

# Multi-Agent Systems

## Introduction to Reinforcement Learning

### Model-free Methods: Monte Carlo, SARSA and Q-learning

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# Reading

- Sutton & Barto: chapters 5 & 6

# Outline

Model-based versus Model-free

Monte Carlo for Model-free Policy Evaluation

Temporal Difference Methods for Model-free Prediction

SARSA and Q-Learning for Model-free Control

Integrating Planning and Learning

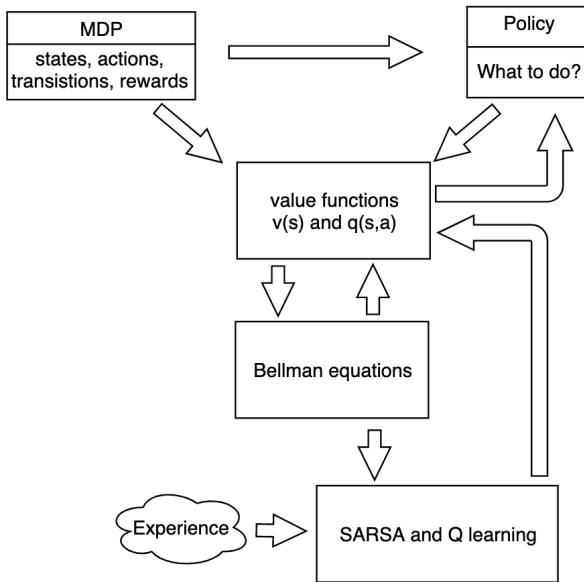
## Model-based versus Model-free

	<b>Prediction</b> <i>Estimation:</i> <i>Given <math>\pi</math>, what is <math>v</math>?</i>	<b>(Optimal) Control</b> <i>Optimisation:</i> <i>What is optimal <math>\pi</math>?</i>
model-based (MDP given)	Policy evaluation using Dyn. Programming (DP)	Policy improvement (+ Policy evaluation) = Policy iteration
model-free (MDP unknown)	Monte Carlo (MC) Temporal Diff <sup>ing</sup> (TD) = "impatient MC" <i>bootstrapping!</i>	Q-learning and Generalized Policy Iteration <i>"simultaneous"</i>

## Solving model-free RL problems

- The agent has **no prior knowledge** about states, actions, rewards, transitions!
- By **acting** in the world, the agent **gains experience**.  
An **experience** can be expressed as a 4-tuple:  $(s, a, r, s')$
- Over time the agent collects a **list of experiences** that he uses to find better policies ... HOW?
  - **Directly:** policy improvement
  - **Indirectly:** estimate value functions, improve policy using greedification;

# Overview



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## Greedification results in policy improvement

- To **improve the policy**, apply successive **iteration steps**:

$$\pi_k \xrightarrow{\text{eval}} v_k, q_k \xrightarrow{\text{greedify}} \pi_{k+1}$$

- Greedification implies **deterministic action choice** at each  $s$ :

$$\pi_{k+1}(s) = a^* \quad \text{iff} \quad q_k(s, a^*) = \max_a q_k(s, a)$$

- Hence **value function increases**, i.e. **policy has been improved**!

$$\begin{aligned} v_{k+1}(s) &= q_k(s, a^*) && (\pi_{k+1} \text{ is deterministic at } s) \\ &= \max_a q_k(s, a) \\ &\geq v_k(s) && (= \sum_{a'} \pi_k(a' | s) q_k(s, a')) \end{aligned}$$



# Greedification requires $q(s, a)$ !

## model-free vs. model-based

**Greedification:**  $\pi(s) := \arg \max_a q(s, a)$

**Model-based:** (a.k.a. **planning, search**)

- Suffices to **estimate value function  $v(s)$** :
- $q(s, a)$  **computed** with Bellman's **one-step look-ahead** (i.e. use back-up diagram):

$$q(s, a) = \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma v(s')]$$

**Model-free:** (a.k.a. **(optimal) control**)

- $q(s, a)$  needs to be **estimated from collected experiences**;
- algorithms shift **focus** from  $v(s)$  to  $q(s, a)$ ;

## Conclusion: for **model-free** RL

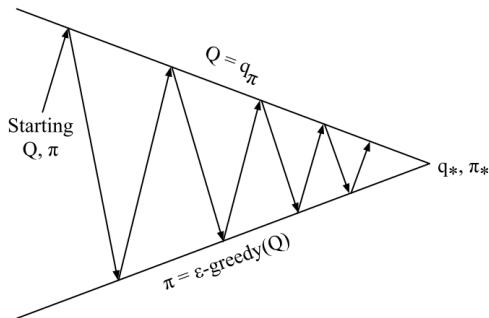
In contrast with to model-based, **model-free** RL is distinct:

- Focus on  $q(s, a)$  rather than  $v(s)$
- Make sure **all state-action pairs  $(s,a)$  are sampled**: importance of **exploration**!

# Policy Iteration (PI) for Model-free RL

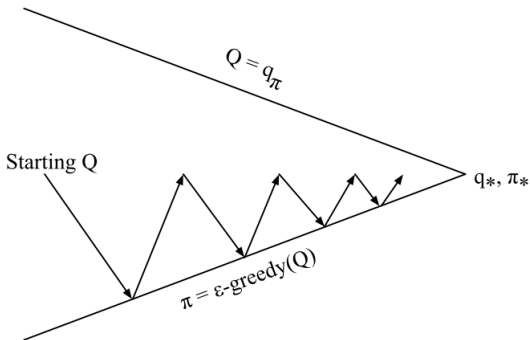
Goal: Find optimal policy  $\pi^*$  (aka optimal control)

- **PI**: Combine **policy evaluation** with **policy improvement**
  - Make  $Q$ -function **consistent with policy**  $\pi$ , i.e.  $Q = q_\pi$ ;
  - **Greedify policy**:  $\pi = \text{greedy}(Q)$ ;
- Introduce **exploration** in policy (e.g.  $\pi = \epsilon\text{-greedy}(Q)$ );

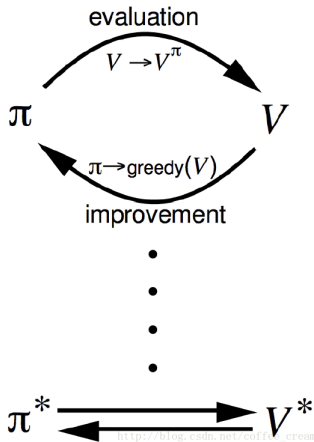
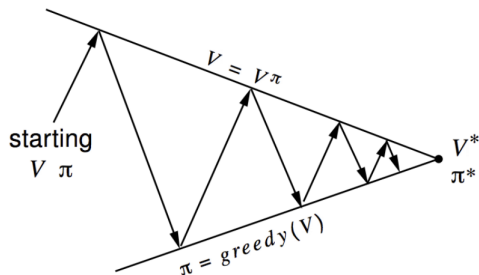


## Generalized Policy Iteration (GPI)

- **Impatient greedification:** No insistence on making  $Q$ -function fully consistent with current policy  $\pi$ ;  
Hence  $Q \approx q_\pi$ , but not necessarily  $Q = q_\pi$ ;
- **Ensure exploration:** use  $\pi = \varepsilon$ -greedy( $Q$ ), **not**  $\pi = \text{greedy}(Q)$ ;
- Works if both processes continue updating all states;



# Generalized Policy Iteration (GPI) schematically



Policy evaluation Estimate  $v_\pi$

Any policy evaluation algorithm

Policy improvement Generate  $\pi' \geq \pi$

Any policy improvement algorithm

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## Monte Carlo methods

- **Monte Carlo methods** are versatile statistical techniques for estimating properties of complex systems via **random sampling**;
- Example 1: if  $X \sim p(x)$  and  $\varphi$  some function, then for any (sufficiently large) i.i.d. sample:  $X_1, X_2, \dots, X_n$ :

$$E(\varphi(X)) = \int \varphi(x) p(x) dx \approx \frac{1}{n} \sum_{X_i \sim p} \varphi(X_i)$$

- Example 2: **Kullback-Leibler**

$$D_{KL}(f; g) = \int f(x) \log \frac{f(x)}{g(x)} dx \approx \frac{1}{n} \sum_{X_i \sim f} \log \frac{f(X_i)}{g(X_i)}$$

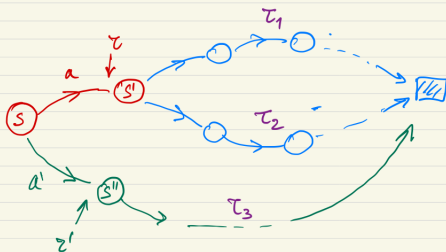
## Model-free **policy evaluation**: MC-based estimation of $v_\pi$ and $q_\pi(s, a)$

- Pick a **starting state**  $s$  (at **random** or systematic **sweep**)
- **Estimation of  $v_\pi(s)$** 
  - Use **policy  $\pi$**  to generate the **initial and all subsequent actions** (till terminal state)
  - Compute total return  $r_{tot} = r_1 + r_2 + \dots + r_T$  along path:
  - $r_{tot}$  yields **one sample value** for  $v_\pi(s)$
- **Estimation of  $q_\pi(s, a)$** 
  - Apply **action  $a$**  in state  $s$ , observe reward  $r_1$  and new state  $s'$
  - Use **policy  $\pi$**  to generate **all subsequent actions** (from  $s'$  till terminal state)
  - Compute total return  $r_{tot} = r_1 + r_2 + \dots + r_T$  along path:
  - $r_{tot}$  yields **one sample value** for  $q_\pi(s, a)$



# Monte-Carlo for model-free **policy** evaluation

Monte Carlo: using **sampled** episodes to estimate  $q(s, a)$ !



$$G(\tau_1) = \sum_{\tau_1} z_i \quad \left. \begin{array}{l} G(\tau_1) = \sum_{\tau_1} z_i \\ G(\tau_2) = \sum_{\tau_2} z_i \end{array} \right\} \rightarrow q_{\pi}(s, a)$$

$$G(\tau_3) = \sum_{\tau_3} z_i \rightarrow q_{\pi}(s, a')$$

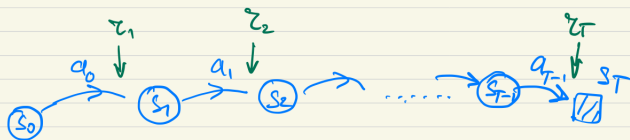
If  $a, a' \sim \pi$ :

$$G(\tau_1), G(\tau_2), G(\tau_3) \rightarrow V_{\pi}(s)$$

## MC estimation of $q(s, a)$ along trajectory

$$\tau = \{ \underbrace{s_0, a_0, r_1}_{\text{trajectory}}, \underbrace{s_1, a_1, r_2, s_2, a_2, \dots}_{\text{trajectory}}, \underbrace{s_{T-1}, a_{T-1}, r_T, s_T}_{\text{trajectory}} \}$$

↳ trajectory

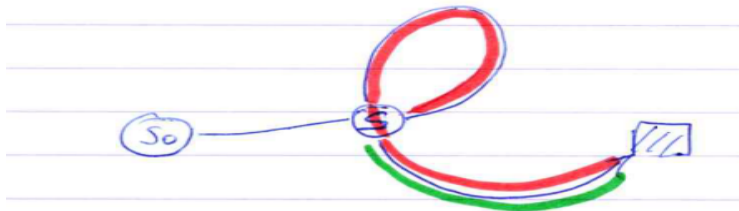


$$\begin{array}{ccccccc} G_0 & \rightarrow & G_1 & \rightarrow & G_2 & \rightarrow & \dots \\ \downarrow & & \downarrow & & \downarrow & & \\ q_{\pi}(s_0, a_0) & & q_{\pi}(s_1, a_1) & & q_{\pi}(s_2, a_2) & & \dots \end{array}$$

## MC policy evaluation: **First** versus **every visit** estimate

???

- **First-visit MC**: average returns only for the first time  $s$  is visited in an episode
- **Every-visit MC**: average returns for every time  $s$  is visited in an episode:
  - More sample efficient: more samples per episode;
  - Samples **no longer independent**
- Both converge asymptotically 渐进的

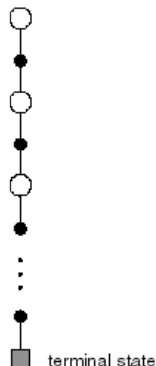


## Monte-Carlo estimation of $q$ -values

- Can learn  $q_\pi$  by averaging returns obtained when following  $\pi$  after taking action  $a$  in state  $s$
- Converges asymptotically if **every  $(s, a)$  visited infinitely often**
  - Requires **explicit exploration** of actions **not favored by  $\pi$**
  - **Possible solutions:**
    - **Exploring starts:** every  $(s, a)$  has a non-zero probability of being the starting pair
    - **Soft policies:**  $\pi(a | s) > 0$  for all  $(s, a)$ . E.g.  $\epsilon$ -greedy

## Monte-Carlo backup diagram

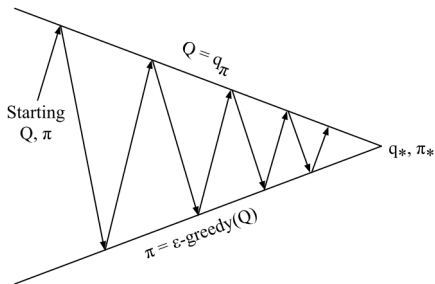
- Unlike dynamic programming, MC has only one choice at each state (i.e. the one actually taken!)
- Unlike dynamic programming, entire episode included: MC does not bootstrap ? ?
- Bellman equations are **NOT** used!



# Model-free MC: From Evaluation to Control

**Control:** Find optimal policy  $\pi^*$ :

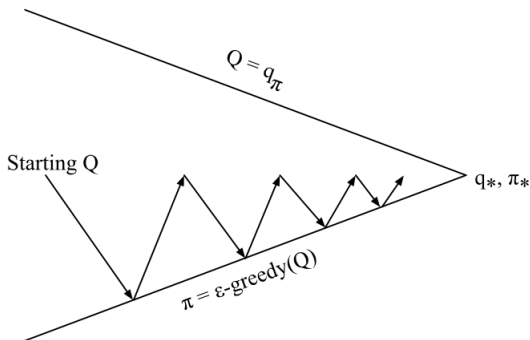
- Combine **MC-based policy evaluation** with **improvement** (e.g. **greedification**)
- Introduce **exploration** in policy
  - (e.g.  $\epsilon$ -greedy)



Policy evaluation Monte-Carlo policy evaluation,  $Q = q_\pi$

Policy improvement  $\epsilon$ -greedy policy improvement

## Monte Carlo for Model-free Control (2)



Every episode:

Policy evaluation Monte-Carlo policy evaluation,  $Q \approx q_\pi$

Policy improvement  $\epsilon$ -greedy policy improvement

# Monte-Carlo for **model-free estimation and control**

- MC provides one way to perform **model-free reinforcement learning** (RL): finding optimal policies without an explicit model for the MDP;
- MC for RL learns from **complete sample returns** in episodic tasks:
- Computes **value functions** using **direct sampling rather than Bellman equations**;



## Convergence condition: Greedy in the Limit with Infinite Exploration (GLIE)

### Def (GLIE)

- All state-action pairs are explored infinitely often:

$$\lim_{t \rightarrow \infty} N_t(s, a) = +\infty.$$

- The policy converges to a greedy policy:

$$\lim_{t \rightarrow \infty} \pi(a | s) = \begin{cases} 1 & \text{if } a = \arg \max_{a'} q(s, a') \\ 0 & \text{otherwise} \end{cases}$$

**Example:**  $\epsilon$ -greedy is GLIE iff  $\epsilon \downarrow 0$ .

## GLIE Monte Carlo Model-free Control

- Sample  $k$ th episode using  $\pi$ :  $\{S_1, A_1, R_2, \dots, S_T\} \sim \pi$
- For each state  $S_t$  and action  $A_t$  in the episode,

$$N(S_t, A_t) \leftarrow N(S_t, A_t) + 1$$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)} (G_t - Q(S_t, A_t))$$

- Improve policy based on new action-value function

$$\epsilon \leftarrow 1/k$$

$$\pi \leftarrow \epsilon\text{-greedy}(Q)$$

### Theorem

*GLIE Monte-Carlo control converges to the optimal action-value function,  $Q(s, a) \rightarrow q_*(s, a)$*

# Control based on MC+exploring starts: Algorithm

## Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize:

$\pi(s) \in \mathcal{A}(s)$  (arbitrarily), for all  $s \in \mathcal{S}$

$Q(s, a) \in \mathbb{R}$  (arbitrarily), for all  $s \in \mathcal{S}$ ,  $a \in \mathcal{A}(s)$

$Returns(s, a) \leftarrow$  empty list, for all  $s \in \mathcal{S}$ ,  $a \in \mathcal{A}(s)$

Loop forever (for each episode):

Choose  $S_0 \in \mathcal{S}$ ,  $A_0 \in \mathcal{A}(S_0)$  randomly such that all pairs have probability  $> 0$

Generate an episode from  $S_0, A_0$ , following  $\pi$ :  $S_0, A_0, R_1, \dots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop for each step of episode,  $t = T-1, T-2, \dots, 0$ :

$G \leftarrow \gamma G + R_{t+1}$

Unless the pair  $S_t, A_t$  appears in  $S_0, A_0, S_1, A_1, \dots, S_{t-1}, A_{t-1}$ :

Append  $G$  to  $Returns(S_t, A_t)$

$Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$

$\pi(S_t) \leftarrow \operatorname{argmax}_a Q(S_t, a)$

**Notice:** exploration based on **exploring starts**, not  $\epsilon$ -greedy!

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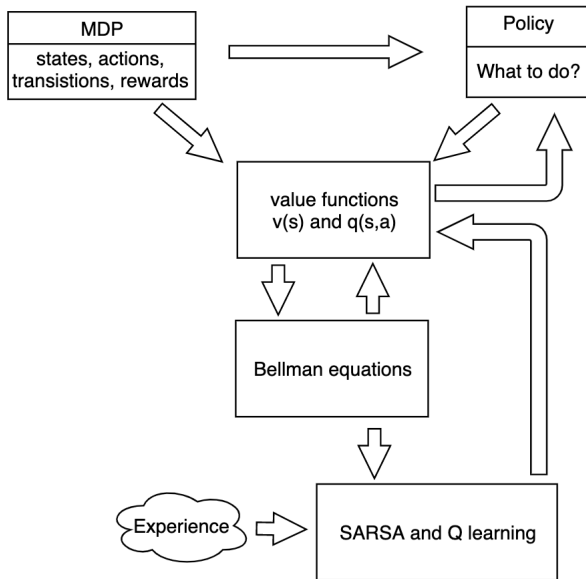
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# Solving model-free RL using Temporal Differencing

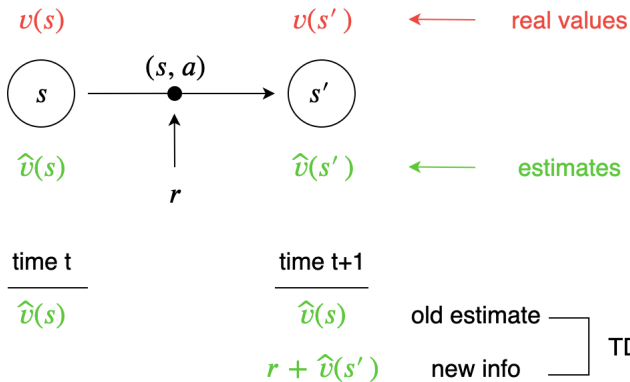
- By **acting in the world**, the agent gains experience.  
An **experience** can be expressed as a 4-tuple:  $(s, a, r, s')$
- Over time the agent collects a list of experiences that he uses to find value functions (and corresponding policy) ... HOW?
- **Key observations:**
  - Bellman eqs. link values in neighbouring states along paths!
  - This **backing-up improves learning** efficiency!
  - Basis for RL algo's (SARSA, Q-learning, etc)

Temporal Differencing (TD): Use Bellman eqs to propagate values!

# Exploiting the Bellman equations



# Temporal-differencing: Exploiting Bellman eqs.



The 1-step reward  $r$  is new information gained from experience.

# Temporal differencing for $v$ : Exploiting Bellman eqs.

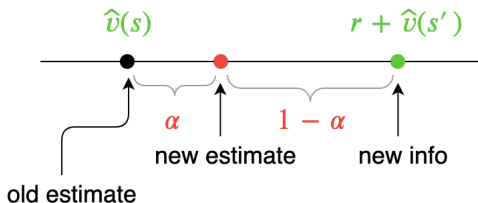
time  $t$  $\hat{v}(s)$ time  $t+1$  $\hat{v}(s)$ 

old estimate

 $r + \hat{v}(s')$ 

new info

TD



$$\hat{v}_{\text{new}}(s) = (1 - \alpha)\hat{v}_{\text{old}}(s) + \alpha(r + \hat{v}(s'))$$



# Temporal Differencing (TD) versus Monte Carlo (MC)

Given **learning rate**  $\alpha$

- **MC update rule:**

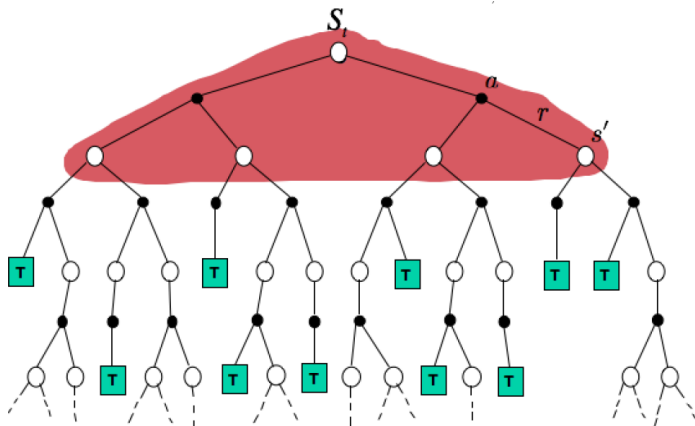
$$\begin{aligned}v_{\pi}^{new}(S_t) &\leftarrow (1 - \alpha)v_{\pi}^{old}(S_t) + \alpha G_t(S_t) \\v_{\pi}^{new}(S_t) &\leftarrow v_{\pi}^{old}(S_t) + \alpha [G_t(S_t) - v_{\pi}^{old}(S_t)]\end{aligned}$$

- **TD update rule:** uses a different **update target**:

$$\begin{aligned}v_{\pi}^{new}(S_t) &\leftarrow (1 - \alpha)v_{\pi}^{old}(S_t) + \alpha [R_{t+1} + \gamma v_{\pi}(S_{t+1})] \\v_{\pi}^{new}(S_t) &\leftarrow v_{\pi}^{old}(S_t) + \alpha [R_{t+1} + \gamma v_{\pi}(S_{t+1}) - v_{\pi}^{old}(S_t)]\end{aligned}$$

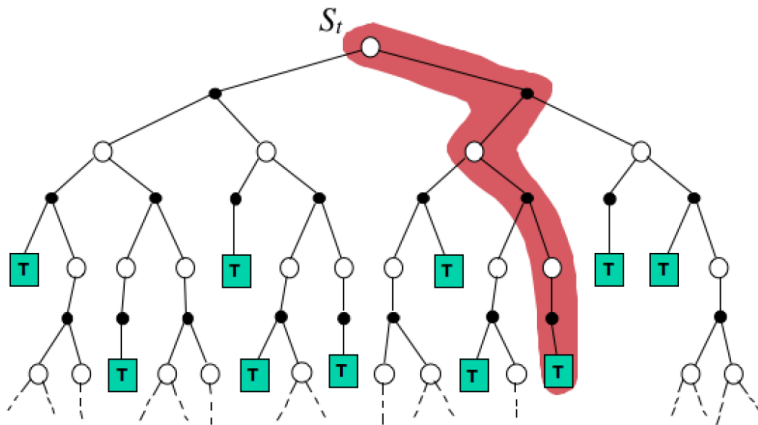
- TD is a **bootstrapping method**:  
bases **updates on existing estimates**, like DP

# Dynamic Programming (DP): Backup diagram



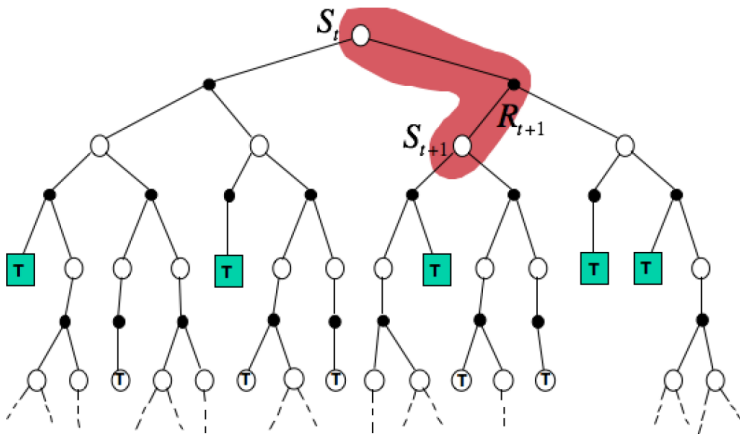
## Monte-Carlo (MC): Backup diagram

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$



# Temporal Difference (TD): Backup diagram

$$V(S_t) \leftarrow V(S_t) + \alpha (R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$



# Temporal-difference (TD) methods

- **DP** exploits Bellman equation but **requires model**
- **MC** doesn't require model but **doesn't exploit Bellman equation**
- **TD methods** can get the best of both worlds: **exploit Bellman equation without requiring a model**
- TD is therefore **core algorithm** for **model-free RL**

## Policy evaluation: Estimating $v_\pi$ using TD(0)

Initialize  $V(s)$  arbitrarily,  $\pi$  to the policy to be evaluated

Repeat (for each episode):

    Initialize  $s$

    Repeat (for each step of episode):

$a \leftarrow$  action given by  $\pi$  for  $s$

        Take action  $a$ ; observe reward,  $r$ , and next state,  $s'$

$V(s) \leftarrow V(s) + \alpha[r + \gamma V(s') - V(s)]$

$s \leftarrow s'$

    until  $s$  is terminal

## TD(0) backup diagram

- Sampling: unlike DP but like MC, only one choice at each state
- Bootstrapping: like DP but unlike MC, use estimate from next state

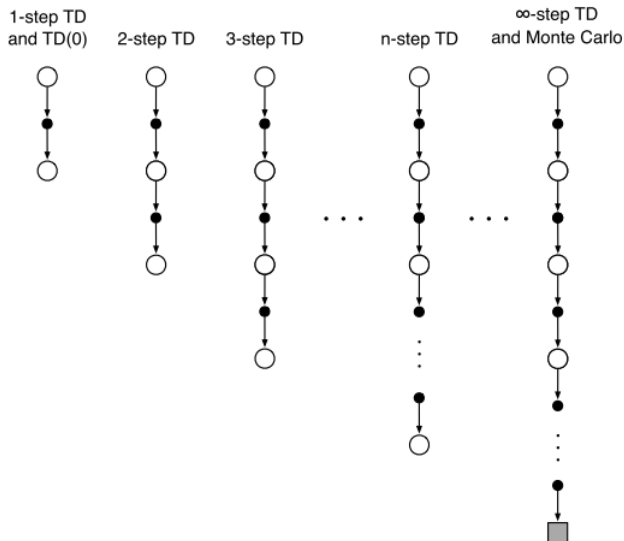


# Advantages of TD prediction methods

- TD methods require **only experience, not a model**
- TD, but not MC, methods can be **fully incremental**
- Learn **before knowing the final outcome**:
  - **Efficient**: less memory and peak computation
  - Learn from **incomplete sequences**
- Both MC and TD converge but TD tends to be faster



## n-step TD: Spectrum between TD(0) and Monte Carlo



## Issues when addressing model-free RL problems

### 1. We need to compute $q(s, a)$ rather than $v(s)$

- To **improve policy**, for every  $s$ , need to know  $\max_a q(s, a)$ :

construct  $\pi : s \mapsto a_{\max}$       where       $q(s, a_{\max}) = \max_{a'} q(s, a')$

- **Model-based:**  $q(s, a)$  can be computed from  $v(s)$ :

$$q(s, a) = r(s, a, s') + v(s')$$

- **Model-free:**  $q(s, a)$  needs to be estimated explicitly!

Hence: SARSA and Q-learning focus on  $q(s, a)$

### 2. Keep exploring!

- Balance exploration versus exploitation
- E.g.  $\epsilon$ -greedy, soft-max, etc.

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## Apply TD methods for estimating $q(s, a)$

**TD methods** Exploit correlations between **successor** states:

- **State value function**  $v_\pi(s)$

$$v_\pi(S_t) \leftarrow v_\pi(S_t) + \underbrace{\alpha(R_{t+1} + \gamma v_\pi(S_{t+1}) - v_\pi(S_t))}_{\text{new info}}$$

- **State value function**  $q_\pi(s, a)$ :

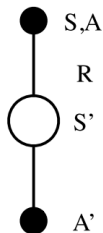
- **SARSA**: policy evaluation

$$q_\pi(s, a) \leftarrow ??$$

- **Q-learning**: **optimal** state-action value

$$q^*(s, a) \leftarrow ??$$

## SARSA: TD estimation of Q-values



- **SARSA** In state  $s$ , take action  $a$ , transition to state  $s'$ , use policy  $\pi$  to select next action  $a'$
- **Bellman:** Two estimate for state-action value:  
 $q_{\pi}(s, a)$     and     $r(s, a, s') + q_{\pi}(s', a')$

## SARSA: TD for action values

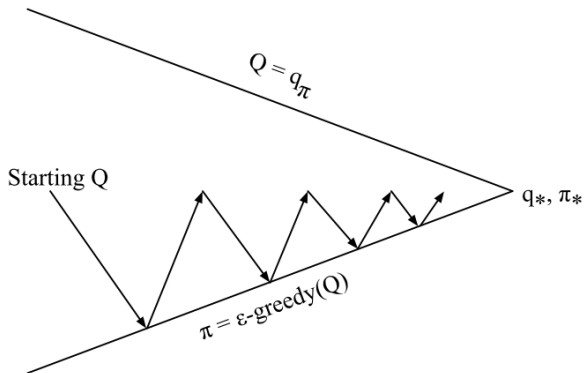
- **TD:** bootstrap update for **value function**  $v_\pi$

$$v_\pi(s) \leftarrow v_\pi(s) + \alpha \underbrace{(r(s, a, s') + \gamma v_\pi(s'))}_{\text{new info}} - v_\pi(s)$$

- **SARSA:** bootstrap update for **action value function**  $q_\pi$

$$q_\pi(s, a) \leftarrow q_\pi(s, a) + \alpha \underbrace{(r(s, a, s') + q_\pi(s', a'))}_{\text{new info}} - q_\pi(s, a)$$

# SARSA model-free control: Schematically



Every **time-step**:

Policy evaluation **Sarsa**,  $Q \approx q_\pi$

Policy improvement  $\epsilon$ -greedy policy improvement

# SARSA: Pseudo-code for model-free control

Initialize  $Q(s, a)$  arbitrarily

Repeat (for each episode):

    Initialize  $s$

    Choose  $a$  from  $s$  using policy derived from  $Q$  (e.g.,  $\epsilon$ -greedy)

    Repeat (for each step of episode):

        Take action  $a$ , observe  $r, s'$

        Choose  $a'$  from  $s'$  using policy derived from  $Q$  (e.g.,  $\epsilon$ -greedy)

$Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma Q(s', a') - Q(s, a)]$

$s \leftarrow s'; a \leftarrow a';$

    until  $s$  is terminal



# SARSA convergence

## Theorem

*Sarsa converges to the optimal action-value function,  $Q(s, a) \rightarrow q_*(s, a)$ , under the following conditions:*

- *GLIE sequence of policies  $\pi_t(a|s)$*
- *Robbins-Monro sequence of step-sizes  $\alpha_t$*

$$\sum_{t=1}^{\infty} \alpha_t = \infty$$

$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

## Applying TD to model-free RL problems

**Recall:** **Model-free**, hence focus on  $q(s, a)$  and use Bellman!

### 1. **SARSA:** **Evaluating** a given policy $\pi$

- $q_{\pi}(s, a) = \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \sum_{a'} \pi(a' | s') q_{\pi}(s', a') \right]$
- **Sample:**  $q_{\pi}(s, a) = r(s, a, s') + q_{\pi}(s', a')$
- **SARSA update rule** (sample-based):

$$q_{\pi}(s, a) \leftarrow (1 - \alpha) q_{\pi}(s, a) + \alpha (r(s, a, s') + q_{\pi}(s', a'))$$

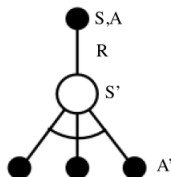
### 2. **Q-learning:** Estimating the **optimal** value function $q^*(s, a)$

- $q^*(s, a) = \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \max_{a'} q^*(s', a') \right]$
- **Sample:**  $q^*(s, a) = r(s, a, s') + \max_{a'} q^*(s', a')$
- **Q-learning update rule** (sample-based):

$$q^*(s, a) \leftarrow (1 - \alpha) q^*(s, a) + \alpha (r(s, a, s') + \max_{a'} q^*(s', a'))$$

# Q-learning:

Bootstrap with **best** action, not actual action



$$Q(S, A) \leftarrow Q(S, A) + \alpha \left( R + \gamma \max_{a'} Q(S', a') - Q(S, A) \right)$$

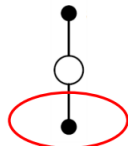
Convergence when TD-error vanishes:

Recall: optimality equation for  $q^*$ :

$$q^*(s, a) = \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \gamma \max_{a'} q^*(s', a') \right]$$

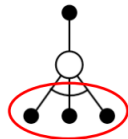
# SARSA vs. Q-learning: On-Policy vs. Off-Policy

Sarsa:



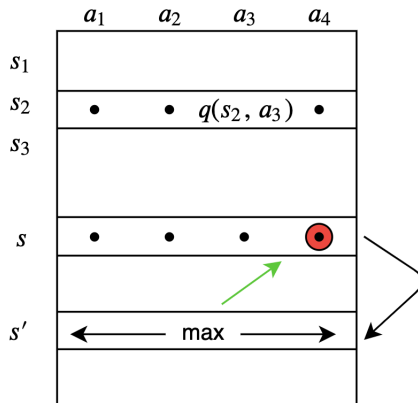
$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

Q-learning:



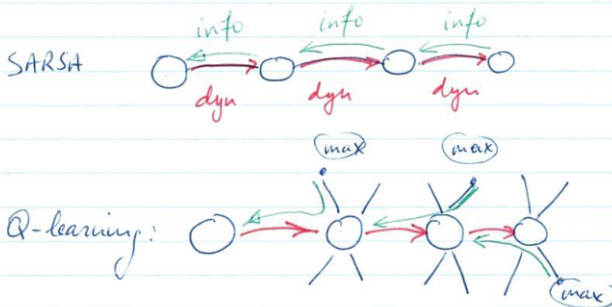
$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

# Q-learning



# SARSA (on-policy) vs. Q-Learning (off-policy)

SARSA vs. Q-learning  
(ON- vs OFF-Policy)



**Q-learning:** **dynamics** policy different from **backup** policy

# ON-policy vs. OFF-policy

## ON-policy

- Info gathered to improve policy, depends on that policy;

Feedback loop: policy  $\longleftrightarrow$  data

- **Sample inefficient:** experiences  $(s, a, r, s', a')$  cannot be re-used when policy changes since  $a'$  depends on actual policy!

## OFF-policy

- **Improved sample efficiency:** can re-use all samples for training;
- E.g. in Q-learning: use **data generated by dynamics policy  $\pi$**  to learn **value function for optimal policy  $\pi^*$** .

## Q-learning ALGO: off-policy TD control

Make TD off-policy: bootstrap with best action, not actual action:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Initialize  $Q(s, a)$  arbitrarily

Repeat (for each episode):

    Initialize  $s$

    Repeat (for each step of episode):

        Choose  $a$  from  $s$  using policy derived from  $Q$  (e.g.,  $\varepsilon$ -greedy)

        Take action  $a$ , observe  $r, s'$

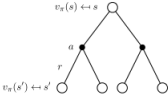

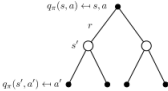

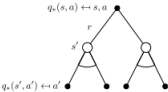
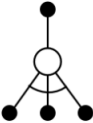
$Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$

$s \leftarrow s'$ ;

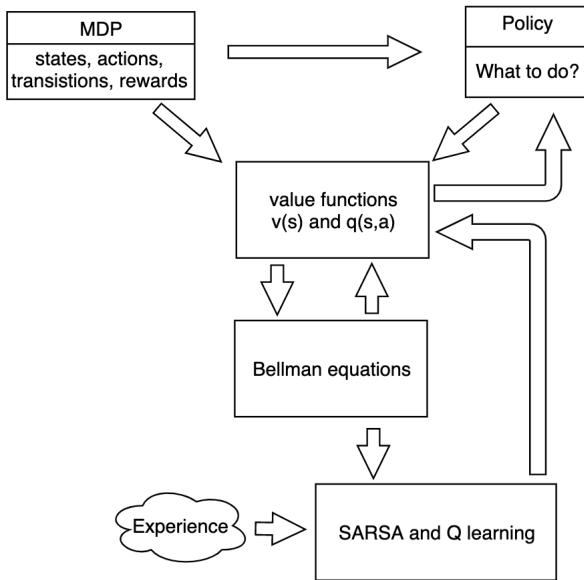
    until  $s$  is terminal



# Dynamic Program<sup>ing</sup> (DP) vs. Temporal Diff<sup>ing</sup> (TD)

	Full Backup (DP)	Sample Backup (TD)
Bellman Expectation Equation for $v_{\pi}(s)$	 <p>Iterative Policy Evaluation</p>	 <p>TD Learning</p>
Bellman Expectation Equation for $q_{\pi}(s, a)$	 <p>Q-Policy Iteration</p>	 <p>Sarsa</p>
Bellman Optimality Equation for $q_{*}(s, a)$	 <p>Q-Value Iteration</p>	 <p>Q-Learning</p>

## Summary



## Summary: Model-based versus Model-free

	<b>Prediction</b> <i>Estimation:</i> <i>Given <math>\pi</math>, what is <math>v</math>?</i>	<b>(Optimal) Control</b> <i>Optimisation:</i> <i>What is optimal <math>\pi</math>?</i>
model-based (MDP given)	Policy evaluation using Dyn. Programming (DP)	Policy improvement (+ Policy evaluation) = Policy iteration
model-free (MDP unknown)	Monte Carlo (MC) Temporal Diff <sup>ing</sup> (TD) = "impatient MC" <i>bootstrapping!</i>	Generalized Policy Iteration <i>"simultaneous"</i>

# Outline

Model-based versus Model-free

Monte Carlo for Model-free Policy Evaluation

Temporal Difference Methods for Model-free Prediction

SARSA and Q-Learning for Model-free Control

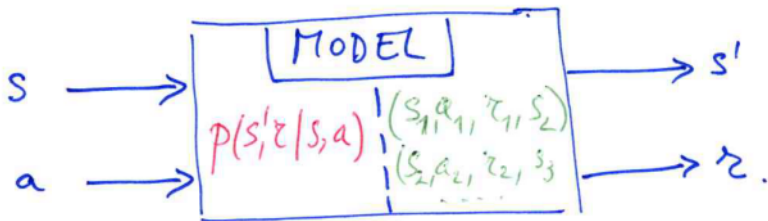
Integrating Planning and Learning

## Dyna-Q; Integrating planning and learning

**Model:** tells agent what will happen next ...

- **Model-based:** planning
- **Model-free:** learning

### Distributional vs. Sample-based Model

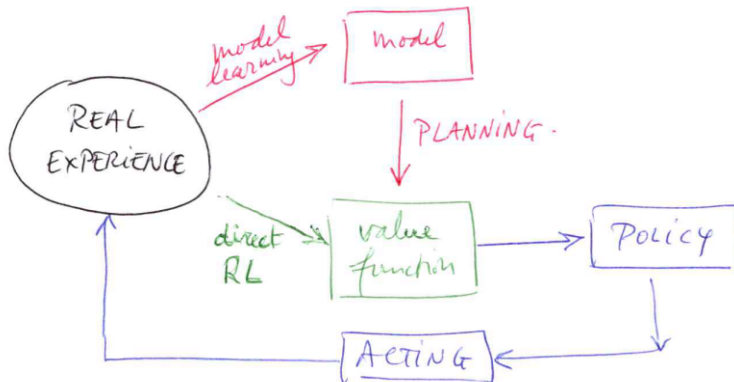


## Model-based learning

- Until now: **Model = fully specified MDP!**
- More generally: **anything that helps the agent to plan:**
- More **accurate models** are **more effective** (myths vs. science):
  - **Folklore** and weather saying:  
*A wet and windy May fills the barn with corn and hay.*
  - Meteorological models running on **supercomputer**

# Dyna-Q; Integrating planning and learning

Real experience can be used in two ways:



## DYNA-Q: Algorithm

Initialize  $Q(s, a)$  and  $Model(s, a)$  for all  $s \in \mathcal{S}$  and  $a \in \mathcal{A}(s)$

Do forever:

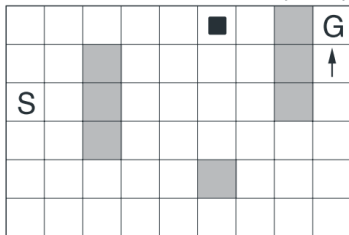
- (a)  $S \leftarrow$  current (nonterminal) state
- (b)  $A \leftarrow \varepsilon$ -greedy( $S, Q$ )
- (c) Execute action  $A$ ; observe resultant reward,  $R$ , and state,  $S'$
- (d)  $Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$
- (e)  $Model(S, A) \leftarrow R, S'$  (assuming deterministic environment)
- (f) Repeat  $n$  times:
  - $S \leftarrow$  random previously observed state
  - $A \leftarrow$  random action previously taken in  $S$
  - $R, S' \leftarrow Model(S, A)$
  - $Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$



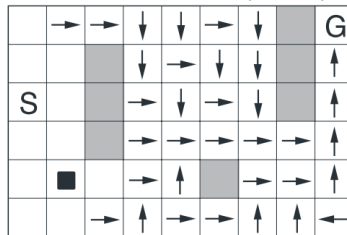
## DYNA-Q: Maze example

Greedy policy midway through 2nd episode:

WITHOUT PLANNING ( $n=0$ )



WITH PLANNING ( $n=50$ )



More info: Sutton and Barto, sections 8.2-8.3