Multi-Agent Systems Introduction to Reinforcement Learning

Model-free Methods:

Monte Carlo, SARSA and Q-learning

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Reading

• Sutton & Barto: chapters 5 & 6

Outline

Model-based versus Model-free

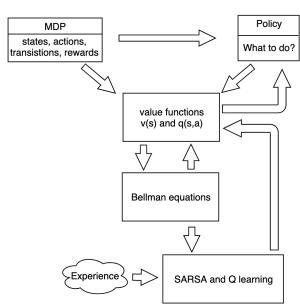
Model-based versus Model-free

	Prediction Estimation:	(Optimal) Control Optimisation:
	Given π , what is v?	What is optimal π ?
model-based	Policy evaluation	Policy improvement
(MDP given)	using	(+ Policy evaluation)
	Dyn. Programming (DP)	= Policy iteration
model-free	Monte Carlo (MC)	Q-learning and
(MDP unknown)	Temporal Diff ^{ing} (TD)	Generalized
	= "impatient MC"	Policy Iteration
	bootstrapping!	"simultaneous"

Solving model-free RL problems

- The agent has no prior knowledge about states, actions, rewards, transitions!
- By acting in the world, the agent gains experience.
 An experience can be expressed as a 4-tuple: (s, a, r, s')
- Over time the agent collects a list of experiences that he uses to find better policies ... HOW?
 - **Directly:** policy improvement
 - Indirectly: estimate value functions, improve policy using greedification;

Overview



Definition of Greedification

• For a given action-value function q(s, a), a corresponding greedy policy is a *deterministic* policy that picks (one of the) actions that maximise the action value:

$$\pi_g(s) = a^* := \arg\max_a q(s, a).$$

Greedification results in policy improvement

• To improve the policy, apply successive iteration steps:

$$\pi_k \stackrel{eval}{\longrightarrow} v_k, q_k \stackrel{greedify}{\longrightarrow} \pi_{k+1}$$

Greedification implies deterministic action choice at each s:

$$\pi_{k+1}(s) = a^*$$
 iff $q_k(s, a^*) = \max_a q_k(s, a)$

Hence value function increases, i.e. policy has been improved!

$$v_{k+1}(s) = q_k(s, a^*)$$
 $(\pi_{k+1} \text{ is deterministic at } s)$

$$= \max_a q_k(s, a)$$

$$\geq v_k(s) \qquad (= \sum_{a'} \pi_k(a' \mid s) q_k(s, a'))$$

Greedification requires q(s, a)!

model-free vs. model-based

Greedification: $\pi(s) := \arg \max_a q(s, a)$

Model-based: (a.k.a. planning, search)

- Suffices to estimate value function v(s):
- q(s, a) computed with Bellman's one-step look-ahead (i.e. use back-up diagram):

$$q(s,a) = \sum_{s'} p(s' \mid s,a) \left[r(s,a,s') + \gamma v(s') \right]$$

Model-free: (a.k.a. (optimal) control)

- q(s, a) needs to be estimated from collected experiences;
- algorithms shift focus from v(s) to q(s, a);

Conclusion: for model-free RL

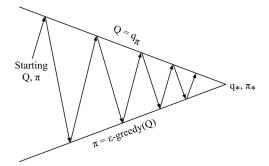
In contrast with to model-based, model-free RL is distinct:

- Focus on q(s, a) rather than v(s) Why??
- Make sure all state-action pairs (s,a) are sampled: importance of exploration!

Policy Iteration (PI) for Model-free RL

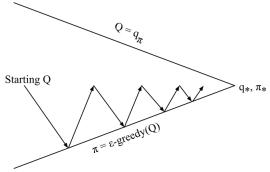
Goal: Find optimal policy π^* (aka optimal control)

- PI: Combine policy evaluation with policy improvement
 - Make Q-function consistent with policy π , i.e. $Q=q_{\pi}$;
 - Greedify policy: $\pi = \text{greedy}(Q)$;
- Introduce exploration in policy (e.g. $\pi = \varepsilon$ -greedy(Q));

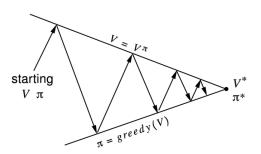


Generalized Policy Iteration (GPI)

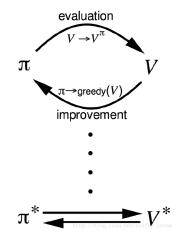
- Impatient greedification: No insistence on making Q-function fully consistent with current policy π ; Hence $Q \approx q_{\pi}$, but not necessarily $Q = q_{\pi}$;
- Ensure exploration: use $\pi = \varepsilon$ -greedy(Q), not $\pi = \text{greedy}(Q)$;
- Works if both processes continue updating all states;



Generalized Policy Iteration (GPI) schematically



Policy evaluation Estimate v_{π} Any policy evaluation algorithm Policy improvement Generate $\pi' > \pi$ Any policy improvement algorithm



Outline

Model-based versus Model-free

Monte Carlo for Model-free Policy Evaluation

Temporal Difference Methods for Model-free Prediction

SARSA and Q-Learning for Model-free Control

Integrating Planning and Learning

Monte Carlo methods

- Monte Carlo methods are versatile statistical techniques for estimating properties of complex systems via random sampling;
- Example 1: if $X \sim p(x)$ and φ some function, then for any (sufficiently large) i.i.d. sample: X_1, X_2, \dots, X_n :

$$E(\varphi(X)) = \int \varphi(x) p(x) dx \approx \frac{1}{n} \sum_{X_i \sim p} \varphi(X_i)$$

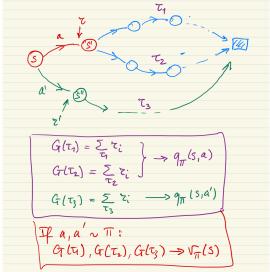
Example 2: Kullback-Leibler

$$D_{KL}(f;g) = \int f(x) \log \frac{f(x)}{g(x)} dx \approx \frac{1}{n} \sum_{X_i \sim f} \log \frac{f(X_i)}{g(X_i)}$$

Model-free **policy evaluation**: MC-based estimation of v_{π} and $q_{\pi}(s, a)$

- Pick a starting state s (at random or systematic sweep)
- Estimation of $v_{\pi}(s)$
 - Use policy π to generate the initial and all subsequent actions (till terminal state)
 - Compute total return $r_{tot} = r_1 + r_2 + ... + r_T$ along path:
 - r_{tot} yields one sample value for $v_{\pi}(s)$
- Estimation of $q_{\pi}(s, a)$
 - Apply action a in state s, observe reward r_1 and new state s'
 - Use policy π to generate all subsequent actions (from s' till terminal state)
 - Compute total return $r_{tot} = r_1 + r_2 + ... + r_T$ along path:
 - r_{tot} yields one sample value for $q_{\pi}(s, a)$

Monte-Carlo for model-free policy evaluation Monte Carlo: using **sampled** episodes to estimate q(s, a)!



MC estimation of q(s, a) along trajectory

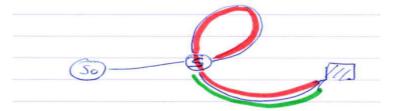
$$T = \begin{cases} S_{01} a_{01} \tau_{11} S_{11} a_{11}, \tau_{12}, S_{21} a_{22} \dots S_{7-1}, a_{7-1}, \tau_{7}, S_{7} \end{cases}$$

$$\Rightarrow \text{trajectry}$$

$$\begin{cases} T_{11} & T_{12} & T_{13} \\ T_{11} & T_{12} & T_{13} \\ T_{12} & T_{13} & T_{13} \\ T_{13} & T_{14} & T_{14} \\ T_{15} & T_{15} & T_{15} \\ T_{15} & T_{15}$$

MC policy evaluation: First versus every visit estimate

- **First-visit MC**: average returns only for the first time *s* is visited in an episode
- **Every-visit MC**: average returns for every time *s* is visited in an episode:
 - More sample efficient: more samples per episode;
 - Samples no longer independent
- Both converge asymptotically 渐进的

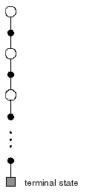


Monte-Carlo estimation of *q*-values

- Can learn q_{π} by averaging returns obtained when following π after taking action a in state s
- Converges asymptotically if every (s, a) visited infinitely often
 - Requires explicit exploration of actions not favored by π
 - Possible solutions:
 - Exploring starts: every (s, a) has a non-zero probability of being the starting pair
 - **Soft policies**: $\pi(a|s) > 0$ for all (s, a). E.g. ϵ -greedy

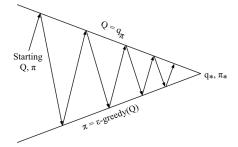
Monte-Carlo backup diagram

- Unlike dynamic programming, MC has only one choice at each state (i.e. the one actually taken!)
- Unlike dynamic programming, entire episode included: MC does not bootstrap ??
- Bellman equations are NOT used!



Model-free MC: From **Evaluation** to **Control Control**: Find optimal policy π^* :

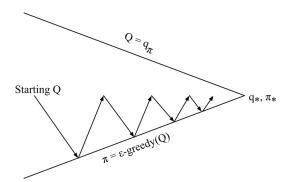
- Combine MC-based policy evaluation with improvement (e.g. greedification)
- Introduce exploration in policy
 - (e.g. ϵ -greedy)



Policy evaluation Monte-Carlo policy evaluation, $Q=q_\pi$

Policy improvement *e*-greedy policy improvement

Monte Carlo for Model-free Control (2)



Every episode:

Policy evaluation Monte-Carlo policy evaluation, $Q \approx q_{\pi}$ Policy improvement ϵ -greedy policy improvement

Monte-Carlo for model-free estimation and control

- MC provides one way to perform model-free reinforcement learning (RL): <u>finding optimal policies without an explicit</u> model for the MDP;
- MC for RL learns from complete sample returns in episodic tasks:
- Computes value functions using direct sampling rather than Bellman equations;

Convergence condition:

Greedy in the Limit with Infinite Exploration (GLIE)

Def (GLIE)

All state-action pairs are explored infinitely often:

$$\lim_{t\to\infty}N_t(s,a)=+\infty.$$

The policy converges to a greedy policy:

$$\lim_{t \to \infty} \pi(a \, | \, s) = \left\{ egin{array}{ll} 1 & ext{ if } \quad a = rg \max_{a'} q(s, a') \\ 0 & ext{ otherwise} \end{array}
ight.$$

Example: ϵ -greedy is GLIE iff $\epsilon \downarrow 0$.

GLIE Monte Carlo Model-free Control

- Sample kth episode using π : $\{S_1, A_1, R_2, ..., S_T\} \sim \pi$
- For each state S_t and action A_t in the episode,

$$egin{aligned} & \mathcal{N}(S_t, A_t) \leftarrow \mathcal{N}(S_t, A_t) + 1 \ & Q(S_t, A_t) \leftarrow Q(S_t, A_t) + rac{1}{\mathcal{N}(S_t, A_t)} \left(G_t - Q(S_t, A_t)
ight) \end{aligned}$$

Improve policy based on new action-value function

$$\epsilon \leftarrow 1/k$$
 $\pi \leftarrow \epsilon$ -greedy(Q)

Theorem

GLIE Monte-Carlo control converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$

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Control based on MC+exploring starts: Algorithm

```
Monte Carlo ES (Exploring Starts), for estimating \pi \approx \pi_*
Initialize:
    \pi(s) \in \mathcal{A}(s) (arbitrarily), for all s \in \mathcal{S}
    Q(s,a) \in \mathbb{R} (arbitrarily), for all s \in S, a \in A(s)
     Returns(s, a) \leftarrow \text{empty list, for all } s \in S, a \in \mathcal{A}(s)
Loop forever (for each episode):
     Choose S_0 \in \mathcal{S}, A_0 \in \mathcal{A}(S_0) randomly such that all pairs have probability > 0
    Generate an episode from S_0, A_0, following \pi: S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T
    G \leftarrow 0
     Loop for each step of episode, t = T-1, T-2, \ldots, 0:
          G \leftarrow \gamma G + R_{t+1}
          Unless the pair S_t, A_t appears in S_0, A_0, S_1, A_1, ..., S_{t-1}, A_{t-1}:
               Append G to Returns(S_t, A_t)
               Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))
               \pi(S_t) \leftarrow \operatorname{arg\,max}_a Q(S_t, a)
```

Notice: exploration based on exploring starts, not ϵ -greedy!

Outline

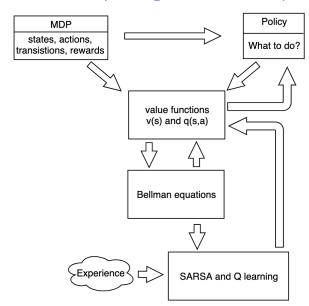
Temporal Difference Methods for Model-free Prediction

Solving model-free RL using **Temporal Differencing**

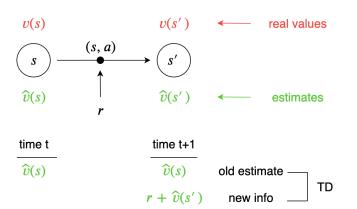
- By acting in the world, the agent gains experience.
 An experience can be expressed as a 4-tuple: (s, a, r, s')
- Over time the agent collects a list of experiences that he uses to find value functions (and corresponding policy) ... HOW?
- Key observations:
 - Bellman eqs. link values in neighbouring states along paths!
 - This backing-up improves learning efficiency!
 - Basis for RL algo's (SARSA, Q-learning, etc)

Temporal Differencing (TD): Use Bellman eqs to propagate values!

Exploiting the Bellman equations



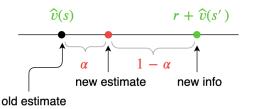
Temporal-differencing: Exploiting Bellman eqs.



The 1-step reward r is new information gained from experience.

Temporal differencing for v: Exploiting Bellman eqs.





$$\widehat{v}_{\text{new}}(s) = (1 - \alpha)\widehat{v}_{old}(s) + \alpha(r + \widehat{v}(s'))$$

Temperal Differencing (TD) versus Monte Carlo (MC)

Given learning rate α

• MC update rule:

$$v_{\pi}^{new}(S_t) \leftarrow (1-\alpha)v_{\pi}^{old}(S_t) + \alpha G_t(S_t)$$

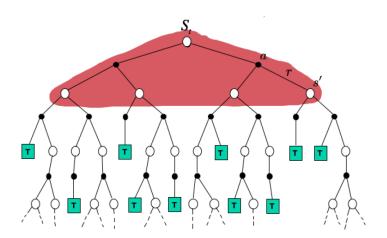
$$v_{\pi}^{new}(S_t) \leftarrow v_{\pi}^{old}(S_t) + \alpha \left[G_t(S_t) - v_{\pi}^{old}(S_t)\right]$$

TD update rule: uses a different update target:

$$v_{\pi}^{new}(S_t) \leftarrow (1 - \alpha)v_{\pi}^{old}(S_t) + \alpha[R_{t+1} + \gamma v_{\pi}(S_{t+1})]$$
$$v_{\pi}^{new}(S_t) \leftarrow v_{\pi}^{old}(S_t) + \alpha[R_{t+1} + \gamma v_{\pi}(S_{t+1}) - v_{\pi}^{old}(S_t)]$$

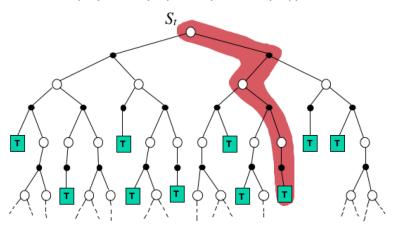
 TD is a bootstrapping method: bases updates on existing estimates, like DP

Dynamic Programming (DP): Backup diagram



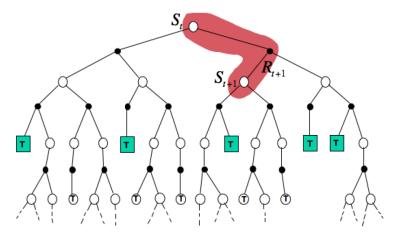
Monte-Carlo (MC): Backup diagram

$$V(S_t) \leftarrow V(S_t) + \alpha \left(\frac{G_t}{G_t} - V(S_t) \right)$$



Temporal Difference (TD): Backup diagram

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$



- DP exploits Bellman equation but requires model
- MC doesn't require model but doesn't exploit Bellman equation
- TD methods can get the best of both worlds: exploit Bellman equation without requiring a model
- TD is therefore core algorithm for model-free RL

Policy evaluation: Estimating v_{π} using TD(0)

Initialize V(s) arbitrarily, π to the policy to be evaluated Repeat (for each episode):

Initialize s

Repeat (for each step of episode):

 $a \leftarrow \text{action given by } \pi \text{ for } s$

Take action a; observe reward, r, and next state, s'

$$V(s) \leftarrow V(s) + \alpha \big[r + \gamma V(s') - V(s) \big]$$

 $s \leftarrow s'$

until s is terminal

TD(0) backup diagram

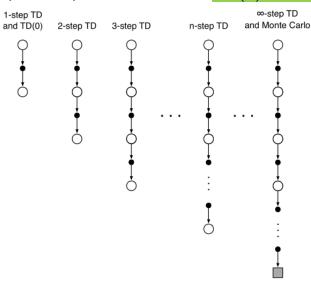
- Sampling: unlike DP but like MC, only one choice at each state
- Bootstrapping: like DP but unlike MC, use estimate from next state



Advantages of TD prediction methods

- TD methods require only experience, not a model
- TD, but not MC, methods can be fully incremental
- Learn before knowing the final outcome:
 - Efficient: less memory and peak computation
 - Learn from incomplete sequences
- Both MC and TD converge but TD tends to be faster

n-step TD: Spectrum between TD(0) and Monte Carlo



Issues when addressing model-free RL problems

- 1. We need to compute q(s, a) rather than v(s)
 - To **improve policy**, for every s, need to know max_a q(s, a):

construct
$$\pi: s \mapsto a_{max}$$
 where $q(s, a_{max}) = \max_{a'} q(s, a')$

• Model-based: q(s, a) can be computed from v(s):

$$q(s,a) = r(s,a,s') + v(s')$$

• Model-free: q(s, a) needs to be estimated explicitly!

Hence: SARSA and Q-learning focus on q(s, a)

- 2. Keep exploring!
 - Balance exploration versus exploitation
 - E.g. ε-greedy, soft-max, etc.

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Monte Carlo for Model-free Policy Evaluation

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SARSA and Q-Learning for Model-free Control

Integrating Planning and Learning

Apply TD methods for estimating q(s, a)

TD methods Exploit correlations between successor states:

• State value function $v_{\pi}(s)$

$$v_{\pi}(S_t) \leftarrow v_{\pi}(S_t) + \alpha \underbrace{(R_{t+1} + \gamma v_{\pi}(S_{t+1})}_{\text{new info}} - v_{\pi}(S_t))$$

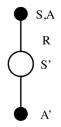
- State value function $q_{\pi}(s, a)$:
 - SARSA: policy evaluation

$$q_{\pi}(s,a) \leftarrow ??$$

• Q-learning: optimal state-action value

$$q^*(s, a) \leftarrow ??$$

SARSA: TD estimation of Q-values



- SARSA In state s, take action a, transition to state s', use policy π to select next action a'
- **Bellman:** Two estimate for state-action value: $q_{\pi}(s,a)$ and $r(s,a,s')+q_{\pi}(s',a')$

SARSA: TD for action values

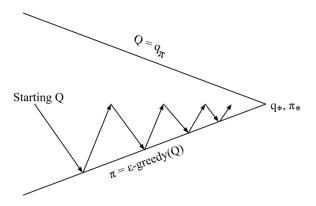
• TD: bootstrap update for value function v_{π}

$$v_{\pi}(s) \leftarrow v_{\pi}(s) + \alpha \underbrace{(r(s, a, s') + \gamma v_{\pi}(s')}_{\text{new info}} - v_{\pi}(s))$$

• SARSA: bootstrap update for action value function q_{π}

$$q_{\pi}(s, a) \leftarrow q_{\pi}(s, a) + \alpha(\underbrace{r(s, a, s') + q_{\pi}(s', a')}_{\text{new info}} - q_{\pi}(s, a))$$

SARSA model-free control: Schematically



Every time-step:

Policy evaluation Sarsa, $Q pprox q_{\pi}$

Policy improvement ϵ -greedy policy improvement

SARSA: Pseudo-code for model-free control

```
Initialize Q(s,a) arbitrarily Repeat (for each episode):

Initialize s
Choose a from s using policy derived from Q (e.g., \varepsilon-greedy)
Repeat (for each step of episode):

Take action a, observe r, s'
Choose a' from s' using policy derived from Q (e.g., \varepsilon-greedy)
Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma Q(s',a') - Q(s,a)]
s \leftarrow s'; \ a \leftarrow a';
until s is terminal
```

SARSA convergence

$\mathsf{Theorem}$

Sarsa converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$, under the following conditions:

- GLIE sequence of policies $\pi_t(a|s)$
- Robbins-Monro sequence of step-sizes α_t

$$\sum_{t=1}^{\infty} \alpha_t = \infty$$

$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

discount factor is assumed to be 1 Applying TD to model-free RL problems

Recall: Model-free, hence focus on q(s, a) and use Bellman!

- 1. **SARSA**: Evaluating a given policy π
 - $q_{\pi}(s, a) = \sum_{s'} p(s' | s, a) \left[r(s, a, s') + \sum_{a'} \pi(a' | s') q_{\pi}(s', a') \right]$
 - Sample: $q_{\pi}(s, a) = r(s, a, s') + q_{\pi}(s', a')$
 - SARSA update rule (sample-based):

$$q_{\pi}(s, a) \leftarrow (1 - \alpha)q_{\pi}(s, a) + \alpha(r(s, a, s') + q_{\pi}(s', a'))$$

- 2. **Q-learning:** Estimating the **optimal** value function $q^*(s, a)$
 - $q^*(s,a) = \sum_{s'} p(s' \mid s,a) \Big[r(s,a,s') + \max_{a'} q^*(s',a') \Big]$
 - Sample: $q^*(s, a) = r(s, a, s') + \max_{a'} q^*(s', a')$
 - Q-learning update rule (sample-based):

$$q^*(s, a) \leftarrow (1 - \alpha)q^*(s, a) + \alpha(r(s, a, s') + \max_{a'} q^*(s', a'))$$

Q-learning:

Bootstrap with **best** action, not actual action



$$Q(S,A) \leftarrow Q(S,A) + \alpha \left(R + \gamma \max_{a'} Q(S',a') - Q(S,A)\right)$$

Convergence when TD-error vanishes:

Recall: optimality equation for q^* :

$$q^*(s, a) = \sum_{s'} p(s' | s, a) \left[r(s, a, s') + \gamma \max_{a'} q^*(s', a') \right]$$

SARSA vs. Q-learning: On-Policy vs. Off-Policy

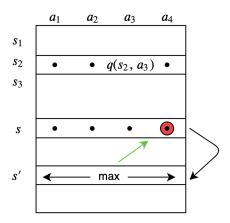
Sarsa:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

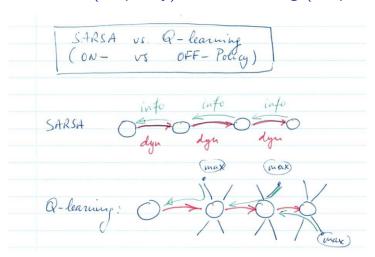
Q-learning:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma \max_{a} Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

Q-learning



SARSA (on-policy) vs. Q-Learning (off-policy)



Q-learning: dynamics policy different from backup policy

ON-policy vs. OFF-policy

ON-policy

Info gathered to improve policy, depends on that policy;

Feedback loop: $policy \longleftrightarrow data$

• Sample inefficient: experiences (s, a, r, s', a') cannot be re-used when policy changes since a' depends on actual policy!

OFF-policy

- Improved sample efficiency: can re-use all samples for training;
- E.g. in Q-learning: use data generated by dynamics policy π to learn value function for optimal policy π^* .

Q-learning ALGO: off-policy TD control

Make TD off-policy: bootstrap with best action, not actual action:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_{a} Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Initialize Q(s, a) arbitrarily

Repeat (for each episode):

Initialize s

Repeat (for each step of episode):

Choose a from s using policy derived from Q (e.g., ε -greedy) Take action a, observe r, s'

$$Q(s, a) \leftarrow Q(s, a) + \alpha [r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$$

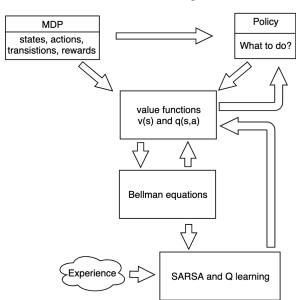
 $s \leftarrow s'$:

until s is terminal

Dynamic Program^{ing} (DP) vs. Temporal Diff^{ing} (TD)

	Full Backup (DP)	Sample Backup (TD)
Bellman Expectation	$v_{\pi}(s) \leftarrow s$ $v_{\pi}(s') \leftarrow s$ $v_{\pi}(s') \leftarrow s'$	
Equation for $v_\pi(s)$	Iterative Policy Evaluation	TD Learning
Bellman Expectation	$q_a(s, a) \leftrightarrow s, a$ r $q_a(s', a') \leftrightarrow a'$	S.A R S'
Equation for $q_{\pi}(s, a)$	Q-Policy Iteration	Sarsa
Bellman Optimality Equation for $q_*(s, a)$	$q_{\sigma}(s,a) \leftrightarrow s,a$ $q_{\sigma}(s',a') \leftrightarrow a'$ Q-Value Iteration	Q-Learning

Summary



Summary: Model-based versus Model-free

	Prediction Estimation: Given π, what is ν?	(Optimal) Control Optimisation: What is optimal π ?
model-based	Policy evaluation	Policy improvement
(MDP given)	using	(+ Policy evaluation)
,	Dyn. Programming (DP)	= Policy iteration
model-free	Monte Carlo (MC)	
(MDP unknown)	Temporal Diff ^{ing} (TD)	Generalized
	= "impatient MC"	Policy Iteration
	bootstrapping!	"simultaneous"

Outline

Model-based versus Model-free

Monte Carlo for Model-free Policy Evaluation

Temporal Difference Methods for Model-free Prediction

SARSA and Q-Learning for Model-free Control

Integrating Planning and Learning

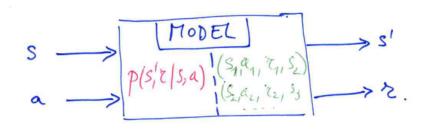
Dyna-Q; Integrating planning and learning

Model: tells agent what will happen next ...

Model-based: planning

• Model-free: learning

Distributional vs. Sample-based Model

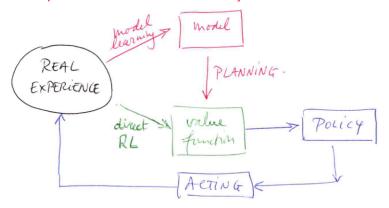


Model-based learning

- Until now: Model = fully specified MDP!
- More generally: anything that helps the agent to plan:
- More accurate models are more effective (myths vs. science):
 - Folklore and weather saying:
 A wet and windy May fills the barn with corn and hay.
 - Meteorological models running on supercomputer

Dyna-Q; Integrating planning and learning

Real experience can be used in two ways:



DYNA-Q: Algorithm

Initialize Q(s, a) and Model(s, a) for all $s \in S$ and $a \in A(s)$ Do forever:

- (a) $S \leftarrow \text{current (nonterminal) state}$
- (b) $A \leftarrow \varepsilon$ -greedy(S, Q)
- (c) Execute action A; observe resultant reward, R, and state, S'
- (d) $Q(S, A) \leftarrow Q(S, A) + \alpha \left[R + \gamma \max_{a} Q(S', a) Q(S, A) \right]$
- (e) $Model(S, A) \leftarrow R, S'$ (assuming deterministic environment)
- (f) Repeat n times:

 $S \leftarrow \text{random previously observed state}$

 $A \leftarrow$ random action previously taken in S

$$R, S' \leftarrow Model(S, A)$$

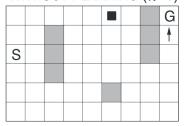
$$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_{a} Q(S', a) - Q(S, A)]$$



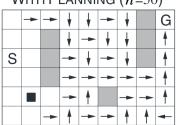
DYNA-Q: Maze example

Greedy policy midway through 2nd episode:

WITHOUT PLANNING (n=0)



WITH PLANNING (n=50)



More info: Sutton and Barto, sections 8.2-8.3