Lab Assignment (Runge-Kutta and multi-step method)

1. Given the initial-value problem

$$y' = \frac{1}{t^2} - \frac{y}{t} - y^2$$
$$1 \leq t \leq 2, y(1) = -1$$

with exact solution $y(t) = -1/t$.

(a) Use Euler's method with $h = 0.05$ to approximate the solution and compare it with the actual values of y.

(b) Use the answers generated in part (a) and linear interpolation to approximate the following values of y and compare them to the actual values.
(1) y(1.052) (2) y(1.555) (3) y(1.978)

(c) Use modified Euler method with $h = 0.05$ to approximate the solution and compare it with the actual values of y.

(d) Use the answers generated in part (c) and linear interpolation to approximate the following values of y and compare them to the actual values.
(1) y(1.052) (2) y(1.555) (3) y(1.978)

(e) Use Runge-Kutta 4th order method with $h = 0.05$ to approximate the solution and compare it with the actual values of y.

(f) Use the answers generated in part (e) and linear interpolation to approximate the following values of y and compare them to the actual values.
(1) y(1.052) (2) y(1.555) (3) y(1.978)

- Construct subroutines/functions for the above numerical methods
- Provide visual graphs showing that your subroutines are working as expected.

2. Consider

$$\frac{dy}{dx} = y - x^2$$

The initial condition is $y(0) = 1$ and the given step size is $\Delta h = 0.1$. The exact solution is $y = 2+2x+x^2-e^x$. Compare the three numerical methods, Runge-Kutta 4th order, Adams-Bashforth and Adams-Moulton, in the x-domain $[0, 3.3]$. For multi-step methods, you can use the exact solutions for initial multi-values.

- Construct subroutines/functions for the three methods.
- Provide a figure showing the three approximations with the exact one.