Digital Signal Processing

Solved HW for Day 6

Question 1: DFT in matrix form

- Express the DFT and inverse DFT (IDFT) formulas (analysis and synthesis) as a matrix - vector multiplication.
- Is the DFT matrix hermitian?

Solution of question 1

Q: Express the DFT and inverse DFT (IDFT) formulas (analysis and synthesis) as a matrix - vector multiplication.

- Recall the DFT (analysis) formula:

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}.$$

- We can define an $N \times N$ square matrix $\mathbf{W}$ by stacking the conjugates of $\{\mathbf{w}^{(k)}\}_k$:

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}^{*(0)} \\ \mathbf{w}^{*(1)} \\ \mathbf{w}^{*(2)} \\ \dots \\ \mathbf{w}^{*(N-1)} \end{bmatrix}$$

Solution of question 1

- And get the analysis formula in matrix - vector multiplication form:

$$\mathbf{X} \;=\; \mathbf{W}\mathbf{x}.$$

- Then, knowing that:

$$\mathbf{W}\mathbf{W}^H = \mathbf{W}^H\mathbf{W} = N\mathbf{I}$$

- we obtain the synthesis formula in matrix-vector multiplication form:

$$\mathbf{x} \;=\; \frac{1}{N}\mathbf{W}^H\mathbf{X}.$$

Solution of question 1

Q: Is the DFT matrix hermitian?

- A matrix $\mathbf{A}$ is hermitian when

$$\mathbf{A} = \mathbf{A}^H$$

- Therefore, for $\mathbf{W}$ to be hermitian, we would need:

$$\mathbf{W}_{nk} = \mathbf{W}^*_{kn}$$

for all $n, k \in \{0, 1, \dots N-1\}$.

Solution of question 1

- This translates to having:

$$e^{-j\frac{2\pi}{N}nk} = e^{j\frac{2\pi}{N}kn}.$$

for all $n, k \in \{0, 1, \ldots N-1\}$, which is generally not the case.

- Consider, for example, the case when $n = k = 1$. We would need to have:

$$e^{-j\frac{2\pi}{N}} = e^{j\frac{2\pi}{N}}$$

which is equivalent to:

$$e^{j\frac{4\pi}{N}} = 1.$$

- Clearly, this can only happen when $N = 2$.