

Lab Assignment (Error Analysis)

1. The two quantities x and y have an uncertainty x and y . Calculate the error in the following cases:

- (a) $x - 2y$
- (b) $4x - 5y$
- (c) $3xy$
- (d) $\frac{x^3}{y^5}$
- (e) \sqrt{x}
- (f) $\sqrt{x^3}$
- (g) $\sqrt{2xy}$
- (h) $2\sqrt{\frac{x}{3y}}$
- (i) $2\sqrt{\frac{x^3}{3y}}$
- (j) $\frac{x}{y} + \frac{y}{x}$
- (k) $x^2 + y^2$

Comment: Less than 5 correct answers means "No answer", between 5 and 9 correct answers "Below expectation" and 10 above "Meets expectation".

2. The radius of a sphere is $R = (22.2 \pm 0.1)cm$. The radius of the base of a cylinder is $r = (12.0 \pm 1.2)cm$, and its height is $h = (24.4 \pm 1.1)cm$. What is the total volume occupied by the sphere and by the cylinder (including the error)?

3. Suppose

$$F = \frac{P\pi a^4}{8lw} \quad (1)$$

Write down an expression for the maximum percentage error in F . Suppose further that $\frac{\delta P}{P} = 0.5\%$, $\frac{\delta a}{a} = 0.5\%$, $\frac{\delta l}{l} = 0.1\%$, and $\frac{\delta w}{w} = 1\%$, calculate the percent error in the result.

4. Solve the system

$$5x_1 + 7x_2 = b_1 \quad (2)$$

$$7x_1 + 10x_2 = b_2 \quad (3)$$

Using Gaussian elimination method to obtain the solution x_1 when $b_T = (b_1, b_2) = (0.7, 1)$. Also solve the above system with $b_A = (b_1, b_2) = (0.69, 1.01)$ using the same method to obtain the solution x_2 . Show that

$$\frac{\|x_1 - x_2\|_2}{\|x_1\|_2} \leq \|A\|_2 \|A^{-1}\|_2 \frac{\|b_T - b_A\|_2}{\|b_T\|_2} \quad (4)$$

where A is the 2×2 coefficient matrix of the above system and the norm in the above inequality is the Euclidean norm for vector and the corresponding induced norm for the matrix.

5. Show by an example that $\|\cdot\|_M$ defined by $\|A\|_M = \max_{\{1 \leq i, j \leq n\}} |a_{ij}|$, does not define an induced matrix norm.

6. Show that $\kappa(A) \geq 1$ for any non-singular matrix A .

7. For any two $n \times n$ non-singular matrices A and B , show that $\kappa(AB) \leq \kappa(A)\kappa(B)$.