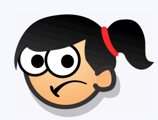
Error Correcting Codes
and
Cryptography

Group 10

Chengze He 

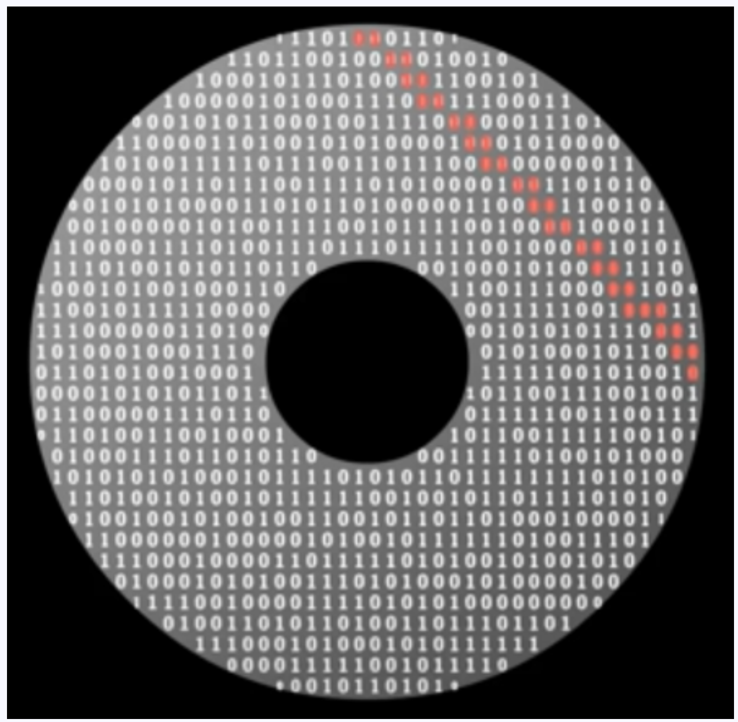
 Qinhang Wu

Yong Li 

 Yuxuan Xia

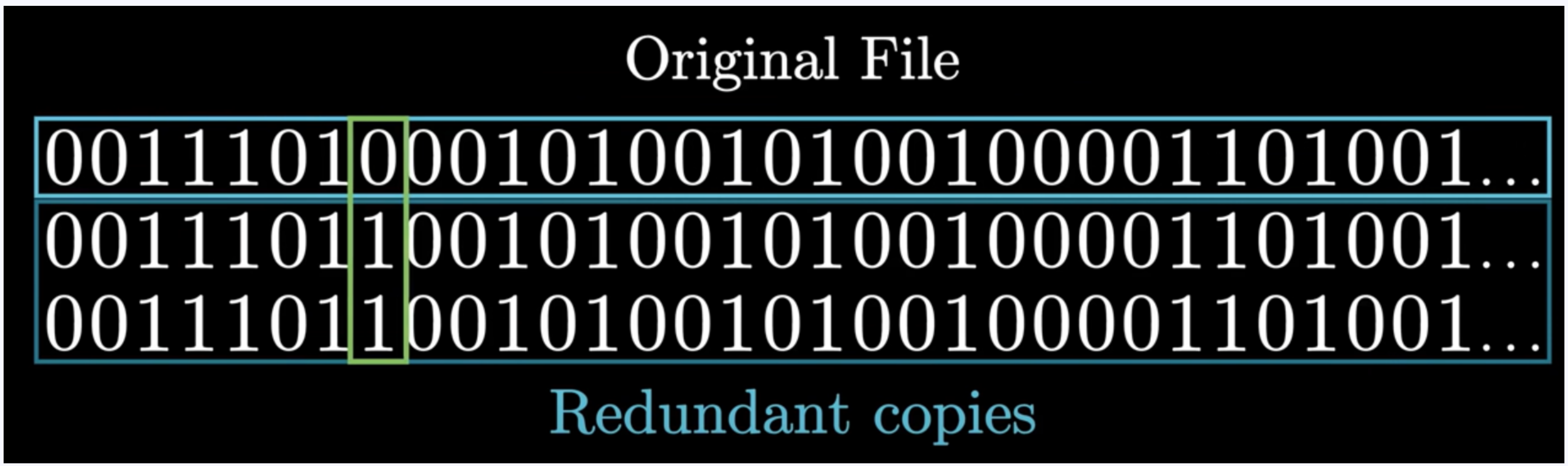
ECC and cryptography Part I

Qinhang Wu



Intro. to Error Correcting Code

- Preliminary Goal: detect partial error during transmission and correct them as an encoding scheme
- Naïve example: repetition

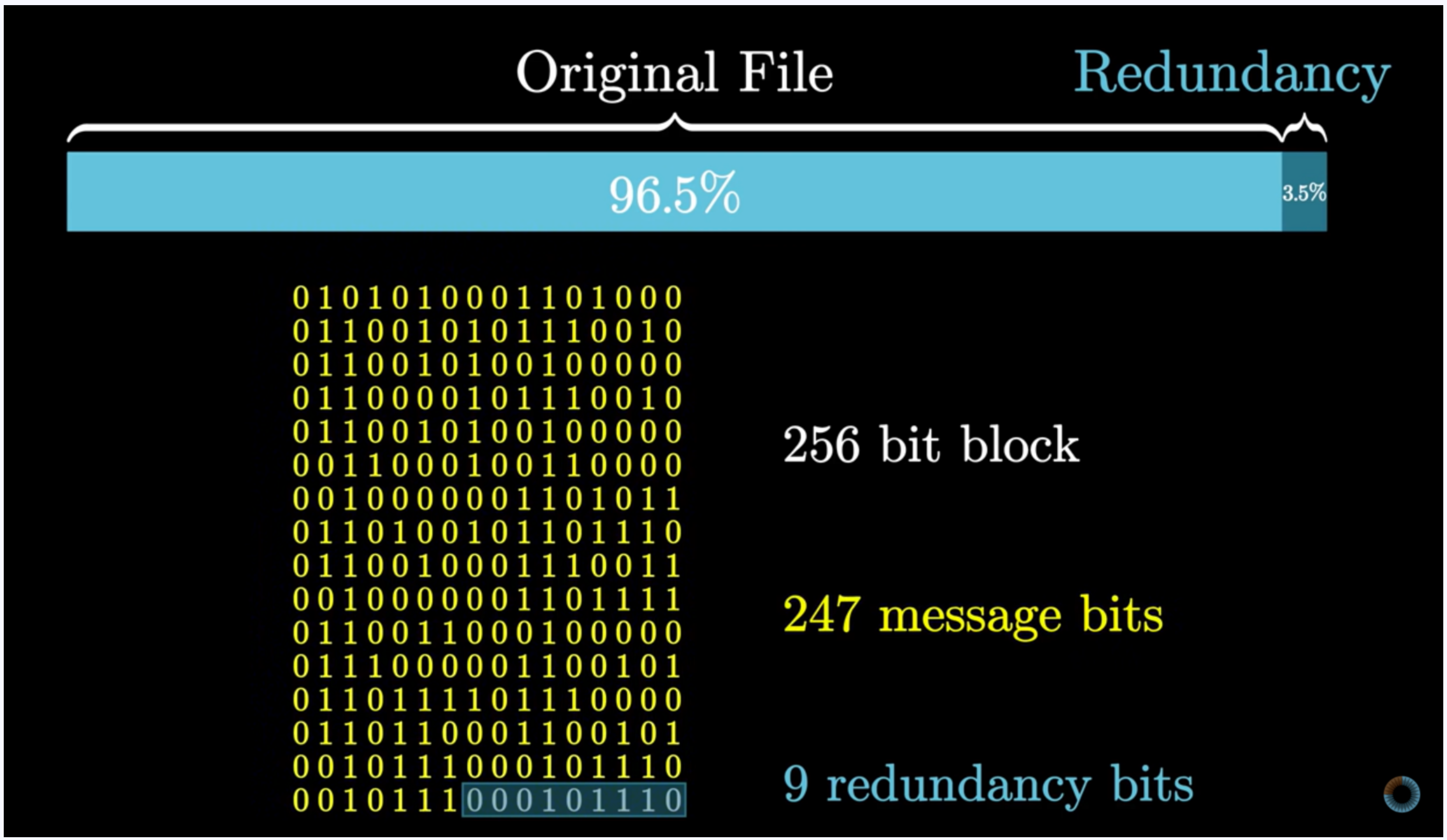


Intro. to Error Correcting Code

- Revised Goal: detect partial error during transmission and correct them as an encoding scheme, while ensuring …
  - 1) **large hamming distance** for code words, and
  - 2) **large transmission rate** for the code
- Hamming Distance: number of positions that two codewords disagree
- Transmission Rate: $\log(M)\ /n$, M as # of possible messages, n as length of each codeword

Intro. to Error Correcting Code

- Parity Checking
- Tradeoff: Redundancy vs. Tolerance



Linear Error Correcting Code

- Hamming Code (1940s)
- Reed-Solomon Code (1960s)
  - Basis of QR Code
- Goppa Code (1970s)
  - McEliece Cryptosystem
- Shor Code (1990s)
  - Quantum error correction

Hamming Code: General Algorithm

- Number the bits starting from 1: bit 1, 2, 3, 4, 5, 6, 7, etc.
- Write the bit numbers in binary: 1, 10, 11, 100, 101, 110, 111, etc.
- All bit positions that are powers of two (have a single 1 bit in the binary form of their position) are parity bits: 1, 2, 4, 8, etc. (1, 10, 100, 1000)
- All other bit positions, with two or more 1 bits in the binary form of their position, are data bits.
- Each data bit is included in a unique set of 2 or more parity bits, as determined by the binary form of its bit position.
- Parity bit 1 covers all bit positions which have the least significant bit set: bit 1 (the parity bit itself), 3, 5, 7, 9, etc.
- Parity bit 2 covers all bit positions which have the second least significant bit set: bits 2-3, 6-7, 10-11, etc.
- Parity bit 4 covers all bit positions which have the third least significant bit set: bits 4–7, 12–15, 20–23, etc.
- Parity bit 8 covers all bit positions which have the fourth least significant bit set: bits 8–15, 24–31, 40–47, etc.
- In general each parity bit covers all bits where the bitwise AND of the parity position and the bit position is non-zero.

Hamming Code: an Example

- 4 bits for redundancy, 1 extra parity bit on the top

			1
	0	1	0
	0	1	0
1	0	0	1

	0		1
	0	1	0
	0	1	0
1	0	0	1

	0	0	1
	0	1	0
	0	1	0
1	0	0	1

	0	0	1
1	0	1	0
	0	1	0
1	0	0	1

	0	0	1
1	0	1	0
1	0	1	0
1	0	0	1

Hamming Code: an Example

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			1
	0	1	0
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1	0	0	1

1	0	0	1
1	0	1	0
1	0	1	0
1	0	0	1

Hamming Code: an Example

- 4 bits for redundancy, 1 extra parity bit on the top

1	0	0	1
1	0	1	0
1	0	1	0
1	0	0	1

noise $\longrightarrow$

1	0	0	1
1	0	1	1
1	0	1	0
1	0	0	1

Hamming Code: an Example

- 4 bits for redundancy, 1 extra parity bit on the top

1	0	0	1
1	0	1	1
1	0	1	0
1	0	0	1

1	0	0	1
1	0	1	1
1	0	1	0
1	0	0	1

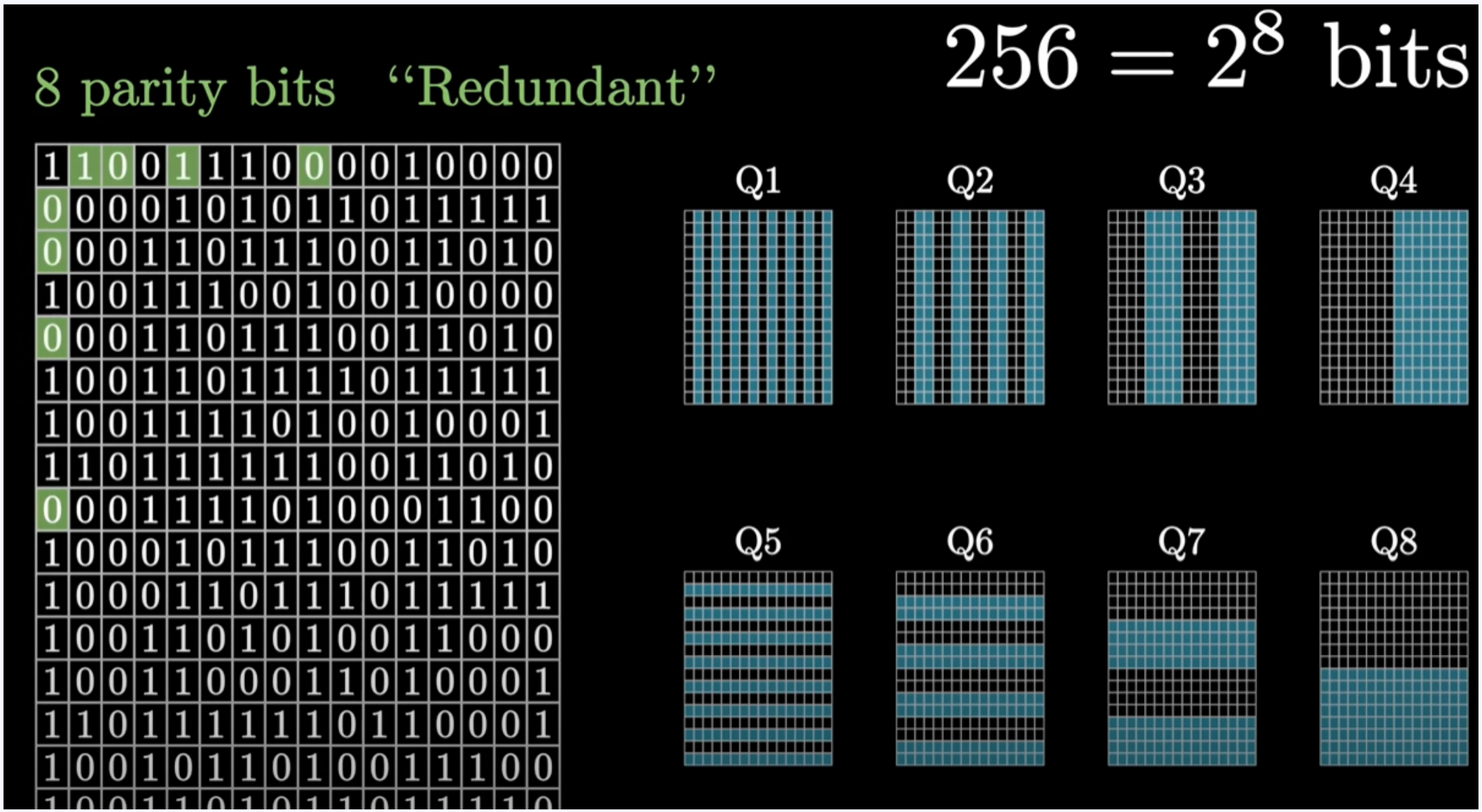
1	0	0	1
1	0	1	1
1	0	1	0
1	0	0	1

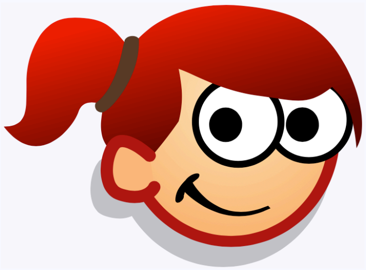
1	0	0	1
1	0	1	1
1	0	1	0
1	0	0	1

1	0	0	1
1	0	1	1
1	0	1	0
1	0	0	1

Hamming Code: an Example

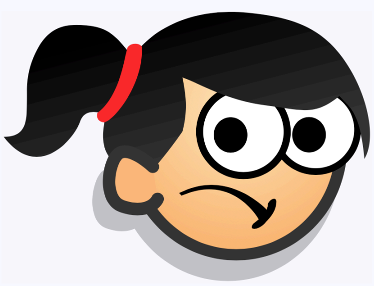
- Extended to 2^8 bits





ECC and cryptography Part II

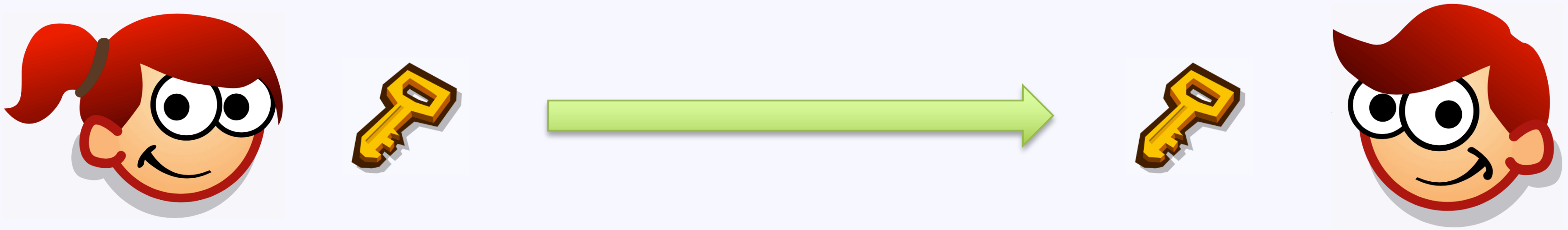
Chengze He



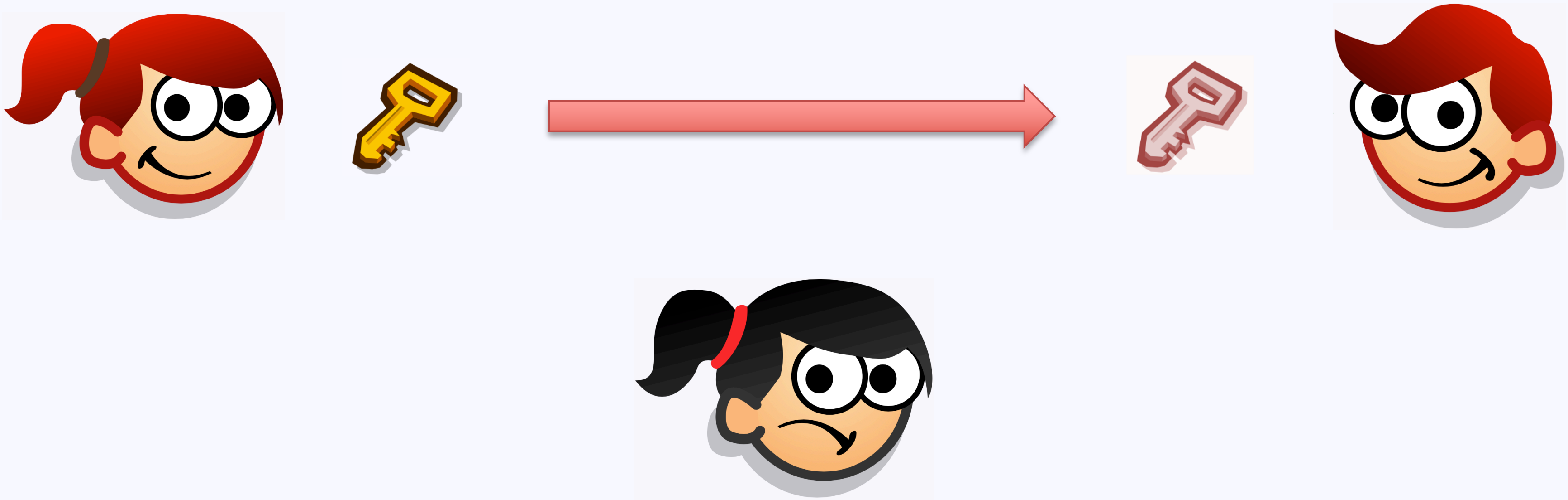
Applications of ECC in cryptography

- Quantum cryptography
- Prevent corruption of encrypted messages
  - Parity check in DES keys
- Cryptosystems based on ECC designs
  - McEliece cryptosystem
  - Neiderreiter cryptosystem

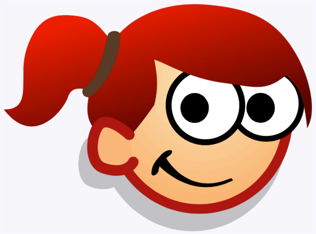
Quantum key distribution



Quantum key distribution



Corruption prevention



We have too much homework for VE475!

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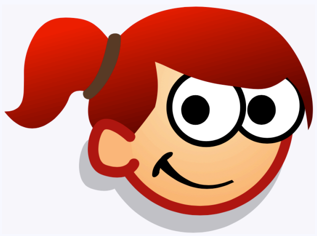
Corruption prevention

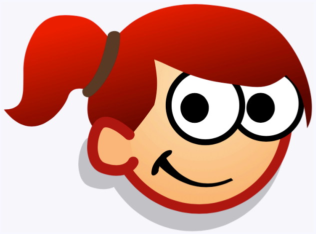


We have too much homework for VE475!

noise

We have too much hlcework for VE475!

Corruption prevention



We have too much homework for VE475!

encryption

942dd7970986c7a3ae34a3c071f6c5678936cbc605bd02b57004da5631a70b0
49b64154d4e9feee232bc4713b1ac0a8f97e9b4fbe6d7f7f4808a3f17201debcd

942dd7970986c7a3ae34a3c071f6c5678936cbc605bd02b57004da5631a70b0
49b64154d4e9feee232bc4713b1ac0a8f97e9b4fbe6d7f7f4808a3f17201debcd

decryption

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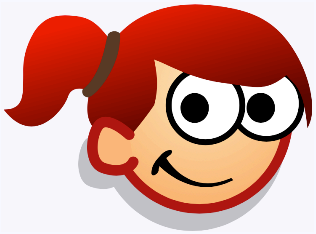
Corruption prevention



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encryption

942dd7970986c7a3ae34a3c071f6c5678936cbc605bd02b57004da5631a70b0
49b64154d4e9feee232bc4713b1ac0a8f97e9b4fbe6d7f7f4808a3f17201debcd

noise

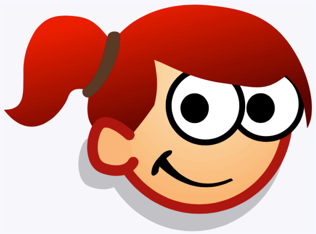
can be fixed using ECC

942dd7970986c7a3ae34a3c071f6c5678936abc605bd02b57004da5631a70b0
49b64154d4e9feee232bc4713b1ac0a8f97e9b4fbe6d7f7f4808a3f17201debcd

decryption

C_]ﺗ͂zynpa~q6ﻫhb

Making use of “noise”?



We have too much homework for VE475!

encryption

942dd7970986c7a3ae34a3c071f6c5678936cbc605bd02b57004da5631a70b0
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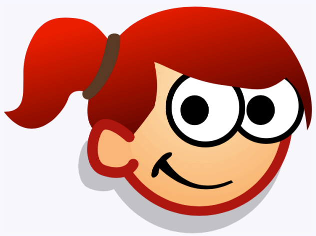
noise

942dd7970986c7a3ae34a3c071f6c5678936abc605bd02b57004da5631a70b0
49b64154d4e9feee232bc4713b1ac0a8f97e9b4fbe6d7f7f4808a3f17201debcd

decryption

C_]ؿzynpa~q6وِhb

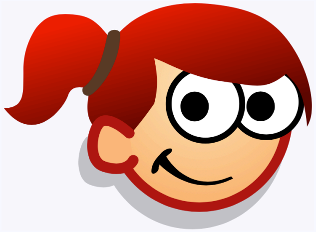
Building a ECC-based cryptosystem



Agree on one ECC scheme

- Introduce artificial noise when encrypting
- Combine encryption and disturbance

Artificial noise



We have too much homework for VE475!

encryption

942dd7970986c7a3ae34a3c071f6c5678936abc605bd02b57004da5631a70b0
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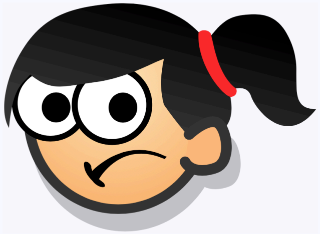


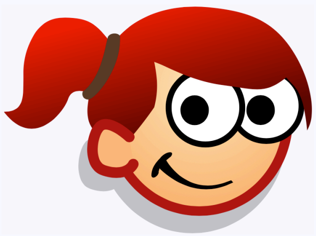
fix

942dd7970986c7a3ae34a3c071f6c5678936cbc605bd02b57004da5631a70b0
49b64154d4e9feee232bc4713b1ac0a8f97e9b4fbe6d7f7f4808a3f17201debcd

decryption

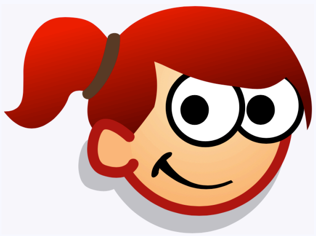
We have too much homework for VE475!

Building a ECC-based cryptosystem



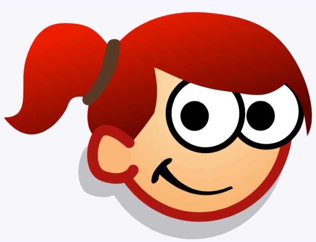
Agree on one ECC scheme

ECC-based public key infrastructure: (Simplified) McEliece cryptosystem



Encoding matrix E
Decoding function D, $D(mE + z) = m$ if z is "small"
Random matrix W
Private key $X = \langle D, W^{-1} \rangle$

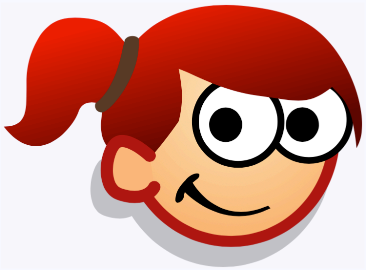
Public key $F = EW$ from Alice
Message m
"small" random noise z
Encryption: $c = mF + z$



Ciphertext c from Bob
Decryption: $D(cW^{-1}) = D((mEW + z)W^{-1}) =$
$D(mEWW^{-1} + zW^{-1}) = D(mE + z') = m$

(Simplified) McEliece cryptosystem

- Randomness in the ciphertexts
  - Encrypting the same message yields different ciphertexts
- Eve can infer neither D nor W^{-1} from F
- Only Alice can correct the errors
  - Eve cannot locate the errors (consider the One Time Pad)
  - Eve does not know the decoding algorithm
- Randomness is important



Thank you