

## VE477

### Introduction to Algorithms

#### Homework 3

Manuel — UM-JI (Fall 2021)

#### Reminders

- Write in a neat and legible handwriting or use  $\text{\LaTeX}$
- Clearly explain the reasoning process
- Write in a complete style (subject, verb, and object)
- Be critical on your results

*Questions preceded by a \* are optional. Although they can be skipped without any deduction, it is important to know and understand the results they contain.*

#### Ex. 1 — Hamiltonian path

- \* 1. Explain and present Depth-First Search (DFS).
- \* 2. Explain and present topological sorting.
- 3. Write the pseudo-code of a polynomial time algorithm which decides if a directed acyclic graph contains a Hamiltonian path.
- 4. Prove its complexity.
- 5. To what complexity class does the Hamiltonian path problem belong to?

#### Ex. 2 — Critical thinking

- 1. Is the function  $\lceil \log n \rceil!$  bounded by a polynomial?
- 2. Is the function  $\log^* \log n$  asymptotically larger than  $\log \log^* n$ ?
- 3. Given eight balls of similar size but where one is lighter, detect which one it is, while minimizing the number of weighting. Provide the pseudocode.

#### Ex. 3 — Rubik's Cube

In about half a page explain the game and at least two algorithms to solve it. Provide references.

#### Ex. 4 — The $\mathcal{NP}$ classe

Prove that the following problems are in  $\mathcal{NP}$ .

- 1. Does a given graph have a simple path?
- \* 2. Is a given integer composite?
- 3. Does a given graph have a vertex cover of size  $k$ , for some integer  $k$ ?

#### Ex. 5 — PRIMES is in $\mathcal{P}$

The PRIMES problem consists in deciding if a given integer  $n$  is prime. A simple algorithm to solve PRIMES is trial division which runs in time  $\mathcal{O}(n)$ . Is it sufficient to prove that PRIMES is in  $\mathcal{P}$ ? Explain.

*Hint:* use the Prime Number Theorem.