Lab Assignment (Iterative method and Gradient descent method)

1. Solve the below linear equation using Gaussian elimination.

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3 \end{aligned}$$

- Provide step-by-step procedures for the answer

2. Make a subroutine or a function for the Jacobi method.
Input : A $n \times n$ matrix A, a vector b
Output : A solution x

3. Apply your subroutine or function to solve the above problem and compare it with your answer based on the Gaussian Elimination.

4. Consider the minimization of the given function $f(X) = (a - x_1)^2 + b(x_2 - x_1^2)^2$, where a and b are parameters that determine the "difficulty" of the problem. Here we choose $b = 5$, and $a = 1$. The gradient of this function is

$$\nabla f(X) = [-4bx_1(x_2 - x_1^2) - 2(a - x_1), 2b(x_2 - x_1^2)]. \tag{1}$$

Find a minimum point of the given function $f(X)$ using the Gradient Descent method starting from $[-1.4, 2]$. As you see, the answer is $[1, 1]$.

- Make a subroutine or a function for the Gradient Descent method
- Input : a given function, the gradient of the function, ϵ (precision), M (max. iteration) and γ (learning step size)
- Output: Minimum value, Minimum point (x^*, y^*)
- Test your subroutine/function for the above problem.