Dynamic Modeling for a Tractor-Trailer System

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Abstract

This report presents a dynamic model for a tractor-trailer system.

Nomenclature

Variables

α_i	Lumped tire slip angle of the ith axle ($i = 1, 2, ..., 5$)	rad
Δ	Articulation angle between tractor and trailer	rad
δ	Lumped steering angle	rad
$\dot{x}_1$	Longitudinal velocity of tractor in the tractor's frame	m/s
$\dot{x}_2$	Longitudinal velocity of trailer in the trailer's frame	m/s
$\dot{y}_1$	Lateral velocity of tractor in the tractor's frame	m/s
$\dot{y}_2$	Lateral velocity of trailer in the trailer's frame	m/s
ψ_1	Yaw angle of the tractor	rad
ψ_2	Yaw angle of the trailer	rad
a_{x1}	Longitudinal acceleration of the tractor	m/s^2
a_{x2}	Longitudinal acceleration of the trailer	m/s^2
a_{y1}	Lateral acceleration of the tractor	m/s^2
a_{y2}	Lateral acceleration of the trailer	m/s^2
F_a	Aerodynamic drag force	N

F_f	Rolling resistance force	N
F_g	Component of gravitational force on the slope	N
F_r	Lumped resistance force	N
F_{xi}	Longitudinal force of the ith axle ($i = 1, 2, ..., 5$)	N
F_{yi}	Lateral force of the ith axle ($i = 1, 2, ..., 5$)	N
M_{z1}	Yaw moment on the tractor	Nm
M_{z2}	Yaw moment on the trailer	Nm

Constants

g	Gravitational Acceleration	$9.81\,m/s^2$
I_1	Tractor yaw moment of inertia	$52,000\,kg \cdot m^2$
I_2	Trailer yaw moment of inertia	$39,290\,kg \cdot m^2$
l_1	Distance from tractor CG to the steering axle	$2.59\,m$
l_2	Distance from tractor CG to the front tandem axle	$2.70\,m$
l_3	Distance from tractor CG to the rear tandem axle	$4.02\,m$
l_4	Distance from trailer CG to the first trailer rear axle	$4.17\,m$
l_5	Distance from trailer CG to the second trailer rear axle	$5.41\,m$
l_6	Distance from the 5th wheel hitch to tractor CG	$3.36\,m$
l_7	Distance from the 5th wheel hitch to trailer CG	$6.32\,m$
m_1	Tractor mass	$9,000\,kg$
m_2	Trailer mass	$6,800\,kg$

1 Introduction

To derive the dynamic model for a tractor-trailer system (as defined in 1.2):

1.1 Objectives

1.2 Definitions

Tractor-Trailer The vehicle system as shown in Fig.1 is composed of two parts: tractor in the front to provide towing power, and trailer in the back to carry freight.

Degree of Freedom (DOF) 6-DOF to 4-DOF due to hitch constraints.

Coordinate System Global frame (inertial frame)

Vehicle local frame (non-inertial frame). Default is forward-left-up.

Tire frame (non-inertial frame).

State-Space Representation Used in control design, especially for linear systems.

Lagrange's Equation

Path-Following Control In contrast, there is trajectory-tracking control.

1.3 Assumptions

Small Angle Approximation Angle smaller than 10 degrees can use small angle approximation [3].

Bicycle Model Left and right wheels are lumped when deriving lateral tire forces.

2 Modeling

2.1 Vehicle Dynamics

Equations of the longitudinal motions are

$$\begin{aligned} m_1 a_{x1} &= m_1(\ddot{x}_1 - \dot{y}_1\dot{\psi}_1) = F_{x1} + F_{x2} + F_{x3} - F_{xh} - F_{r1}, && (1) \\ m_2 a_{x2} &= m_2(\ddot{x}_2 - \dot{y}_2\dot{\psi}_2) = F_{x4} + F_{x5} + F_{xh} - F_{r2}, && (2) \end{aligned}$$

where, F_{r1} and F_{r2} are lumped resistance forces on the tractor and the trailer, respectively.

$$F_r = F_f + F_a + F_g \tag{3}$$

Equations of the tractor's lateral and yaw motions:

$$\begin{aligned} m_1 a_{y1} &= m_1(\ddot{y}_1 + \dot{x}_1\dot{\psi}_1) = F_{y1} + F_{y2} + F_{y3} - F_{yh} && (4) \\ I_1\ddot{\psi}_1 &= l_1 F_{y1} - l_2 F_{y2} - l_3 F_{y3} + l_6 F_{yh} + M_{z1} && (5) \end{aligned}$$

Equations of the trailer's lateral and yaw motions:

$$\begin{aligned} m_2 a_{y2} &= m_2(\ddot{y}_2 + \dot{x}_2\dot{\psi}_2) = F_{y4} + F_{y5} + F_{yh} && (6) \\ I_2\ddot{\psi}_2 &= -l_4 F_{y4} - l_5 F_{y5} + l_7 F_{yh} + M_{z2} && (7) \end{aligned}$$

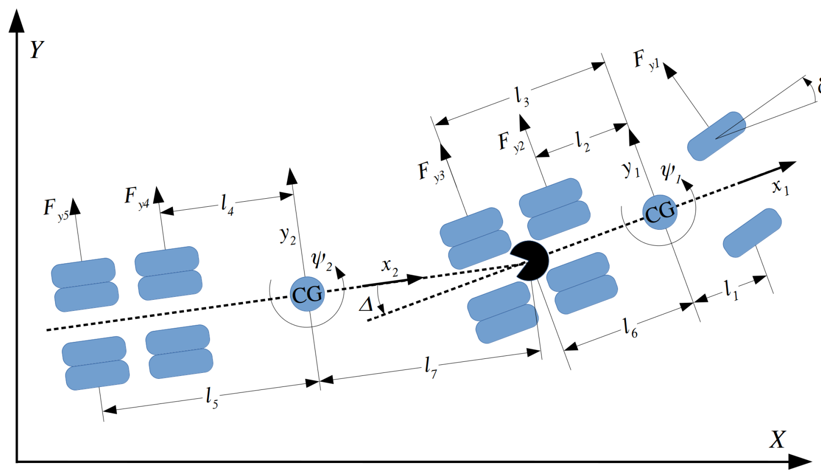

Figure 1: Tractor-Trailer Force System

Note that the additional yaw moment terms M_{zi} are generated by differential braking, active Limited Slip Differential (LSD), or other devices.
Remark: What are the directions of the hitch forces F_{hx} and F_{hy}? They are defined in this report to be aligned with the tractor's coordinate frame. Note that in this case hitch point can be viewed as a steering axle for the trailer.

2.2 Kinematics

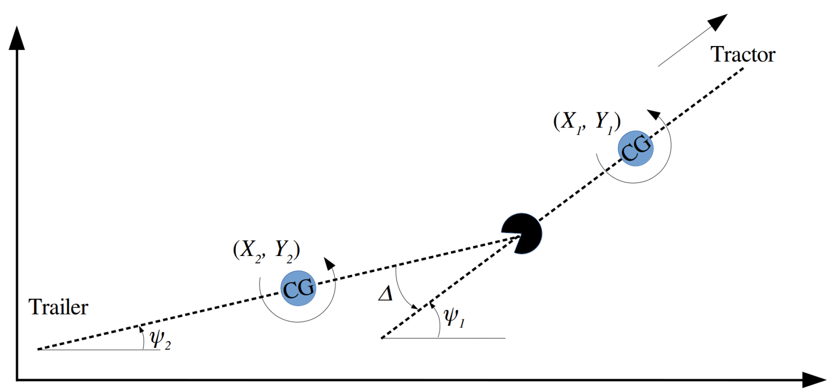

Figure 2: Tractor-Trailer Positions

For path following or trajectory tracking, we need to get the location of the tractor-trailer. The CG positions of both the tractor and the trailer in the global frame can be obtained from their velocities and yaw angles, as shown in Fig.2. (i stands for 1: tractor, or 2: trailer)

$$\dot{X}_i = \dot{x}_i \cos\psi_i - \dot{y}_i \sin\psi_i \tag{8}$$

$$\dot{Y}_i = \dot{x}_i \sin\psi_i + \dot{y}_i \cos\psi_i \tag{9}$$

There is not much difference between the tractor-trailer system and a regular car, other than that the CG locations are correlated by the hitch connection like a robot arm. The geometric constraints imposed by the hitch position are summarized in the following:

$$X_1 - l_6 \cos\psi_1 = X_2 + l_7 \cos\psi_2 \tag{10}$$

$$Y_1 - l_6 \sin\psi_1 = Y_2 + l_7 \sin\psi_2 \tag{11}$$

$$\Delta = \psi_1 - \psi_2 \tag{12}$$

Taking derivatives for both sides of (10) and (11), and substituting in the expressions in (8), (9), (12) results in:

$$\dot{x}_1 = \dot{x}_2 \cos\Delta + (\dot{y}_2 + l_7\dot{\psi}_2)\sin\Delta$$
$$\dot{y}_2 = -l_7\dot{\psi}_2 + (\dot{y}_1 - l_6\dot{\psi}_1)\cos\Delta + \dot{x}_1 \sin\Delta$$

Small angle approximation renders the following equations.

$$\dot{x}_1 \approx \dot{x}_2 \tag{13}$$

$$\dot{y}_2 = -l_7\dot{\psi}_2 + \dot{y}_1 - l_6\dot{\psi}_1 + \dot{x}_1\Delta \tag{14}$$

$$\ddot{y}_2 + \dot{x}_2\dot{\psi}_2 = \ddot{y}_1 + \dot{x}_1\dot{\psi}_1 - l_6\ddot{\psi}_1 - l_7\ddot{\psi}_2 \tag{15}$$

where, the velocities terms are all in the tractor's and the trailer's local frames. It can be seen that (13) restricts the longitudinal motion, and (14) restricts the lateral/yaw motions of the tractor-trailer system, respectively. Lateral acceleration constraint (15) is derived following the similar philosophy as (14), yet with further simplification by neglecting the a_{x1} term, since a_{x1} is usually small for a heavy-duty tractor-trailer system [1][2].
To avoid obstacles, certain ”critical” points, e.g., rear left and rear right corners of the trailer, may need to be defined and their trajectories tracked.

2.3 Tire Model

Lumping the left and right wheels and applying small angle approximation, tire slip angles for all five axles are calculated in the following.
Tractor front (steering) axle (as shown in Fig.3):

$$\alpha_1 = \delta - \frac{\dot{y}_1 + l_1\dot{\psi}_1}{\dot{x}_1} \tag{16}$$

There is another enhanced model for the tractor front axle, as introduced in [1]. In this case, the tire slip angle and its derivative of the tractor front axle will be treated as another two states.

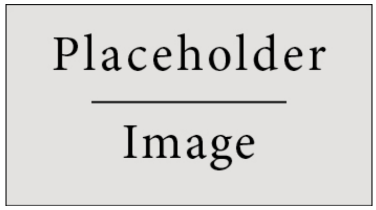

Figure 3: Placeholder for Side-Slip Angle

Tractor rear axles:

$$\alpha_2 = -\frac{\dot{y}_1 - l_2\dot{\psi}_1}{\dot{x}_1} \tag{17}$$

$$\alpha_3 = -\frac{\dot{y}_1 - l_3\dot{\psi}_1}{\dot{x}_1} \tag{18}$$

Trailer axles:

$$\alpha_4 = -\frac{\dot{y}_2 - l_4\dot{\psi}_2}{\dot{x}_2} \tag{19}$$

$$\alpha_5 = -\frac{\dot{y}_2 - l_5\dot{\psi}_2}{\dot{x}_2} \tag{20}$$

The lateral forces are calculated as:

$$F_{yi} = C_i\alpha_i, i = 1, 2, ..., 5 \tag{21}$$

2.4 State-Space Formulations

We try to further simplify and rearrange all the above equations and come up with the following linear system represented by state-space equations:

$$\boldsymbol{M}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{22}$$

$$\boldsymbol{z} = \boldsymbol{C}\boldsymbol{x} \tag{23}$$

Therefore, we will have to neglect the longitudinal dynamics, and only consider lateral and yaw motions of the tractor-trailer system, which now has three degrees of freedom. The longitudinal velocity is a time-varying parameter, which makes the system linear time-varying (LTV).

2.4.1 A Popular Choice of States

The state vector defined in many previous work is $\boldsymbol{x} = \begin{bmatrix}\dot{y}_1 & \dot{\psi}_1 & \dot{\Delta} & \Delta\end{bmatrix}$. In this case, the main control objective is vehicle stabilization, while the path-following part is accomplished by the human driver.

$$
\begin{aligned}
\boldsymbol{M} &= \begin{bmatrix} m_1+m_2 & -m_2(l_6+l_7) & m_2 l_7 & 0 \\ * & * & * & * \\ * & * & * & * \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\boldsymbol{A} &= \begin{bmatrix} -\frac{\sum C_i}{\dot{x}_1} & -(m_1+m_2)\dot{x}_1 - \frac{a_{12}}{\dot{x}_1} & -\frac{a_{13}}{\dot{x}_1} & -(C_4+C_5) \\ * & * & * & * \\ * & * & * & * \\ 0 & 0 & 1 & 0 \end{bmatrix} \\
a_{12} =& C_1 l_1 - C_2 l_2 - C_3 l_3 - C_4 l_4 - C_5 l_5 - (C_4+C_5)(l_6+l_7) \\
a_{13} =& C_4 l_4 + C_5 l_5 + C_4 l_7 + C_5 l_7
\end{aligned}
$$

* denote entries to be further derived.

2.4.2 Augmented System

For autonomous trucks, the path-following (or trajectory tracking) problem is to be solved on-line by the computer programs. We need to augment the system with the path-following errors, i.e., $\boldsymbol{x} = \begin{bmatrix} \dot{y}_1 & \dot{\psi}_1 & \dot{\Delta} & \Delta & e_{y1} & e_{\psi 1} \end{bmatrix}$.

3 Control System

3.1 Control Objectives

(1) Minimize the lateral offsets for both tractor and trailer
(2) Maintain vehicle stability
(3) Keep articulation angle and its rate of change small
(4) Track desired speed profile for fuel efficiency (optional)

3.2 Problem Formulation

(1) Given the target path information, control the steering angle to satisfy the objectives?

4 Simulations

5 Experiments

6 Conclusions

References

[1] L. Alexander, M. Donath, M. Hennessey, V. Morellas, and C. Shankwitz. A lateral dynamic model of a tractor-trailer: experimental validation. Technical report, University of Minnesota, 1996.

[2] A. Hac, D. Fulk, and H. Chen. Stability and control considerations of vehicle-trailer combination. *SAE International Journal of Passenger Cars-Mechanical Systems*, 1:925–937, 2008.

[3] R. Rajamani. *Vehicle dynamics and control*, volume 1. Springer Science & Business Media, 2011.