# Multi-Agent Systems

## Homework Assignment 3

## MSc AI, VU

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## 3 Sequential Games with Perfect Information

### 3.1 Reduced centipede game

Consider a sequential 2-player game with the following game-tree: at each decision node the associated player needs to decide whether to continue (c) or stop (s). The tree (including utilities) and the players' rationality are common knowledge.

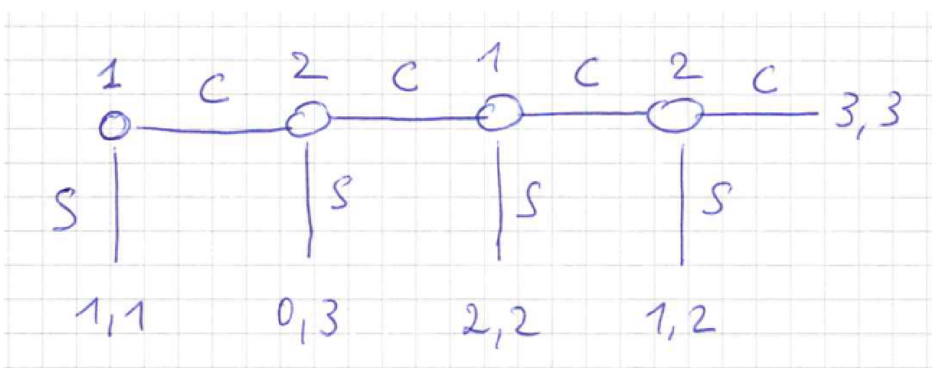



Figure 1: Shortened version of the centipede game.

#### Questions

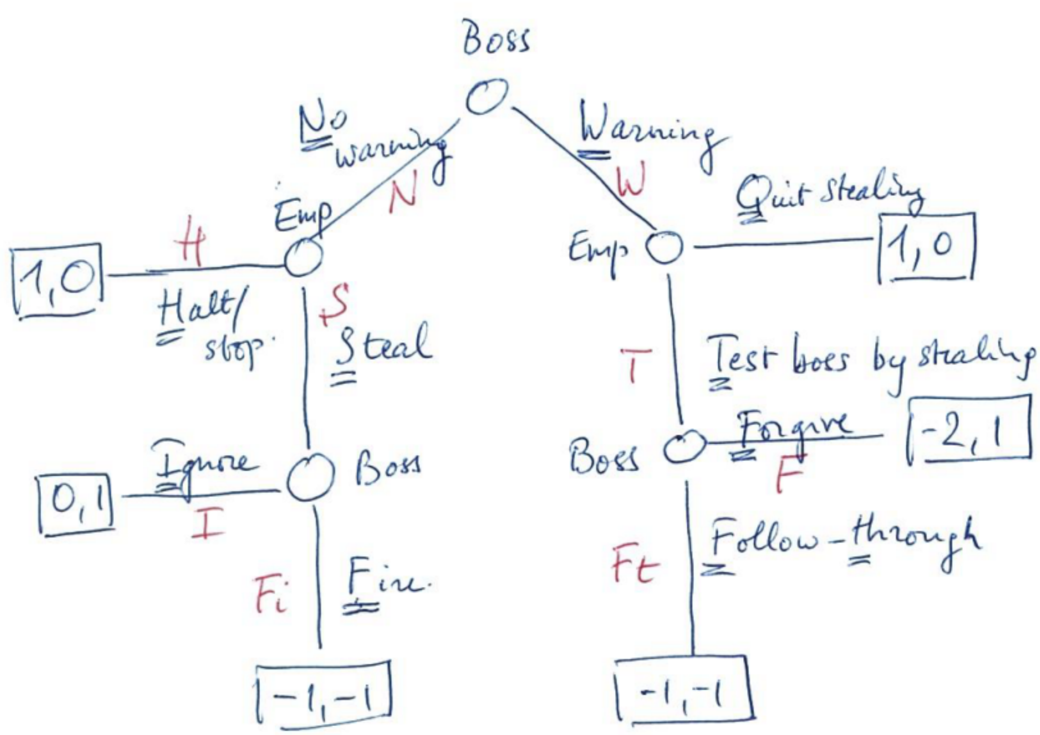
- Write the normal form for this game and find all Nash equilibria in pure strategies (PNEs).
- List all subgames and determine which of these PNEs are also subgame-perfect?
- Can you solve this game using backward induction? Discuss.

## 3.2 Boss and stealing employee

A boss notices that one of her employers has been stealing company material lately. The material was not all that valuable, so she is inclined to let it pass, preferring to keep the employee around rather than firing him and having to hire and retrain a replacement. Nevertheless she wants the stealing to stop.

She is therefore thinking to issue a warning at the next company meeting: the next person caught stealing company property will be fired immediately. She envisages the following game tree with pay-offs (see fig below).

1. Analyse this game using backward induction.
2. What are the pure actions for the two players (boss and employee)? Construct the normal form matrix.
3. Use this matrix to identify all the pure Nash equilibria of the normal form game.
4. Determine the subgame-perfect equilibrium (equilibria?) by eliminating all the Nash equilibria that fail to induce a NE in subgames.
5. Compare to the solution based on backward induction.




## 3.3 Stackelberg's Duopoly Model

Stackelberg's duopoly model is a sequential version of Cournot's duopoly model. There are two firms that produce some bland product (e.g. fertilizer). Firm 1 moves first and decides to produce a total quantity $q_1$ (think of this as a continuous variable). Firm 2 observes this move and then decides to produce a quantity $q_2$. The market price (per unit) depends (linearly) on the total amount produced:

$$P(q_1, q_2) = \alpha - \beta(q_1 + q_2)$$

where $\alpha, \beta > 0$ are known (positive) constants. Assume that both firms can produce the product at a fixed unit cost $c$. Hence the pay-off for each firm equals:

$$u_i(q_1, q_2) = P(q_1, q_2)q_i - cq_i.$$

1. Use backward induction to determine the optimal quantities for both firms.

2. Compare your results to the once obtained for the Cournot (simultaneous) model. Is there a "first mover" advantage?