Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence

$$T(n) = \begin{cases} 2 & \text{if } n = 2, \\ 2T(n/2) + n & \text{if } n = 2^k, \text{ for } k > 1 \end{cases}$$

is $T(n) = n \lg n$.

Solution.

Induction Basis: When $n = 2$, $T(n) = n \lg n$ holds.

Inductive Step: Suppose that $T(n) = n \lg n$ holds for $n = 2^k$, i.e. $T(2^k) = 2^k \lg 2^k = k \cdot 2^k$. Then $T(2^{k+1}) = 2T(2^k) + 2^{k+1} = k \cdot 2^{k+1} + 2^{k+1} = (k+1) \cdot 2^{k+1} = 2^{k+1} \lg 2^{k+1}$, i.e. $T(n) = n \lg n$ holds for $n = 2^{k+1}$.