Digital Signal Processing

Solved HW for Day 13

Question 1

Consider a stationary i.i.d. random process $x[n]$ whose samples are uniformly distributed over the $[-1, 1]$ interval. Consider a quantizer $\mathcal{Q}\{\cdot\}$ with the following characteristic:

$$\mathcal{Q}\{x\} = \begin{cases} -1 & \text{if } -1 \leq x < -0.5 \\ 0 & \text{if } -0.5 \leq x \leq 0.5 \\ 1 & \text{if } 0.5 < x \leq 1 \end{cases}$$

The quantized process $y[n] = \mathcal{Q}\{x[n]\}$ is still i.i.d.; compute its error energy.

Solution of question 1

- In this quantizer, $K = 4$, $B = 1$, $A = -1$ and the input is uniformly distributed.
- Thus $P_e = \frac{4}{2} \int_0^{0.5} \tau^2 \, \mathrm{d}\tau = \frac{\Delta^2}{3} = (0.25)/3$

Question 2

Consider a stationary i.i.d. random process $x[n]$ whose samples are uniformly distributed over the $[-1, 2]$ interval.The process is uniformly quantized with a 1-bit quantizer $\mathcal{Q}\{\cdot\}$ with the following characteristic:

$$\mathcal{Q}\{x\} = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

Compute the signal to noise ratio at the output of the quantizer

Solution of question 2

- In this quantizer, $K = 3$, $B = -1$, $A = 2$.
- Thus $P_e = \frac{3}{3}\int_0^1 \tau^2 \,\mathrm{d}\tau = \frac{\Delta^2}{3} = (1)/3$
- $P_x = \frac{(B-A)^2}{12} = 0.25$
- Thus $SNR = \frac{P_x}{P_e} = \frac{9}{4}$