Aalto University
School of Electrical
Engineering

Communication acoustics
Ch 2: Physics of sound – Acoustics

Ville Pulkki and Matti Karjalainen

Department of Signal Processing and Acoustics
Aalto University, Finland

September 11, 2017

Physics of sound

- Basic quantities
- Vibration – generation of sound
- Resonance, resonators
- Sound radiation
- Sound propagation
- Reflection, absorption,
- Diffraction, refraction
- Modeling of acoustics

Sound pressure

- Listeners are sensitive to sound pressure waves
- Atmospheric pressure p_a, can be assumed constant with time
- Audible sounds can be caused by pressure disturbances in air that have frequencies between 20Hz and 20kHz
- Instantaneous sound pressure $p(t)$ [Pa]
- Effective value (rms value) of sound pressure

$$p_{\mathrm{rms}} = \frac{1}{t_2 - t_1} \sqrt{\int p(t)^2 \mathrm{d}t}$$

Sound pressure level, Decibel

- Effective sound pressure p [Pa]
- Sound pressure level $L_p = 20\log_{10}(p/p_0)$
- Reference level $p_0 = 20 \cdot 10^{-6}$ Pa

ratio	decibels	ratio	decibels
1/1	0		
$\sqrt{2} \approx 1.41$	$\approx 3.01 \approx 3$	$\sqrt{1/2} \approx 0.71$	$\approx -3.01 \approx -3$
2/1	$\approx 6.02 \approx 6$	1/2	$\approx -6.02 \approx -6$
$\sqrt{10} \approx 3.16$	10	$\sqrt{1/10} \approx 0.316$	-10
10/1	20	1/10	-20
100/1	40	1/100	-40
1000/1	60	1/1000	-60

Other relevant quantities

- Particle velocity, $\mathbf{v}(t)$, expressed as 3D vector
- Volume velocity q effective rate of particle flow through area in unit time
- Sound velocity (speed of sound) c about 340 m/s, scalar value describing speed of moving sound waves in any direction
- Intensity $\mathbf{I}(t) = p(t)\mathbf{v}(t)$, net flow of energy through measurement position
- Sound energy is estimated often as p^2, better term would be

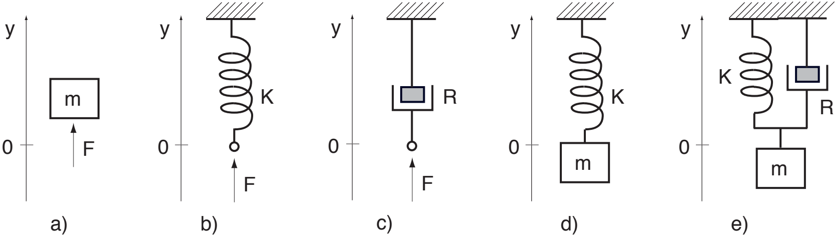
$$E = \frac{\rho_0}{4}||\mathbf{v}||^2 + \frac{1}{4\rho_0 c^2}|p|^2$$

- Sound power, property of sound source radiating sound energy along with the sound wave it creates
- Sound power level L_W
- Impedances Z_a and Z_0, acoustic and and characteristic impedances, relations between pressure and either particle velocity or volume velocity, respectively

Vibration – generation of sound

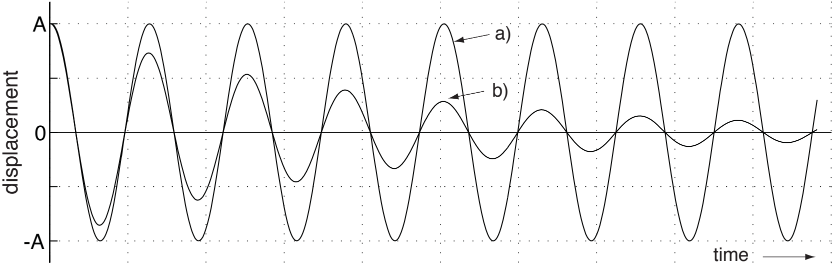
- Most of physical sounds in natural environment are caused by vibrating objects
- Frequency range 20 Hz – 20 kHz (audible frequencies)
- Impact sounds, water, animal sounds, human voice, musical instruments
- Exception: electric sparks, thunder

Vibrating systems



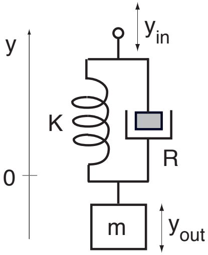
- Simple vibration: mass-spring system

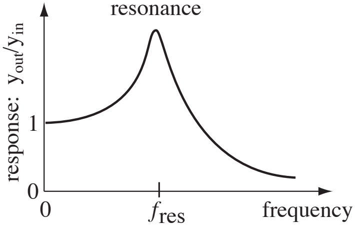
Vibrating systems



- $y(t) = Ae^{-\alpha t}\cos(\omega_{\mathrm{d}} t + \phi) = A(t)\cos(\psi(t))$

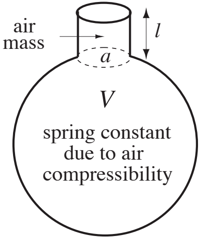
Resonance





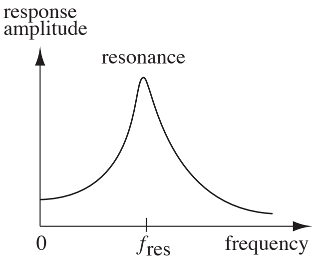
- Mass-spring resonator
- Single mass, single resonance, single mode

Resonance



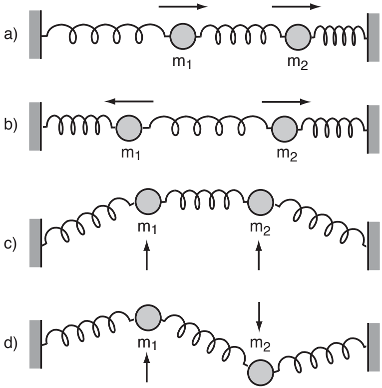
$$f_{res} = \frac{c}{2\pi}\sqrt{\frac{a}{Vl}}$$

$$c = 340 \text{ m/s}$$



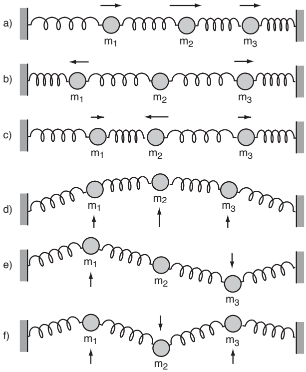
- Helmholtz-resonator

Two-mass vibrating systems



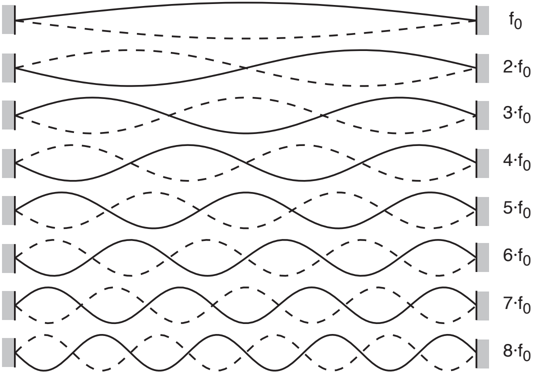
- Transversal and longitudinal vibration of a two-mass system
- Each case forms a mode (a resonance)
- Two modes in transversal vibration
- Two modes in longitudinal vibration

Three-mass vibrating systems



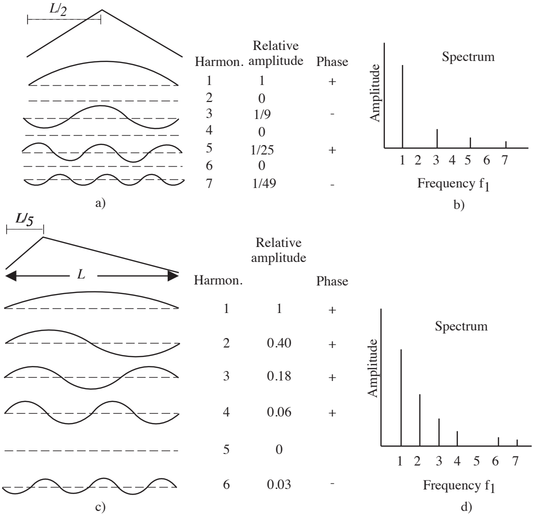
- Transversal and longitudinal vibration of a three-mass system

Vibration modes of a string

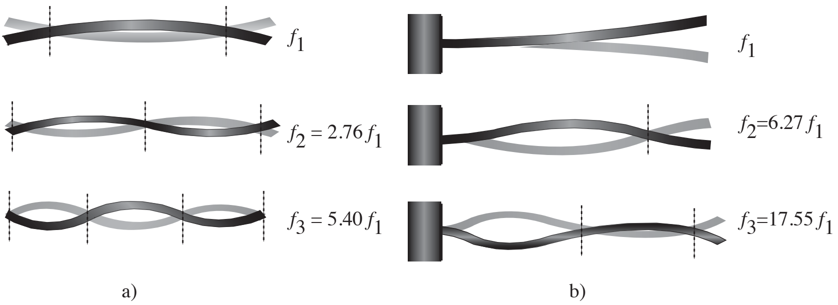


- String is continuous, infinite number of masses
- Infinite number of modes, whose resonance frequencies are integer multiples of fundamental frequency
- Harmonic spectrum
- (demo with guitar string)

Modes and spectral content in string vibration

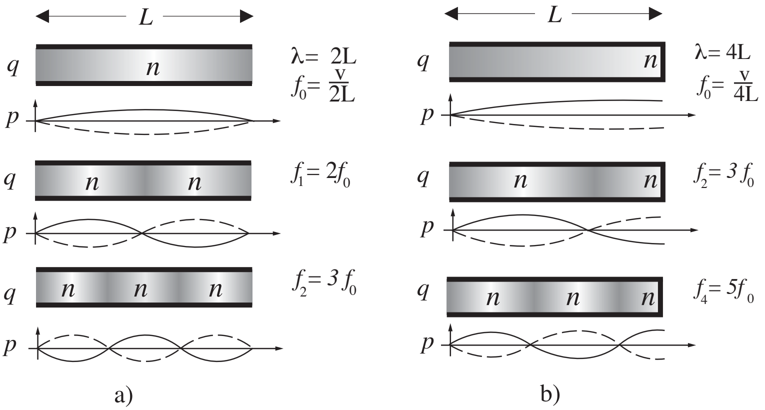


Modes in bars



- Stiffness of bar makes frequencies to travel with different speeds
- Modes are not related to each other with integer relations
- Inharmonic spectrum
- (demo)

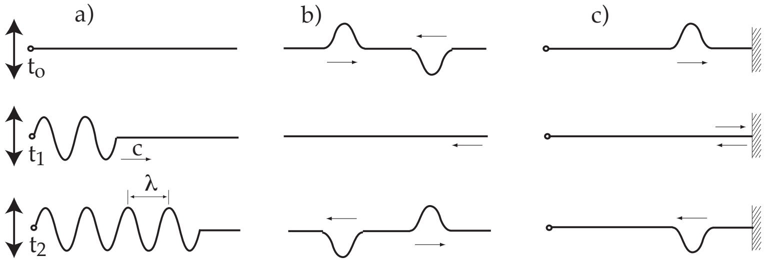
Resonance modes in tube



- Harmonic spectrum
- Spectrum: a) all harmonics b) only odd harmonics
- (demo with tube)

1D wave propagation

- Wave equation $\ddot{y} = c^2 y''$
- D'Alember 1D solution $y(t, x) = g_1(ct - x) + g_2(ct + x)$
- $\lambda = c/f$



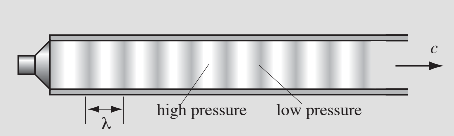
Wave propagation animations

Click for animation

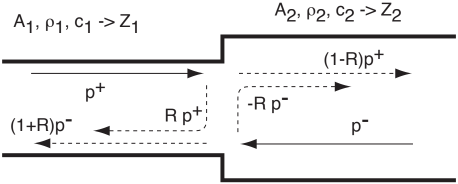
Animation courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State

- Speed of wave propagation c

Wave phenomena: plane wave in tube



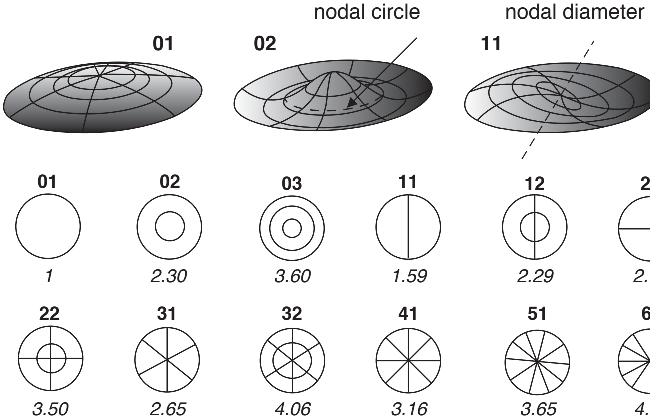
- Reflection and transmission. $R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$



2D and 3D propagation of vibration

- If vibration can propagate in 2D or 3D, it will do that
- Waves in the sea, vibration on a drum, earthquakes (2D)
- Propagation of sound in air, radio waves (3D)

Modes in 2D membranes



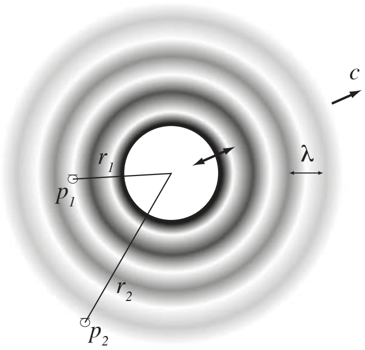
- Complex distribution of modes → inharmonic spectrum

Radiation from sound source

- Sound is caused by mechanical vibrations in audible range of frequencies
- Sound source is coupled to air
- Some of energy of source vibration emanates as sound
- Radiation has often directional pattern, sound is radiated to different directions with different strengths

Spherical wave propagation

- Sound velocity in air $c_{\mathrm{air}}(T) = 331.3 + 0.6T$
- Longitudinal wave, moving rarefactions and compressions
- Spherical wave:
  - Energy is constant in each spherical wave
  - Area of the wave $\propto r^2$ -> energy decays with $1/r^2$ law
  - Amplitude decays with 1/r law
  - $p_1 r_1 = p_2 r_2$, $p_2 = p_1(r_1/r_2)$



Directional patterns

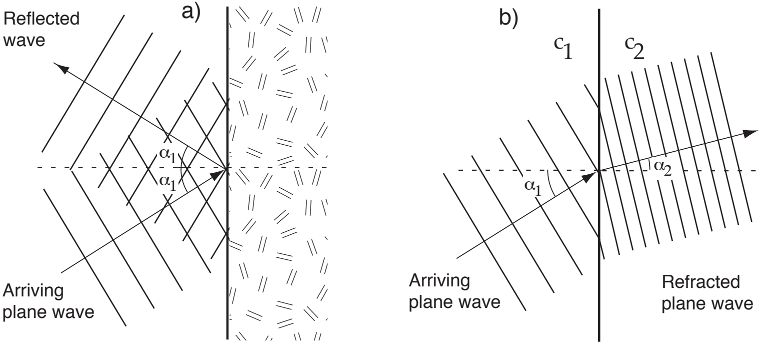
- Omnidirectional radiates with amplitude coefficient c_0 to all directions
- Dipole $c_1 \cos\theta$
- Quadrupole $c_2 \cos 2\theta$
- When source is large compared to wave length, the radiation pattern is affected a lot
- The directionality is often irregular, as sources are typically irregular in shape
- In practise sound source directivities are complex frequency-dependent combinations $\sum_n c_n \cos n\theta$

▸ Link to dipole radiation video

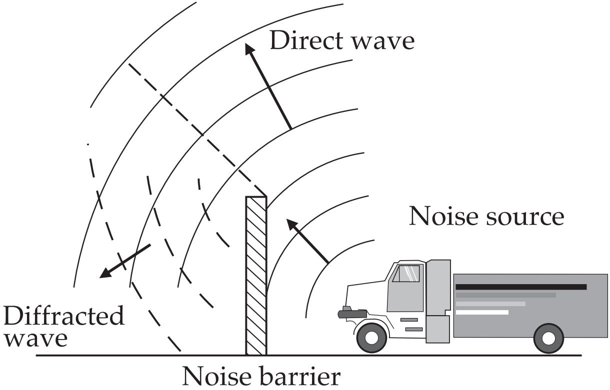
▸ Link to video on acoustical directivities

Reflection and refraction

- After radiation, sound wave continues straight ahead, if medium is still and has constant density
- In cases of obstacles, or changes in medium, sound changes its direction

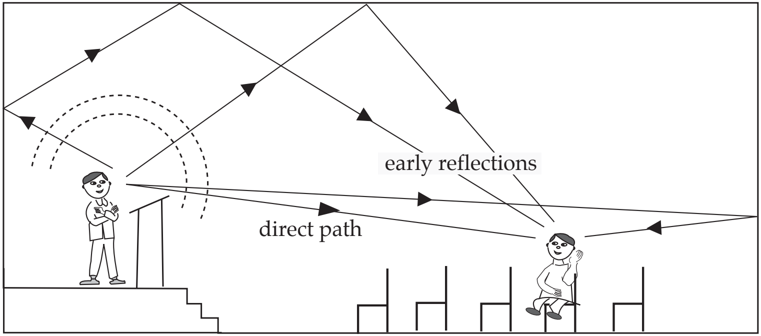


Diffraction

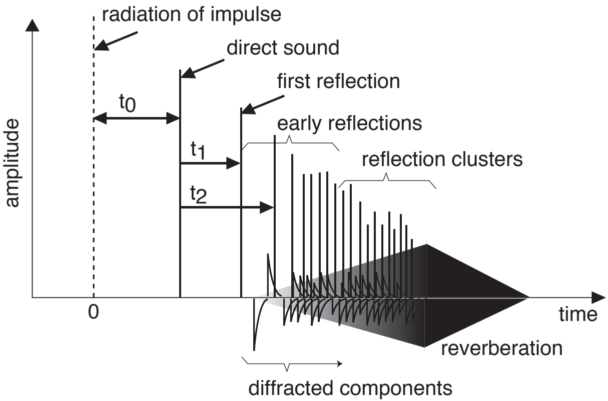


- non-planar surfaces and especially edges (other than 90 degrees corners) act as secondary sources

Sound propagation paths in a room

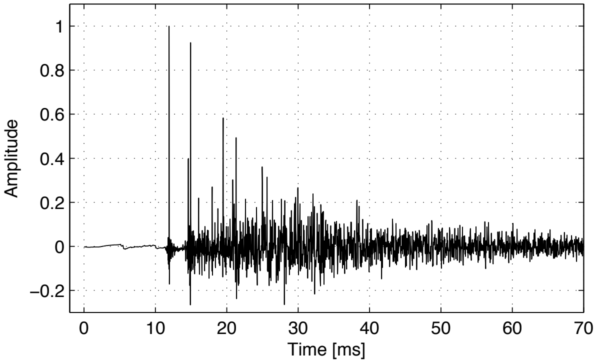


Impulse response of a room



- Theoretic response to an ideal impulse
- Instantaneous amplitude of $p(t)$ is plotted

Impulse response of a room



Real measured response of a listening room to a laser-induced spark source

3D propagation of sound visualized in 2D plane

Click for animation

Animation courtesy of Tapio Lokki, Aalto University

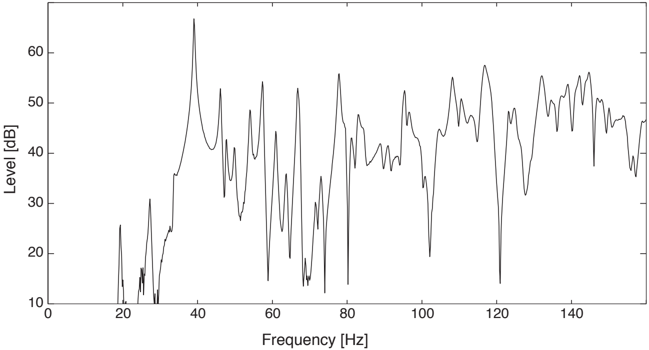
Diffuse field, room modes

- Diffuse field: sound with equal frequency content arrives evenly from all directions with random phase relations
- Late reverberation produces diffuse field in many rooms
- Rooms have also resonances
- Standing waves can be evoked with sinusoidal stimulus
- Room modes: spatial distribution of pressure [or velocity] maxima

▸ Link to room mode visualization

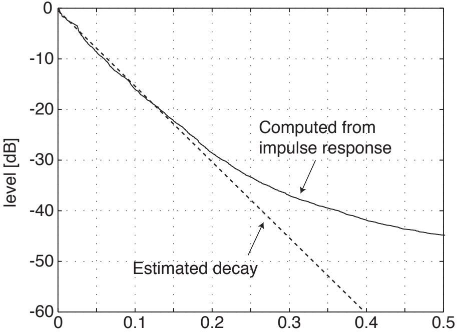
Modes in rectangular room

Mode frequencies $f(n_x, n_y, n_z) = \frac{c}{2}\sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2}$



Reverberation time

- The time that it takes sound to decay 60dB after offset
- Can be measured from impulse response
- In reverberant rooms similar values found for different positions
- In rooms with absorbents, value may change a lot depending on position

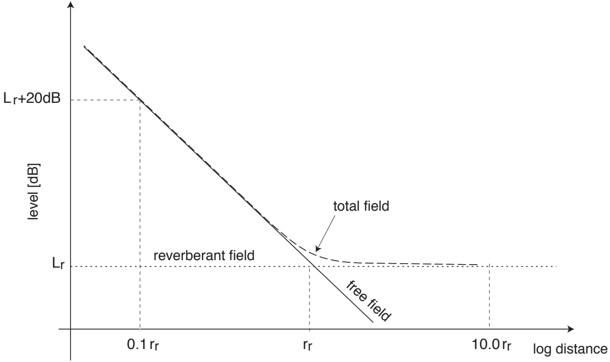


Estimation of reverberation time

- Sabine's formula for simple estimation from room geometry: $T_{60} = 0.161 V/S$
- Volume of room V [m^3]
- Absorption area of room S
- $S = \sum a_i A_i$, a_i is absorption coefficient (e.g., table below)
- A_i is surface area [m^2]

Frequency	125	250	500	1000	2000	4000
Glass window	0.35	0.25	0.18	0.12	0.07	0.04
Painted concrete	0.10	0.05	0.06	0.07	0.09	0.08
Wooden floor	0.15	0.11	0.10	0.07	0.06	0.07

Level of sound field in room as function of distance from source



$$L_p = L_W + 10\log_{10}\left(\frac{Q}{4\pi r^2} + \frac{4}{S}\right), L_\mathrm{r} = L_W + 10\log_{10}\left(\frac{4}{S}\right)$$

- radius of reverberation r_r
- $r_r = \frac{1}{4}\sqrt{\frac{QS}{\pi}}$

Sound pressure caused by multiple sources

- Sound pressure is always measured in single position with a microphone (or listened to with ear)
- Often microphone captures sound (almost) equally from all directions
- How can we compute the sound pressure caused by multiple sources?

$$p_{\mathrm{rms}} = \frac{1}{t_2 - t_1}\sqrt{\int (p_1(t) + p_2(t))^2 \mathrm{d}t}$$

$$p_{\mathrm{rms}} = \frac{1}{t_2 - t_1}\sqrt{\int (p_1^2(t) + 2p_1(t)p_2(t) + p_2^2(t)) \mathrm{d}t}$$

$2p_1(t)p_2(t)$ has mean value of zero if $p_1(t)$ and $p_2(t)$ are uncorrelated

$$p_{\mathrm{rms}} = \frac{1}{t_2 - t_1}\sqrt{\int (p_1^2(t) + p_2^2(t)) \mathrm{d}t}$$

Sound pressure caused by multiple sources

- If coherent sound arrives from multiple directions to microphone (reflections, stereophonic sound)
  - Two sources: $p_{\text{tot}}(t) = p(t) + p(t) = 2p(t)$
  - 6dB increase in p_{rms}
- If incoherent sound arrives from multiple directions to microphone (multiple concurrent sources)
  - N sources: $p_{\text{tot}_{\text{rms}}} = \sqrt{\sum p^2_{\text{n}_{\text{rms}}}}$
  - Two sources: $L_{\text{tot}} = 10 \log_{10} \left(10^{L_1/10} + 10^{L_2/10} \right)$
  - 3dB increase in p_{rms}

Example 1, radiation from loudspeaker cone

A loudspeaker radiates sound with good efficiency, if the dimensions of the radiating surface are of the same order with sound being radiated. Too small radiating surface causes only air movement in vicinity of loudspeaker, and no sound is radiated far field. What can you say about radiation from a loudspeaker cone? What diameter should be needed for radiation at 30Hz, 1kHz or 2kHz.
$\lambda = c/f$

Example 1

Lets assume $c = 331.5\text{m/s}$
$\lambda = c/f$

30Hz: $\lambda = \frac{331.5\mathrm{m/s}}{30\ 1/\mathrm{s}} = 11.05\text{m} \approx 11$ m.

Example 1

Lets assume $c = 331.5$m/s
$\lambda = c/f$

30Hz: $\lambda = \frac{331.5\mathrm{m/s}}{30\ 1/\mathrm{s}} = 11.05$m ≈ 11 m. 1kHz:

$\lambda = \frac{331.5\mathrm{m/s}}{1000\ 1/\mathrm{s}} = 0.332$ m ≈ 33 cm.

Example 1

Lets assume $c = 331.5\text{m/s}$
$\lambda = c/f$

30Hz: $\lambda = \frac{331.5\mathrm{m/s}}{30\ 1/\mathrm{s}} = 11.05\text{m} \approx 11$ m. 1kHz:

$\lambda = \frac{331.5\mathrm{m/s}}{1000\ 1/\mathrm{s}} = 0.332$ m ≈ 33 cm. 10kHz:

$\lambda = \frac{331.5\mathrm{m/s}}{10000\ 1/\mathrm{s}} = 0.0332$ m ≈ 3.3 cm.

In practise, even largest cones are too small for 30Hz.

Example 2, loudspeaker cone coupled to a vented box

The efficiency of a cone in free air can be made better by attaching it to a box, and even better radiation is obtained when the resonance frequency of the box adjusted to match with desired frequency. A loudspeaker with bass reflex principle is in fact a Helmholtz resonator.
A typical small loudspeaker has volume V of 5500 cm^3, and a reflex tube with length l and opening area of a=8cm^2. What would be the correct tube length for 45Hz resonance frequency? Helmholtz resonance frequency can be computed

$$f = \frac{c}{2\pi}\sqrt{\frac{a}{Vl}} \Leftrightarrow l = a\frac{c^2}{V4\pi^2 f^2}$$

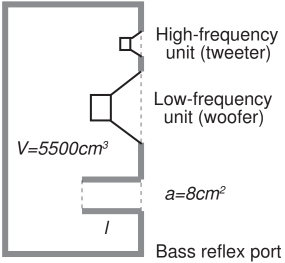
Example 2, loudspeaker cone coupled to a vented box

Helmholtz resonance frequency can be computed

$$f = \frac{c}{2\pi}\sqrt{\frac{a}{Vl}} \Leftrightarrow l = a\frac{c^2}{V4\pi^2 f^2}$$

Numerical values:

$$l = 8 * \frac{(340 * 100 \text{ cm/s})^2}{5500 * 4\pi^2 45^2} = 21 \text{ cm}$$



Example 3, reverberation time of a room

The dimensions of an empty right-angled room are 10 m$\times$6 m$\times$3 m. The absorption coefficients at 500Hz are in floor 0.06, ceiling 0.17 and walls 0.20. There are no windows neither doors in the room.

1. Compute the T_{60} of the room
2. What absorption coefficient should the walls have to obtain 0.7s for T_{60}?

Sabine's formula to estimate (more or less roughly) reverberation time:

$$T_{60} = 0.161 \frac{V}{S}$$

where V = volume, S = absorption area = $\sum_i a_i A_i$, A_i = area of a surface i, and a_i = absorption coefficient of surface i

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6$ m^2

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6$ m^2
   ceiling: $S_c = 60 * 0.17 = 10.2$ m^2

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6$ m^2
   ceiling: $S_c = 60 * 0.17 = 10.2$ m^2
   walls: $S_w = 2 * 30 * 0.20 + 2 * 0.2 * 18 = 19.2$ m^2

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6$ m^2
   ceiling: $S_c = 60 * 0.17 = 10.2$ m^2
   walls: $S_w = 2 * 30 * 0.20 + 2 * 0.2 * 18 = 19.2$ m^2
   Reverberation time is computed from total absorption area:

$$S = 3.6 + 10.2 + 19.2 = 33.0 \ \mathrm{m}^2 \tag{1}$$

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6$ m^2
   ceiling: $S_c = 60 * 0.17 = 10.2$ m^2
   walls: $S_w = 2 * 30 * 0.20 + 2 * 0.2 * 18 = 19.2$ m^2
   Reverberation time is computed from total absorption area:

$$S = 3.6 + 10.2 + 19.2 = 33.0 \text{ m}^2 \tag{1}$$

$$T_{60} = 0.161 \frac{V}{S} = 0.161 * \frac{180}{33} = 0.878 \text{ s}$$

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6$ m^2
   ceiling: $S_c = 60 * 0.17 = 10.2$ m^2
   walls: $S_w = 2 * 30 * 0.20 + 2 * 0.2 * 18 = 19.2$ m^2
   Reverberation time is computed from total absorption area:

$$S = 3.6 + 10.2 + 19.2 = 33.0\ \mathrm{m}^2 \tag{1}$$

$$T_{60} = 0.161 \frac{V}{S} = 0.161 * \frac{180}{33} = 0.878\ \mathrm{s}$$

2. Absorption coefficient for walls.

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180$ m^3

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6$ m^2
   ceiling: $S_c = 60 * 0.17 = 10.2$ m^2
   walls: $S_w = 2 * 30 * 0.20 + 2 * 0.2 * 18 = 19.2$ m^2
   Reverberation time is computed from total absorption area:

$$S = 3.6 + 10.2 + 19.2 = 33.0\ \mathrm{m}^2 \tag{1}$$

$$T_{60} = 0.161 \frac{V}{S} = 0.161 * \frac{180}{33} = 0.878\ \mathrm{s}$$

2. Absorption coefficient for walls.
   Absorption area: $S_w = K \frac{V}{T_{60}} - S_f - S_c$

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180\ \text{m}^3$

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6\ \text{m}^2$
   ceiling: $S_c = 60 * 0.17 = 10.2\ \text{m}^2$
   walls: $S_w = 2 * 30 * 0.20 + 2 * 0.2 * 18 = 19.2\ \text{m}^2$
   Reverberation time is computed from total absorption area:

$$S = 3.6 + 10.2 + 19.2 = 33.0\ \mathrm{m}^2 \tag{1}$$

$$T_{60} = 0.161 \frac{V}{S} = 0.161 * \frac{180}{33} = 0.878\ \mathrm{s}$$

2. Absorption coefficient for walls.
   Absorption area: $S_w = K \frac{V}{T_{60}} - S_f - S_c$
   Substitute numeric values $S_w = 0.161 \frac{180}{0.7} - 3.6 - 10.2 = 27.6\ \text{m}^2$

Example 3, reverberation time of a room

Volume of the room: $V = 10 * 6 * 3 = 180 \text{ m}^3$

1. Reverberation time. Total absorption area: $S = S_f + S_c + S_w$
   floor: $S_f = 60 * 0.06 = 3.6 \text{ m}^2$
   ceiling: $S_c = 60 * 0.17 = 10.2 \text{ m}^2$
   walls: $S_w = 2 * 30 * 0.20 + 2 * 0.2 * 18 = 19.2 \text{ m}^2$
   Reverberation time is computed from total absorption area:

$$S = 3.6 + 10.2 + 19.2 = 33.0 \text{ m}^2 \tag{1}$$

$$T_{60} = 0.161 \frac{V}{S} = 0.161 * \frac{180}{33} = 0.878 \text{ s}$$

2. Absorption coefficient for walls.
   Absorption area: $S_w = K \frac{V}{T_{60}} - S_f - S_c$
   Substitute numeric values $S_w = 0.161 \frac{180}{0.7} - 3.6 - 10.2 = 27.6 \text{ m}^2$
   The absorption coefficient should then be: $a = \frac{S_w}{A_w} = \frac{27.6}{2*30+2*18} = 0.29$

Example 4, SPL generated by multiple sources

Average talking produces 60dB SPL in 1m distance. What would be the SPL when

1. two persons are talking?
2. 10 persons are talking?
3. two persons with distances 1m and 3m are talking?

Assume that all of them are in 1m distance from the position of measurement.
Sound arriving from each source is incoherent, which means that the effective pressure p will be summed as quadratic.

$$p = \sqrt{(p_1^2 + p_2^2)}$$

Example 4, SPL generated by multiple sources

Effective sound pressure caused by one talker is p_1.
Lets compute SPLs:

1. Two talkers
   The pressure for two talkers is $p_2 = \sqrt{p_1^2 + p_1^2} = p_1\sqrt{2}$. $\Rightarrow$
   $L = 20\lg(p_2) = 20\lg(p_1\sqrt{2}) = L_1 + 3.01\mathrm{dB} = 63$ dB.

Example 4, SPL generated by multiple sources

Effective sound pressure caused by one talker is p_1.
Lets compute SPLs:

1. Two talkers
   The pressure for two talkers is $p_2 = \sqrt{p_1^2 + p_1^2} = p_1\sqrt{2}$. $\Rightarrow$
   $L = 20\lg(p_2) = 20\lg(p_1\sqrt{2}) = L_1 + 3.01\mathrm{dB} = 63$ dB.
2. Ten talkers: $\Rightarrow L = 20\lg(\sqrt{10} * p_1) = L_1 + 10$ dB = 70 dB

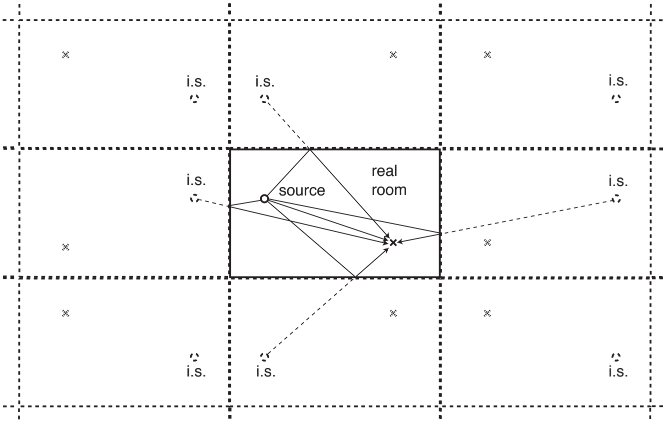
Example 4, SPL generated by multiple sources

Effective sound pressure caused by one talker is p_1.
Lets compute SPLs:

1. Two talkers
   The pressure for two talkers is $p_2 = \sqrt{p_1^2 + p_1^2} = p_1\sqrt{2}$. $\Rightarrow$
   $L = 20\lg(p_2) = 20\lg(p_1\sqrt{2}) = L_1 + 3.01\mathrm{dB} = 63$ dB.
2. Ten talkers: $\Rightarrow L = 20\lg(\sqrt{10} * p_1) = L_1 + 10$ dB = 70 dB
3. Two talkers with distances of 1 and 3 meters:
   $p_t = \sqrt{p_1^2 + (p_1(r_1/r_2))^2} = \sqrt{p_1^2 + (p_1 1/3)^2} = \sqrt{p_1^2 + 1/9(p_1)^2} = p_1\sqrt{10/9}$

$$L = 20\lg(\sqrt{10/9} * p_1) = L_1 + 0.5dB = 60.5\ \mathrm{dB} \tag{2}$$

Image source model of room acoustics



Each surface mirrors the source, and recursively generated mirror images

Other modeling methods for room acoustics

- *Ray tracing*, cast a number of rays and follow them until they hit the listener
- *Finite element method (FEM)*, fill the space with small connected elements, and compute the field in each element
- *Boundary-element method (BEM)*, model the elements on the boundaries of a space

All modeling methods have their pros and cons. They are useful in design of the acoustics of rooms, however, at least currently they can not provide perceptually authentic auralization of modeled rooms.

References

These slides follow corresponding chapter in: Pulkki, V. and Karjalainen, M. Communication Acoustics: An Introduction to Speech, Audio and Psychoacoustics. John Wiley & Sons, 2015, *where also a more complete list of references can be found*.