Automata Theory

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1. Basic Concepts
2. Deterministic Finite State Automata (DFA)
3. Non-Deterministic Finite State Automata(NFA)
4. Equivalence of DFA and NFA
5. Summary

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Why do we study automata theory?

- Automata is *an abstract computing devices*.
- It's the plural form of automaton which means "something that works automatically"
- Application of finite automata
  - Modeling and verifying finite state systems, such as communication protocols
  - Compiler for designing its lexical analyzer
  - Searching for keywords in a text
  - Etc

Basic Concepts

- alphabet: a finite, non-empty set of symbols.
- We indicate an alphabet by Σ,

Example 1

$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is decimal alphabet.
$\Sigma = \{a, b, c, ..., z\}$ the set of all lower case letters.

- A string: a finite sequence of symbols chosen from an alphabet,

Example 2

236 is a string over decimal alphabet.

- The empty string, ϵ, is a string with zero occurrence of symbols
- The length of a string denoted by $|w|$

Basic Concepts

- Power of an Alphabet: Σ^k is the set of strings with length k.
  $\Sigma^2 = 00, 01, ..., 10, 11, 12, ..., 99$
  $\Sigma^0 = \epsilon$
- The set of all strings is denoted by $\Sigma^* = \Sigma^0 \cup \Sigma^1.....$
- The set of non-empty strings is represented by Σ^+.
- Concatenation: if a and b are two strings, then ab is also a string.
  $a = 23$ and $b = 54$ imply that 2354 is a string
  Is ϵa a string?

Basic Concepts

- A (possibly infinite) language L defined over Σ is a subset of Σ^*.
- Examples
  - The set of all decimal numbers less than 1000
  - All English verbs. What is the alphabet?
  - All statements. What is the alphabet?
  - The set of Java programs

BonusPoint 1

- *What does the language* $\{0,100,1000,1100,10000,10100,11000,111000, ...\}$ *defined over the alphabet* $\{0,1\}$ *represent?*
- *What is(are) the common language(s) of two arbitrary alphabets* Σ_1 *and* Σ_2*?*

- The empty language is $\{\epsilon\} \neq \emptyset$.
- **Problem:** Does a given string s belong to a language L?
  e.g. is a Java program syntactically correct?
  or 1111110 is a member of L_b?

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Deterministic Finite State Automata (DFA)

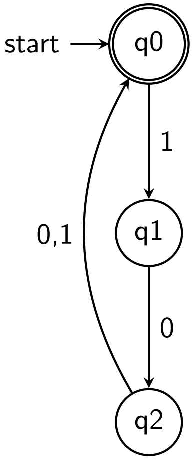
Definition 3

A DFA is a quintuple $A = (Q, \Sigma, \delta, q0, F)$ where

- Q is a finite set of states
- Σ is a finite alphabet (=input symbols)
- $\delta : Q \times \Sigma \rightarrow Q$ is a transition function and a transition $(q, a) \rightarrow p$ states that the DFA goes to q from p when the input action a is received,
- $q0 \in Q$ is the start state
- $F \subseteq Q$ is a set of final states

Representation of DFA-transition diagrams

- Nodes are states
- Edges are transitions labeled with the corresponding action
- Initial state is marked with $\rightarrow$
- Final states are double-lined nodes
- $\Sigma = \{0, 1\}$



Representation of DFA-transition tables

- Rows denote states and columns are the alphabet symbols
- Initial states are marked with the $\rightarrow$ symbol
- Final states are marked with the $*$ symbol
- Let $\Sigma = \{0, 1\}$

Example 4

state	0	1
* $\rightarrow$ q0		q1
q1	q2	
q2	q0	q0

(D)FA Language

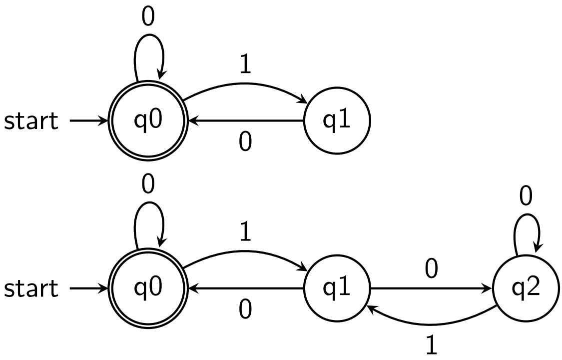
- A DFA accepts a string $s = a_1 a_2 \ldots a_n$ if there is a path in its transition diagram that
  - Starts at a start state
  - Ends in a final state
  - Has the sequence of observed labels $a_1 a_2 \ldots a_n$ from the start start to the final state
- The language of an automaton A is represented by $L(A)$
- $L(A)$ is the set of strings labeling the accepting paths.
- How to find the language of an automaton?

Find all the paths from the start state to the final states. The language of the DFA will be the labels of all such paths.

Example 5

$\{100, 101, 100100, 100101, 101100, 101101, \ldots\}$

Example



BonusPoint 2

- *What is the language of the first automaton?*
- *What is the relationship (e.g., same, subset, superset etc) between the two languages? The second one is a NFA.*

Basic Modelling: Vending Machine Operations

- The user inserts money, the amount is checked by the VM.
- If the money is enough, the operation buttons become active to choose the products.Otherwise, the money is returned back to the user.
- The user chooses the product, the product is delivered, and the change is returned.
- If the selected product is unavailable, VM will reject the service

Basic Modelling: Vending Machine Operations

- Alphabets are the valid actions/interactions

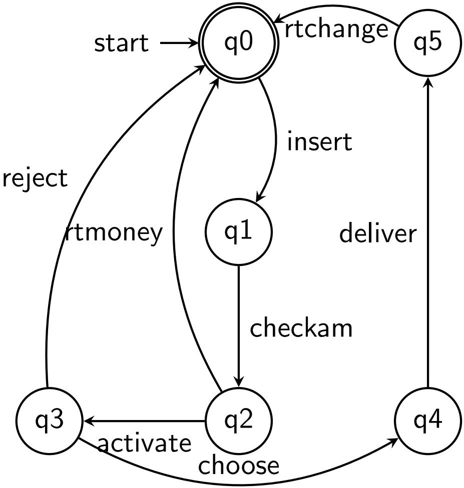
$$\Sigma = \{\mathit{insert}, \mathit{checkam}, \mathit{activate}, \mathit{choose}, \mathit{rtmoney}, \mathit{deliver}, \mathit{rtchange}, \mathit{reject}\}$$

- Its language consists of the set of valid sequence of events, e.g. the successful purchase is the following sequence:

$$\mathit{insert}\ \mathit{checkam}\ \mathit{activate}\ \mathit{choose}\ \mathit{deliver}\ \mathit{rtchange}$$

- **Problem** Can it happen that the vending machine eats the money?

Basic Modelling: Vending Machine Operations



Basic Modelling: Vending Machine Operations

Example 6

$$s_1 = \textit{insert checkam activate choose deliver rtchange}$$

$$s_2 = \textit{insert checkam activate reject}$$

$$s_3 = \textit{insert checkam rtmoney}$$

$$L = \{s_1, s_2, s_3, s_1 s_1, s_1 s_2, s_1 s_3, s_2 s_1, s_2 s_2, s_2 s_3, \ldots\}$$

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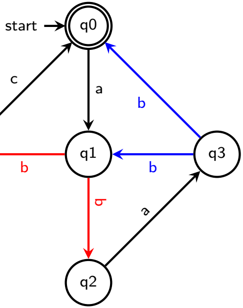
NFA

An NFA is an automaton which has several transitions with the same labels from a state.



NFA

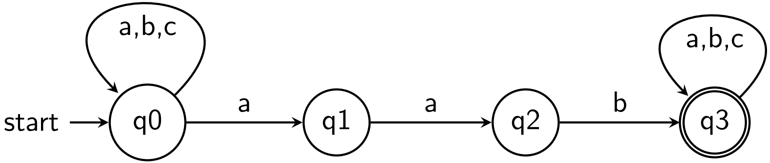
Definition 7

A NFA is a quintuple $A = (Q, \Sigma, \delta, q0, F)$ where

- Q is a finite set of states
- Σ is a finite alphabet (=input symbols)
- $\delta : Q \times \Sigma \rightarrow 2^Q$ is a transition function,
- $q0 \in Q$ is the start state
- $F \subseteq Q$ is a set of final states

The language of NFA is defined in the same way as that of DFA.

- Let $\Sigma = \{a, b, c\}$
- Design an NFA that accepts strings with *aab* as substring



How would you design it as a DFA?

FA with ϵ transitions

Definition 8

An ϵ-NFA is a quintuple $(Q, \Sigma, \delta, q0, F)$ where δ is a function from $Q \times (\Sigma \cup \{\epsilon\})$ to the powerset of Q.

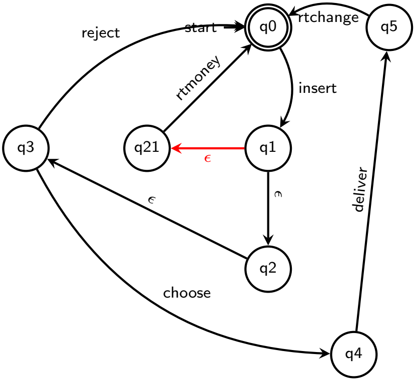
Definition 9

Let *eClosure* be a function that returns all states reachable from a state with only ϵ transitions, defined as follows:

$$q \in eClosure(q)$$

$$p \in eClosure(q) \wedge r \in \delta(p, \epsilon) \implies r \in eClosure(q).$$

NFA with ϵ transitions

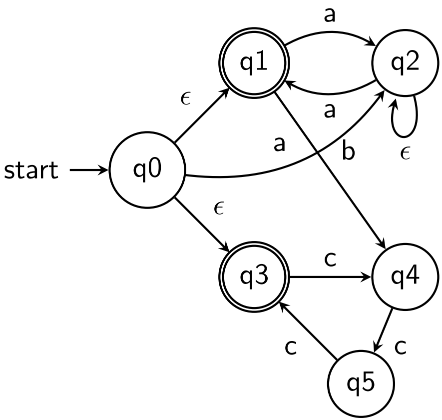


We only observe our interaction with the vending machine, i.e. it is modelled as a black box component

$eClosure(q1)$?

ϵ-NFA's language

$\Sigma = \{a, b, c\}$



BonusPoint 3

What is the language?

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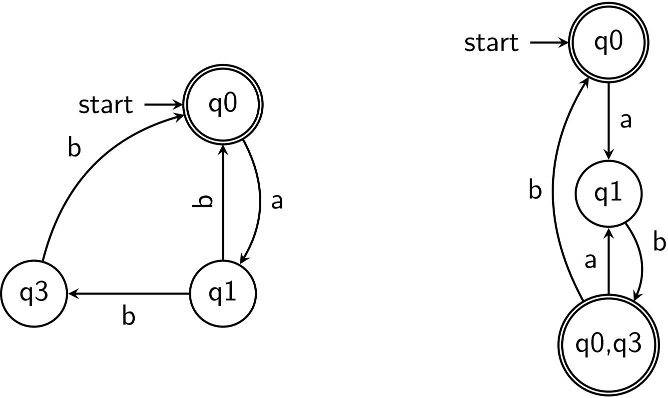
Equivalence of DFA and NFA

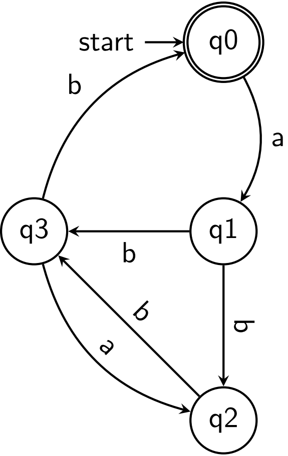
- NFA are usually easier to use, e.g. to model a system.
- A DFA is a NFA but not the opposite.
- We show that they are equivalent:

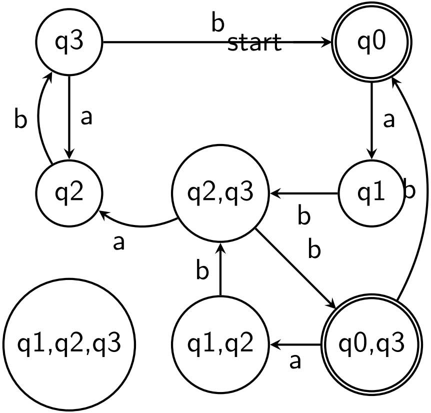
  For any NFA N there is a DFA A such that $L(N) = L(A)$, and vise versa.

From NFA to DFA-Example



From NFA to DFA-Example

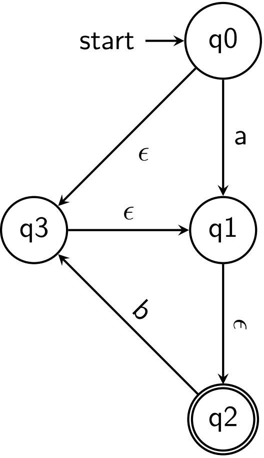


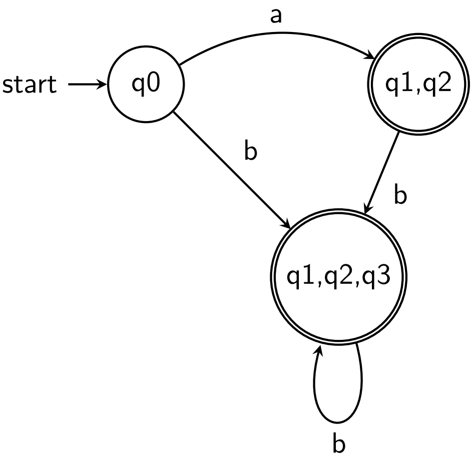


From NFA to DFA

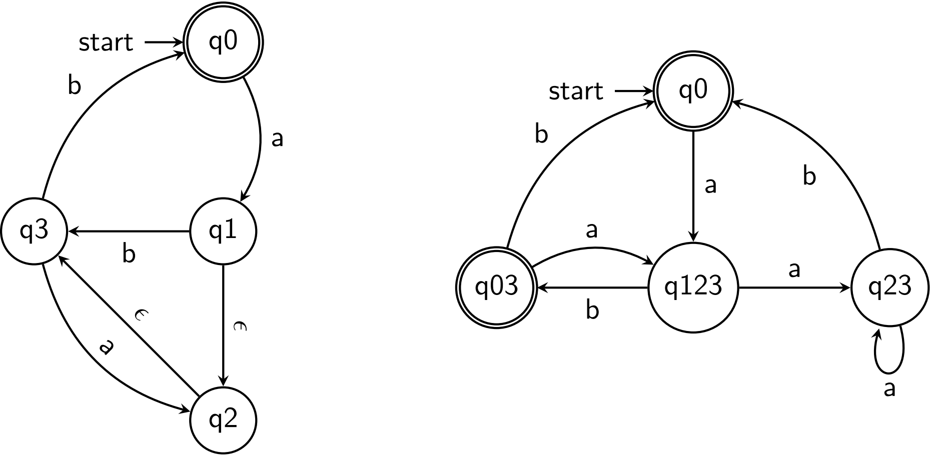
- **Problem** Given an NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$, construct a DFA $D = (Q_D, \Sigma, \delta_D, q_0, F_D)$ such that $L(D) = L(N)$.
- $Q_D = \{S : S \subseteq Q_N\}$, i.e. the DFA states are all possible subsets of the states in NFA.
- A state is final in DFA, if one of its consisting states is final in NFA, i.e. $F_D = \{S \in Q_D : S \cap F_N \neq \emptyset\}$.
- For every state $S \subseteq Q_N$ and $a \in \Sigma$, $\delta_D(S, a) = \bigcup_{p \in S} \delta_N(p, a)$, the target state is the union of targets of the outgoing transitions labeled a from the states in S.

From ϵ-NFA to DFA-Example





From ϵ-NFA to DFA-Example



From ϵ-NFA to DFA

Problem Given an ϵ-NFA $N = (Q_N, \Sigma, \delta_N, q_{0,N}, F_N)$, construct a DFA $D = (Q_D, \Sigma, \delta_D, q_{0,D}, F_D)$ such that $L(D) = L(N)$.

- $Q_D = \{S : S \subseteq Q_N \wedge S \in eClosure(S') \wedge S' \in Q_N\}$
- $q_{0,D} = eClosure(q_{0,N})$
- $F_D = \{S \in Q_D : S \cap F_N \neq \emptyset\}$.
- For every state $S \subseteq Q_N$ and $a \in \Sigma$,
  $\delta_D(S, a) = \bigcup\{eClosure(p) : p \in \delta_N(t, a)$ for some $t \in S\}$.

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Summary and Upcoming Events

- Summary
  - Deterministic Finite Automata
  - Non-Deterministic Finite Automata
  - Converting NFA to DFA
- Next lecture on regular expressions