

# Multi-Agent Systems

## Introduction to Multi-Agent Reinforcement Learning (MARL)

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# Outline

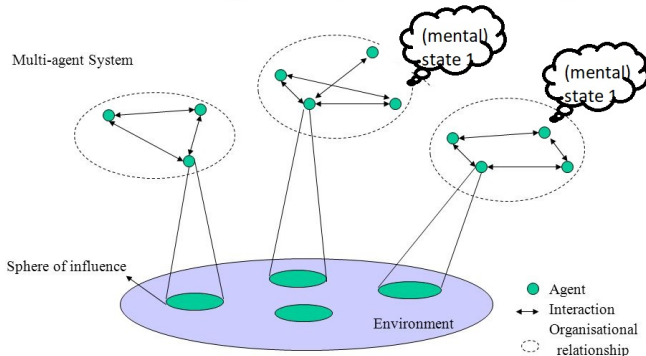
## Multi-Agent Reinforcement Learning (MARL)

# Reading

- P. Hernandez-Leal, M. Kaisers, T. Baarslag, E. Munoz de Cote: Survey of Learning in MultiAgent Environments: Dealing with Non-Stationarity. arXiv:1707.09183v2

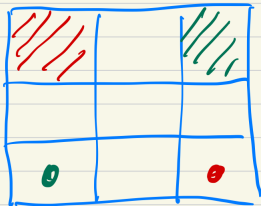
# MultiAgent Systems: Overview

## Multi-agent Systems (MAS)



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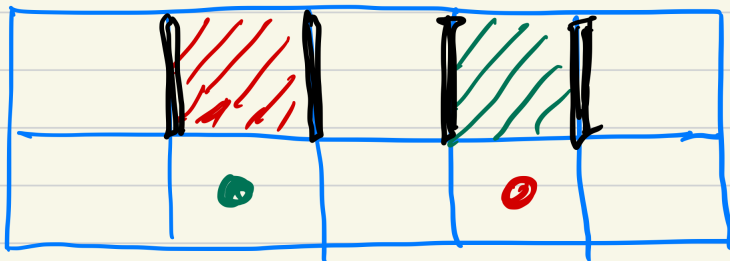
## Simple two agent game



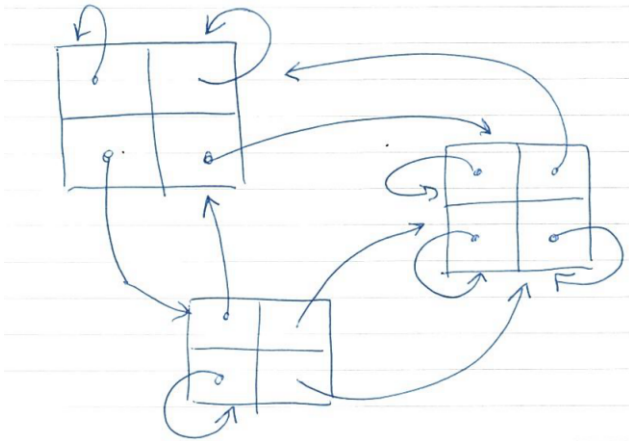
Goal:    ● →  +10  
               ● →  +10

- ① game ends as soon as  
  ✓ one goal is reached.
- ② game ends when there  
  is a collision.

## Simple two agent game

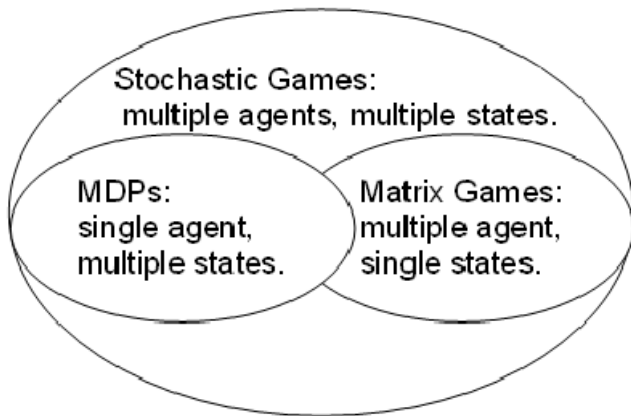


# Stochastic Games (Markov Games)



## Stochastic Games (Markov Games)

Stochastic games generalise **MDP** and **repeated matrix games**





# MARL Problem Setting

- Several agents share environment and act on it **concurrently**:
- **Environment dynamics**: how does the environment change as a consequence of the **concurrent** actions by agents;
- **Non-stationary Problem**:
  - Environment is changing
  - **Learning**: you are **adapting your behaviour** in response to other agents;
  - **Teaching**:
    - Other agents might be learning and **adapting to your behaviour**!
    - They might be learning to take advantage of you!

## Analogue for Bellman eqs. in MARL setting

- Bellman for  $q^*$  for single agent

$$\begin{aligned}
 q^*(s, a) &= \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma v^*(s')] \\
 &= \underbrace{\sum_{s'} p(s' | s, a) r(s, a, s')}_{R(s, a)} + \gamma \sum_{s'} p(s' | s, a) v^*(s') \\
 &= R(s, a) + \gamma \sum_{s'} T(s, a, s') \underbrace{\max_{a'} q^*(s', a')}_{\text{summary op.}} \\
 &= R(s, a) + \gamma \sum_{s'} T(s, a, s') \underbrace{H_{a'} q^*(s', a')}_{\text{summary op.}}
 \end{aligned}$$

## Analogue for Bellman eqs. in MARL setting

- Bellman for  $q^*$  for **single agent**

$$q^*(s, a) = R(s, a) + \gamma \sum_{s'} T(s, a, s') \underbrace{H_{a'} q^*(s', a')}_{\text{summary op.}}$$

- Bellman for  $q^*$  for **two agents**

$$s = (s_1, s_2) \xrightarrow{(a', b')} s' = (s'_1, s'_2)$$

$$q_1^*(s, (a, b)) = R_1(s, (a, b)) + \gamma \sum_{s'} T(s, (a, b), s') H_{(a', b')} q_1^*(s', (a', b'))$$

- Appropriate **summarisation** depends on **assumptions about opponents**;

## Example: Maximin Q-learning for zero-sum games

- Agent (a) and Opponent (o);
- (Zero-sum game  $u_a(a, o) = -u_o(a, o)$ )
- Value  $v(s)$  of a state ( $s$ ) is safety level: using maximin strategy:

$$v(s) = \max_{\pi_a} \min_{o \in O} \sum_{a \in A} q(s, a, o) \pi_a$$

- Corresponding Q-learning rule:

$$q(s, a, o) = r(s, a, o) + \gamma \sum_{s'} p(s' | s, a, o) v(s')$$

## Example: Nash-Q

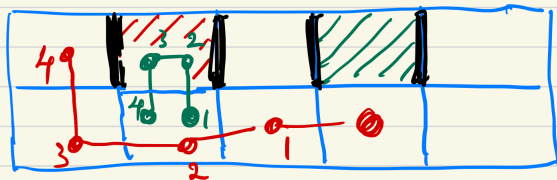
- **Nash Q-value**: represents an agent's expected future cumulative reward when, after choosing a specific joint action, all agents follow a joint Nash Equilibrium policy.

$$q_i(s, (a, b)) = R(s, (a, b)) + \gamma \sum_{s'} T(s, (a, b), s') v(s', \pi_1^*, \pi_2^*)$$

where \* indicates Nash policy.

## Simple two agent game

Nash-eq. (mutual best response)



## Different ways to handle non-stationarity

- **Ignore** Pretend environment is stationary;
  - Q-learning (single agent) or Fictitious Play
- **Forget** Favour more recent over older information;
- **Respond to target opponents** assume that opponent adheres to one of a class of well-defined strategies;
  - Friend-Q, Minimax-Q (Foe-Q), Nash-Q
- **Learn** model opponent and use this to plan;
- **Theory of Mind** Assume opponent is modelling you, and respond to that;

P. Hernandez-Leal, et al. Survey of Learning in MultiAgent Environments: Dealing with Non-Stationarity.

arXiv:1707.09183v2

## Example of *Ignore*: Fictitious Play

- **Ignore the changes** in opponent strategies;
- **Model-based learning:** Opponent is assumed to be playing **stationary** but **unknown mixed strategy**;
- **Empirical:** Each agent uses **observed action frequencies** to estimate mixed strategy;
- Each player plays **best response to current estimate** of opponent's strategy



# Fictitious Play: Convergence

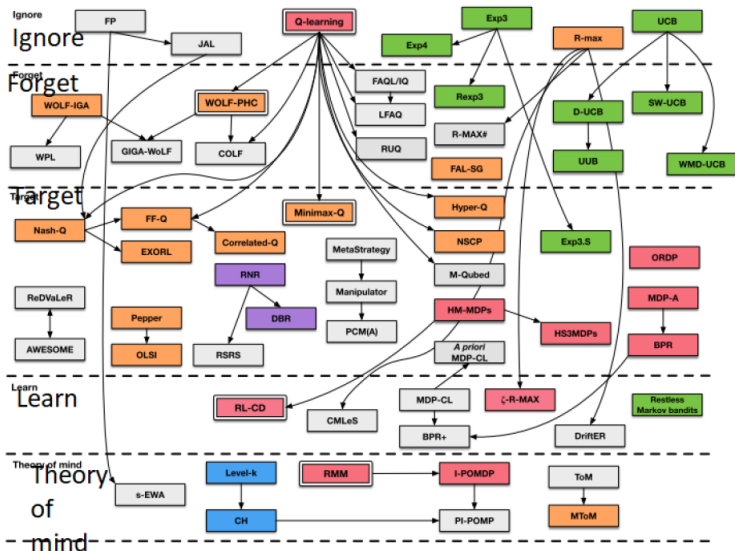
## Convergence in Fictitious Play

If the empirical distribution of each player's strategy converge, then they converge to a Nash equilibrium.

**Sufficient conditions** for convergence in 2-player finite games:

- Zero sum game;
- Game is solvable using elimination of strictly dominated strategies;
- Potential game;

## Overview of MARL algo's



## Image credit and further reading:

- P. Hernandez-Leal, et al. Survey of Learning in MultiAgent Environments: Dealing with Non-Stationarity.  
arXiv:1707.09183v2

L. Buşoniu, R. Babuška, and B. De Schutter, “Multi-agent reinforcement learning: An overview,” Chapter 7 in *Innovations in Multi-Agent Systems and Applications – 1* (D. Srinivasan and L.C. Jain, eds.), vol. 310 of *Studies in Computational Intelligence*, Berlin, Germany: Springer, pp. 183–221, 2010.