Lab Assignment (interpolation and ODE basic)

1. Interpolate the data from $x = 1$ to 10 at increments of 0.5 using the following two methods. Plot the interpolated values and data together using each of the following methods (create a new plot with the data for each method).

(a) linear interpolation
(b) Lagrange interpolating polynomial

TABLE I: Data

x	1	2	3	4	6	8	10
y	2	2.5	7	10.5	12.75	13	13

- Construct two subroutines/functions for linear and Lagrange interpolating polynomial.
- The subroutine/function should be generally constructed in this case.
- Inputs : the given data, x and y, and query points, x_q. (those are vectors)
- Output: a vector of interpolated values s
- Provide graphs showing your results based on the two subroutines/functions
- Do not forget providing comments in your code.
- You must submit your code and pictures

2. Consider the initial-value problem

$$y' = y - x$$
$$y(0) = \frac{1}{2}.$$

Use Euler's method with (a) $h = 0.1$, (b) $h = 0.05$ and (c) $h = 0.01$ to obtain an approximation to $y(1)$. Given that the exact solution to the initial-value problem is

$$y(x) = x + 1 - \frac{1}{2}e^x,$$

compare the errors in the three approximations to $y(1)$.

- Construct a subroutine/function for the Euler method.
- Provide a figure showing the three approximations with the exact one.