Digital Signal Processing
List of Transforms

Coursera - February 2013

Discrete Fourier Transform (DFT)	
used for:	finite support signals ($x[n] \in \mathbb{C}^N$)
analysis formula:	$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}, \quad k = 0, \ldots, N-1$
synthesis formula:	$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-nk}, \quad n = 0, \ldots, N-1$
symmetries:	$x[-n \mod N] \overset{\text{DFT}}{\longleftrightarrow} X[-k \mod N]$
	$x^*[n] \overset{\text{DFT}}{\longleftrightarrow} X^*[-k \mod N]$
shifts:	$x[(n-n_0) \mod N] \overset{\text{DFT}}{\longleftrightarrow} W_N^{kn_0} X[k]$
	$W_N^{-nk_0} x[n] \overset{\text{DFT}}{\longleftrightarrow} X[(k-k_0) \mod N]$
Parseval:	$\sum_{n=0}^{N-1} \|x[n]\|^2 = \frac{1}{N} \sum_{k=0}^{N-1} \|X[k]\|^2$

Some DFT pairs for length-N signals:	$(n, k = 0, 1, \ldots, N-1)$
$x[n] = \delta[n-M]$	$X[k] = e^{-j\frac{2\pi}{N}Mk}$
$x[n] = 1$	$X[k] = N\delta[k]$
$x[n] = e^{j\frac{2\pi}{N}L}$	$X[k] = N\delta[k-L]$
$x[n] = \cos(\frac{2\pi}{N}Ln + \phi)$	$X[k] = (N/2)[e^{j\phi}\delta[k-L] + e^{-j\phi}\delta[k-N+L)]]$
$x[n] = \sin(\frac{2\pi}{N}Ln + \phi)$	$X[k] = (-jN/2)[e^{j\phi}\delta[k-L] - e^{-j\phi}\delta[k-N+L]]$
$x[n] = \begin{cases} 1 & \text{for } n \leq M-1 \\ 0 & \text{for } M \leq n \leq N-1 \end{cases}$	$X[k] = \frac{\sin((\pi/N)Mk)}{\sin((\pi/N)k)} e^{-j\frac{\pi}{N}(M-1)k}$

Discrete-Time Fourier Transform (DTFT)

used for:	infinite, two sided signals ($x[n] \in \ell_2(\mathbb{Z})$)
analysis formula:	$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
synthesis formula:	$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$
symmetries:	$x[-n] \overset{\text{DTFT}}{\longleftrightarrow} X(e^{-j\omega})$
	$x^*[n] \overset{\text{DTFT}}{\longleftrightarrow} X^*(e^{-j\omega})$
shifts:	$x[n-n_0] \overset{\text{DTFT}}{\longleftrightarrow} e^{-j\omega n_0}X(e^{j\omega})$
	$e^{j\omega_0 n}x[n] \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j(\omega-\omega_0)})$
Parseval:	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$

Some DTFT pairs:

$x[n] = \delta[n-k]$	$X(e^{j\omega}) = e^{-j\omega k}$
$x[n] = 1$	$X(e^{j\omega}) = \tilde{\delta}(\omega)$
$x[n] = u[n]$	$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}} + \frac{1}{2}\tilde{\delta}(\omega)$
$x[n] = a^n u[n] \quad \lvert a\rvert < 1$	$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$
$x[n] = e^{j\omega_0 n}$	$X(e^{j\omega}) = \tilde{\delta}(\omega-\omega_0)$
$x[n] = \cos(\omega_0 n + \phi)$	$X(e^{j\omega}) = (1/2)[e^{j\phi}\tilde{\delta}(\omega-\omega_0) + e^{-j\phi}\tilde{\delta}(\omega+\omega_0)]$
$x[n] = \sin(\omega_0 n + \phi)$	$X(e^{j\omega}) = (-j/2)[e^{j\phi}\tilde{\delta}(\omega-\omega_0) - e^{-j\phi}\tilde{\delta}(\omega+\omega_0)]$
$x[n] = \begin{cases} 1 & \text{for } 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$	$X(e^{j\omega}) = \frac{\sin((N/2)\omega)}{\sin(\omega/2)}\, e^{-j\frac{N-1}{2}\omega}$