QF 620 Stochastic Modelling in Finance

Group Project

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Part I (Analytical Option Formula)

1. Black-Scholes Model

In this model, the underlying stock is valued at a risk neutral probability with a risk-free bond used as the numeraire security.

SDE Derivation:

Given: $$dS_t = rS_t dt + \sigma S_t dW_t$$

Let: $$X_t = \log(S_t) = f(S_t)$$

Using Itô's Formula:

$$\begin{aligned} dX_t &= f'(S_t)dS_t + \tfrac{1}{2}f''(S_t)(dS_t)^2 \\ &= \tfrac{1}{S_t}(rS_t dt + \sigma S_t dW_t) - \tfrac{1}{2}\tfrac{1}{S_t^2}(\sigma^2 S_t^2 dt) \\ &= \left(r - \tfrac{1}{2}\sigma^2\right)dt + \sigma dW_t \end{aligned}$$

Integrating:

$$\begin{aligned} \int_0^T dX_t &= \left(r - \tfrac{1}{2}\sigma^2\right)\int_0^T dt + \sigma\int_0^T dW_t \\ \log\left(\tfrac{S_T}{S_0}\right) &= \left(r - \tfrac{1}{2}\sigma^2\right)T + \sigma W_T \\ S_T &= S_0 \exp\left[\left(r - \tfrac{1}{2}\sigma^2\right)T + \sigma W_T\right] \end{aligned}$$

$$W_T = \sqrt{T}X_t \sim N(0,t),\ where\ X_t \sim N(0,1)$$

Find x* Where S_T = K:

$$\begin{aligned} S_T &= K \\ K &= S_0 \exp\left[\left(r - \tfrac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}x\right] \\ \log\left(\tfrac{K}{S_0}\right) &= \left(r - \tfrac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}x \\ x^* &= \frac{\log\left(\frac{K}{S_0}\right) - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \end{aligned}$$

Valuation Formula:

Options	Pricing Formula
Vanilla Call	$S_0\Phi\left(\frac{\log\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) - Ke^{-rT}\Phi\left(\frac{\log\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$
Vanilla Put	$Ke^{-rT}\Phi\left(\frac{\log\left(\frac{K}{S_0}\right) - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) - S_0\Phi\left(\frac{\log\left(\frac{K}{S_0}\right) - \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$
Digital Cash-or-Nothing Call	$e^{-rT}\Phi\left(\frac{\log\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$
Digital Cash-or-Nothing Put	$e^{-rT}\Phi\left(\frac{\log\left(\frac{K}{S_0}\right) - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$
Digital Asset-or-Nothing Call	$S_0\Phi\left(\frac{\log\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$
Digital Asset-or-Nothing Put	$S_0\Phi\left(\frac{\log\left(\frac{K}{S_0}\right) - \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$

2. Bachelier Model

Compared to other price models, the Bachelier model does not rely on logarithms, thus it could represent negative values.

SDE Derivation (*assuming r = 0*)**:**

Given: $$dS_t = \sigma S_0 dW_t$$

Integrating:

$$\int_0^T dS_t = \sigma S_0 \int_0^T dW_t$$
$$S_T - S_0 = \sigma S_0 W_T$$
$$S_T = S_0(1 + \sigma W_T)$$

$$W_T = \sqrt{T} X_t \sim N(0, t), \ where \ X_t \sim N(0,1)$$

Find x^* where $S_T = K$:

$$S_T = K$$
$$K = S_0 + S_0 \sigma \sqrt{T} x$$
$$x^* = \frac{K - S_0}{S_0 \sigma \sqrt{T}}$$

Valuation Formula:

Options	Pricing Formula
Vanilla Call	$(S_0 - K)\Phi\left(\frac{S_0 - K}{S_0\sigma\sqrt{T}}\right) + S_0\sigma\sqrt{T}\phi\left(\frac{S_0 - K}{S_0\sigma\sqrt{T}}\right)$
Vanilla Put	$(K - S_0)\Phi\left(\frac{K - S_0}{S_0\sigma\sqrt{T}}\right) + S_0\sigma\sqrt{T}\phi\left(\frac{K - S_0}{S_0\sigma\sqrt{T}}\right)$
Digital Cash-or-Nothing Call	$\Phi\left(\frac{S_0 - K}{S_0\sigma\sqrt{T}}\right)$
Digital Cash-or-Nothing Put	$\Phi\left(\frac{K - S_0}{S_0\sigma\sqrt{T}}\right)$
Digital Asset-or-Nothing Call	$S_0\left[\Phi\left(\frac{S_0 - K}{S_0\sigma\sqrt{T}}\right) + \sigma\sqrt{T}\phi\left(\frac{S_0 - K}{S_0\sigma\sqrt{T}}\right)\right]$
Digital Asset-or-Nothing Put	$S_0\left[\Phi\left(\frac{K - S_0}{S_0\sigma\sqrt{T}}\right) - \sigma\sqrt{T}\phi\left(\frac{K - S_0}{S_0\sigma\sqrt{T}}\right)\right]$

3. Black76 Model

The Black76 Lognormal model is almost identical with the Black-Scholes model in structure. The only difference being that Black76 uses forward prices to model the value of a futures option at maturity versus the spot prices Black-Scholes used. Hence, the SDE for forward prices is more compact and driftless.

SDE Derivation:

Given:

$$dF_t = \sigma F_t dW_t$$

Let:

$$X_t = \log(F_t) = f(F_t)$$

Where:

$$F_t = S_t e^{r(T-t)}$$

Using Itô's Formula:

$$\begin{aligned} dX_t &= f'(F_t)dF_t + \tfrac{1}{2}f''(F_t)(dF_t)^2 \\ &= \tfrac{1}{F_t}(\sigma F_t dW_t) - \tfrac{1}{2}\tfrac{1}{F_t^2}(\sigma^2 F_t^2 dt) \\ &= -\tfrac{1}{2}\sigma^2 dt + \sigma dW_t \end{aligned}$$

Integrating:

$$\begin{aligned} \int_0^T dX_t &= -\tfrac{1}{2}\sigma^2 \int_0^T dt + \sigma \int_0^T dW_t \\ \log\left(\tfrac{F_T}{F_0}\right) &= -\tfrac{1}{2}\sigma^2 T + \sigma W_T \\ F_T &= F_0 \exp\left[-\tfrac{1}{2}\sigma^2 T + \sigma W_T\right] \end{aligned}$$

$$W_T = \sqrt{T}X_t \sim N(0,t),\ where\ X_t \sim N(0,1)$$

Find x^* where $F_T = K$:

$$\begin{aligned} F_T &= K \\ K &= F_0 \exp\left[-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}x\right] \\ \log\left(\frac{K}{F_0}\right) &= -\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}x \\ x^* &= \frac{\log\left(\frac{K}{F_0}\right) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \end{aligned}$$

Valuation Formula:

Options	Pricing Formula
Vanilla Call	$e^{-rT}\left[F_0\Phi\left(\frac{\log\left(\frac{F_0}{K}\right)+\frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}\right) - K\Phi\left(\frac{\log\left(\frac{F_0}{K}\right)-\frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}\right)\right]$
Vanilla Put	$e^{-rT}\left[K\Phi\left(\frac{\log\left(\frac{K}{F_0}\right)+\frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}\right) - F_0\Phi\left(\frac{\log\left(\frac{K}{F_0}\right)-\frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}\right)\right]$
Digital Cash-or-Nothing Call	$e^{-rT}\Phi\left(\frac{\log\left(\frac{F_0}{K}\right)-\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}\right)$
Digital Cash-or-Nothing Put	$e^{-rT}\Phi\left(\frac{\log\left(\frac{K}{F_0}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}\right)$
Digital Asset-or-Nothing Call	$e^{-rT}F_0\Phi\left(\frac{\log\left(\frac{F_0}{K}\right)-\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}\right)$
Digital Asset-or-Nothing Put	$e^{-rT}F_0\Phi\left(\frac{\log\left(\frac{K}{F_0}\right)-\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}\right)$

4.The Displaced-Diffusion Model

SDE Derivation:

Given:

$$dF_t = \sigma[\beta F_t + (1-\beta)F_0]dW_t$$

Let:

$$X_t = \log\left(\beta F_t + (1-\beta)F_0\right) = f(F_t)$$

Using Itô's Formula:

$$\begin{aligned}
dX_t &= f'(F_t)dF_t + \frac{1}{2}f''(F_t)(dF_t)^2 \\
&= \frac{\beta}{\beta F_t + (1-\beta)F_0}\left(\sigma(\beta F_t + (1-\beta)F_0)dW_t\right) \\
&\quad -\frac{1}{2}\frac{\beta^2}{(\beta F_t + (1-\beta)F_0)^2}\left(\sigma^2(\beta F_t + (1-\beta)F_0)^2 d_t\right) \\
&= \beta\sigma dW_t - \frac{1}{2}\beta^2\sigma^2 dt
\end{aligned}$$

Integrating:

$$\begin{aligned}
\int_0^T dX_t &= \beta\sigma\int_0^T dW_t - \frac{1}{2}\beta^2\sigma^2\int_0^T dt \\
X_T - X_0 &= \beta\sigma W_T - \frac{1}{2}\beta^2\sigma^2 T \\
\log\left(\frac{\beta F_T + (1-\beta)F_0}{\beta F_0 + (1-\beta)F_0}\right) &= \beta\sigma W_T - \frac{1}{2}\beta^2\sigma^2 T \\
\frac{\beta F_T + (1-\beta)F_0}{F_0} &= e^{\beta\sigma W_T - \frac{1}{2}\beta^2\sigma^2 T} \\
F_T &= \frac{F_0}{\beta}e^{\beta\sigma W_T - \frac{1}{2}\beta^2\sigma^2 T} - \frac{1-\beta}{\beta}F_0
\end{aligned}$$

$$W_T = \sqrt{T}X_t \sim N(0,t),\ where\ X_t \sim N(0,1)$$

Find x^* where $F_T = K$:

$$x^* = \frac{\log\left(\frac{K + (1-\beta)/(\beta)F_0}{F_0/\beta}\right) + \frac{(\beta\sigma)^2 T}{2}}{\beta\sigma\sqrt{T}}$$

Valuation Formula:

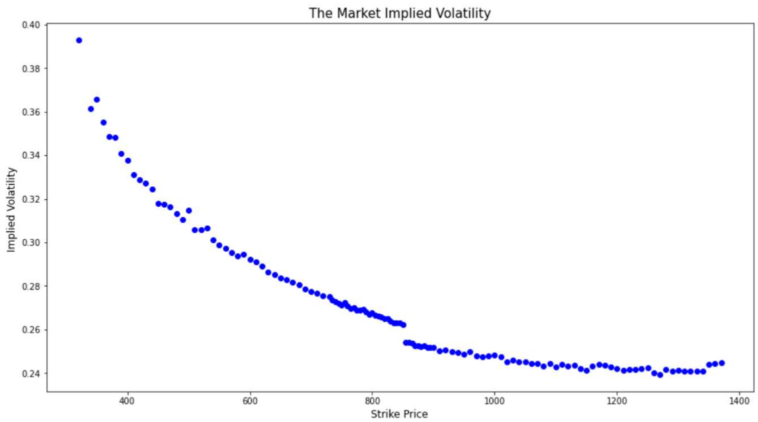
Options	Pricing Formula
Vanilla Call	$e^{-rT}\left[\frac{F_0}{\beta}\Phi\left(\frac{\log\left(\frac{F_0/\beta}{K+F_0((1-\beta)/\beta)}\right)+\frac{(\beta\sigma)^2T}{2}}{\beta\sigma\sqrt{T}}\right)-\left(K+F_0\frac{1-\beta}{\beta}\right)\Phi\left(\frac{\log\left(\frac{F_0/\beta}{K+F_0((1-\beta)/\beta)}-\frac{(\beta\sigma)^2T}{2}\right)}{\beta\sigma\sqrt{T}}\right)\right]$
Vanilla Put	$e^{-rT}\left[\left(K+F_0\frac{1-\beta}{\beta}\right)\Phi\left(\frac{\log\left(\frac{K+F_0((1-\beta)/\beta)}{F_0/\beta}\right)+\frac{(\beta\sigma)^2T}{2}}{\beta\sigma\sqrt{T}}\right)-\frac{F_0}{\beta}\Phi\left(\frac{\log\left(\frac{K+F_0((1-\beta)/\beta)}{F_0/\beta}\right)-\frac{(\beta\sigma)^2T}{2}}{\beta\sigma\sqrt{T}}\right)\right]$
Digital Cash-or-Nothing Call	$e^{-rT}\Phi\left(\frac{\log\left(\frac{F_0/\beta}{K+F_0((1-\beta)/\beta)}\right)-\frac{(\beta\sigma)^2T}{2}}{\beta\sigma\sqrt{T}}\right)$
Digital Cash-or-Nothing Put	$e^{-rT}\Phi\left(\frac{\log\left(\frac{K+F_0((1-\beta)/\beta)}{F_0/\beta}\right)+\frac{(\beta\sigma)^2T}{2}}{\beta\sigma\sqrt{T}}\right)$
Digital Asset-or-Nothing Call	$e^{-rT}\frac{F_0}{\beta}\Phi\left(\frac{\log\left(\frac{F_0/\beta}{K+F_0((1-\beta)/\beta}\right)+\frac{(\beta\sigma)^2T}{2}}{\beta\sigma\sqrt{T}}\right)$
Digital Asset-or-Nothing Put	$e^{-rT}\frac{F_0}{\beta}\Phi\left(\frac{\log\left(\frac{K+F_0((1-\beta)/\beta)}{F_0/\beta}\right)-\frac{(\beta\sigma)^2T}{2}}{\beta\sigma\sqrt{T}}\right)$

Part II Model Calibration:

Calibration of displaced diffusion model and SBAR model to match market implied volatility derived from Black-76 Lognormal option pricing model

1. Market Implied Volatility

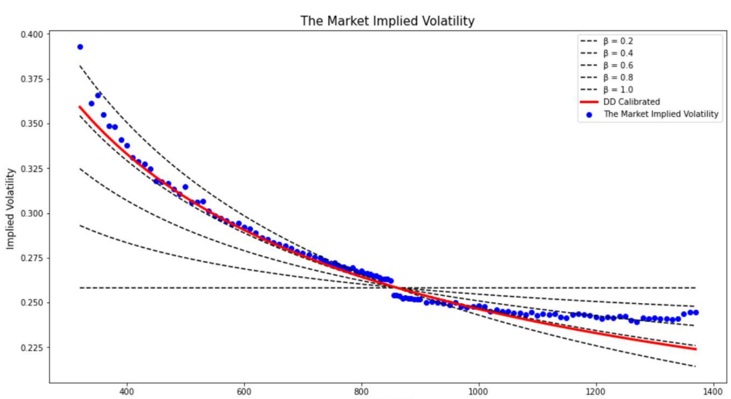
Implied volatility from market prices were extracted using the Black-76 Lognormal option pricing model, and the implied volatility across different strikes are plotted as per below chart:



2. Model Calibration using Displaced-Diffusion ("DD") Model

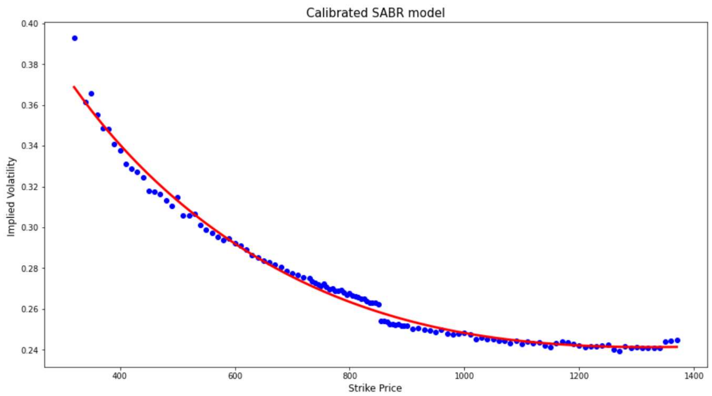
The DD model adds a beta variable to the original black76model. To calibrate and match the market volatility, we first need to determine the sigma value. To select an appropriate sigma value, we first selected the 4 prices (2 above and 2 below) closest to at-the-money ("ATM") strike F (expected future stock price under the risk-neutral measure given by S_0e^{rT}). For the 2 prices above F, we computed the average price, from which we derived the 1st implied volatility of this average price above F. We then did likewise for the 2 prices below F, getting the 2nd implied volatility. After which, we take the mid-point of the 2 implied volatilities as the value of our ATM sigma (0.2583).

To calibrate the curve, we first plug different Beta values into the DD model to determine which would most closely represent the implied volatility observed in the market. We then employed the least-squares method to find the Beta most suitable, which is approximately 0.3658 (red line in the chart below).



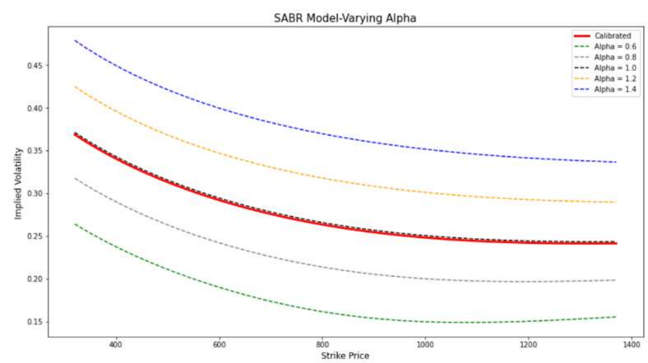
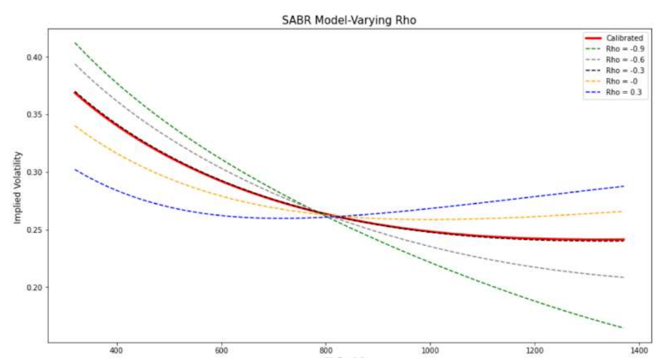
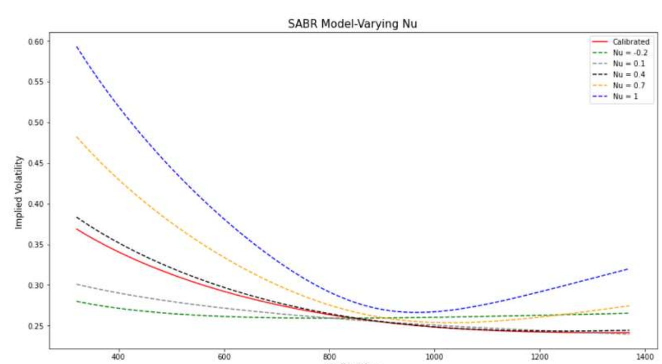
While the DD model does a good job at matching at-the-money ("ATM") strike, it is inadequate for deriving the implied volatilities for those strikes not near the ATM price at the extreme ends. The most challenging aspect is that we are unable to change the skewness and kurtosis of the curve, without affecting the implied volatility of the strikes near both ends of the curve. The main reason for this is that the market is pricing in much higher implied volatility for lower strikes, and much lower implied volatility for higher strikes than the above curves suggest. In the case of deep-in-the-money options or deep-out-of-the- money options, even though the probability is very small, tend to make investors overvalue or undervalue options respectively, resulting in the skew of volatility. We thus need a model that is able to incorporate all these additional factors, which is where the SABR model comes in.

3. Model Calibration using SABR model (fix β = 0.8)



Unlike the DD model which uses only 1 Beta parameter, the SABR model uses more parameters to try to capture the intricacy of the volatility smile in the derivatives markets. We calibrated the SABR model using the least squares method and to obtain the following parameters of Alpha, Rho and Nu, while keeping Beta fixed at 0.8. The calibrated SABR model fits the market implied volatility very well, due to the additional parameters which allow us to simulate the shape of the curve even better.

Calibrated Results: **Alpha** = 0.9908 , **Rho** = -0.2851, **Nu** = 0.3522

Alpha (α): Alpha (α) is a volatility-like parameter for the volatility and controls the height of the implied volatility level. From the chart on the right, it can be seen that the higher the Alpha value, the higher the implied volatility.	
Rho (ρ): Rho (ρ) is the instantaneous correlation between the underlying and its volatility. The correlation parameter ρ is proportional to the skewness of stock returns. From the chart on the right, it can be seen that the higher the Rho value, the lower the slope of the implied volatility.	
Nu (v): Nu (v) is the volatility of volatility. Increasing volatility-of-volatility has the effect of increasing the kurtosis of return. From the chart on the right, it can be seen that the higher the Nu value, the higher the kurtosis of the implied volatility curve.	

Part III Static Replication:

Evaluate the payoff of $\boldsymbol{S_T^3 \times 10^{-8} + 0.5 \times \log(S_T) + 10}$ using Black-Scholes Model and Bachelier Model

Black Scholes Model:

$$S_T = S_0 \exp\left[\left(r - \frac{1}{2}\sigma^2\right)T + \sigma W_T\right]$$

Payoff Function under Black Scholes Model:

$$V_0 = e^{-r} \times E\left[\,S_T^3 \times 10^{-8} + 0.5 \times \log(S_T) + 10\,\right]$$

$$V_0 = e^{-rT} \times E\left[\,S_0^3 \times \exp\left[3 \times \left(r - \frac{1}{2}\sigma^2\right)T + 3\sigma W_T\right] \times 10^{-8} + 0.5 \times \log\left(S_0 \exp\left[\left(r - \frac{1}{2}\sigma^2\right)T + \sigma W_T\right]\right) + 10\,\right]$$

$$V_0 = e^{-rT} \times \left[S_0^3 \times \exp[3 \times (r + \sigma^2)T] \times 10^{-8} + 0.5 \times \log(S_0) + 0.5 \times \exp\left(r - \frac{1}{2}\sigma^2\right)T + 10\right]$$

The assumption of Black Scholes model is that the implied volatility are the same across all strikes. Therefore, we use at-the-money sigma in the calculation of option price, because at the money options have the highest liquidity and best representation of the sigma.

$S_0 =$ 846.90	$T = 1.38356$	$\sigma = 0.25827$	$r = 0.00405$

The price of the derivative contract under Black Scholes Model is:

$V_0 =$ 21.378

Bachelier Model: *(Assuming r = 0)*

$$S_T = S_0(\,1 + \sigma W_T)$$

Payoff Function under Bachelier Model:

$$V_0 = E\,[S_T^3 \times 10^{-8} + 0.5 \times \log(S_T) + 10.0\,]$$

$$V_0 = E\,[[S_0(\,1 + \sigma W_T)]^3] \times 10^{-8} + 0.5 \times E\,[\,\log[\,S_0(\,1 + \sigma W_T)\,]] + 10$$

$$V_0 = E\,[S_0^3 + S_0^3\sigma^3 W_T^3 + 3S_0^3\sigma W_T + 3S_0^3\sigma^2 W_T^2\,] \times 10^{-8} + 0.5 \times E\,[\log(S_0) + \log(\,1 + \sigma W_T)\,] + 10$$

$$V_0 = (S_0^3 + 3S_0^3\sigma^2 T) \times 10^{-8} + 0.5 \times E\,[\,\log(S_0)] + 0.5 \times E[\log(\,1 + \sigma W_T)] + 10$$

$$V_0 = (S_0^3 + 3S_0^3\sigma^2 T) \times 10^{-8} + 0.5 \times \log(S_0) + 0.5 \times \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \log\left(1 + \sigma\sqrt[2]{T}x\right)dx + 10$$

The assumption of Bachelier model is that the implied volatility are the same across all strikes. Therefore, we use at-the-money sigma in the calculation of the option price because at the money option has the highest liquidity and best representation of the sigma.

$S_0 =$ 846.90	$T = 1.38356$	$\sigma = 0.25827$

The price of the derivative contract under Bachelier Model is:

$V_0 = 21.099$

Part IV Dynamic Hedging

Given: S_0= \$100, σ = 0.2, r = 5%, T = $\frac{1}{12}$ year and K = \$100

Using Black-Scholes model to simulate the stock price, sell at-the-money call option, and hedge N times during the life of the call option to test the final profit and loss of our dynamic hedging strategy. (Assume there are 21 trading days over the month.)

<u>Black Scholes Model:</u>

Brownian Motion:

$$W_n(t) = \sqrt{t}\left(\frac{\sum_{i=1}^{n} X_i}{\sqrt{n}}\right)$$

Stock price process:

$$S_T = S_0 \exp\left[\left(r - \frac{1}{2}\,\sigma^2\right)T + \sigma W_T\right]$$

Vanilla European call option pricing formula:

$$V_0^c = S_0\Phi\left(\frac{\log\frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) - Ke^{-rT}\Phi\left(\frac{\log\frac{S_0}{K} + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$$

The Dynamic Hedging strategy for an option:

$C_t = \phi_t S_t - \psi_t B_t$,

where

$\phi_t = \frac{\partial C}{\partial S} = \Phi\left(\frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}\right)$, and

$\psi_t B_t = -Ke^{-r(T-t)}\Phi\left(\frac{\log\left(\frac{S_t}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sqrt{T-t}}\right)$.

The final P&L of the dynamic hedging strategy:

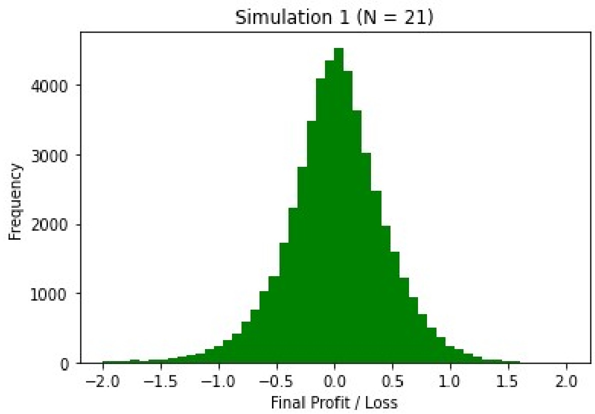
Hedging Error $= (\phi_T S_T - \psi_T B_T) - max\{S_T - K, 0\}$

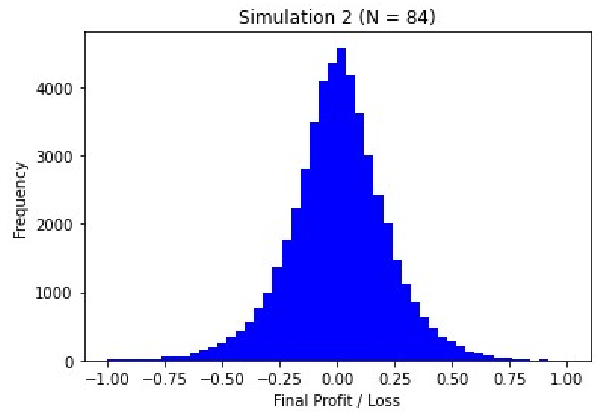
Dynamic Hedging Simulation Results

We generated a total of 50,000 paths across a period of one month to simulate dynamic hedging, with the price ranging between 80-130. Dynamic hedging was done at timesteps for (N=21) and (N=84) respectively.

Expected values of the final P&L for both are quite close to each other but the standard deviations of hedging error when N=84 is about half that of when N=21. What this shows is that hedging more frequently actually reduces the standard deviation of the P&L, roughly halving the standard deviation when hedging is done at 4 times the frequency. Thus, there is a risk of large variations in the P&L when hedging is done at a much lower frequency. This would be detrimental especially if sudden spikes were to occur in the underlying prices in between rebalancing trades.

Below are the histograms of the hedging errors generated for (N=21) and (N=84).





	N = 21	N = 84
Mean	0.00912	0.00371
Standard deviation	0.42716	0.21812

Insights

The Black-Scholes replication strategy assumes we are able to delta-hedge continuously until the option's maturity, but this impractical due to the limitations in the real-world. Thus practically, most market participants could only do discrete delta hedging by shorting or longing the underlying assets to remain delta-neutral at each time step. This, however, also presents its own challenges.

For example, intermittent hedging is prone to larger replication errors and consequently P&L fluctuations, even when taking into account all other option market parameters are known. To reduce this particular error, one either have to is to rebalance more frequently, or to avoid the need for rebalancing by running a more closely matched book whose gamma is close to zero. We have to be mindful that the advantage gained by increased re-hedging are not offset by the additional transaction costs and potentially adverse market impact.

As a sampling of the underlying prices of the stock would be taken intermittently for discrete delta-hedging at every time step, thus only an approximate of the true underlying volatility could be obtained. Moreover, assuming a known constant volatility and no spikes to underlying asset prices, due to the nature of statistical fluctuations, measured volatility across some paths would still deviate away from the 20% constant volatility. This makes delta-hedging challenging due to the introduction of replication error.

We could mitigate the impact of replication errors by increasing the hedging frequency to get a better approximate of the true volatility of the underlying price with which to hedge. As shown by our simulation, just increasing the hedging frequency by 4 times will reduce the standard deviation risk by about half. We have to strike a balance between the benefits of increasing the frequency of hedging and the costs that will be incurred due to increased hedging frequency.