Natural Language Processing

Lecture 11: Hidden Markov Models

Finding POS Tags

Bill directed plays about English kings

Running Example

Bill	directed	plays	about	English	kings
PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

Running Example

Bill directed plays about English kings

Bill	directed	plays	about	English	kings
PropN Verb Noun	Adj Verb	Verb PlN	Prep Adv Part	Adj Noun	PlN Verb

		p(t|Bill)
PropN	41	0.118
Verb	2	0.006
Noun	303	0.870

		p(t|directed)
Adj	0	0.000
Verb	10	1.000

		p(t|plays)
Verb	18	0.750
PlN	6	0.250

		p(t|about)
Prep	1546	0.750
Adv	502	0.244
Part	12	0.006

Running Example: POS

Bill directed plays about English kings

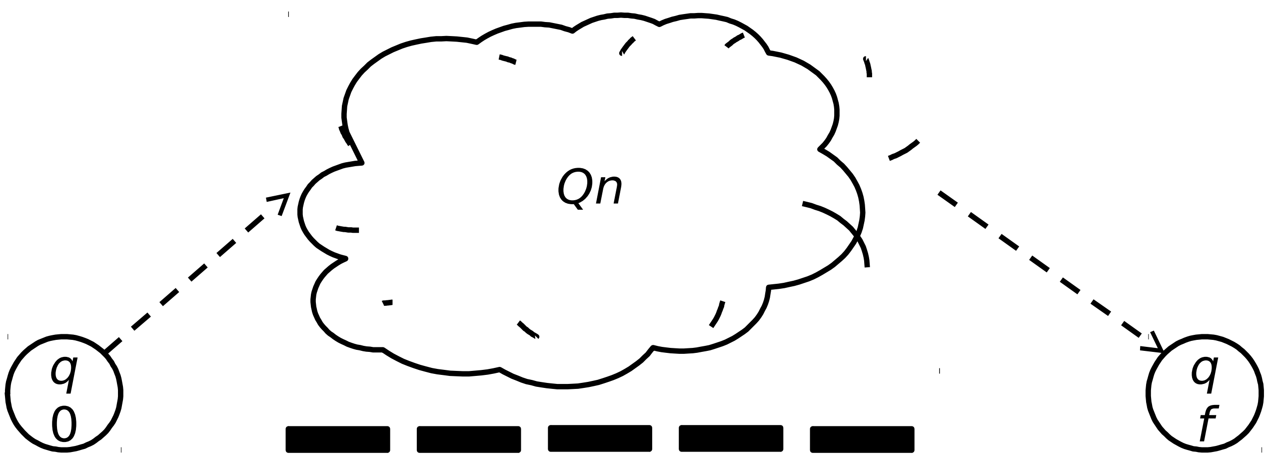
Bill	directed	plays	about	English	kings
PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

		p(t |English)
Adj	11	0.344
Noun	21	0.656

		p(t |kings)
PlN	3	1.000
Verb	0	0.000

Hidden Markov Model

- $q0$: start state (“silent”)
- qf: final state (“silent”)
- Q: set of “normal” states (excludes $q0$ and final qf)
- Σ: vocabulary of observable symbols
- $\gamma i,j$: probability of transitioning to qj given current state qi
- $\eta i,w$: probability of emitting $w \in \Sigma$ given current state qi



HMM as a Noisy Channel

$p(x \mid y)$ using $\{\eta i,w\}$

$p(y)$ using $\{\gamma i,j\}$

sourc
e

y (tags)

x (words)

channel

decode

States vs. Tags

Running Example

Bill directed plays about English kings

PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

p(PropN | <S> <S>)	0.202
p(Verb | <S> <S>)	0.023
p(Noun | <S> <S>)	0.040

Running Example

Bill directed plays about English kings

PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

p(PropN | <S> <S>)	0.202	p(Adj | <S> PropN)	0.004	0.00081
		p(Verb | <S> PropN)	0.139	0.02808
p(Verb | <S> <S>)	0.023	p(Adj | <S> Verb)	0.062	0.00143
		p(Verb | <S> Verb)	0.032	0.00074
p(Noun | <S> <S>)	0.040	p(Adj | <S> Noun)	0.005	0.00020
		p(Verb | <S> Noun)	0.222	0.00888

Running Example

Bill directed plays about English kings

PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

p(Adj | <S> PropN)	0.00081	p(Verb | PropN Adj)	0.011	0.00001
		p(PlN | PropN Adj)	0.157	0.00013
p(Verb | <S> PropN)	0.02808	p(Verb | PropN Verb)	0.162	**0.00455**
		p(PlN | PropN Verb)	0.022	0.00062
p(Adj | <S> Verb)	0.00143	p(Verb | Verb Adj)	0.009	0.00001
		p(PlN | Verb Adj)	0.246	0.00035
p(Verb | <S> Verb)	0.00074	p(Verb | Verb Verb)	0.078	0.00006
		p(PlN | Verb Verb)	0.034	0.00003
p(Adj | <S> Noun)	0.00020	p(Verb | Noun Adj)	0.020	0.00000
		p(PlN | Noun Adj)	0.103	0.00002
p(Verb | <S> Noun)	0.00888	p(Verb | Noun Verb)	0.176	0.00156
		p(PlN | Noun Verb)	0.018	0.00016

Running Example

Bill directed plays about English kings

PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

		p(t |Bill)	p(Bill | t)
PropN	41	0.118	0.00044
Verb	2	0.006	0.00002
Noun	303	0.870	0.00228

Running Example

Bill directed plays about English kings

PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

		p(t |directed)	p(directed |t)
Adj	0	0.000	0.00000
Verb	10	1.000	0.00008

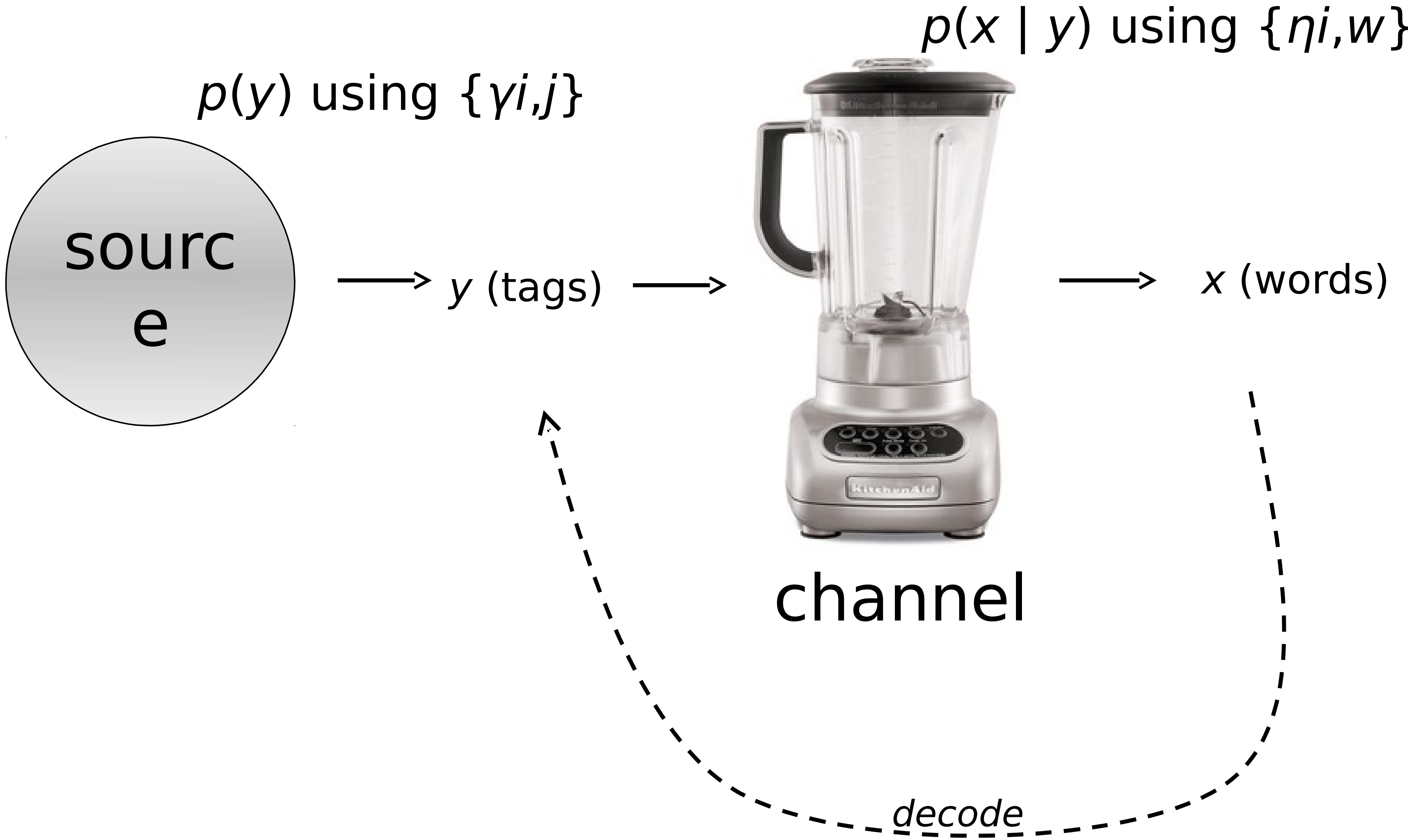
Running Example

Bill directed plays about English kings

PropN	Adj	Verb	Prep	Adj	PlN
Verb	Verb	PlN	Adv	Noun	Verb
Noun			Part		

		p(t |plays)	p(plays |t)
Verb	18	0.750	0.00014
PlN	6	0.250	0.00010

HMM as a Noisy Channel



Part-of-Speech Tagging Task

- Input: a sequence of word tokens $\boldsymbol{x}$
- Output: a sequence of part-of-speech tags $\boldsymbol{y}$, one per word

HMM solution: find the most likely tag sequence, given the word sequence.

If I knew the best state sequence for words $x_1 \ldots x_{n-1}$, then I could figure out the last state.

That decision would depend only on state $n - 1$.

$$
\begin{aligned}
y_n^* &= \arg\max_{q_i \in Q} p(Y_1 = y_1^*, \ldots, Y_{n-1} = y_{n-1}^*, Y_n = q_i \mid \boldsymbol{x}) \\
&= \arg\max_{q_i \in Q} V[n-1, y_{n-1}^*] \cdot \gamma_{y_{n-1}^*, i} \cdot \eta_{i, x_n} \cdot \gamma_{i, f} \\
&= \arg\max_{q_i \in Q} \gamma_{y_{n-1}^*, i} \cdot \eta_{i, x_n} \cdot \gamma_{i, f}
\end{aligned}
$$

I don't know that best sequence, but there are only $|Q|$ options at $n - 1$.

So I only need the score of the best sequence up to $n - 1$, ending in each possible state at $n - 1$. Call this $V[n - 1, q]$ for $q \in Q$.

Ditto, at every other timestep $n - 2$, $n - 3$, ... 1.

Viterbi Algorithm (Recursive Equations)

$$V[0, q_0] = 1$$

$$V[t, q_j] = \max_{q_i \in Q \cup \{q_0\}} V[t-1, q_i] \cdot \gamma_{i,j} \cdot \eta_{j,x_t}$$

$$\text{goal} = \max_{q_i \in Q} V[n, q_i] \cdot \gamma_{i,f}$$

Viterbi Algorithm (Procedure)

$V[*, *] \leftarrow 0$

$V[0, q0] \leftarrow 1$

for $t = 1 \ldots n$

  foreach qj

    foreach qi

      $V[t, qj] \leftarrow \max\{V[t, qj]\ , V[t - 1, qi] \times \gamma i,j \times \eta i,xt\}$

foreach qi

  goal $\leftarrow \max\{$ goal, $V[n, qi] \times \gamma i,f\ \}$

return goal

Running Example

Bill directed plays about English kings

$q0$	1						
$q1$							
$q2$							
$q3$							
$q4$							
...							
$q\|Q\|$							
qf							

Running Example

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