dominates $G\big(\cdot \mid \psi^*(B)\big)$.*

Lemma 3 *If $F(\psi \mid B)$ has an increasing hazard rate, then*

$$H(c) = \mathbb{E}\big(\psi \mid \psi > c\big) \; - \; \mathbb{E}\big(\psi \mid \psi < c\big)$$

is strictly increasing in c.

Combining Lemmas 2 and 3, we see that as $\psi^*(B)$ increases, $G^{-1}(\kappa_2)$ decrease, $\mathbb{E}\big(\psi \mid \psi > \psi^*(B)\big) - \mathbb{E}\big(\psi \mid \psi < \psi^*(B)\big)$ increase, thus the denominator $D\big(\psi^*(B)\big)$ increases monotonically. Because $T(\cdot)$ is strictly increasing, the inverse $T^{-1}(\cdot)$ is strictly increasing, and therefore the entire right-hand side is strictly decreasing in $\psi^*(B)$.

Since the mapping is monotone and maps a compact interval into itself, by Tarski's fixed point theorem there exists at least one solution $\psi^*(B)$. This completes the existence argument. ■

A.1.1 Proof for Lemma 1

Proof. The utility of doing the predoc for a student with type ψ is

$$U^{\text{predoc}}(\psi) = \Pr\big(q > \delta_1 \mid B, k = 1\big)\, T(\psi, h_1) - c(B, 1),$$

and the utility of not doing the predoc is

$$U^{\text{no predoc}}(\psi) = \Pr\big(q > \delta_1 \mid B, k = 0\big)\, T(\psi, h_1).$$

If the student with type ψ chooses to do the predoc, then

$$U^{\text{predoc}}(\psi) \geq U^{\text{no predoc}}(\psi),$$

which can be rearranged as

$$\Big[\Pr\big(q > \delta_1 \mid B, k = 1\big) - \Pr\big(q > \delta_1 \mid B, k = 0\big)\Big]\, T(\psi, h_1) \geq c(B, 1).$$

Since $T(\psi, h_1)$ is strictly increasing in ψ, it follows that

$$T(\psi', h_1) > T(\psi, h_1).$$

Therefore,

$$\Big[\Pr\big(q > \delta_1 \mid B, k=1\big) - \Pr\big(q > \delta_1 \mid B, k=0\big)\Big]\, T(\psi', h_1)$$
$$> \Big[\Pr\big(q > \delta_1 \mid B, k=1\big) - \Pr\big(q > \delta_1 \mid B, k=0\big)\Big]\, T(\psi, h_1) \geq c(B,1).$$

This implies

$$U^{\text{predoc}}(\psi') - U^{\text{no predoc}}(\psi') = \Big[\Pr\big(q > \delta_1 \mid B, k=1\big) - \Pr\big(q > \delta_1 \mid B, k=0\big)\Big]\, T(\psi', h_1) - c(B,1) > 0,$$

so the student with type ψ' strictly prefers doing the predoc. ■

A.1.2 Proof for Lemma 2

Proof. Recall that:

$$G(q) = \iint \sum_k \Pr(k \mid B)\, \mathbb{1}\big\{\mathbb{E}\big[S_k(\psi) \mid B, k\big] \;+\; \eta \leq q\big\}\, dF_B(B)\, dF_\eta(\eta).$$

Under the cutoff strategy, for each B, there is a threshold $\psi^*(B)$ such that students with $\psi < \psi^*(B)$ choose not to do the predoc, and those with $\psi \geq \psi^*(B)$ choose to do the predoc. Therefore, we can write:

$$G(q) = \iint \Pr\big(\psi < \psi^*(B) \mid B\big)\, \mathbb{1}\big\{\mathbb{E}\big(\psi \mid \psi < \psi^*(B), B\big) + \eta \leq q\big\}$$
$$+\; \Pr\big(\psi \geq \psi^*(B) \mid B\big)\, \mathbb{1}\big\{\mathbb{E}\big[S(\psi) \mid \psi \geq \psi^*(B), B\big] + \eta \leq q\big\}\, dF_B(B)\, dF_\eta(\eta).$$

We change the integration order B and η and get:

$$G(q) = \int \Big[F_\eta\big[q - \mathbb{E}\big(\psi \mid \psi < \psi^*(B), B\big)\big]\; \Pr\big(\psi < \psi^*(B) \mid B\big)$$
$$+\; F_\eta\big[q - \mathbb{E}\big(S(\psi) \mid \psi \geq \psi^*(B), B\big)\big]\; \Pr\big(\psi \geq \psi^*(B) \mid B\big)\Big]\, dF_B(B).$$

Given $\eta \sim U[-\bar{\eta}, \bar{\eta}]$,

$$\begin{aligned}
G &= \int \frac{1}{2\bar{\eta}}\, [q - E\left(\psi \mid \psi < \psi^*(B), B\right) F\left(\psi^*(B) \mid B\right) \\
&\qquad - E(S(\psi) \mid \psi > \psi^*(B), B)\left(1 - F\left(\psi^*(B)\right)\right)] + 1 dF_B(B) \\
&= \int \frac{1}{2\bar{\eta}}\, [q - E(\psi \mid B) - E(S(\psi) - \psi \mid \psi > \psi^*(B), B)\left(1 - F\left(\psi^*(B)\right)\right)] + 1 dF_B(B) \\
&= \int \frac{1}{2\bar{\eta}} \left[q - E(\psi \mid B) - \int_{\psi^*(B)}^{\infty} S(\psi) - \psi dF(\psi \mid B)\right] + 1 dF_B(B)
\end{aligned}$$

Observe that when the cutoff $\psi^*(B)$ increases to $\widetilde{\psi}^*(B)$, then:

$$\int_{\widetilde{\psi}^*(B)}^{\infty} \big[S(\psi) - \psi\big]\, dF(\psi \mid B) < \int_{\psi^*(B)}^{\infty} \big[S(\psi) - \psi\big]\, dF(\psi \mid B),$$

This means that for all q, the CDF $G(q \mid \widetilde{\psi}^*(B))$ lies strictly below $G(q \mid \psi^*(B))$, i.e.,

$$G\big(q \mid \widetilde{\psi}^*(B)\big) < G\big(q \mid \psi^*(B)\big),$$

which is the definition of first-order stochastic dominance. ■

A.1.3 Proof for Lemma 3

Proof. Notice that,

$$H(c) = \frac{1}{1 - F(c)} \int_{c}^{\infty} \psi\, f(\psi)\, d\psi - \frac{1}{F(c)} \int_{-\infty}^{c} \psi\, f(\psi)\, d\psi.$$

We take the derivative with respect to c:

$$\frac{dH}{dc} = \frac{d}{dc}\Big[\frac{1}{1 - F(c)} \int_{c}^{\infty} \psi f(\psi)\, d\psi\Big] \; - \; \frac{d}{dc}\Big[\frac{1}{F(c)} \int_{-\infty}^{c} \psi f(\psi)\, d\psi\Big].$$

Rearranging and we get

$$\begin{aligned}
\frac{dH}{dc} &= \frac{f(c)}{1 - F(c)}\Big[\mathbb{E}(\psi \mid \psi > c) - c\Big] - \frac{f(c)}{F(c)}\Big[c - \mathbb{E}(\psi \mid \psi < c)\Big]. \\
&= \frac{f(c)}{[1 - F(c)]F(c)}[F(c)E(\psi \mid \psi > c) + (1 - F(c))E(\psi \mid \psi < c) - c] \\
&= \frac{f(c)}{[1 - F(c)]F^2(c)}[(2F(c) - 1)E(\psi \mid \psi > c) + (1 - F(c))E(\psi) - F(c)c] \\
&= \frac{f(c)}{[1 - F(c)]F^2(c)}\,[(1 - F(c))[E(\psi) - E(\psi - c \mid \psi > c)] + F(c)(E(\psi \mid \psi > c) - c)]
\end{aligned}$$

From increasing hazard rate, $E(\psi) - E(\psi - c \mid \psi > c) > 0$ and $E(\psi \mid \psi > c) - c > 0$. Thus we have $\frac{dH}{dc} > 0$. That is, $H(c)$ is strictly increasing in c. ■

A.2 Proof for Proposition 3

Proof.

A.2.1 Program Quality and Predoc Adoption

From the indifference condition,

$$\begin{aligned} L = &\Big[E\big(S(\psi) \mid \psi > \psi^*(B), B\big) - G^{-1}\big(1-\kappa_1\big)\Big] \cdot T\big(S(\psi^*(B)), h_1\big) \\ -&\Big[E\big(\psi \mid \psi < \psi^*(B), B\big) - G^{-1}\big(1-\kappa_1\big)\Big] \cdot T\big(\psi^*(B), h_1\big) - c = 0. \end{aligned}$$

We compute the comparative statics in turn.

Derivative with respect to ψ^*.

$$\begin{aligned} \frac{\partial L}{\partial \psi^*} = &\frac{\partial E\big(S(\psi) \mid \psi > \psi^*, B\big)}{\partial \psi^*} \cdot T\big(S(\psi^*), h_1\big) \\ &+ \Pr\big(q > 1-\kappa_1 \mid \psi > \psi^*\big) \cdot \frac{\partial T\big(S(\psi^*), h_1\big)}{\partial S(\psi^*)} \cdot \frac{\partial S}{\partial \psi^*} \\ &- \frac{\partial E\big(\psi \mid \psi < \psi^*, B\big)}{\partial \psi^*} \cdot T\big(\psi^*, h_1\big) \\ &+ \Pr\big(q > 1-\kappa_1 \mid \psi < \psi^*\big) \cdot \frac{\partial T\big(\psi^*, h_1\big)}{\partial \psi^*}. \end{aligned}$$

From the supermodularity of T,

$$\frac{\partial T\big(S(\psi^*), h_1\big)}{\partial S(\psi^*)} > \frac{\partial T\big(\psi^*, h_1\big)}{\partial \psi^*}.$$

From Lemma 3,

$$\frac{\partial E\big(S(\psi) \mid \psi > \psi^*\big)}{\partial \psi^*} > \frac{\partial E\big(\psi \mid \psi < \psi^*\big)}{\partial \psi^*}.$$

Moreover,

$$T\big(S(\psi^*), h_1\big) > T\big(\psi^*, h_1\big), \quad \Pr\big(q > 1-\kappa_1 \mid \psi > \psi^*\big) > \Pr\big(q > 1-\kappa_1 \mid \psi < \psi^*\big), \quad \frac{\partial S}{\partial \psi^*} > 0.$$

Therefore,

$$\frac{\partial L}{\partial \psi^*} > 0.$$

Derivative with respect to h_1.

$$\frac{\partial L}{\partial h_1} = \Pr\bigl(q > 1-\kappa_1 \mid B, k=1\bigr) \cdot \frac{\partial T\bigl(S(\psi^*), h_1\bigr)}{\partial h_1} - \Pr\bigl(q > 1-\kappa_1 \mid B, k=0\bigr) \cdot \frac{\partial T\bigl(\psi^*, h_1\bigr)}{\partial h_1}.$$

Since T is supermodular and $S(\psi^*) > \psi^*$,

$$\frac{\partial T\bigl(S(\psi^*), h_1\bigr)}{\partial h_1} > \frac{\partial T\bigl(\psi^*, h_1\bigr)}{\partial h_1},$$

implying

$$\frac{\partial L}{\partial h_1} > 0.$$

Therefore,

$$\frac{\partial \psi^*}{\partial h_1} = -\frac{\frac{\partial L}{\partial h_1}}{\frac{\partial L}{\partial \psi^*}} < 0.$$

So as h_1 increases, ψ^* decreases, meaning more students choose to do a predoc.

A.2.2 Capacity and Predoc Adoption

Let the capacity of the top Ph.D. program be $\kappa \in (0,1)$. In equilibrium, predoc take-up is increasing in private research ability ψ; thus predoc choice follows a cutoff rule: there exists $\psi^*(\kappa)$ such that a student takes a predoc if and only if $\psi \geq \psi^*(\kappa)$. Hence the aggregate predoc participation rate is

$$\Psi(\kappa) = \Pr(\psi \geq \psi^*(\kappa) \mid B) = 1 - F(\psi^*(\kappa) \mid B),$$

so that

$$\Psi'(\kappa) = -f(\psi^*)\, \psi^{*\prime}(\kappa),$$

where $f(\cdot)$ is the type density, strictly positive on the interior.

The cutoff $\psi^*(\kappa)$ is determined by the indifference condition of the marginal student, which can be rewritten as

$$1 - \kappa = G\bigl(\psi^*,\ H(\psi^*)\bigr), \tag{8}$$

where

$$H(\psi^*) = \frac{\mathbb{E}[S(\psi) \mid \psi > \psi^*, B]\ T\bigl(S(\psi^*), h_1\bigr) - \mathbb{E}[\psi \mid \psi < \psi^*, B]\ T\bigl(\psi^*, h_1\bigr)}{T\bigl(S(\psi^*), h_1\bigr) - T\bigl(\psi^*, h_1\bigr)}. \tag{9}$$

Here G denotes the cumulative distribution function of the posterior index used in admissions:

$$G(q) = \iint \sum_k \Pr(k \mid B)\, \mathbf{1}\big\{\mathbb{E}\big[S_k(\psi) \mid B, k\big] + \eta \le q\big\}\ dF_B(B)\, dF_\eta(\eta),$$

which depends both on the cutoff ψ^* and on the composite statistic $H(\psi^*)$.

Step 1. Differentiating the indifference condition. Taking total derivatives of equation (8) with respect to ψ^* yields

$$-\frac{d\kappa}{d\psi^*} = G_1\big(\psi^*, H(\psi^*)\big) + G_2\big(\psi^*, H(\psi^*)\big)\, H'(\psi^*), \tag{10}$$

where $G_1 = \partial G/\partial \psi^*$ and $G_2 = \partial G/\partial H$. Under the assumptions of the model:

$$G_1(\psi^*, H) < 0, \qquad G_2(\psi^*, H) > 0, \qquad H'(\psi^*) > 0.$$

The first inequality reflects that tightening the cutoff ψ^* reduces the admission mass; the second follows because a higher H shifts the distribution of signals to the right; and the third follows from monotone $S(\psi)$ and the supermodularity of $T(\cdot, \cdot)$.

Step 2. Limiting regimes. We examine (10) at two extremes of κ.

(i) Extremely small κ (tight capacity). When $\kappa \to 0$, only the very top types are admitted. In this limit, $H(\psi^*) \to 0$ and the first term in (10) becomes negligible relative to the second:

$$-\frac{d\kappa}{d\psi^*} \approx G_2\, H'(\psi^*) > 0.$$

Thus $\frac{d\kappa}{d\psi^*} < 0$, implying $\psi^{*\prime}(\kappa) < 0$ and hence

$$\Psi'(\kappa) = -f(\psi^*)\, \psi^{*\prime}(\kappa) > 0.$$

That is, when the top-program capacity is extremely small, further reducing κ decreases the incentive to take a predoc.

(ii) Large κ (loose capacity). When $\kappa \to 1$, the admission threshold ψ^* falls, and the difference between predoc and non-predoc groups becomes negligible. In this regime $H'(\psi^*) \to 0$, so the second term in (10) vanishes and

$$-\frac{d\kappa}{d\psi^*} \to G_1(\psi^*, H(\psi^*)) < 0.$$

Hence $\frac{d\kappa}{d\psi^*} > 0$ and $\psi^{*\prime}(\kappa) > 0$, which gives

$$\Psi'(\kappa) = -f(\psi^*)\,\psi^{*\prime}(\kappa) < 0.$$

That is, when the top program is large, reducing κ increases predoc participation.

Step 3. Existence of a threshold κ^*. Define $\Phi(\kappa) = \Psi'(\kappa) = -f(\psi^*(\kappa))\,\psi^{*\prime}(\kappa)$. Since $f(\psi^*) > 0$, the sign of $\Phi(\kappa)$ coincides with that of $-\psi^{*\prime}(\kappa)$. From the two limits above, $\Phi(\kappa) > 0$ for $\kappa \to 0$ and $\Phi(\kappa) < 0$ for $\kappa \to 1$. Because equation (8) defines $\psi^*(\kappa)$ implicitly and $G_1 + G_2 H' \neq 0$ by assumption, $\psi^*(\kappa)$ (and hence $\Phi(\kappa)$) is continuous in κ.

By the Intermediate Value Theorem, there exists $\kappa^* \in (0, 1)$ such that $\Phi(\kappa^*) = 0$, i.e.,

$$\exists\, \kappa^* \in (0,1) \quad \text{s.t.} \quad \begin{cases} \Psi'(\kappa) < 0, & \kappa > \kappa^*, \\ \Psi'(\kappa) > 0, & \kappa < \kappa^*. \end{cases}$$

■

A.3 Proof for Proposition 2

Proof. Suppose that $\tilde{F}(\psi \mid B)$ second-order stochastically dominates $F(\psi \mid B)$. Recall G function is:

$$G(q) = \int \frac{1}{2\bar{\eta}} \Big[q - E(\psi \mid B) - \int_{\psi^*(B)}^{\infty} \big(S(\psi) - \psi\big)\, dF(\psi \mid B) \Big] + 1\, dF_B(B).$$

This can be expressed as

$$G(q) = q - E(\psi) - \int_{\psi^*(B)}^{\infty} \big(S(\psi) - \psi\big)\, dF(\psi \mid B).$$

Keeping $\psi^*(B)$ fixed, note that replacing F with the more informative distribution $\tilde{F}$ increases the dispersion of ψ. Because F SOSD $\tilde{F}$, the upper tail mass decreases under $\tilde{F}$. Consequently, for any given cutoff ψ^*, we have

$$\tilde{G}(q) < G(q),$$

which implies that $\tilde{G}(q)$ first-order stochastically dominates $G(q)$.

Recall the indifference condition characterizing the cutoff $\tilde{\psi}^*(B)$ under $\tilde{F}$:

$$\begin{aligned}&\big[E\big(S(\psi) \mid \psi > \tilde{\psi}^*(B), B\big) - \tilde{G}^{-1}\big(1-\kappa_1\big)\big] \cdot T\big(S(\tilde{\psi}^*(B)), h_1\big) \\ &- \big[E\big(\psi \mid \psi < \tilde{\psi}^*(B), B\big) - \tilde{G}^{-1}\big(1-\kappa_1\big)\big] \cdot T\big(\tilde{\psi}^*(B), h_1\big) \quad - \ c = 0.\end{aligned}$$

Assume, for the sake of contradiction, that

$$\tilde{\psi}^*(B) \ \leq \ \psi^*(B).$$

Since $\tilde{G}$ FOSD G, it follows that

$$\tilde{G}^{-1}\big(1-\kappa_1\big) \ > \ G^{-1}\big(1-\kappa_1\big).$$

Evaluating the indifference expression at $\psi^*(B)$ instead of $\tilde{\psi}^*(B)$ yields

$$\begin{aligned}&\big[E\big(S(\psi) \mid \psi > \psi^*(B), B\big) - \tilde{G}^{-1}\big(1-\kappa_1\big)\big] \cdot T\big(S(\psi^*(B)), h_1\big) \\ &- \big[E\big(\psi \mid \psi < \psi^*(B), B\big) - \tilde{G}^{-1}\big(1-\kappa_1\big)\big] \cdot T\big(\psi^*(B), h_1\big) \quad - \ c \ < \ 0.\end{aligned}$$

This inequality contradicts the definition of $\tilde{\psi}^*(B)$ as the unique threshold satisfying the indifference condition. Therefore, the assumption must be false, implying

$$\tilde{\psi}^*(B) \ > \ \psi^*(B).$$

■

A.4 First Stage Identification

Primitives, observables, and equilibrium objects. Students are indexed by cells $c = (B, k, t)$; programs by j with quality h_j, attributes Z_j, and quotas κ_j^t. Type parameters $\boldsymbol{\theta_\psi} = (\psi_L, \Delta\psi, \gamma_B, \gamma_k, \gamma_t)$ imply the belief

$$x_c := \Pr(\psi_H \mid B, k, t) = \Phi(\gamma_B B + \gamma_k k + \gamma_t) \in (0, 1).$$

Training parameters are $\boldsymbol{\theta_{tr}} = (\tau_1, \tau_2, \{s_k\}_k)$. Matching parameters are $\boldsymbol{\theta_m} = (\beta, \sigma_\eta, \sigma_\zeta)$. Program evaluation (admissions index) is

$$q_c = \psi_L + \Delta\psi \, x_c + \eta, \qquad \eta \sim N(0, \sigma_\eta^2),$$

with program cutoff τ_{jt} chosen to satisfy quota κ_j^t. Conditional on offers, student utility is

$$U_{jc} = \tau_1 h_j + \tau_2(\psi + s_k)h_j + \beta^\top Z_j + \zeta_j, \qquad \zeta_j \sim N(0, \sigma_\zeta^2).$$

Let $A_{jc} = \Pr(\text{admit } j \mid c)$ and P_{jc}^{match} denote the observed match share. The outcome equation for cell (B, t) is

$$\begin{aligned}
E[y \mid B, t] = \psi_L + \sum_k s_k \Pr(k \mid B, t) + (\tau_1 + \tau_2 \psi_L) \sum_j h_j \Pr(j \mid B, t) \\
+ \tau_2 \sum_k s_k \sum_j h_j \Pr(j, k \mid B, t) + R(B, k, t), \\
R(B, k, t) = \Delta\psi \sum_{j,k} (h_j + 1)\, F_j\big(\Phi(\gamma_B B + \gamma_k k + \gamma_t); \boldsymbol{\theta_m}\big) \Pr(j, k \mid B, t).
\end{aligned}$$

Assumptions.

1. **Exclusions.** Z_j enters only student utility (via β) and is excluded from admissions q_c. (B, k, t) enter admissions only through x_c and are excluded from Z_j.

2. **Support/Rank.** (i) Z_j has full-rank variation across programs; h_j varies across programs and time. (ii) (B, k, t) have nondegenerate support so x_c varies on an interval within $(0, 1)$. (iii) Across j, $\{F_j(\cdot)\}$ are uniquely pinned down by parameter θ_m and not all affine with a common slope on the support of x_c. (iv) The regressors in the linear part of $E[y \mid B, t]$ are linearly independent of the nuisance family generated by $\{(h_j + 1)F_j(\Phi(\cdot); \theta_m) \Pr(j, k \mid B, t)\}$.

3. **Independence.** ζ_j are i.i.d. across j and independent of (B, k, t, h, Z); η is independent of (B, k, t) and of ζ.

4. **Equilibrium regularity and uniqueness.** For each t, the market clears in a unique stable matching; given $(\boldsymbol{\theta_\psi}, \boldsymbol{\theta_{tr}}, \boldsymbol{\theta_m})$ and quotas $\{\kappa_j^t\}$, admission cutoffs $\{\tau_{jt}\}$ and acceptance rules are uniquely determined; $F_j(\cdot)$ is monotone in its index.

Claim 1 *Under (A1)–(A5), the mapping from*

$$\theta := (\boldsymbol{\theta_\psi}, \boldsymbol{\theta_{tr}}, \boldsymbol{\theta_m})$$

to the joint distribution of observed matching $\{P_{jc}^{match}\}$ (given quotas $\{\kappa_j^t\}$) and outcome moments $\{E[y \mid B, t]\}$ is injective.

Sketch of Proof. We show that if θ and θ' generate the same observables, then $\theta = \theta'$.

Step 1 (Choice parameters β up to scale). Fix a cell $c = (B, k, t)$ so x_c is constant. Conditional on admissible sets, variation in Z_j (excluded from admissions by (A1)) shifts only student utilities. Standard multinomial choice identification with σ_η normalized in (A4) implies $\beta' = \beta$.

Step 2 (Admissions inversion $\Rightarrow$ latent index z_c). Let $z_c := \psi_L + \Delta\psi\, x_c$. With unique stable matching (A5), the observed assignment matrix $\{P_{jc}^{\text{match}}\}$, capacities $\{\kappa_j^t\}$, and known demand elasticities (from Step 1) uniquely invert to admission probabilities A_{jc} and program cutoffs $\{\tau_{jt}\}$. The probit relation

$$A_{jc} = \Phi(z_c - \tau_{jt})$$

and known τ_{jt} recover z_c in levels for every cell c. Thus $z_c' = z_c$ pointwise.

Step 3 (Outcomes $\Rightarrow$ $(\psi_L, \Delta\psi, \tau_1, \tau_2, \{s_k\})$). Write the outcome equation as

$$E[y \mid B, t] = L(B, k, t; \psi_L, \tau_1, \tau_2, \{s_k\}) \;+\; R(B, k, t; \Delta\psi, \gamma, \theta_m),$$

where $L(\cdot)$ is linear in $(\psi_L, \tau_1, \tau_2, \{s_k\})$ and

$$R(B, k, t) = \Delta\psi \sum_{j,k} (h_j + 1) F_j(x_c; \theta_m) \Pr(j, k \mid B, t).$$

By Step 2, $\{A_{jc}\}$ and hence $\Pr(j, k \mid B, t)$ are pinned down by the recovered matching, so the only unknowns in R are $(\Delta\psi, \gamma, \theta_m)$ through x_c and $F_j(\cdot)$. Equality of outcome moments under θ and θ' implies

$$L(\cdot; \psi_L, \tau_1, \tau_2, \{s_k\}) - L(\cdot; \psi_L', \tau_1', \tau_2', \{s_k'\}) = R(\cdot; \Delta\psi', \gamma', \theta_m') - R(\cdot; \Delta\psi, \gamma, \theta_m).$$

Assumption (A2)(iv) (rank: linear regressors not in the span of the nuisance family) and the functional-form wedge in (A2)(iii) (not all F_j affine with common slope) imply the unique solution is

$$\psi_L' = \psi_L, \quad \tau_1' = \tau_1, \quad \tau_2' = \tau_2, \quad \{s_k'\} = \{s_k\}, \quad \Delta\psi' = \Delta\psi.$$

(Otherwise the linear part would need to be replicated by the nonlinear R across the support, contradicting (A2)(iii)-(iv).)

Step 4 (Recover x_c and identify γ). With ψ_L and $\Delta\psi$ matched, we have $x_c = (z_c - \psi_L)/\Delta\psi$ for all c, and thus $x_c' = x_c$. Since $x_c = \Phi(\gamma_B B + \gamma_k k + \gamma_t)$ with known link Φ and normalization in (A4), standard single-index identification yields $\gamma' = \gamma$.

Step 5 (Matching functions F_j and remaining matching scales). Given $(\beta, \psi_L, \Delta\psi, \gamma)$ and observed capacities, the equilibrium acceptance rules $F_j(\cdot)$ that rationalize $\{P_{jc}^{\text{match}}\}$ are unique by (A5); hence $\theta'_m = \theta_m$. Therefore $\theta' = \theta$. ■

Remarks. Dispersion moments (dispersion of assigned h_j across z_c) strengthen identification because, under the two-type mixture, they scale like $(\Delta\psi)^2\, F_j(x_c)\big(1 - F_j(x_c)\big)$, adding curvature distinct from means.

B Computation and Monte Carlo Simulations

B.1 Computation of Equilibrium

Before proceeding with the computation, two additional assumptions are imposed to ensure tractability of the model. First, I assume that the signal function $S_k(\cdot)$ takes the additive form

$$S_k(\psi) = \psi + \delta_k,$$

where τ_k is a known constant shift associated with predoc option k. This specification simplifies the mapping between underlying research ability and the signal observed by programs. Second, I assume that the idiosyncratic preference shocks ε_{ik} over predoc options are independent and identically distributed according to the Type I Extreme Value distribution. This assumption yields the standard multinomial logit form for students' choice probabilities.

To compute the equilibrium of the model, note that the distribution $H(u)$ is determined exogenously by the fixed program capacities and common student preferences. In contrast, the distribution of posterior means $G(q)$ depends endogenously on students' predoc choices and must be solved for in equilibrium. Accordingly, the equilibrium requires jointly determining $G(q)$, the implied admission thresholds, the admission probabilities, and the students' optimal predoc decisions in a manner that is mutually consistent.

The computational procedure proceeds iteratively as follows:

1. Make the initial guess of $E(\psi|B,k)$ and compute $E(S_k(\psi)|B,k)$;

2. Compute the posterior mean distribution $G(q_i)$ using equation 6;

3. Compute the admission criteria δ_j via simulated matching;

4. Compute expected payoff $E(U|B,k)$ using equation 7;

5. Given $E(U|B,k)$, compute the probability type (B) student choose to do predoc k:

$$Pr(k|B,\psi) = \frac{exp(E(U_i|\psi,B,k) - c(B,k)))}{1 + \sum_{k'} exp(E(U_i|\psi,B,k') - c(B,k'))};$$

6. Compute the conditional distribution and expectation of ξ conditional on j, X_i:

$$Pr(\psi|B,k) = \frac{Pr(k|B,\psi)f(\psi|B)}{\int Pr(k|B,\psi)f(\psi|B)d\psi},$$

$$E(\psi|B,k) = \int \psi Pr(\psi|B,k)d\psi;$$

7. Iterate from 2 to 6 until $E(\psi|B,k)$ converge.

B.2 Monte Carlo Simulations for Comparative statics

This appendix complements the theoretical results in Propositions 3 and 2 by illustrating the corresponding comparative statics in a simulated version of the model. The goal is to demonstrate that the equilibrium patterns predicted by the propositions emerge even when we drop the double sorting, $J = 2$ Ph.D. program and $K = 1$ predoc assumptions.

B.2.1 Simulation Setup

We simulate an economy with $N = 2000$ students and $J = 5$ Ph.D. programs. Each student has a vector of binary observable characteristics $B \in \{0,1\}^3$ drawn independently from Bernoulli(0.5), and an unobservable research ability ψ that is positively correlated with B. Specifically,

$$\psi = 3 \cdot \frac{B\alpha + \varepsilon_\psi - \psi_{min}}{\psi_{max} - \psi_{min}},$$

where $\alpha = (0.3, 0.3, 0.3)$, and ε_ψ is Gaussian noise with mean zero and variance varied in some experiments. Students may choose $k \in \{0,1\}$ years of predoc training. Training yields a type-specific productivity gain T_k (set to 0 for $k = 0$ and 1 for $k = 1$) and costs C_k (varied in some experiments). Students also draw idiosyncratic costs ε_k from a Gumbel distribution.

Each program j has observable quality h_j and characteristics z_j; the match utility for student i at program j is

$$u_{ij} = \psi_i + h_j + \psi_i h_j + \beta_i \cdot z_j + \xi_j,$$

with β_i determined by B_i and an individual taste shock η_i. Program admissions are modeled as a capacity-constrained serial dictatorship on a scalar "school score" derived from predicted ψ and training effects.

The equilibrium is computed as a fixed point in which students' k-choices and programs' match probabilities are mutually consistent.

In all exercises, we hold fixed the random draws $(B, \varepsilon_\psi, \eta, \varepsilon_k, \beta, z, \xi, \zeta)$ except for the parameter being varied, so the comparative statics isolate the intended mechanism.

B.2.2 Predoc Cost Variation

We first vary the fixed cost C_1 of undertaking predoc from 2 to 8 and record the equilibrium fraction choosing $k = 1$. The results are shown in Figure 10. As expected, higher

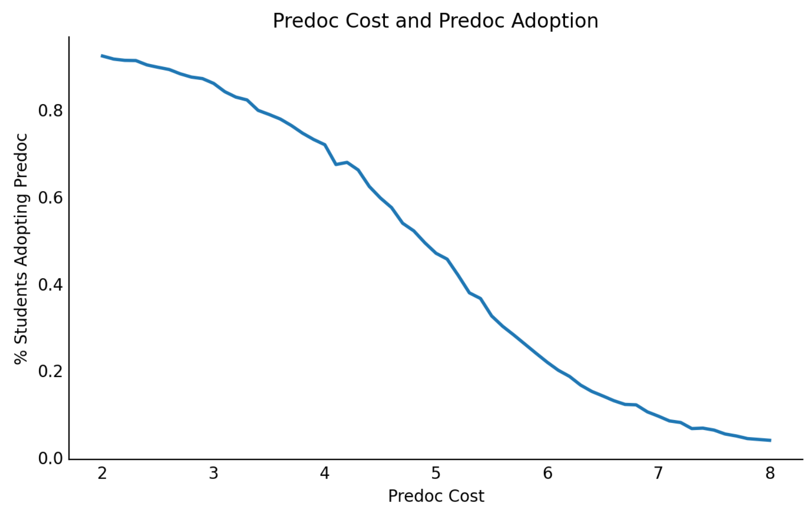

Figure 10: **Predoc cost variation.** The figure plots the number of students choosing $k = 1$ against the fixed cost C_1. Higher costs reduce participation, holding all other parameters constant.

costs monotonically reduce predoc take-up, consistent with the basic monotonicity of best responses in costs.

B.2.3 Grade Inflation

To illustrate Proposition 2, we vary the variance of ε_ψ, which makes the signal B noisier about ψ. Specifically, we multiply a fixed $\mathcal{N}(0, 1)$ noise draw by a scaling factor $\sigma \in [0.05, 1.0]$ (fine grid) or larger in robustness checks. The results are shown in Figure 11. Higher σ compresses the informativeness of B in the admission process. As predicted, noisier signals lead to higher predoc participation: students invest in predoc training to compensate for the diminished ability to separate from peers through B alone.

B.2.4 Program Quotas (Slot Reduction)

To illustrate the quota effect in Proposition 3, we fix $h = (0, 1, 2, 3, 4)$ and set z identical across schools to isolate the h channel. We then reduce the capacities of the top two programs ($h = 3$ and $h = 4$) by fractions $f \in [0, 0.6]$, either (i) reallocating the removed slots evenly to lower-h programs or (ii) removing them entirely. The results are shown in Figure 12. In both cases, shrinking top-program quotas reduces the incentive to choose $k = 1$, consistent with the proposition's second part. The effect is more pronounced without reallocation, as overall admissions become tighter.

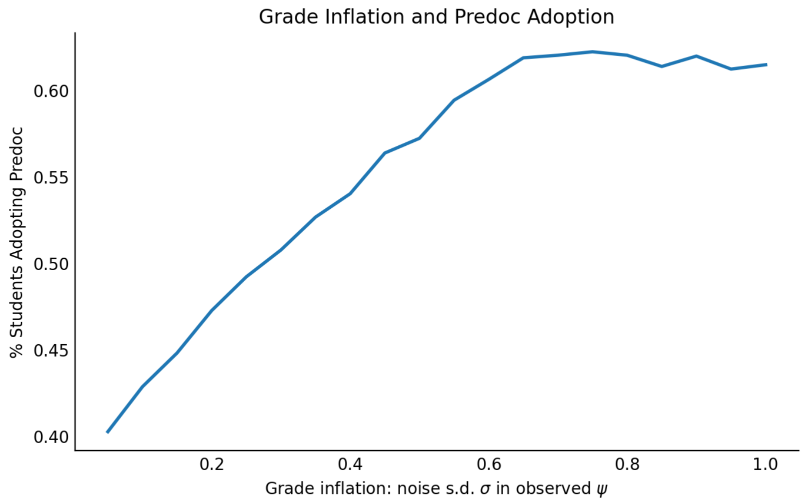

Figure 11: **Grade inflation / noise variation.** The figure plots the number of students choosing $k = 1$ as the standard deviation σ of the noise in ψ increases. Noisier exogenous signals increase predoc take-up, consistent with Proposition 2.

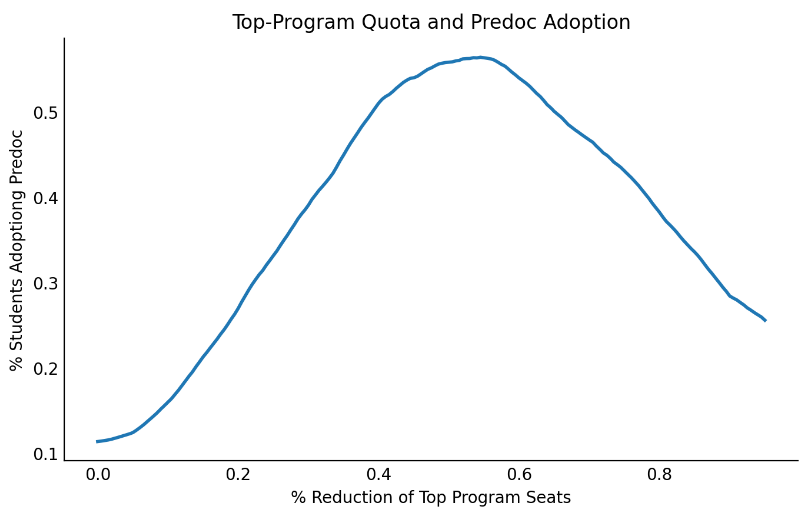

Figure 12: **Program quota reduction.** The figure shows the number of students choosing $k = 1$ as the fraction of seats removed from the two highest-h programs increases. Solid line: removed seats reallocated to lower-h programs; dashed line: seats removed from the market. Both cases show reduced predoc take-up, consistent with Proposition 3.

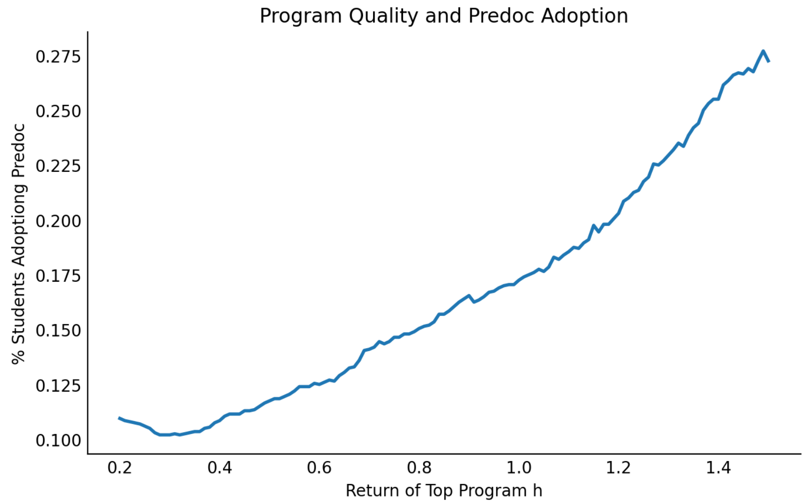

Figure 13: **Program quality scaling.** The figure plots the number of students choosing $k = 1$ as the common multiplier on h increases. Higher relative returns to elite programs increase predoc take-up, consistent with Proposition 3.

B.2.5 Program Quality Scaling

Finally, to illustrate the first part of Proposition 3, we scale the entire h vector by a multiplier $s \in [0.2, 2.0]$, holding capacities and z fixed. The results are shown in Figure 13. This changes the return differential between high- and low-quality programs without altering quotas. As s increases, the payoff to reaching the top programs rises, and more students select into predoc, confirming the proposition's prediction.

C Details of Simulated Method of Moments

C.1 Model and Notation

Let i index students, j Ph.D. programs with quality index h_j, $k \in \{0, 1, \ldots, K\}$ predoc types, and t years. Denote observed student traits by B_i, predoc assignment by k_i, program by j_i, and job outcome by y_i. The outcome equation is

$$\mu(B_i, k_i, j_i, t; \vartheta) = (\psi_\ell + s_{k_i}) + \big(b_1 + b_2(\psi_\ell + s_{k_i})\big) h_{j_i} + (\Delta\psi + b_2 \Delta\psi \, h_{j_i}) \, P_h(B_i, k_i, j_i, t; \gamma),$$

$$P_h(B, k, j, t; \gamma) = F_m\Big(\underbrace{\Phi(\gamma_B^\top B + \gamma_k + \gamma_t)}_{P_h(B,k,t;\gamma)}; \ \kappa_t, \theta_m \Big), \tag{11}$$

with parameter vector $\vartheta = \{\psi_\ell, \Delta\psi, b_1, b_2, (s_k)_{k=0}^K, \gamma_B, \gamma_k, \gamma_t, \theta_m\}$. We normalize $s_{k=0} = 0$. Quotas κ_t are observed.

Simulation. Given ϑ, we: (i) compute $P_h(B_i, k, t; \gamma)$ and then $P_h(B_i, k, j, t; \gamma)$ via $F_m(\cdot; \kappa_t, \theta_m)$; (ii) simulate matching realizations j_i where needed (using common random numbers); (iii) form model-implied cell means using (11).

C.2 Moment Families

We stack five families of outcome/matching moments (A–E) and three standard matching structure moments.

(A) Cell-Mean Fit (Outcome Moments)

For each populated (B, k, j, t):

$$m^A_{Bkjt}(\vartheta) \ = \ \bar{y}(B, k, j, t) \ - \ \mu(B, k, j, t; \vartheta).$$

Identification role: variation across h_j (within B, k, t) identifies (b_1, b_2); across k (within B, j, t) identifies (s_k); through κ_t and F_m the induced change in P_h identifies $(\gamma, \Delta\psi)$.

(B) Quota-Shift Differences Over Time

For programs j and predoc type k, pick (t_1, t_2) with $\kappa_{t_1} \neq \kappa_{t_2}$. Let

$$\Delta\bar{y}_{kj}(t_1 \to t_2) \;=\; \bar{y}(\cdot, k, j, t_2) - \bar{y}(\cdot, k, j, t_1), \quad \Delta\overline{P}_{hkj}(t_1 \to t_2) \;=\; \overline{P_h}(\cdot, k, j, t_2) - \overline{P_h}(\cdot, k, j, t_1).$$

Moment:

$$m^B_{kj,t_1,t_2}(\vartheta) \;=\; \Delta\bar{y}_{kj}(t_1 \to t_2) - \big(\Delta\psi + b_2 \Delta\psi\, h_j\big)\, \Delta\overline{P}_{hkj}(t_1 \to t_2).$$

Identification role: differencing purges level terms $(\psi_\ell + s_k) + (b_1 + b_2(\psi_\ell + s_k))h_j$; remaining variation maps quota-driven selection changes to $(\gamma, \Delta\psi)$.

(C) Unconditional Within (B, t)

$$\bar{y}(B, t) \equiv \sum_{k,j} \bar{y}(B, k, j, t)\, \Pr(j \mid B, k, t)\pi_k(B, t),$$

and model-implied

$$\overline{P_h}(B, t; \gamma) = \sum_{k,j} P_h(B, k, j, t; \gamma)\, \Pr(j \mid B, k, t)\pi_k(B, t), \qquad \overline{s}(B, t) = \sum_k \pi_k(B, t)s_k.$$

Moment:

$$m^C_{Bt}(\vartheta) = \bar{y}(B, t) - \Big[\psi_\ell + (b_1 + b_2\psi_\ell)\bar{h}(B, t) + \Delta\psi\, \overline{P_h}(B, t; \gamma) + \overline{s}(B, t) + b_2\bar{h}(B, t)\, \overline{s}(B, t)\Big].$$

Identification role: forces the unconditional mean to be explained by (i) baseline and Ph.D. training via $\bar{h}$ (pins b_1, b_2, ψ_ℓ), (ii) average selection $\overline{P_h}$ (pins γ jointly with $\Delta\psi$), and (iii) average predoc training $\overline{s}$ (pins levels of s_k up to the normalization).

(D) Within-Program Predoc Share

For any pair k, k' within the same (B, j, t):

$$m^D_{Bjt,kk'}(\vartheta) = \big[\bar{y}(B, k, j, t) - \bar{y}(B, k', j, t)\big] - \Big[(s_k - s_{k'}) + b_2 h_j (s_k - s_{k'}) + (\Delta\psi + b_2\Delta\psi h_j)\big(P_h(B, k, j, t) - P_h(B, k', j, t)\big)\Big].$$

Identification role: isolates $s_k - s_{k'}$ from selection differences $P_h(\cdot)$ at fixed h_j, sharpening the decomposition of training vs. signaling.

(E) Matching Structure Moments

To discipline γ (and optionally θ_m) further, we include standard assignment moments:

$$\begin{aligned}
\textbf{Student–Program Cross-Moments:} \quad M_t^{xz}(\text{data}) &= \frac{1}{M_t} \sum_{(i,j)\in\mathcal{M}_t} B_i Z_j^\top, \\
\textbf{Within-Program Student Means:} \quad M_{jt}^{ov}(\text{data}) &= \frac{1}{n_{jt}} \sum_{i\in\mathcal{I}_{jt}} B_i, \\
\textbf{Within-Program Second Moments:} \quad M_t^{w}(\text{data}) &= \sum_j \frac{n_{jt}}{N_t} \Big(\frac{1}{n_{jt}} \sum_{i\in\mathcal{I}_{jt}} B_i B_i^\top \Big).
\end{aligned}$$

Let $M_t^{xz}(\vartheta), M_{jt}^{ov}(\vartheta), M_t^{w}(\vartheta)$ denote their model-implied counterparts (via simulated assignments using P_h and F_m). Moments:

$$m_t^{E1}(\vartheta) = \text{vec}(M_t^{xz}(\text{data}) - M_t^{xz}(\vartheta))\,, \quad m_{jt}^{E2}(\vartheta) = M_{jt}^{ov}(\text{data}) - M_{jt}^{ov}(\vartheta), \quad m_t^{E3}(\vartheta) = \text{vec}(M_t^{w}(\text{data}) - M_t^{w}($$

Identification role: these match the sorting structure between B and Z_j induced by γ and $F_m(\cdot;\kappa_t,\theta_m)$, ensuring that the selection engine that feeds into (11) is empirically correct.

C.3 Stacked Objective and Weighting

Let $m(\vartheta)$ be the stacked vector of sample moments

$$m(\vartheta) = \big(m^A,\, m^B,\, m^C,\, m^D,\, m^{E1},\, m^{E2},\, m^{E3}\big).$$

The SMM estimator solves

$$\hat{\vartheta} = \arg\min_{\vartheta} \left[m(\vartheta)\right]^\top \hat{W} \left[m(\vartheta)\right],$$

with a diagonal $\hat{W}^{(0)}$ proportional to cell sizes for the first step and the optimal two-step weight $\hat{W}^{(2)} = \hat{S}^{-1}$ in the second step where $S = Var(\sqrt{N} m(\vartheta_0))$ We use a two-stage procedure: (i) coarse global search (multi-start), then (ii) a gradient-based local solver (BFGS/L-BFGS-B) with analytic or automatic-differentiation Jacobians of $m(\vartheta)$.

D Structural Parameter Estimates

This appendix reports the full set of structural parameter estimates from the model in Section 7. The main text discusses the key coefficients and economic intuition; here we provide the complete estimates, their standard errors, and brief notes on identification.

D.1 Private Type Distribution

Table 9 report the estimates for type distribution. The private-type parameters characterize the unobserved heterogeneity in students' underlying research ability. The model assumes a binary latent type $\psi \in \{\psi_L, \psi_H\}$ capturing students' long-run research potential that is not directly observable to Ph.D. programs. The estimated values $\psi_L = 0.113$ and $\psi_H = 0.449$ imply that high-type students are, on average, roughly four times more likely to succeed in job market, the value will not be severe in our estimation as we does not estimate individual research type, but probability so from program's perspective students' distribution is continuous, while this difference will influence Ph.D. program preference significantly. The estimated type-probability parameters γ govern how observable background and predoc experience shift the probability that a student is high-type:

$$\Pr(\psi_H|B, k) = \Phi(\gamma_0 + \gamma_B B + \gamma_k k).$$

Positive coefficients on U.S. Top Undergraduate institutions and Economics major indicate that these characteristics strongly predict higher latent research ability, consistent with the idea that selective undergraduate programs and disciplinary training screen for ability. In contrast, the negative coefficient on "Grade Inflation" suggests that in environments where grades are less informative, the predicted share of high types falls, which motivates students to acquire alternative signals such as predoc experience. Finally, the positive γ_k estimates show that completing a predoc increases the inferred probability of being high-type, confirming the signaling interpretation of predoc participation in the admission market.

D.2 Training Parameters

Table 10 report the estimates for training parameters. The training parameters (τ_1, τ_2, s_k) govern how students' human capital accumulates through both Ph.D. training and predoc experience. The production function

$$T(\tilde{\psi}_i, h_j) = \tilde{\psi}_i + \tau_1 h_j + \tau_2 \tilde{\psi}_i h_j, \quad \tilde{\psi}_i = \psi_i + s_k,$$

Table 9: Estimates of Type Distribution Parameters

Parameter	Estimate	s.e.
ψ_L	0.113	(0.034)
ψ_H	0.449	(0.141)
γ_0	-0.640	(0.420)
Background effects:		
U.S. Top UG	0.310	(0.110)
Non-U.S. Top UG	0.105	(0.054)
U.S. Other UG	0.052	(0.009)
Male	0.034	(0.013)
UG Econ Major	0.032	(0.012)
Grade Inflation	-0.146	(0.062)
Predoc-specific effects:		
U.S. Non-Univ. Research	0.300	(0.130)
U.S. Univ. Research	0.550	(0.230)
U.S. Master's	0.300	(0.130)
Non-U.S. Master's	0.420	(0.210)
Non-U.S. Research	0.050	(0.180)

Notes: Notes: Parameters are estimated from cross-program variation in student composition. ψ_H and ψ_L denote high and low latent research ability. γ_B and γ_k capture the influence of background and predoc choice on type probability, $\Pr(\psi_H|B,k) = \Phi(\gamma_0 + \gamma_B B + \gamma_k k + \gamma_t)$.

implies that τ_1 measures the direct productivity of program quality, while τ_2 captures the complementarity between student ability and program quality—i.e., supermodularity in training. The positive estimate $\tau_2 = 0.563$ indicates that higher-ability students gain disproportionately more from attending top Ph.D. programs, the results come from outcome difference between program conditional on enrolled students characteristics.

The s_k parameters quantify the average human-capital gain attributable to each predoc type, independent of signaling. Estimates range from 0.04 to 0.06, implying that predoc participation raises effective ability by roughly 10–15% of the gap between ψ_L and ψ_H. U.S. university-based and non-university research predocs yield slightly larger training effects than master's programs, consistent with their stronger research exposure. As discussed in identification, this comes from average outcome change overyear controlling the Ph.D. program training. These estimates provide the foundation for separating the training channel of welfare improvement from the signaling channel in the counterfactual analysis.

Table 10: Estimates of Training Parameters

Parameter	Estimate	s.e.
τ_1	0.082	(0.022)
τ_2	0.563	(0.102)
Predoc training increment s_k:		
U.S. Non-Univ. Research	0.055	(0.035)
U.S. Univ. Research	0.039	(0.021)
U.S. Master's	0.038	(0.021)
Non-U.S. Master's	0.063	(0.026)
Non-U.S. Research	0.040	(0.040)

Notes: Parameters τ_1 and τ_2 govern the complementarity between student ability and Ph.D. program quality. s_k measures incremental training from each predoc type.

Table 11: Training Intensity Weights on Program Characteristics

	Econ Top5	Fin Top3	Acc Top3	Mgmt Top5	Tilburg BE	IDEAS Score	USNews Rank
Estimate	0.06	0.09	0.05	0.07	0.55	0.08	0.07
s.e.	(0.03)	(0.05)	(0.03)	(0.04)	(0.09)	(0.04)	(0.04)

Notes: Each weight w_l reflects the relative contribution of field and ranking indicators to Ph.D. program quality h_j. Estimates are identified from cross-program variation in placement outcomes.

D.3 Matching Parameters

The matching parameters describe how program characteristics influence the equilibrium pairing between students and Ph.D. programs. They are identified from both within-program and across-program variation in observed student characteristics. Across-program differences in average student composition (e.g., stronger students concentrating in urban, high-rent, or private universities) identify the mean preference coefficients, while within-program dispersion in student traits around those means identifies the heterogeneity terms ($\sigma_{\text{Rent}}, \sigma_{\text{Public}}, \sigma_{\text{Rural}}$). The positive coefficient on Log Rent suggests that programs located in more expensive cities attract relatively stronger students, likely reflecting higher institutional prestige and local amenities. Public universities have a mild positive loading, while rural location carries a negative sign, consistent with weaker attraction of top students to rural programs.

Table 12: Estimates of Matching Parameters

Variable	Mean	s.e.
Log Rent	0.015	(0.005)
σ_{Rent}	0.014	(0.008)
Public	0.022	(0.009)
σ_{Public}	0.020	(0.013)
Rural	-0.036	(0.012)
σ_{Rural}	0.063	(0.032)

Notes: Estimated from observed sorting between students and Ph.D. programs. Mean effects capture average preference weight; σ parameters measure heterogeneity across programs.

D.4 Predoc Cost Parameters

Tables 13 and 14 report the estimated parameters governing students' cost of undertaking each predoc type. These parameters are identified from the observed distribution of predoc choices conditional on student background characteristics. The baseline cost parameters in Table 13 reflect the mean opportunity or tuition cost of each predoc type, while the interaction terms in Table 14 capture heterogeneity in costs across student characteristics.

The scale parameter $\alpha = 9.09$ indicates that students are highly responsive to expected utility differentials, consistent with steep participation elasticities. Among baseline categories, U.S. university and non-university research predocs have the highest implied costs, followed by non-U.S. research positions and master's programs. Despite higher tuition, master's programs exhibit lower effective cost parameters, suggesting that they are perceived as less risky and more predictable pathways into Ph.D. programs.

Heterogeneity estimates reveal systematic differences in access and affordability. Students from top U.S. undergraduate institutions face significantly lower costs for domestic research predocs (negative coefficients), while those from non-U.S. institutions or majors outside economics face higher implicit costs. The negative coefficient on "Grade Inflation" implies that when undergraduate grades are less informative, students perceive the relative cost of signaling through predocs to be lower—making predoc adoption more attractive. Together, these parameters quantify how financial and informational frictions shape the extensive margin of predoc participation, which feeds into the welfare decomposition between efficient information acquisition and excessive signaling.

Table 13: Baseline Predoc Cost Parameters

	α	U.S. Non-Univ. Res	U.S. Univ. Res	U.S. Master's	Non-U.S. Master's	Non-U.S. Res
Estimate	9.09	5.20	4.91	2.77	3.02	4.52
s.e.	(1.33)	(0.87)	(1.09)	(0.85)	(0.77)	(1.19)

Notes: Estimated using a multinomial logit model of predoc choice. α scales expected utility, while λ_k^0 determines baseline costs for each predoc category.

Table 14: Heterogeneity in Predoc Cost by Student Background

	U.S. Top UG	Non-US Top UG	US Other UG	Male	UG Econ Major	Grade Inflation
U.S. Non-Univ. Research	-2.12 (0.96)	-0.03 (1.00)	-0.51 (0.96)	-0.92 (0.92)	-0.59 (0.91)	-0.09 (0.97)
U.S. Univ. Research	-0.88 (1.03)	-0.16 (1.03)	-0.06 (1.01)	0.57 (0.97)	0.39 (0.96)	0.17 (1.01)
U.S. Master's	0.09 (1.00)	0.11 (1.04)	0.55 (0.99)	0.12 (0.90)	-0.17 (0.96)	-0.16 (0.95)
Non-U.S. Master's	2.81 (0.97)	-0.37 (0.99)	3.01 (1.06)	-0.02 (0.79)	-0.69 (0.88)	-0.93 (0.92)
Non-U.S. Research	–	0.69 (1.01)	–	-0.28 (0.99)	-0.54 (1.01)	0.23 (1.00)

Notes: Coefficients represent interaction terms λ_k^B in the cost function $C_k(B) = \lambda_k^0 + \lambda_k^B B + \lambda_k^t$. Negative coefficients imply lower cost. Standard errors in parentheses.

Summary. The estimates show that U.S. university research predocs have the largest signaling component, while U.S. top undergraduate students face systematically lower costs. Non-U.S. master's programs provide the most accessible signaling route for international students, consistent with observed participation patterns.