

1. Using the truth table to prove DeMorgan's law.

①

x	y	$(xy)'$	$x'+y'$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

$$\Rightarrow (xy)' = x' + y'$$

②

x	y	$(x+y)'$	$x'y'$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

$$\Rightarrow (x+y)' = x'y'$$

proved

2.

$$\begin{aligned}
 F &= a'b(c+d') + a(b'+c) + a(b+d)c \\
 &= a'bc + a'bd' + ab' + ac + abc + adc \\
 &= (a'+a)bc + ac + a'bd' + ab' + abc \\
 &= bc + ac + a'bd' + a(b'+c) \\
 &= bc + ac + a'bd' + ab'
 \end{aligned}$$

3. $F = ac' + abd' + acd.$

$$\begin{aligned} F' &= (ac' + abd' + acd)' = (ac' + abd')' (acd)' = \cancel{(ac')'} (abd')' (acd)' \\ &= (a' + c)(a' + b' + d)(a' + c' + d') \\ &= (a' + a'b' + a'd + a'c + b'c + cd)(a' + c' + d') \\ &= (a' + b'c + cd)(a' + c' + d') \\ &= a' + a'b'c + \cancel{a'd} + b'cd' + a'cd + 0 + 0 \\ &= a' + b'cd' \end{aligned}$$

So $F' = a' + b'cd'$

4.

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$\begin{aligned} \Rightarrow f &= a'b'c + a'bc' + a'bc + ab'c + abc' + abc. \\ &= m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 \\ &= \Sigma(1, 2, 3, 4, 5, 6, 7) \end{aligned}$$

5. $F = a'b'c + a'bc' + a'bc + ab'c + abc' + abc.$

$$\begin{aligned} &= \cancel{a'(b'c + bc)} \\ &= a'c(b' + b) + bc(a' + a) + ac(b' + b) \\ &= a'c + bc' + ac \\ &= c + bc' = abc \end{aligned}$$

6. $F(a,b,c) = abc + ab + a + b + c = a + b + c$

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$\Rightarrow F_1 = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 \\ = \sum (1, 2, 3, 4, 5, 6, 7)$$

7. (a) $F = ab + cd$

$$G = \overline{a(\overline{ab} + \overline{cd})} = ((ab)'(cd)')' \\ = ((a'+b) \cdot (c'+d))' = ((a'+b)' + (c'+d)')' = (ab + cd)' = (a' + b')(c' + d')$$

$\therefore F \neq G$ and $F' = G$.

1b)

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

a	b	c	d	G
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

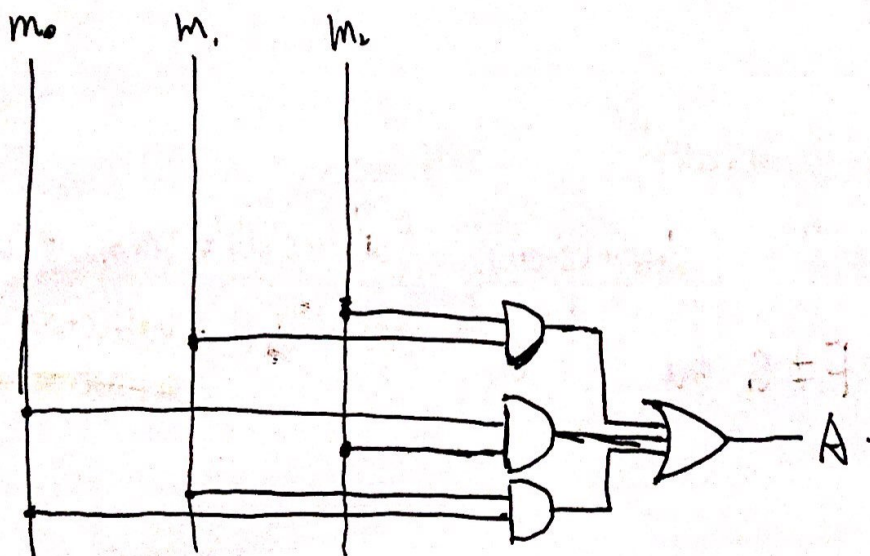
$F \neq G$, $F' = G$.

8.

m₀ m ₀	m ₁	m ₂	A
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$\begin{aligned}
 \Rightarrow A &= m_0' m_1 m_2 + m_0' m_1' m_2 + m_0 m_1 m_2' + m_0 m_1 m_2 \\
 &= m_1 m_2 + m_0 m_1' m_2 + m_0 m_1 m_2' \\
 &= m_1 (m_2 + m_0 m_2') + m_0 m_1' m_2 \\
 &= m_1 (m_0 + m_2) + m_0 m_1' m_2 \\
 &= m_0 m_1 + m_1 m_2 + m_0 m_1' m_2 \\
 &= m_0 m_1 + m_2 (m_0 + m_1) \\
 &= m_0 m_1 + m_0 m_2 + m_1 m_2
 \end{aligned}$$

Circuit:



9. (a) $F(a,b,c) = ab'c + abc + a'bc + abc'$
 $= ac + a'bc + abc'$
 $= c(a+b) + abc' = ac + b(a+c) = ab + ac + bc.$

(b) K-map

	bc	b'c'	b'c	bc	bc'
a'	0	0	1	0	0
a	0	1	1	1	1

$\Rightarrow F = bc + ac + ab.$

10. (a) algebraic method.

$F(a,b,c,d) = ab + a'b'd'$

(b) K-map.

	cd	c'd'	c'd	cd	cd'
a'b'	1	0	0	1	0
a'b	0	0	0	0	0
ab	1	1	1	1	0
ab'	0	0	0	0	0

We can obviously find that F cannot be optimized.

11. Using K-map to derive $F(a,b,c,d) = a'b'cd + ab'cd'$

F

$ab \backslash cd$	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	0	X	0
$a'b$	0	1	X	0
ab	X	X	X	X
ab'	0	0	X	1

$$\Rightarrow F = bd + ac$$

12. (a) $F = a'c + ac + a'b = c + a'b$

F

$ab \backslash c$	00	01	10	11
0	0	1	0	0
1	1	1	1	1

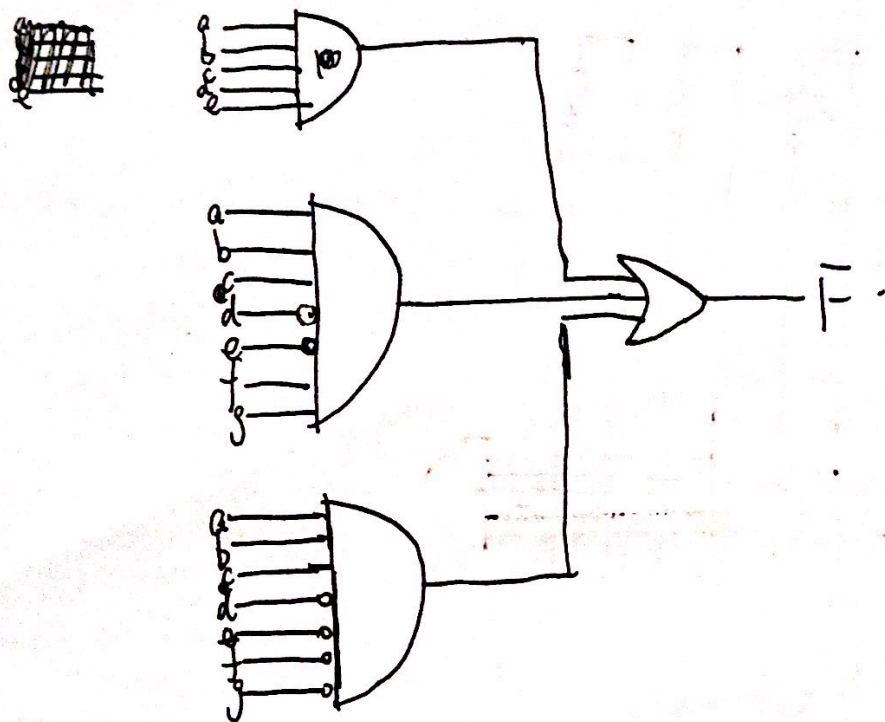
prime implicants are c and $a'b$

The essential prime implicants

are also $b'c$ and $a'bc$
 c and $a'b$.

$$\begin{aligned}
 13. \quad F &= abcde + abcd'e'fg + abc d'e'f'g' \\
 &= abc (de + d'e'fg + d'e'f'g') \\
 &= abc (de + d'e'(fg + f'g')).
 \end{aligned}$$

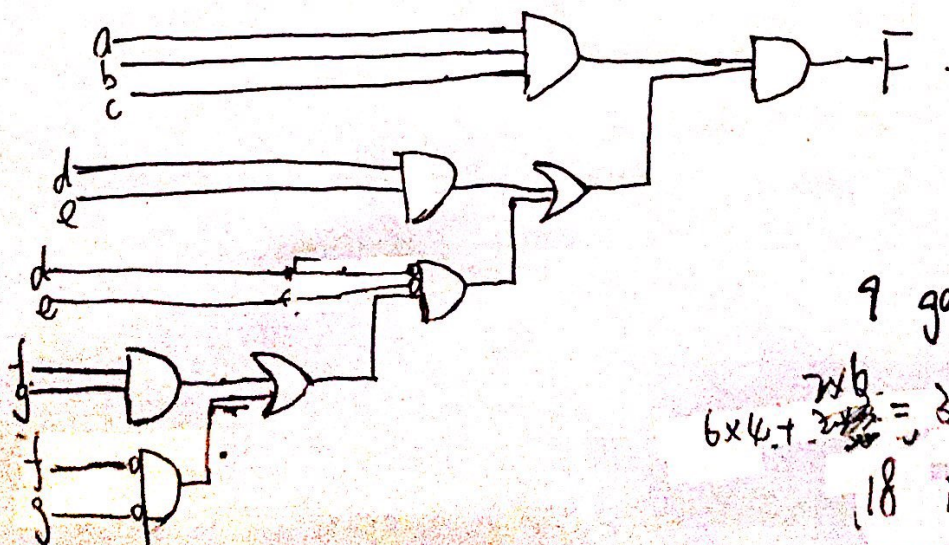
(1) original circuit.



2 gate delays.

10 + 14 + 14 + 6 = 44 transistors
12 inputs.

(2) multilevel.



9 gate delays.

$6 \times 4 + 2 \times 6 = 36$ transistors.
18 inputs