Digital Signal Processing

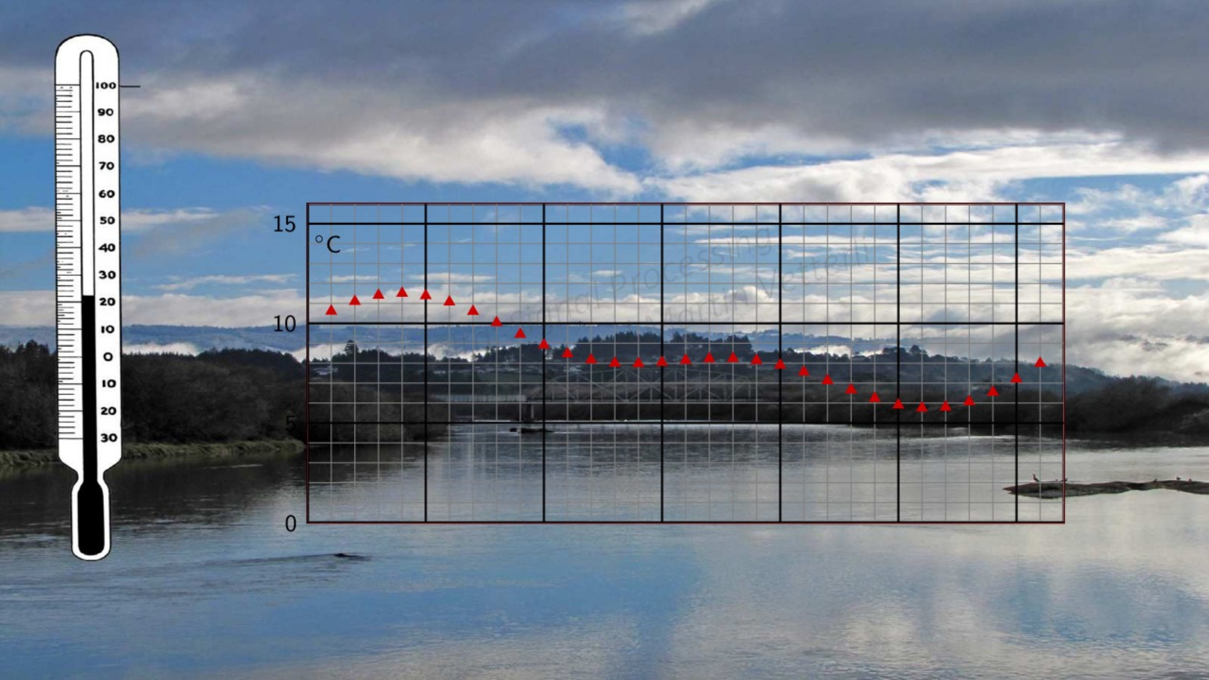
Module 1: Introduction

What is a signal?

Description of the evolution of a physical phenomenon

Examples:

- temperature (weather)
- pressure (sound)
- magnetic deviation (recorded sound)
- gray level on paper (photograph)
- ...

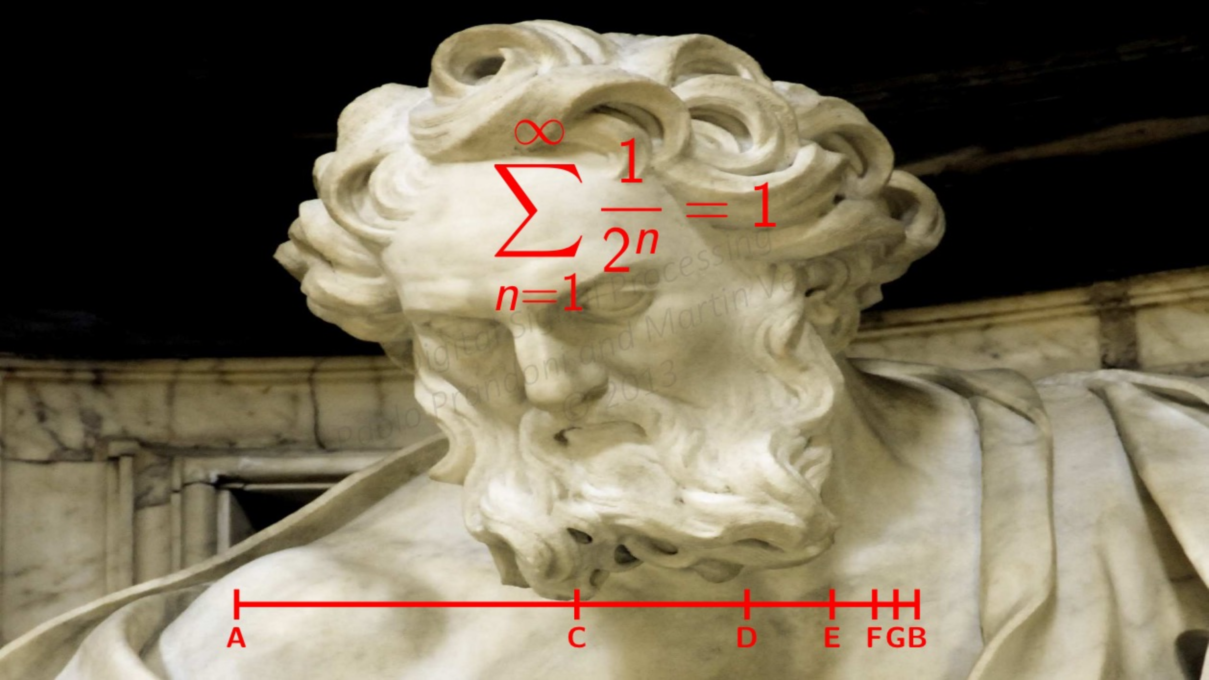

100
90
80
70
60
50
40
30
20
10
0
10
20
30
15
°C
10
5
0

Digital Signals

Key ingredients:

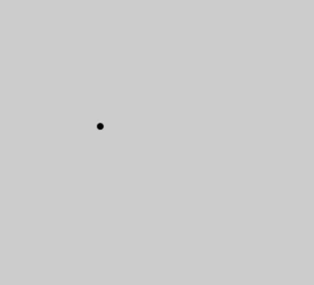
- discrete time
- discrete amplitude

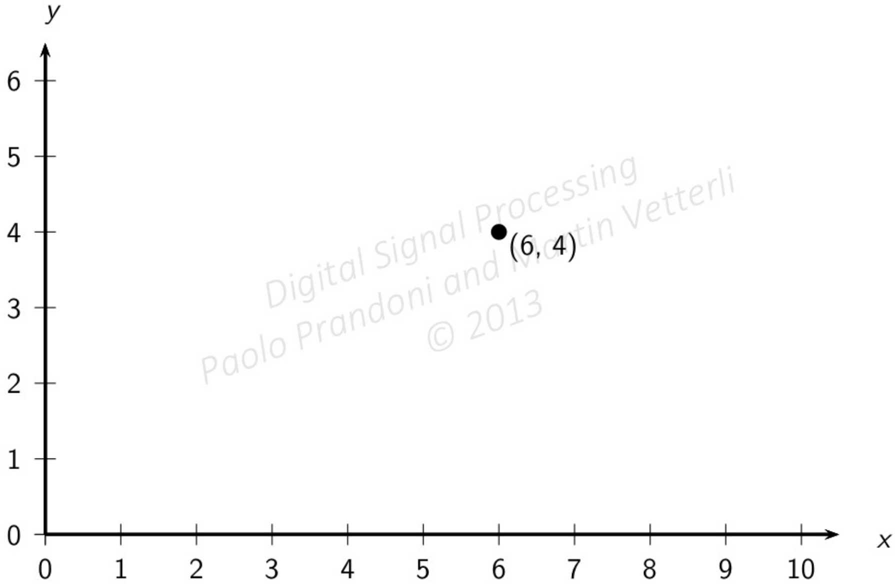
The circle point out (from left to right): Pythagoras, Parmenides, Plato, Euclid. Green for philosophers, red for mathematicians

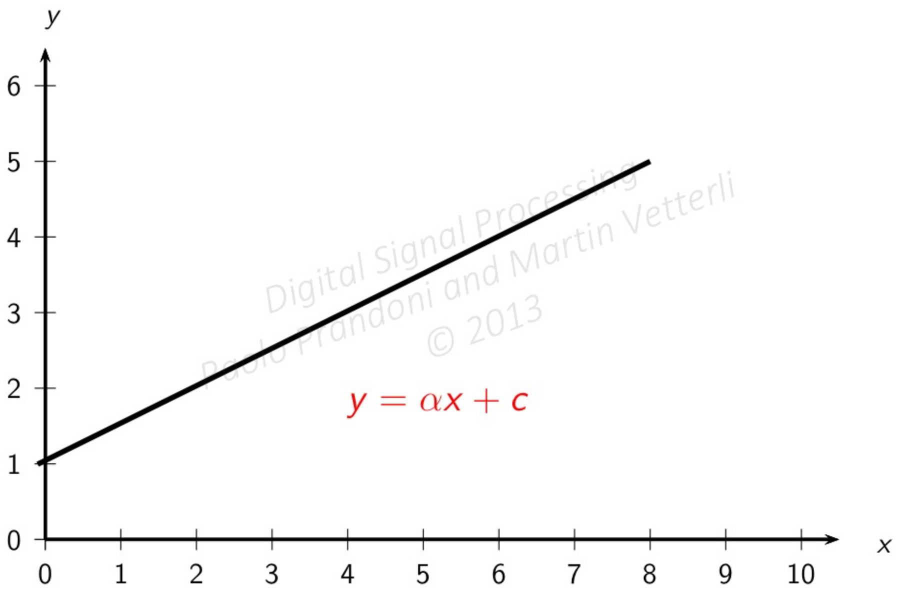


$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$
A
C
D
E
F
G
B

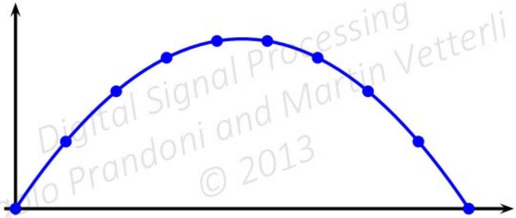

Idea here is: you got a point, nameless geometrical entity. Descartes comes and puts a reference around it, point gets a name. Then more complex things get names (such as lines) and can be described in terms of algebra




y
6
5
4
3
2
1
0
(6, 4)
0
1
2
3
4
5
6
7
8
9
10
x


y
x
0
1
2
3
4
5
6
7
8
9
10
$y = \alpha x + c$

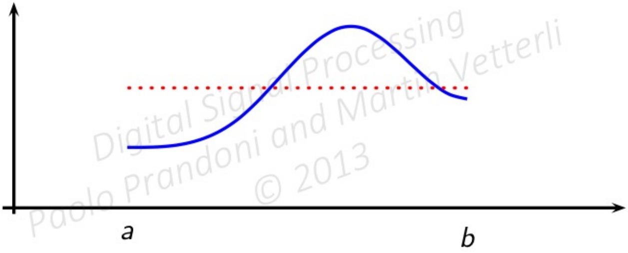
Calculus: ideas at the service of war



$$\vec{x}(t) = \vec{v}_0 t + (1/2)\vec{g}\, t^2$$

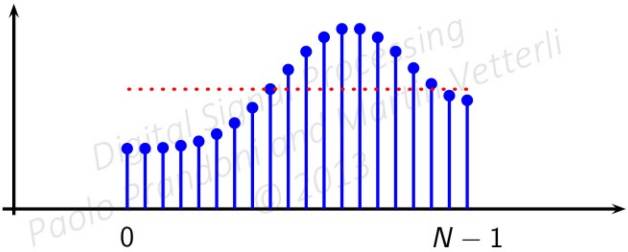
Galileo, 1638

Continuous-time signal processing: the average



$$\bar{x} = \frac{1}{b-a}\int_a^b f(t)dt$$

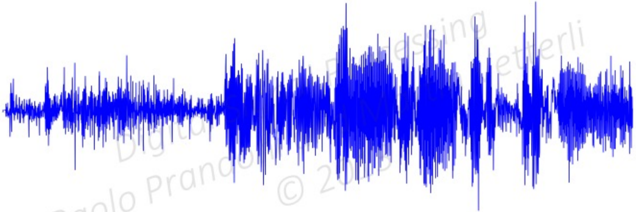
Discrete-time signal processing: the average



$$\bar{x} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]$$

Are we missing pieces?

What if the signal is "too fast"?



$$f(t) = ?$$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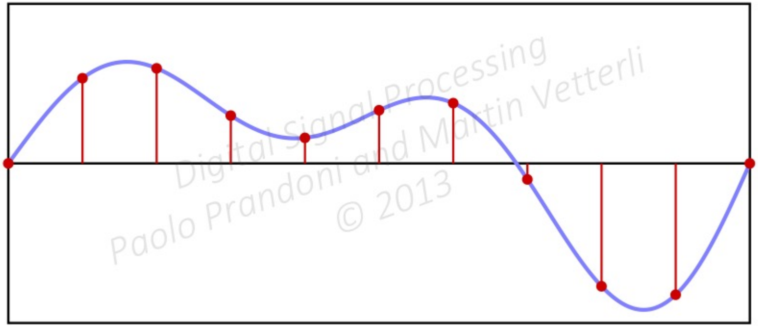
The Sampling Theorem (1920)

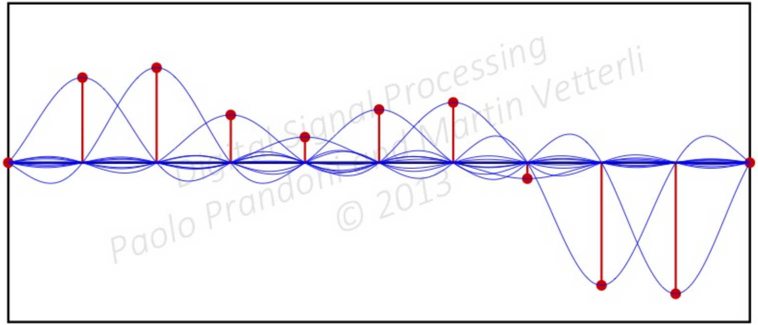
Under appropriate "slowness" conditions for $x(t)$ we have:

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(\pi(t - nT_s)/T_s)}{\pi(t - nT_s)/T_s}$$

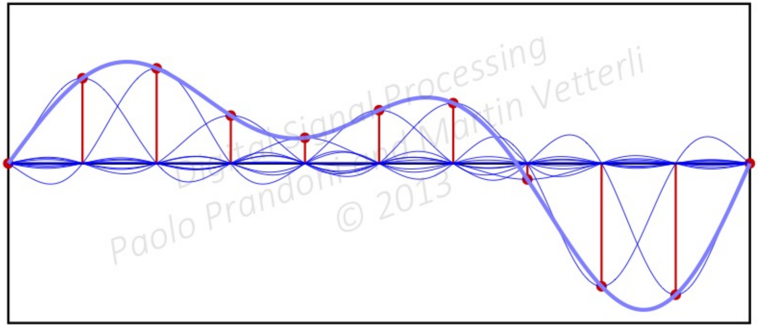
Digital Signals



Digital Signals



Digital Signals

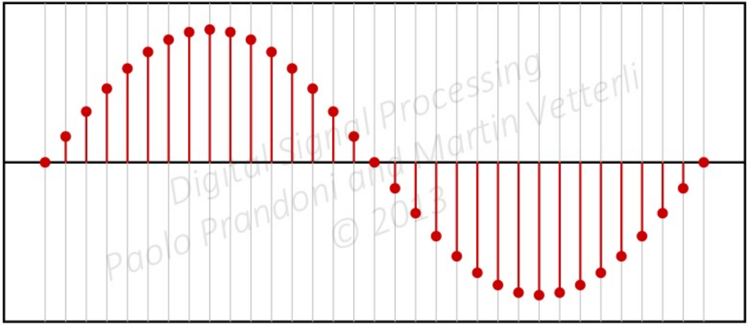


The world is analog, the computer is digital

So, what is resolution, really?

Key ingredients:

- discrete time
- **discrete amplitude**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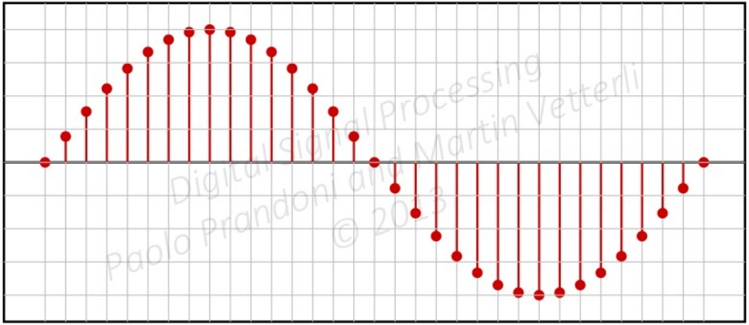
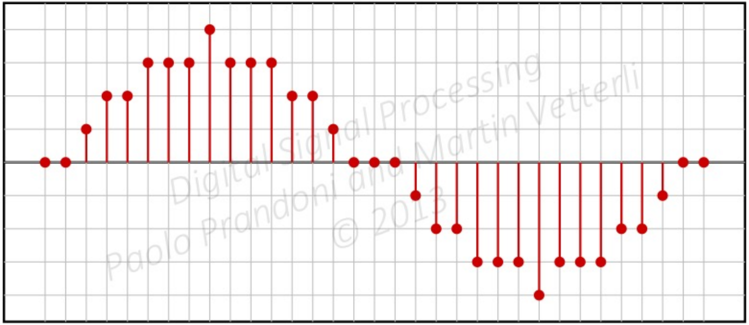
$x[n]$

Digital Signals



$x[n]$

Digital Signals



$\hat{x}[n]$

Digital amplitude

Why it is important:

- storage
- processing
- transmission

Analog storage:

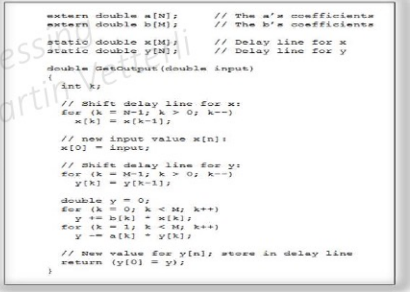
paper, wax cylinders, reel-to-reel, vinyl, compact cassette, VHS, Betamax, silver plates, Kodachrome, Super8, 8-Track, microfilm, ...

Digital storage:

$$\{0, 1\}$$

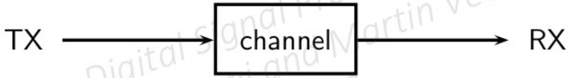

IBM
PRODIGY
Verbatim
DataLifePlus
BeOS
QNX
radius
redhat
TURBOLINUX
MICROSOFT
iMac
COMPAQ
Just one
MicroSD card
stores more than
the rest combined...
25 years
of storage

Processing

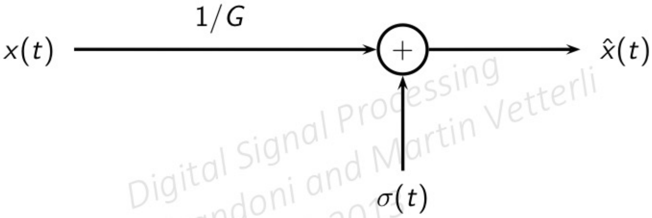


```
extern double a[N];      // The a's coefficients
extern double b[M];      // The b's coefficients

static double x[M];      // Delay line for x
static double y[N];      // Delay line for y

double GetOutput(double input)
{
  int k;

  // Shift delay line for x:
  for (k = N-1; k > 0; k--)
    x[k] = x[k-1];

  // new input value x[n]:
  x[0] = input;

  // Shift delay line for y:
  for (k = M-1; k > 0; k--)
    y[k] = y[k-1];

  double y = 0;
  for (k = 0; k < M; k++)
    y += b[k] * x[k];
  for (k = 1; k < M; k++)
    y -= a[k] * y[k];

  // New value for y[n]; store in delay line
  return (y[0] = y);
}
```

Data transmission

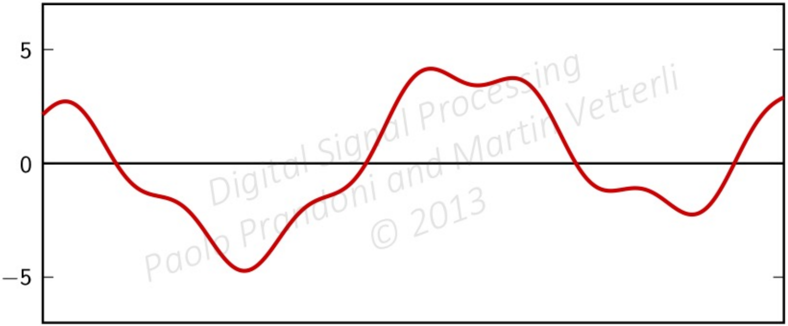


What happens to analog signals



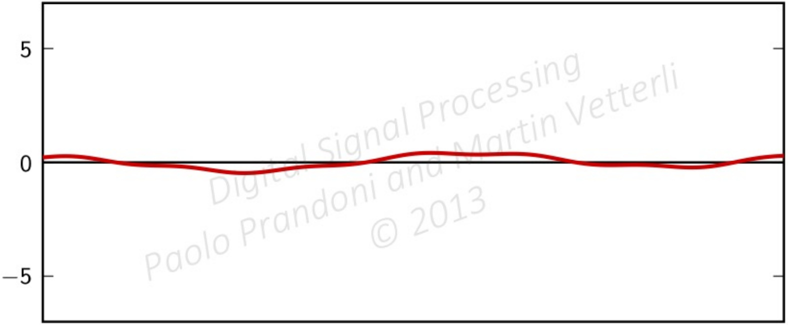
$$\hat{x}(t) = x(t)/G + \sigma(t)$$

What happens to analog signals



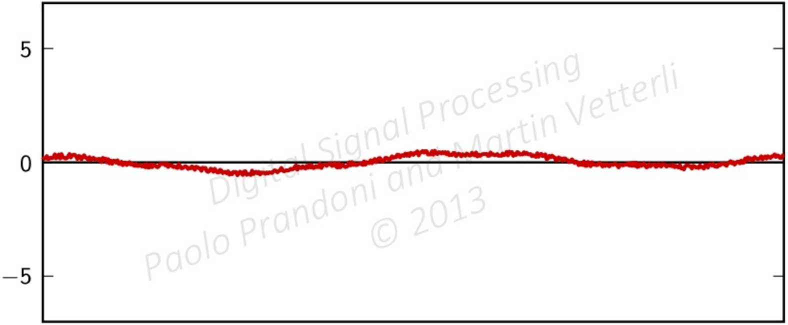
$x(t)$

What happens to analog signals



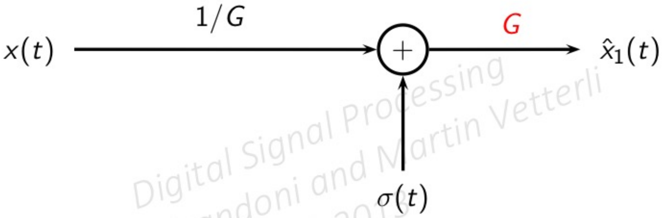
$x(t)/G$

What happens to analog signals



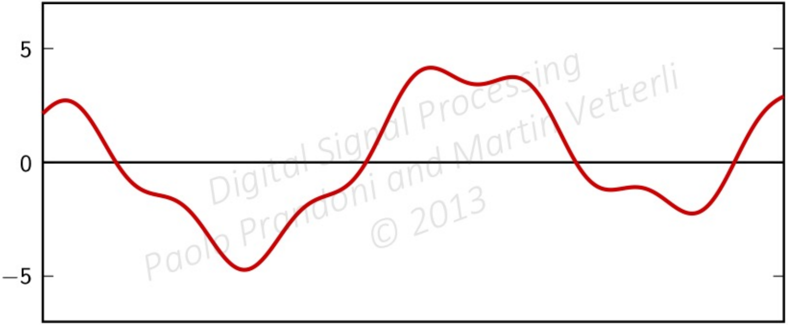
$$x(t)/G + \sigma(t)$$

We can amplify to compensate attenuation

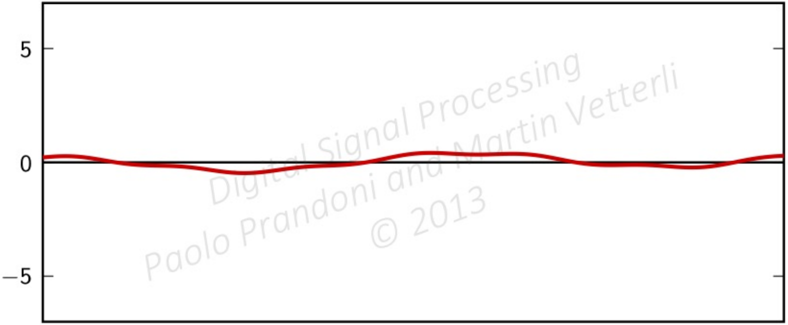


but: $\hat{x}_1(t) = x(t) + G\sigma(t)$

Transmission of analog signals



$x(t)$


5
0
−5
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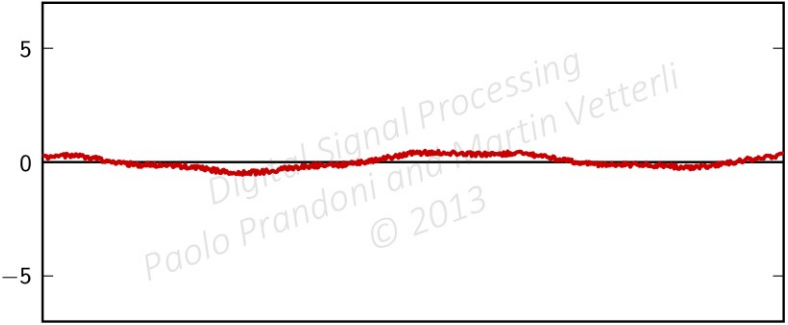
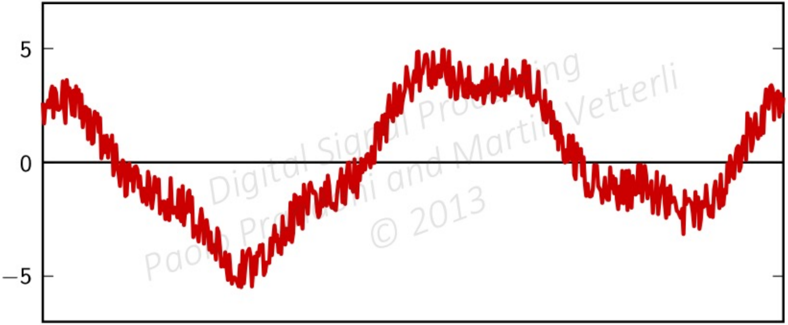
$x(t)/G$

Transmission of analog signals

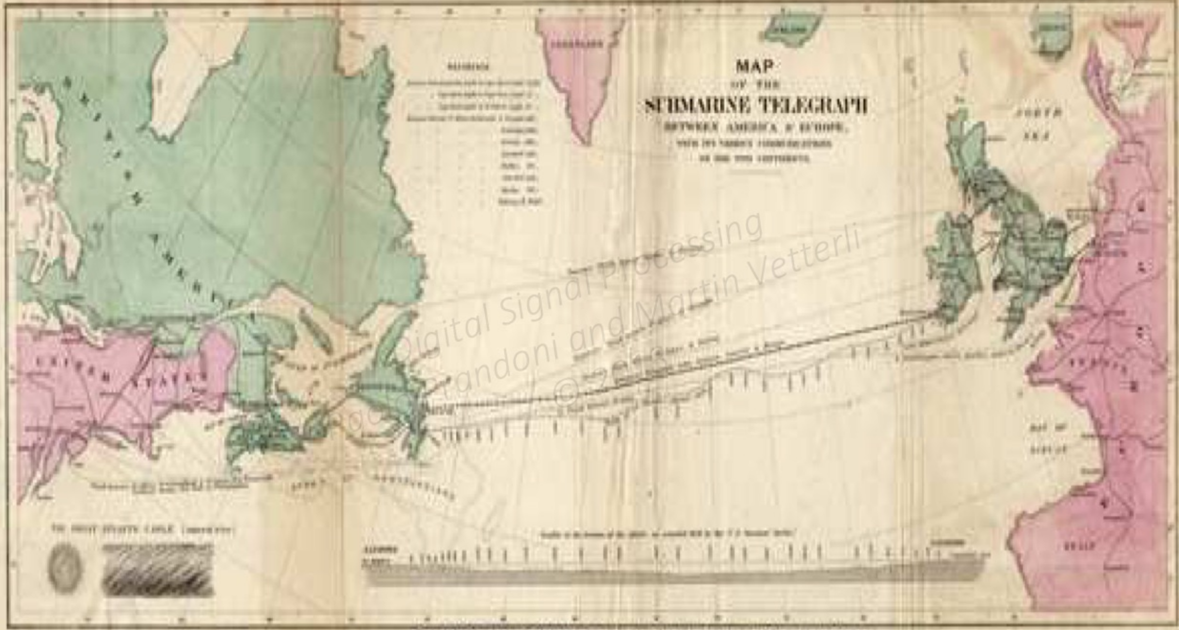


$$x(t)/G + \sigma(t)$$

Transmission of analog signals



$$\hat{x}_1(t) = G[x(t)/G + \sigma(t)] = x(t) + G\sigma(t)$$



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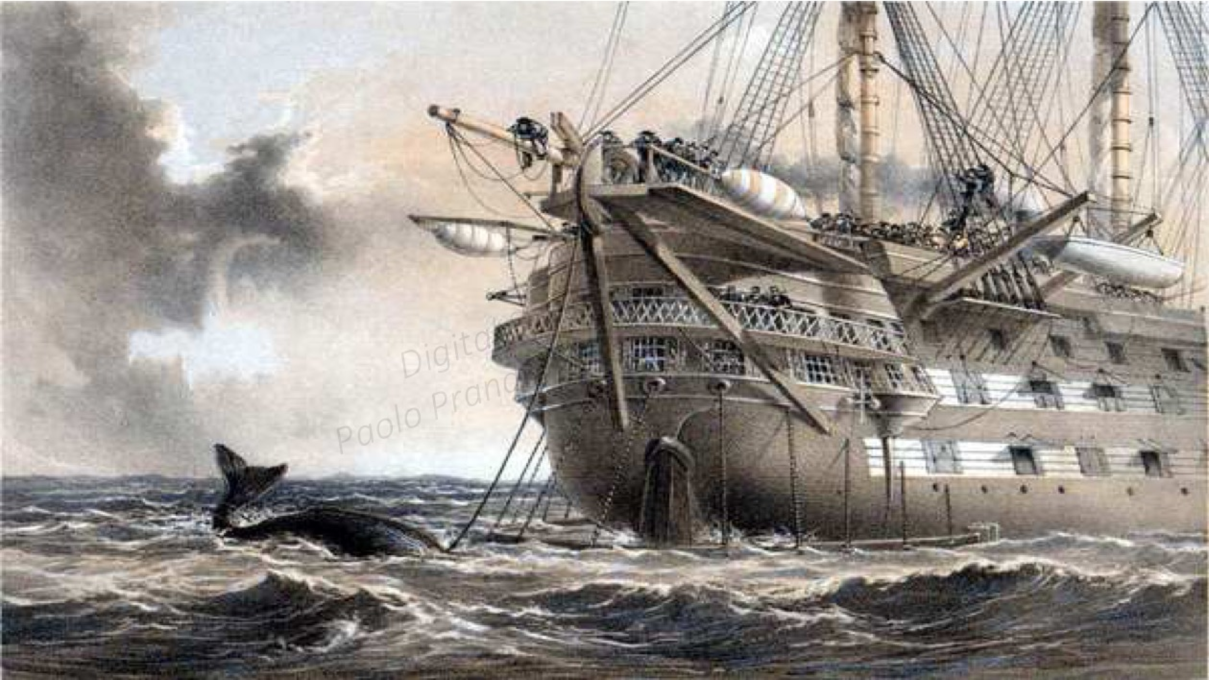
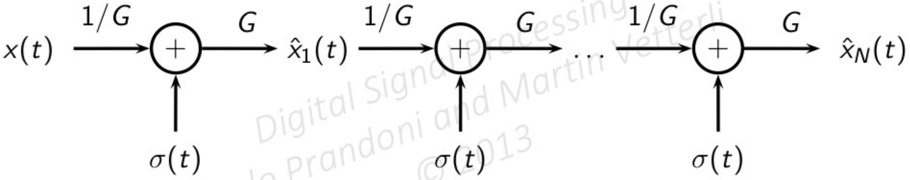
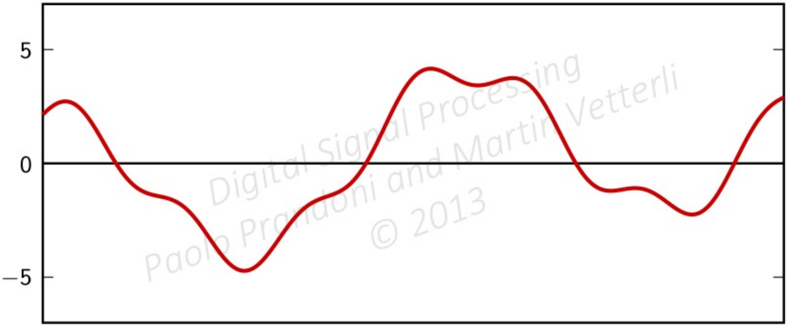


Transmitting a signal overseas

For a long, long channel we need repeaters



$$\hat{x}_N(t) = x(t) + NG\sigma(t)$$


5
0
−5
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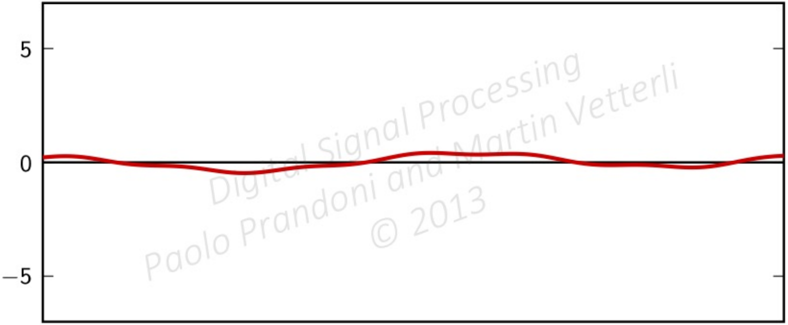
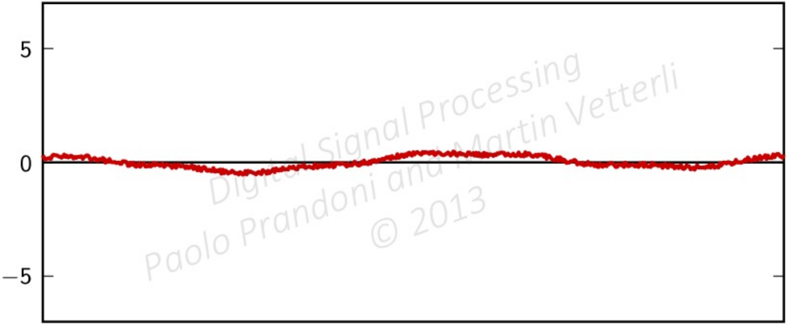
$x(t)$

Transmission of analog signals

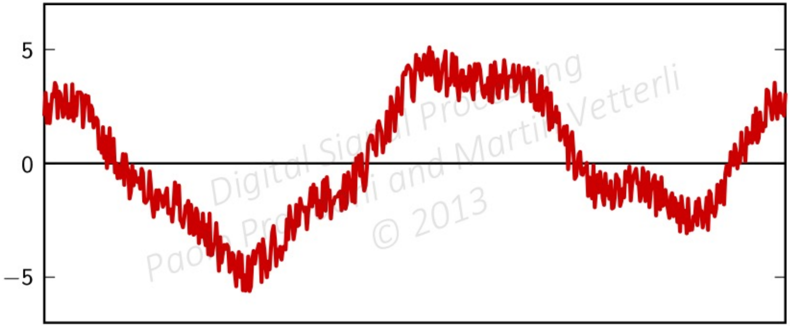


$x(t)/G$

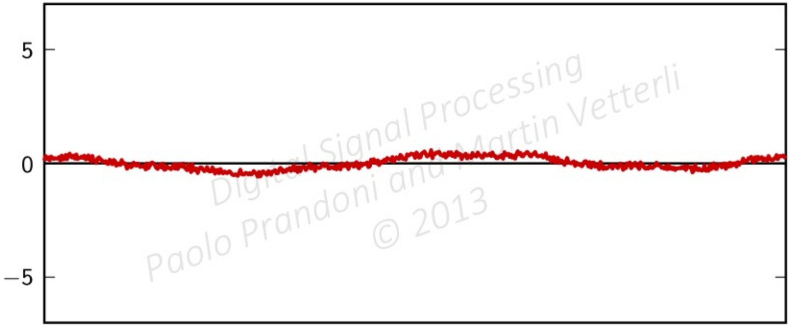
Transmission of analog signals



$$x(t)/G + \sigma(t)$$

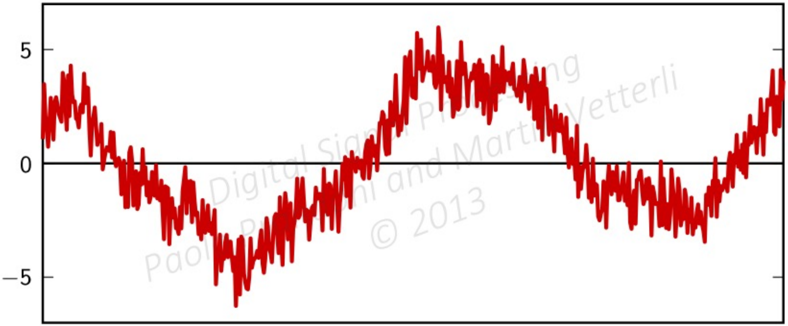


$$\hat{x}_1(t) = G[x(t)/G + \sigma(t)] = x(t) + G\sigma(t)$$



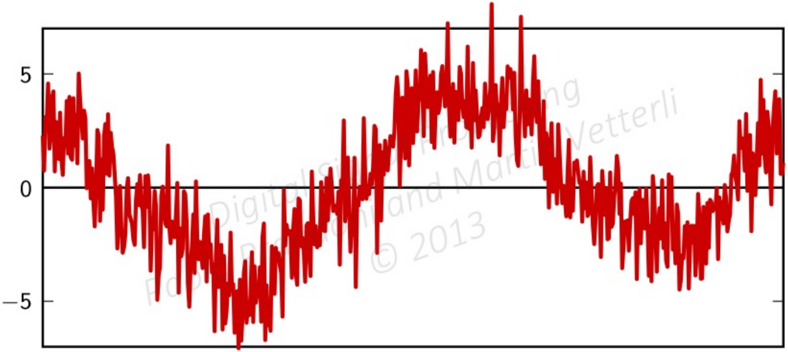
$$\hat{x}_1(t)/G + \sigma(t)$$

Transmission of analog signals



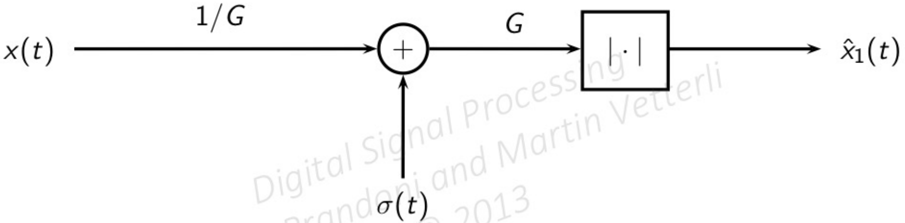
$$\hat{x}_2(t) = G[\hat{x}_1(t)/G + \sigma(t)] = x(t) + 2G\sigma(t)$$

Transmission of analog signals



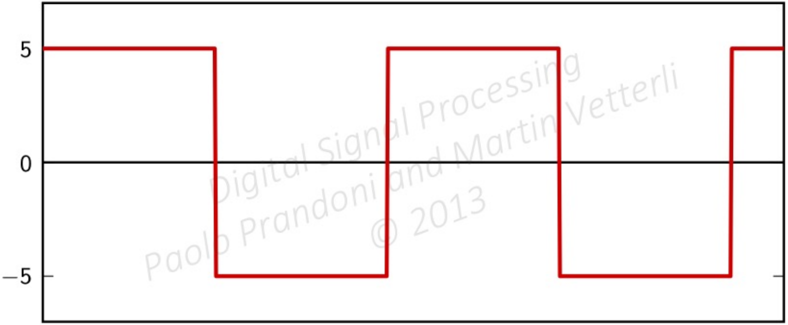
$$\hat{x}_N(t) = x(t) + NG\sigma(t)$$

In digital signals we can threshold



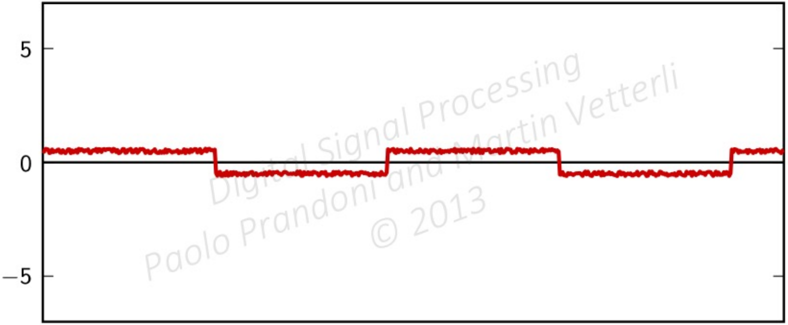
$$\hat{x}_1(t) = \mathrm{sgn}[x(t) + G\sigma(t)]$$

Transmission of quantized signals



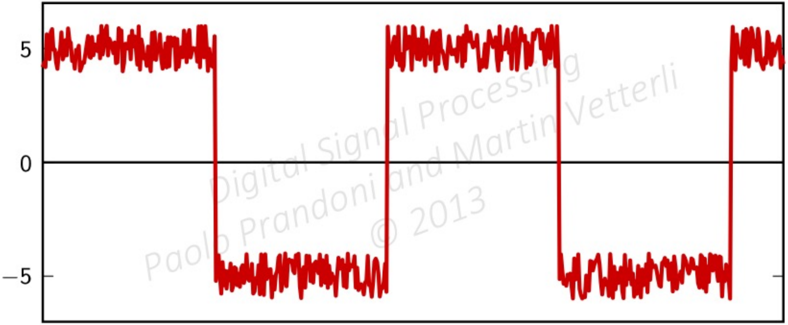
$x(t)$

Transmission of quantized signals



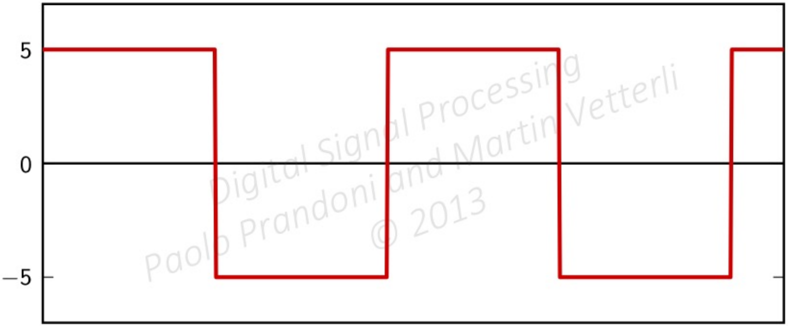
$$x(t)/G + \sigma(t)$$

Transmission of quantized signals



$$G[x(t)/G + \sigma(t)] = x(t) + G\sigma(t)$$

Transmission of quantized signals



$$\hat{x}_1(t) = G\,\mathrm{sgn}[x(t) + G\sigma(t)]$$

Digital data throughputs

- Transatlantic cable:
  - 1866: 8 words per minute ($\approx$5 bps)
  - 1956: AT&T, coax, 48 voice channels ($\approx$3Mbps)
  - 2005: Alcatel Tera10, fiber, 8.4 Tbps (8.4×10^{12} bps)
  - 2012: fiber, 60 Tbps
- Voiceband modems
  - 1950s: Bell 202, 1200 bps
  - 1990s: V90, 56Kbps
  - 2008: ADSL2+, 24Mbps

Final words