

VE477

Introduction to Algorithms

Challenge (Hint 2)

Manuel — UM-JI (Fall 2021)

- Abstract a real life problem
- Find an algorithm to solve a problem
- Prove its correctness and complexity
- Rewarded by a bonus on the final grade

After a letting you think of the problem for a short while Bill wants to ensure you fully get the problem. He has prepared a simple toy example, where trucks leave from Shanghai Pudong airport, stop at Zizhu Science Park, Qingpu, and Kunshan, to finally arrive in Suzhou industrial park, in his simulation a day is only three hours long and trucks can load a maximum of ten packages. He then asks you what else need to be defined. After a quick reflection you list the following items: (i) how many packages are initially at each stop, (ii) how many arrive every hour, and (iii) the size of the warehouse at each place on the way. As Bill sees you are OK with that he carries on, assuming there are two, three, and four packages left from the previous day in Zizhu, Qingpu, and Kunshan, respectively. Still using the same order for the stops he assumes four, three, and two packages arrive every hour, while the warehouse can store no more than ten, nine, and eight packages. He then explains that the first thing to check is that the problem is solvable. As this is indeed the case, he tried with three trucks, since there was three hours. It obviously worked, but as he wanted to know the minimum number of trucks it had to look at two and one. With a unique truck he could not find any solution

- If it leaves before the first hour, then the warehouse will be full very quickly;
- If it leaves after then first hour, then it will take six packages in Zizhu, only four in Qingpu, and none in Kunshan, so again at least a warehouse will be full;
- If it leaves after the second hour, then the truck will be full in Zizhu, so no more package can be taken in other places and the warehouses will be full;

Bill was however able to find a solution with two trucks:

- A first one leaves before the first hour, and collects all the packages from the previous day;
- After the second hour the three warehouses contain eight, six, and four packages, i.e. under their capacity;
- Before the third hour send a truck to collect the packages. Then there are zero, four, and four packages to store;
- At the third hour new packages have arrived such that four, seven, and six packages need to be stored until the next day. Since this is less than the capacity of the various warehouses, the problem can be solved with two trucks;

Bill seems pretty happy with his simple demonstration. Seeing that you look not fully satisfied, he acknowledges that this is not a very large contribution. But at least you can ensure your understanding of the problem is correct before you go any further. He already has a few ideas that could be applied to really solve it, but he wants you to think a bit more before he provides you with a bigger hint.