1. Background

- Net load fluctuations, defined as difference between demand and power output of DERs, causes uncertainty on nodal power injections.

2. AC-CCOPF in radial distribution networks

2.1 Definition of radial distribution system (using LinDistFlow model)

- nodes: active/reactive power demand d_i^P and d_i^Q; voltage magnitude $v_i \in [v_i^{min}, v_i^{max}]$
  - root node 0; non-root nodes $\mathcal{N}^+ := \mathcal{N} \setminus \{0\}$, edges $i \in \mathcal{E}$ are indexed by $\mathcal{N}^+$.
  - substation / DER nodes: active/reactive power generation $g_i^P \in [g_i^{P,min}, g_i^{P,max}]$ and $g_i^Q \in [g_i^{Q,min}, g_i^{Q,max}]$
  - ancestor node A_i: $|A_i| = 1$, $\forall i \in \mathcal{N}^+$
- edges: resistance R_i; reactance X_i; apparent power flow limit S_i^{max}, $i \in \mathcal{E}$;
  - active/reactive power flows f_i^P and f_i^Q, $i \in \mathcal{E}$ is the index of downstream node of edge i.
  - LinDistFlow approximation: (推导)

$$
\begin{cases}
(d_i^p - g_i^p) + \sum_{j \in C_i} f_j^p = f_i^p, \quad p \in \{P, Q\}, \ \forall i \in \mathcal{N}^+ \\
u_{A_i} - 2(f_i^P R_i + f_i^Q X_i) = u_i, \quad \forall i \in \mathcal{N}^+ \\
u_i^{min} \le u_i \le u_i^{max}, \ \forall i \in \mathcal{N}; \quad (f_i^P)^2 + (f_i^Q)^2 \le (S_i^{max})^2, \ \forall i \in \mathcal{E}
\end{cases}
$$

2.2 Chance Constraint OPF for LinDistFlow

- Define the uncertain net load injections as $d_i = \bar{d}_i + \epsilon_i$, given forecast + normally distributed forecast error.
- ϵ_i: mean $\mathbb{E}(\epsilon_i) = \mu_i = 0$, standard-deviation σ_i, variance $Var(\epsilon_i) = \sigma_i^2 \Rightarrow \mathbb{E}(d_i) = \bar{d}_i$; ϵ_i^P and ϵ_i^Q
- generators compensate for load deviations using affine control: $g_i^P = \bar{g}_i^P + \alpha_i \tilde{\epsilon}^P$ ①; $g_i^Q = \bar{g}_i^Q + \alpha_i \tilde{\epsilon}^Q$ ②, $i \in G$; $\tilde{\epsilon}^P = \sum_{i \in \mathcal{N}} \epsilon_i^P$; $\sum_{i \in G} \alpha_i = 1$
- power transmission distribution factors (PTDF) A: $a_{ij} = 1$ if line i is part of the path from root to bus j.
- uncertain edge power flow: $f_i^P = \bar{f}_i^P + a_{i*}(\epsilon^P - \alpha \tilde{\epsilon}^P)$ ③; $f_i^Q = \bar{f}_i^Q + a_{i*}(\epsilon^Q - \alpha \tilde{\epsilon}^Q)$ ④
- uncertain voltage magnitude squared: $u_i = u_{A_i} - 2(R_i f_i^P + X_i f_i^Q) = \bar{u}_i - 2 a_{*i}^T (R \circ A(\epsilon^P - \alpha \tilde{\epsilon}^P) + X \circ A(\epsilon^Q - \alpha \tilde{\epsilon}^Q))$ ⑤

Complete model:

$$\min_{(\bar{g}, \alpha, \bar{f}, \bar{u})} \mathbb{E}[f(\bar{g}^P, \bar{g}^Q, \epsilon^P, \epsilon^Q)]$$

s.t. $\sum_{i \in G} \alpha_i = 1$

Deterministic LinDistFlow

$$
\begin{cases}
(\bar{d}_i^p - \bar{g}_i^p) + \sum_{j \in C_i} \bar{f}_j^p = \bar{f}_i^p, \quad p \in \{P, Q\}, \ \forall i \in \mathcal{E} \\
\bar{u}_{A_i} - 2(\bar{f}_i^P R_i + \bar{f}_i^Q X_i) = \bar{u}_i, \ \forall i \in \mathcal{N}^+
\end{cases}
$$

Chance Constraints

$$
\begin{cases}
\mathbb{P}(g_i^{P,min} \le g_i^P \le g_i^{P,max}) \ge (1 - 2\eta_g), & \forall i \in G_c \\
\mathbb{P}(g_i^{Q,min} \le g_i^Q \le g_i^{Q,max}) \ge (1 - 2\eta_g), & \\
\mathbb{P}(u_i^{min} \le u_i \le u_i^{max}) \ge (1 - 2\eta_v), & \forall i \in \mathcal{N}
\end{cases}
$$

η_g and η_v are confidence level.

① ~ ⑤

2.3 Expected Cost

$$\mathbb{E}[f_p(g^P, \epsilon^P)] = \sum_{i\in G}[c_{i2}\, \alpha_i^2 Var(\tilde{\epsilon}^P) + c_i(\bar{g}_i^P)$$

2.4 Conic Reformulation of Chance Constraints

For normally distributed random variable $X \sim Norm(\mu, \sigma^2)$, $\mathbb{P}(X \le x^{max}) > \eta$ holds if and only if $x^{max} \geq \mu + z_\eta \sigma$; $z_\eta = \phi^{-1}(1-\eta)$ is $(1-\eta)$-quartile of standard normal distribution

$$\begin{cases} g_i^{p,max} \geq \mathbb{E}(g_i^p) + z_{\eta_g}\sqrt{Var(g_i^p)} = \bar{g}_i^p + z_{\eta_g}\alpha_i\sqrt{\sum_{j=1}^{b} Var(\epsilon_j^p)},\ p\in\{P,Q\};\ -g_i^{p,min} \geq -\bar{g}_i^p + z_{\eta_g}\alpha_i\sqrt{\sum_{j=1}^{b} Var(\epsilon_j^p)} \\ u_i^{max} \geq \bar{u}_i + z_{\eta_v}\sqrt{Var(u_i)}\ ;\ -u_i^{min} \geq -\bar{u}_i + z_{\eta_v}\sqrt{Var(u_i)} \\ Var(u_i) = 4\sum_{k=1}^{l} a_{ki}\left[\left(R_k^2\sum_{j=1}^{b} a_{kj}(Var(\epsilon_j^P) + \alpha_j^2 Var(\tilde{\epsilon}^P)\right) + \left(X_k^2\sum_{j=1}^{b} a_{kj}(Var(\epsilon_j^Q) + \alpha_j^2 Var(\tilde{\epsilon}^Q))\right)\right] \end{cases}$$

(电压平方项的方差)

3. Data-Driven DR-AC-CCOPF：在AC-CCOPF基础上加上考虑矩不确定性的DRO

The above formulations are under the assumption of perfect knowledge of the distribution of random variable (mean and variance). In this chapter, true distribution cannot be known. We can only inform by finite observations.

Redefine the AC-CCOPF obj: $\min_x \max_{\mathbb{P}\in U} \mathbb{E}_{\mathbb{P}}[f(x,\epsilon)]$. minimize the worst-case expectation based on the distributional uncertainty set.

3.1 Definition of Uncertainty Set

- $H(\epsilon_i) = \{\hat{\epsilon}_{it}\}_{t\le N}$, $i\in N$ is the set of N observed realizations of forecast error at bus i.
- Zero-mean, normally distributed ⇒ distribution defined by sample variance: $\hat{\sigma}_i^2 = \frac{1}{N}\sum_{t\le N}\hat{\epsilon}_{it}^2$, $i \in N$
- The sample variance follows a Chi-Square (χ^2) distribution → define uncertainty set $U_{\hat{\sigma}_i^2} = [\hat{\varepsilon}_{i,l}, \hat{\varepsilon}_{i,h}]$

  where: $\hat{\varepsilon}_{i,l} = \frac{N\hat{\sigma}_i^2}{\chi^2_{N,(1-\varepsilon)/2}}$, $\hat{\varepsilon}_{i,h} = \frac{N\hat{\sigma}_i^2}{\chi^2_{N,\varepsilon/2}}$. $\chi^2_{N,\varepsilon}$ is the ε-quartile of χ^2-distribution (not symmetric)

3.2 Worst-Case Expectation

$$\sup_{\sigma_i^2\in U_{\hat{\sigma}_i^2},\, i\in N} \mathbb{E}[f_p(\bar{g}^P, \epsilon^P)] = \sup_{\sigma_i^2\in U_{\hat{\sigma}_i^2},\, i\in N} \sum_{i\in G}[c_{i2}(\alpha_i^2 Var(\tilde{\epsilon}^P) + (\bar{g}_i^P)^2) + c_{i0}] = \sum_{i\in G}[c_{i2}(\alpha_i^2\sum_{i\in N}\hat{\varepsilon}_{i,h} + (\bar{g}_i^P)^2) + c_{i1}\bar{g}_i^P + c_{i0}]$$

3.3 DR-AC-CCOPF model

最小化由误差引起的成本

$$\min_{\bar{g},\alpha,\bar{f},\bar{u})} \sum_{i\in\mathcal{G}}\left[c_{i2}^P\left(\alpha_i^2\sum_{i\in\mathcal{N}}\hat{\zeta}_{i,h}^P + (\bar{g}_i^P)^2\right) + c_{i1}\bar{g}_i^P + c_{i0}^P\right] \tag{22a}$$

有功预测误差方差上界(20b)

数据

$$\sum_{i\in\mathcal{G}}\alpha_i = 1 \tag{22b}$$

$$(\bar{d}_i^P - \bar{g}_i^P) + \sum_{j\in\mathcal{C}_i}\bar{f}_j^P = \bar{f}_i^P \quad \forall i\in\mathcal{E}, p\in\{P,Q\} \tag{22c}$$

负荷平衡

$$\bar{u}_{\mathcal{A}_i} - 2(\bar{f}_i^P R_i + \bar{f}_i^Q X_i) = \bar{u}_i \quad \forall i\in\mathcal{N}^+ \tag{22d}$$

电压约束

$$g_i^{p,max} \geq \bar{g}_i^p + z_{\eta_g}\alpha_i\sqrt{\sum_{k=1}^{b}\hat{\zeta}_{i,h}^p}\ \forall i\in\mathcal{G}, p\in\{P,Q\} \tag{22e}$$

发电机功率限制约束

$$-g_i^{p,min} \geq -\bar{g}_i^p + z_{\eta_g}\alpha_i\sqrt{\sum_{k=1}^{b}\hat{\zeta}_{i,h}^p}\ \forall i\in\mathcal{G}, p\in\{P,Q\} \tag{22f}$$

$$u_i^{max} \geq \bar{u}_i + z_{\eta_v}2\sqrt{h_i(\alpha)} \quad \forall i\in\mathcal{N} \tag{22g}$$

$$-u_i^{min} \geq -\bar{u}_i + z_{\eta_v}2\sqrt{h_i(\alpha)} \quad \forall i\in\mathcal{N} \tag{22h}$$

$$h_i(\alpha) = \sum_{k=1}^{l} a_{ki}\left[\left(R_k^2\sum_{j=1}^{b} a_{kj}(\hat{\zeta}_{j,h}^P + \alpha_j^2\sum_{m=1}^{b}\hat{\zeta}_{m,h}^P)\right) + \left(X_k^2\sum_{j=1}^{b} a_{kj}(\hat{\zeta}_{j,h}^Q + \alpha_j^2\sum_{m=1}^{b}\hat{\zeta}_{m,h}^Q)\right)\right] \tag{22i}$$

4. Case Study

4.1 Uncertainty interval derivation: use true distribution to obtain $N = 100$ error samples

Change χ^2-distribution quantile ε and obtain different uncertainty intervals and possible distributions.

4.2 In-Sample Evaluation: use true distribution to generate 750 random samples

Change η_v, obtain different probabilities of voltage constraint violations and costs (trade-off)

4.3 Out-of-Sample Performance: use newly parameterized distributions supported by initial sample data to generate 750 samples

→ new value of σ^2, changed by parameter δ : $\sigma^2+0.2\delta$, $\sigma^2+0.5\delta$, $\sigma^2+0.8\delta$.

Change η_v and ε, get different probabities of voltage violations

Increase of δ will lead to larger distance between OOS distribution and true distribution, which will increase probability of voltage violations