Project 607

we need to use CVXOPT package to solve an optimization problem

Format:

$$\min_{x} \ \frac{1}{2} x^T P x + q^T x$$

subject to $Gx \leq h$

$Ax = b$

we need to figure out matrix <u>P, q, G, h, A, b and x</u> to solve the problem

$$\text{solve for } \underset{(98\times1)}{\vec{x}} = \begin{array}{l} \left.\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{50} \end{bmatrix}\right\} 50 \text{ unit} \\ \left.\begin{bmatrix} P_2 \\ P_3 \\ \vdots \\ P_{49} \end{bmatrix}\right\} 48 \text{ unit} \end{array}$$

given $P_1 = P_{50} = 0$

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constrain 3 and 6 (matrix G and h)

$P_i > 0$ for all P

$-C_i + C_{i+1} \leq 0$ for in $\in \{1, 2, \cdots, 50\}$

$$G\,(97\times98) = \begin{array}{c c} \overbrace{}^{50} & \\ 49\left\{\begin{bmatrix} -1, 1, 0, 0, \cdots 0 & 0 \\ 0, -1, 1, 0, \cdots 0 & 0 \\ 0, 0, -1, 1, \cdots 0 & 0 \\ \vdots & \vdots \\ 0 \ \ 0 \ \ 0 \ \cdots \ -1 & 1 \end{bmatrix}\right. & 0 \\ 0 & \left.\underbrace{\begin{bmatrix} -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & & -1 \end{bmatrix}}_{48}\right\} 48 \end{array}$$

$$h\,(97\times1) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

constrain 4 (matrix A and b)

$$u \cdot \sum P_i = 1$$

$$\overset{A}{\left[\underbrace{0, 0, \cdots 0}_{50} \ \underbrace{u, u, \cdots u}_{48} \right]} \cdot \vec{x} \qquad \overset{b}{\left[1 \right]}$$

constrain 5, (matrix A and b)

$$C_1 = S_0 e^{(r^d - r^f)T} - k_1$$

$$C_{50} = 0$$

$$\overset{A}{\begin{bmatrix} 1, 0, 0 \cdots 0, 0, & 0, 0 \cdots 0, 0 \\ \underbrace{0, 0, 0 \cdots 0, 1,}_{50} & \underbrace{0, 0 \cdots 0, 0}_{48} \end{bmatrix}} \cdot \vec{x} = \overset{b}{\begin{bmatrix} S_0 \cdot e^{(r^d - r^f)T} \cdot k_1 \\ 0 \end{bmatrix}}$$

constrain 1, matrix A and b

$$C_{i+1} + C_{i-1} - 2C_i = \left(\frac{2}{3}P_i + \frac{1}{6}P_{i+1} + \frac{1}{6}P_{i-1}\right)u^2$$

$$48\left\{ \overset{A}{\begin{bmatrix} 1, -2, 1 & 0 & 0 & \cdots & 0, & -\frac{2}{3}u^2, & -\frac{1}{6}u^2, & 0, & 0 & \cdots & 0 \\ 0 & 1, -2, 1 & 0 & \cdots & 0, & -\frac{1}{6}u^2, & -\frac{2}{3}u^2, & -\frac{1}{6}u^2, & 0 & \cdots & 0 \\ 0 & 0 & 1, -2, 1 & \cdots & 0, & 0, & -\frac{1}{6}u^2, & -\frac{2}{3}u^2, & -\frac{1}{6}u^2 & \cdots & 0 \\ 0 & 0 & 0, 1, -2 & \cdots & & & & & & & \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & & & \vdots \\ 0 & 0 & 0 \ 0 \ 0 & \cdots & 1 & 0 & 0 & 0, & 0 & \cdots & -\frac{2}{3}u^2 \end{bmatrix}} \right. \cdot \vec{x} = \overset{B}{\begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}}$$

$$\underbrace{\qquad\qquad\qquad}_{50} \qquad \underbrace{\qquad\qquad\qquad\qquad}_{48}$$

constrain 2 (matrix A and b

$$c(\hat{k}_j) = ac_i + bc_{i+1} + \frac{(a^3-a)u^2}{6} P_i + \frac{(b^3-b)u^2}{6} P_{i+1}$$

$$a = \frac{k_{i+1} - \hat{k}_j}{u} \qquad b = 1-a$$

$[k_i, k_{i+1}]$ represent where $\hat{k}_j$ falls in

Logic:

Since, we have 5 k_j value, since k list is arange (kmin, kmax, u)

we can find k_{i+1}'s index within k list based on k_j's value

once we find the exact index of k_i and k_{i+1},

we calculate a, b, $\underbrace{\frac{(a^3-a)u^2}{6}}_{c}$, $\underbrace{\frac{(b^3-b)u^2}{6}}_{d}$ and place them

in to a matrix

$$\overbrace{5\left\{\begin{bmatrix} \underbrace{0, 0, a, b, \cdots 0}_{50} & \underbrace{X, 0, c, d, \cdots}_{48} \\ & \\ & \\ & \\ & \end{bmatrix}\right.}^{A} \cdot \vec{X} = \begin{array}{cl} \overbrace{\begin{bmatrix} k_j \\ k_j \\ k_j \\ k_j \\ k_j \end{bmatrix}}^{b} & \begin{array}{l} \text{1st } k_j \\ \text{2nd } k_j \\ \text{3rd } k_j \\ \vdots \\ \vdots \end{array} \end{array}$$

we concate all A for constrain 4, 5, 1 and 2

to a single matrix A, and same for b,

Then plug all matrix to the solver, and its done

Lastly, matrix P is

$$\begin{array}{ccc} & \begin{array}{cc} 50 & 48 \end{array} & \\ \begin{array}{c} 50 \\ 48 \end{array} & \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix} & \begin{array}{c} 50 \\ 48 \end{array} \\ & \begin{array}{cc} 50 & 48 \end{array} & \end{array}$$

48×48, fron

R is from constrain 1, refer to project guide for details