GNTSAT: a genetic SAT solver based on WalkSAT

Xiaoyue Chen   Suling Xu   Qinhan Hou

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Abstract

In this paper we developed a genetic algorithm-based solver. The algorithm combined crossover from genetic algorithm and local search from WalkSAT. We performed several experiments using different kinds of crossover methods and taking WalkSAT as the contrast to compare their efficiency and effectiveness. The results showed that our algorithm using uniform crossover performed best over those methods, and our solver performed better than WalkSAT.

Keywords: genetic algorithm, SAT, NP complete problem

1 Introduction

The satisfiability problem (SAT) is an NP-complete problem (Cook, 1971). A SAT problem consists of a set of Boolean variables $x_1, \cdots, x_n$ and a Boolean formula $f : \mathbb{B}^n \to \mathbb{B} = \{0, 1\}$. The question is whether an assignment $x = (x_1, \cdots, x_n)$ exists such that $f(x) = 1$ (Gottlieb, Marchiori, & Rossi, 2002). A wide range of problems can be expressed as SAT problems. This includes verification, planning, scheduling and combinatorial design (Biere, Heule, & van Maaren, 2009). Hence, a SAT solver has been a general purpose platform for solving various real-world problems and has obvious importance.

There are two types of SAT solver technologies: complete methods and incomplete methods. Complete methods can guarantee to eventually report a satisfying assignment or prove not satisfiable, long enough enough time. Complete methods have an exponential time complexity if $P \neq NP$. On the other hand, incomplete methods has no such guarantee and generally use stochastic local search

(Gomes, Kautz, Sabharwal, & Selman, 2008). In many cases where the problem does not require a definitive answer, incomplete methods could outperform complete methods in terms of both speed and memory usage.

Genetic algorithms (GAs) are incomplete methods that have been applied to many NP-complete problems, including SAT (Gottlieb et al., 2002). However, results suggest that classical GAs may not outperform local search algorithms (De Jong & Spears, 1989). Nevertheless, recent results show that GAs can yield good results if combined with other methods (Gottlieb et al., 2002).

This paper proposes a genetic algorithm: GNTSAT, which is based on WalkSAT (Selman, Kautz, & Cohen, 1994) and outperforms WalkSAT in many benchmark instances. This paper introduces the GNTSAT algorithm, makes performance evaluations based on benchmarks, and discusses the possible reasons for the performance difference and their implications.

2 Background

Incomplete methods for SAT can significantly outperform complete methods in many domains. Hence there has been tremendous amount of research done on them since the early 1990's. SAT problems can be effectively solved by using greedy decent if they do not have any local minima, as it could rapidly approach the global minima without exhaustively branching or backtracking. However, most interesting problems do have local minima. Hence the main challenge and opportunity for incomplete SAT methods is escaping local minima (Biere et al., 2009).

WalkSAT (Selman et al., 1994) plays a key role in the success of local search for satisfiability (Biere et al., 2009). It improves its ability of escaping lo-

cal minima by introducing noise into search in the form of uphill moves. It always select a variable to flip from a randomly chosen unsatisfied clause C. If flipping a bit in C does not turn any currently satisfied clauses to unsatisfied, WalkSAT flips the bit (a "freebie" move). Otherwise according to a change, it either makes a "greedy" move (a bit flip that will minimize the number of currently satisfied clauses becoming unsatisfied) or a "random walk" move (flipping a random bit in C). WalkSAT is proved to significantly outperform greedy decent methods on hard satisfiable problems. GNTSAT uses WalkSAT on generated children as its mutation operator.

FlipGA (Marchiori & Rossi, 1999) is a genetic local search algorithm. FlipGA uses a small population size (10 in the original paper). It selects parents with probability proportional to their fitness and generate children with uniform crossover. Its mutation operator has a 0.9 probability to be used on each individual, with a 0.5 probability to flip each gene. FlipGA then perform a greedy local search on the children. It scans the genes in random order and tries to flip each one of them. If a flip results in an increase of satisfied clauses, the flip is accepted. This process is repeated if the obtained chromosome has more satisfied clauses after all the genes have been considered. Benchmark results from Gottlieb et al., 2002 show that FlipGA has similar performance to WalkSAT.

GASAT (Lardeux, Saubion, & Hao, 2006) is another genetic local search algorithm. It combines many local search methods and is more complex than the previous mentioned algorithms. GASAT uses a population scheme where the newly created child is accepted only if it is fitter than a subset of the population. The child is inserted to the population to replace the oldest individual in the population. Lardeux et al. experimented with different crossover operators and remarked that Corrective Clause crossover (CC) and Corrective Clause and Truth Maintenance crossover (CCTM) have better behaviors than other types of crossovers in the sense that they could quickly reduce the number of false clauses while maintaining a higher level of population diversity than other crossovers. GASAT uses tabu search (TS) to help avoid local minima. It also uses TS with diversification to solve the *stumble clause* problem i.e., the last false clause is often the same clause and block to tabu search.

Lardeux et al. (2006) made a comprehensive comparison between GASAT and other algorithms, including WalkSAT and FlipGA. The results show that GASAT has reliable and competitive performance. Inspired by GASAT, GNTSAT uses a similar population management scheme to GASAT. We will also make comparisons on different crossover operators mentioned by Lardeux et al.

3 Methods

3.1 The algorithm
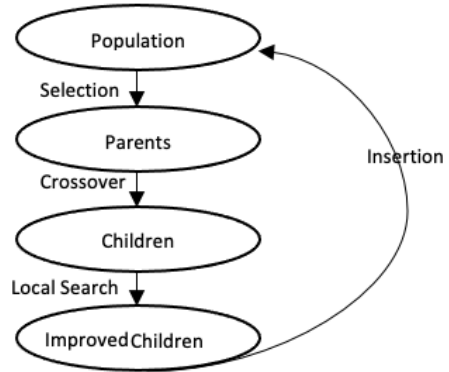
The algorithm scheme is described in Fig 1.

Figure 1: Algorithm scheme. The population is initialized randomly. In each iteration, first using tournament to select some best individuals according to fitness. Then crossover are performed on two individuals among the bests. Each generated child is improved individually using local search, and then new individuals are inserted to the population to replace the old individuals under some conditions. Iterate the process until a solution is found.

3.1.1 Representation

A candidate solution is a string of bits whose length equals to the number of variables of the considered problem instance. Every variable is associated to one bit. This is the most obvious way to represent a solution candidate. It also makes mutation and crossover operations computationally efficient. The most successful evolutionary algorithms for SAT also uses representation (Gottlieb et al.,

2002). Hence the bit string representation is a reasonable starting point.

3.1.2 Fitness function

The fitness value is equivalent to the number of satisfied clauses of the individual.

$$fitness(x) = \sum_{i=1}^{m} c_i(x)$$

where $c_i(x)$ is the truth value of the ith clause, and m is the number of clauses.

3.1.3 Selection operation

GNTSAT uses a modified version of tournament selection. First, a certain number of individuals are randomly select to participate in the tournament. The participating individuals are ranked according to fitness. The most fit individuals will be selected and stored in a best individuals pool. The two parents will be selected randomly from this pool.

3.1.4 Crossover operators

The main goal of the crossover operator in GNTSAT is to create potentially promising new individuals. Using the selected parents, there are many ways for them to crossover, and we consider 5 types: CC (Corrective Clause) Crossover, F&F (Fleurent and Ferland) Crossover, Uniform Crossover, One-point Crossover and Two-point Crossover. We will introduce them respectively below.

- CC (Corrective Clause) Crossover: For each clause c that is unsatisfiable for both parent X and parent Y solutions and for each variables that appears in clause c, find the bit position using the literally absolute value i of the variable that produces the maximum improvement on two parents guided by the improvement evaluation function. The function equals to the number of false clauses which become true by flipping the *ith* bit in one of the solutions minus the number of satisfied clauses which become false. Assign the flipped value applied to one of the parents to the child in the bit position found, and all the bits with

no value of child take the value in corresponding position of parent X or parent Y with the equal probability. CC crossover is illustrated in detail as Algorithm **??**

Algorithm 1: Corrective Clause Algorithm

Data: two parents X and Y
Result: one child Z

all the bits of child Z take the values of parent X or parent Y with the equal probability;
for *each clauses c such that* $\neg sat(X, c) \wedge \neg sat(Y, c)$ **do**
  for *all positions i such that the variable* x_i *appears in c* **do**
    Compute $\sigma = improvement(X, i) + improvement(Y, i)$;
  end
  Set $Z|k = flip(X|k)$ where k is the position such that σ is maximum;
end

- F&F (Fleurent and Ferland) Crossover: For each clause c that is satisfiable for one parent and unsatisfiable for the other parent and for each variables that appears in clause c, the bits positions in the child, corresponding to the literally absolute value i of the variable in parents, are assigned values according to the parent satisfying the identified clause. All the bits with no value of the child take the value in corresponding position of parent X or parent Y with the equal probability. F&F crossover is illustrated in detail as Algorithm **??**

- Uniform Crossover: With equal probability, each bits of the child is chosen from either parent. In that case, one new offspring is generated. Uniform Crossover is illustrated in detail as Algorithm **??**

- One-point Crossover: Randomly select one pivot point (ranging between 0 to length of the bits string) and exchange the substring from that bit point till the end of the string between the two individuals. In that case, two new offspring individual are generated. One-point Crossover is illustrated in detail as Algorithm **??**

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Algorithm 2: Fleurent and Ferland Crossover

Data: two parents X and Y
Result: one child Z

all the bits of child Z take the values of parent X or parent Y with the equal probability;
for *each clauses c such that $sat(X,c) \wedge \neg sat(Y,c)$ (resp. $\neg sat(X,c) \wedge sat(Y,c)$)* **do**
  for *all positions i such that the variable x_i appears in c* **do**
    $Z|i = X|i$ (resp. $Z|i = Y|i$);
  end
  Set $Z|k = flip(X|k)$ where k is the position such that σ is maximum;
end

Algorithm 3: Uniform Crossover

Data: two parents X and Y
Result: one child Z

for *each bit x* **do**
  $Z|x = X|x$ or $Z|x = Y|x$ with equal possibility;
end

Algorithm 4: One-point Crossover

Data: two parents X and Y
Result: two children Z_1 and Z_2

for *each bit x where its position is less than the pivot point* **do**
  $Z_1|x = X|x$;
  $Z_2|x = Y|x$;
end
for *each bit x where its position is larger than the pivot point* **do**
  $Z_1|x = Y|x$;
  $Z_2|x = Y|x$;
end

- Two-point Crossover: Randomly pick two pivot points and the bits in between the two points are swapped between the parent individuals. In that case, two new offspring individual are generated. Two-point Crossover is illustrated in detail as Algorithm **??**

Algorithm 5: Two-point Crossover

Data: two parents X and Y
Result: two children Z_1 and Z_2

for *each bit x where its position is in between the two pivot points* **do**
  $Z_1|x = X|x$;
  $Z_2|x = Y|x$;
end
for *each bit x where the position is not in between the two pivot points* **do**
  $Z_1|x = Y|x$;
  $Z_2|x = Y|x$;
end

3.1.5 Local search

We implement the idea of mutation in genetic algorithm using WalkSat local search method. It's a randomized local search algorithm. WalkSat is illustrated in detail as Algorithm **??**. It attempts to determine the best move by randomly choosing a clause among those that are currently unsatisfied and selecting a variable to flip within it under some conditions. The evaluation function in deciding bits to flip here is called break-count. It equals to the number of satisfied clauses which become unsatisfied by flipping a bit. The maximum number of flips (MAX-Flips) is set to 1000 and the probability of a random move p is set to 0.57 since it's the optimal noise value (Seitz, Alava, & Orponen, 2005). The search stops when one solution is found.

3.2 Implementation

GNTSAT is implemented in C++. GNTSAT takes a DIMACS CNF file as input and output the resulting bit string if found. User could adjust parameters such as population size, tournament size, and crossover operator.

Algorithm 6: WalkSat Local Search

Data: one child Z generated from the crossover above
Result: a solution for the instance

all the bits of child Z take the values of parent X or parent Y with the equal probability;
for $i \leftarrow 1$ *to MAX-Flips* **do**
    if Z *is a solution* **then**
        | return Z;
    end
    Clause C is randomly selected among unsatisfied clauses;
    if $\exists$ *variable* $c \in C$ *with* beak-count $= 0$ **then**
        | $v \leftarrow c$;
    else
        | with probability p: $v \leftarrow$ a variable x in C chosen randomly;
        | with probability 1-p: $v \leftarrow$ a variable x in C with smallest beak-count ;
    end
    Flip v in Z;
end

3.3   Performance measures

After all these processes to generate a solution, we compare our solver using different crossovers and with other existing solvers. Several runs are required for each benchmark instance under consideration and some measuring methods need to be taken for statistically meaningful and also relatively fair results. We take 10 runs and consider three measuring methods: SR (Success rate), AT (Average Time) and AFS (Average Flips to Solution). We will introduce them respectively below.

- SR (Success rate): SR represents the percentage of runs where a solution has been found within a time limit. Since some runs where the time to get a solution is too long or the solver is stuck in the local optimal solution, we use SR to measure the quality of the solver. And the maximum time we set for a successful run is 120 seconds.

- AT (Average Time): AT represents the average seconds taken in successful runs.

- AFS (Average Flips to Solution): Relating the computational costs to the number of the basic moves in the search space for a solution has become the standard measure used for studying the cost of SAT algorithms (Singer, Gent, & Smaill, 2000). AFS represents the average number of bit flips needed to find a solution in successful runs, which was raised by Gottlieb and Voss and used to compare EAs generating new solution candidates by single bit flips (Gottlieb & Voss, 2000).

4   Experiments & Results

4.1   Experiments

Here we performed our algorithms in different datasets with variant clause numbers to determine both its efficiency and effectiveness. We choose datasets with 100, 150, 200 and 250 variables, corresponding to 430, 645, 860 and 1075 clauses respectively. That is to say, the rates of a number of variables and clauses are fixed and equals to 4.3. The datasets we used are all generated randomly. Datasets with 100, 150 and 200 variables are downloaded from SATLIB, an online published by UBC[1]. Datasets with 250 variables are generated by ourselves. All of the datasets are satisfiable for 3-SAT problems. In order to reduce contingency, we run the genetic algorithm-based method and the WalkSAT method in each dataset 10 times, and then we computed the average time used and average flip times to measure the efficiency. In addition, we had set a two minutes timeout for the

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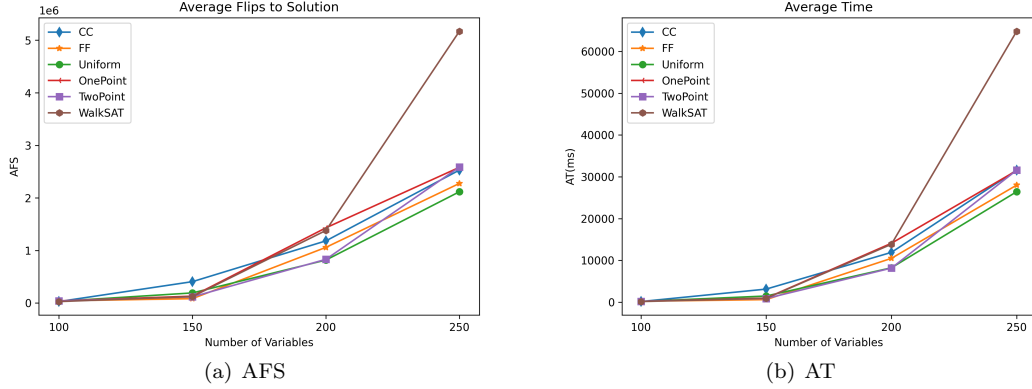[1]https://www.cs.ubc.ca/~hoos/SATLIB/benchm.html

Figure 2: The performance on datasets with different numbers of variables. We computed the average AFS and AT on all the 5 datasets with the same number of variables and then plot them in the figures.

two algorithms, which means that failing to find an available solution in two minutes would seem a failure to solve the dataset. We would count the number of failures to determine the effectiveness of the algorithms. The detailed results could be found in Table.**??**.

In our experiments we tested five kinds of crossover methods for genetic-based algorithm, they were CC, FF, uniform, one-point and two-point crossover respectively. Besides we run Walk-SAT on the same dataset as the contrast to find the difference between our algorithms and local search methods.

4.2 Results

According to Table.**??**, it is obvious that with the variables increasing, the AFS and AT of all the algorithms raised, but the SR decreased. For datasets with 100 and 150 variables, the SRs of all the algorithms were 100%, but algorithms began to fail solving problems when it came to 200 variables (for example, dataset UF-200-068). From Fig.2 it could be found that there were only slight differences between different methods while the number of variables was low. However, while the problems becoming more complex, the difference would show. For those datasets having 250 variables, the Walk-SAT algorithm would take a long time to search the solution, and there would be a high possibility that it could not give a solution. Overall, the genetic-based algorithm using uniform crossover performed

best on the complex 3-SAT problems, both in computation complexity and time consumption aspect. For other kinds of crossover methods, CC, one-point, and two-point crossover performed most the same, while FF had shown a little superiority over them.

5 Discussions

6 Conclusion

In this paper, we have introduced our SAT solver based on a genetic algorithm and its performance on different benchmarks. The experiment results have illustrated that our solver has a better performance comparing with WalkSAT. In addition, we have tried to use different crossover methods in our algorithm and finally uniform crossover got the best result over those methods. Although we cannot say with certainty that this is the best crossover method, at least it gave the best results under the scenario experiment mentioned in this paper.

We believe that we have developed a mature solver to solve the 3-SAT problem. Due to the limited time, we could not have more experiments on it, but we still expect its performance in more complex benchmarks and want to know if it has a better performance comparing with other solvers. Our future work would focus on collecting other solvers' performances and compared them with our solver, thus we may do possible improvement on

our solver. Besides, considering our algorithm is based on an incomplete method, we would like to extend it to solve the MAX-SAT problem to show its superiority.

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