Announcements

- Final exam
  - Aug. 2, 6:20pm-8pm
  - Closed-book, 2 A4 cheat-sheets with your own writing
  - No electronic device, except basic calculator
- Still some questions?
  - Piazza
  - RC class on Thursday, 2-4pm
- Course evaluation

Advice

- Read carefully the problem description
  - Justify when needed
- Problems are independent
- Write clearly

Ve492: Introduction to Artificial Intelligence
Final Review

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UM-SJTU Joint Institute

Slides adapted from http://ai.berkeley.edu, AIMA, UM, CMU

Content

- Probability review
- Probabilistic reasoning
  - Bayes nets
  - Markov models and HMMs
- Machine learning
  - Naive Bayes
  - Perceptron
  - Neural networks
- Logic-based approaches
  - Propositional logic
  - First-order logic
  - Classical planning

Probability

- For each of the following statements, either prove it is true or give a counterexample.
  - If P(a | b, c) = P(b | a, c), then P(a | c) = P(b | c)
  - If P(a | b, c) = P(a), then P(b | c) = P(b)
  - If P(a | b) = P(a), then P(a | b, c) = P(a | c)

1) $P(a|c) = P(a|b,c) \cdot P(b|b)$ $P(b|c) = P(b|a,c) \cdot P(a|c)$

2)

Bayes Rule

$P(A|V)$

Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus.

$P(A|\bar{V})$ $P(V)$

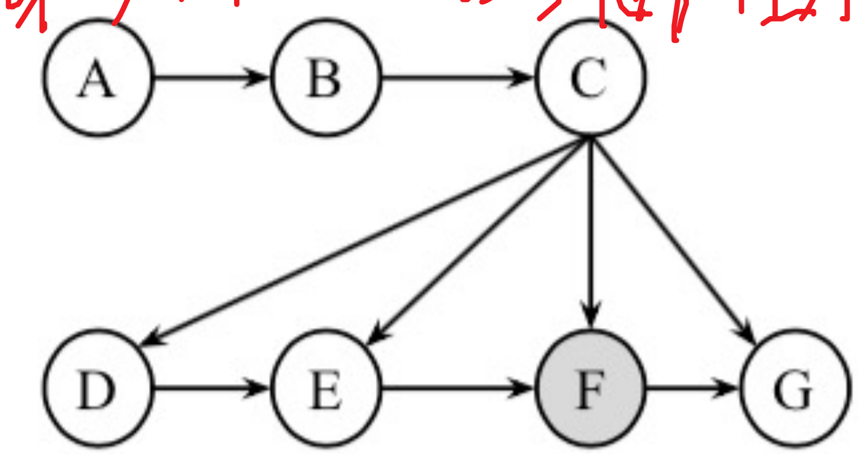
- Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

$$P(V|A) = \frac{P(A|V)\cdot P(V)}{P(A)} = \frac{P(A|V)\cdot P(V)}{P(A|V)\cdot P(V) + P(A|\bar{V})\cdot P(\bar{V})}$$

Bayes' Net

- Write the joint distribution of the following Bayes' net

P(A), P(B|A), P(C|B), P(D|C), P(E|C,D), P(F|C,E), P(G|C,F)



- How many values does the joint distribution have?

2^7

$2^7 - 1$ degrees of freedom

- How many parameters does the Bayes' net have?

degrees of freedom: 1+2+2+2+4+4+4

Values: 2+4+4+4+8+8+8

D-Separation

- Check Bayes applet

Inference

join

A: $P(A), P(B|A) \Rightarrow f_1(A,B)$ 2

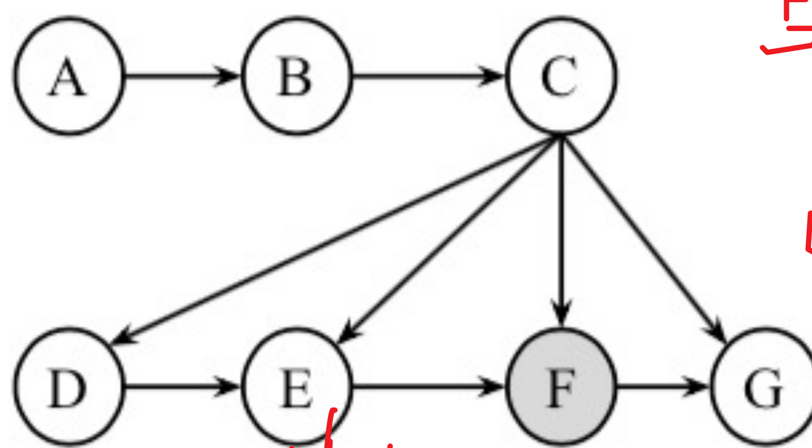
$\rightarrow f_1(B)$ 1

C: $P(C|B), P(D|C), P(E|C,D)$

$P(+f|C,E), P(G|C,+f)$

$\rightarrow f_2(C,D,E,+f,G) \rightarrow f_3(D,E,+f,G)$

4



E: $f_3(D,E,+f,G)$

$\rightarrow f_4(D,+f,G)$ 3

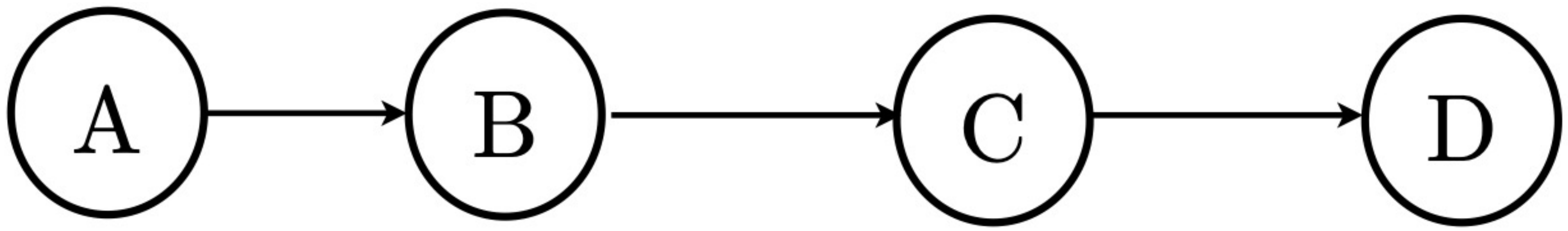
G: $f_4(D,+f,G)$

$\rightarrow f_5(D,+f)$ 1

- Run Variable Elimination to compute $P(B, D| + f)$ with order A, C, E, G
- What is the size of the largest generated factor? 4
- Find the best ordering for Variable Elimination G, E, C, A
- What is the cutset for this graph?

{C}

Sampling



$P(A)$	
$-a$	$3/4$
$+a$	$1/4$

$P(B\|A)$		
$-a$	$-b$	$2/3$
$-a$	$+b$	$1/3$
$+a$	$-b$	$4/5$
$+a$	$+b$	$1/5$

$P(C\|B)$		
$-b$	$-c$	$1/4$
$-b$	$+c$	$3/4$
$+b$	$-c$	$1/2$
$+b$	$+c$	$1/2$

$P(D\|C)$		
$-c$	$-d$	$1/8$
$-c$	$+d$	$7/8$
$+c$	$-d$	$5/6$
$+c$	$+d$	$1/6$

$\frac{1}{3 \times 8}$

Samples

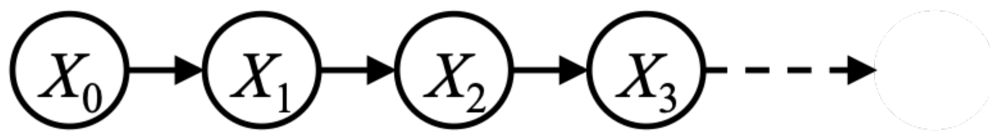
$+a$	$+b$	$-c$	$-d$
$+a$	$-b$	$+c$	$-d$
$-a$	$+b$	$+c$	$-d$
$-a$	$-b$	$+c$	$-d$
$+a$	$-b$	$-c$	$+d$
$+a$	$+b$	$+c$	$-d$
$-a$	$+b$	$-c$	$+d$
$-a$	$-b$	$+c$	$-d$

$+b$ $-d$

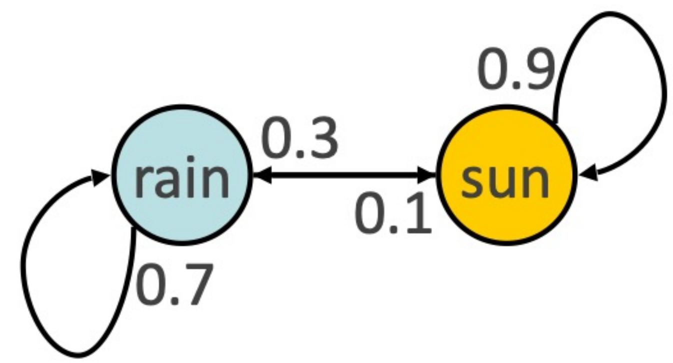
- Estimate $P(+c|+a,-d)$ via rejection sampling $\to \frac{2}{3}$
- Estimate $P(-a|+b,-d)$ via likelihood weighting

$$\frac{\frac{1}{18} \to \frac{5}{18}}{\frac{1}{18}+\frac{5}{18}+\left(\frac{1}{40}+\frac{1}{8}\right)}$$

Markov Chain



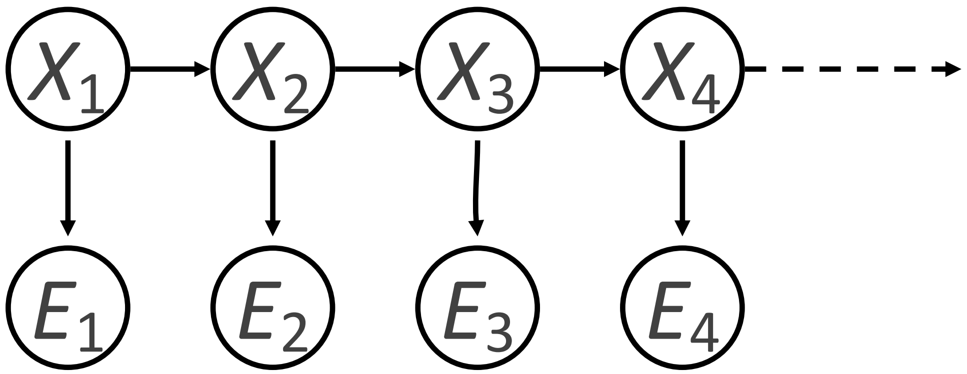
$P(X_0)$ $\qquad$ $P(X_t \mid X_{t-1})$

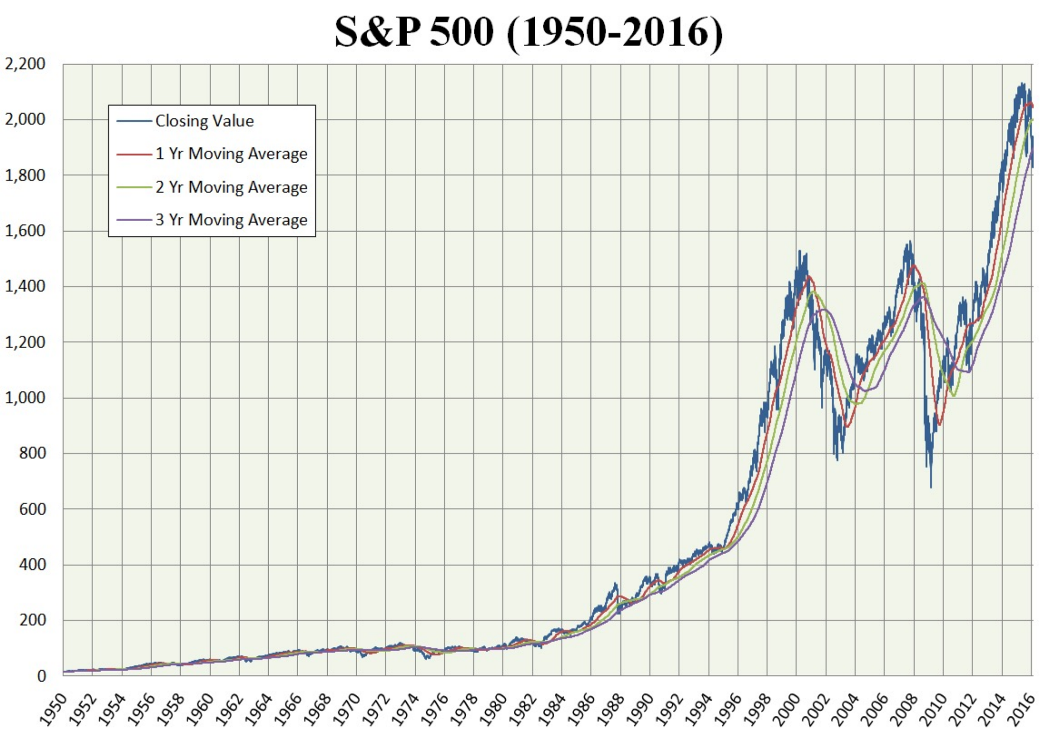


$P(X_t) = \sum_{X_{t-1}} P(X_t \mid X_{t-1}) \cdot P(X_{t-1})$

- What is the probability of $P(X_t)$?
- What is the stationary distribution for the weather example?

Hidden Markov Model





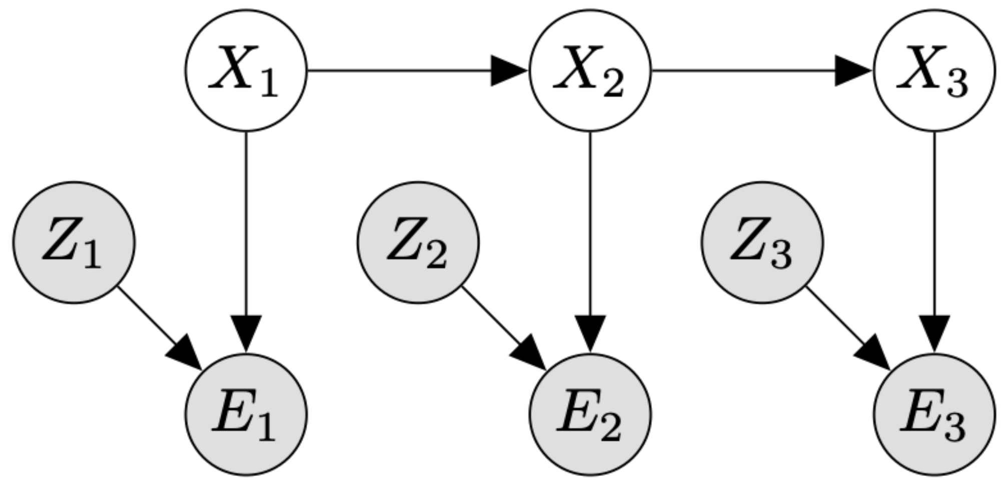
Wikipedia

- Financial investment
  - X = market condition: bull, bear
  - E = price evolution of some index: up, down
- Label past data into bull vs bear
- Use historical data to estimate transition/emission probabilities

Hidden Markov Model

1) $P(X_{t+1} | z_{1:t})$

$\Rightarrow P(X_{t+1} | z_{1:t+1})$



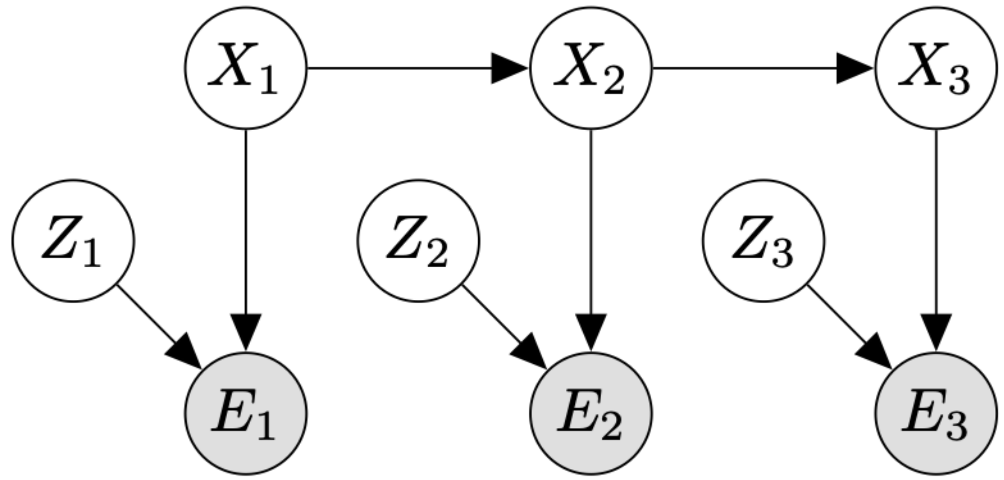
$P(X_t | z_{1:t}, e_{1:t})$ know

- Adapt the forward algorithm to this variant of HMM
  - Predict step $P(X_{t+1} | z_{1:t}, e_{1:t}) = \sum_{x_t} P(X_{t+1} | X_t) P(X_t | z_{1:t}, e_{1:t})$
  - Update

$P(X_{t+1} | z_{1:t}, e_{1:t}, z_{t+1}, e_{t+1}) \propto$

P

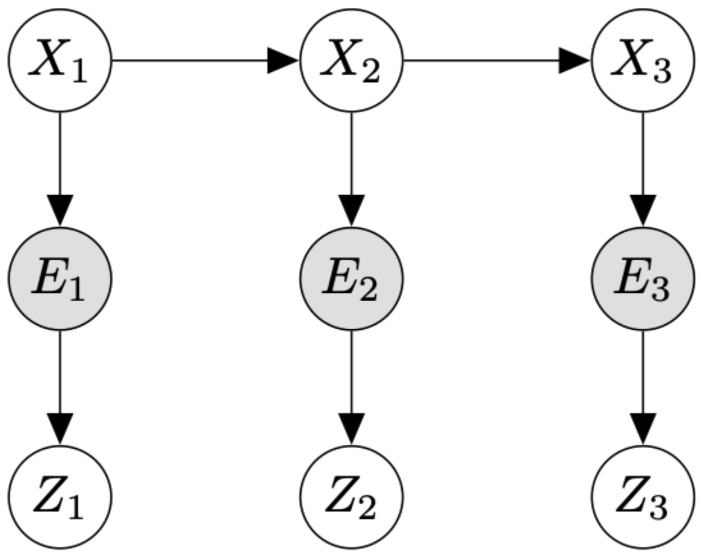
Hidden Markov Model



- Adapt the forward algorithm to this variant of HMM
  - Predict step $P(X_{t+1} \mid e_{1:t}) = \sum_{x_t} P(X_{t+1} \mid x_t) \cdot P(x_t \mid e_{1:t})$
  - Update

$$P(X_{t+1} \mid e_{1:t}, e_{t+1}) \propto \sum_{z_{t+1}} P(z_{t+1}) \cdot P(e_{t+1} \mid X_{t+1}, z_{t+1}) \cdot P(X_{t+1} \mid e_{1:t})$$

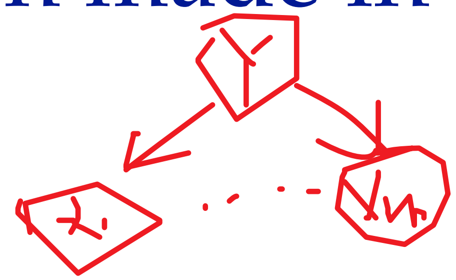
Hidden Markov Model

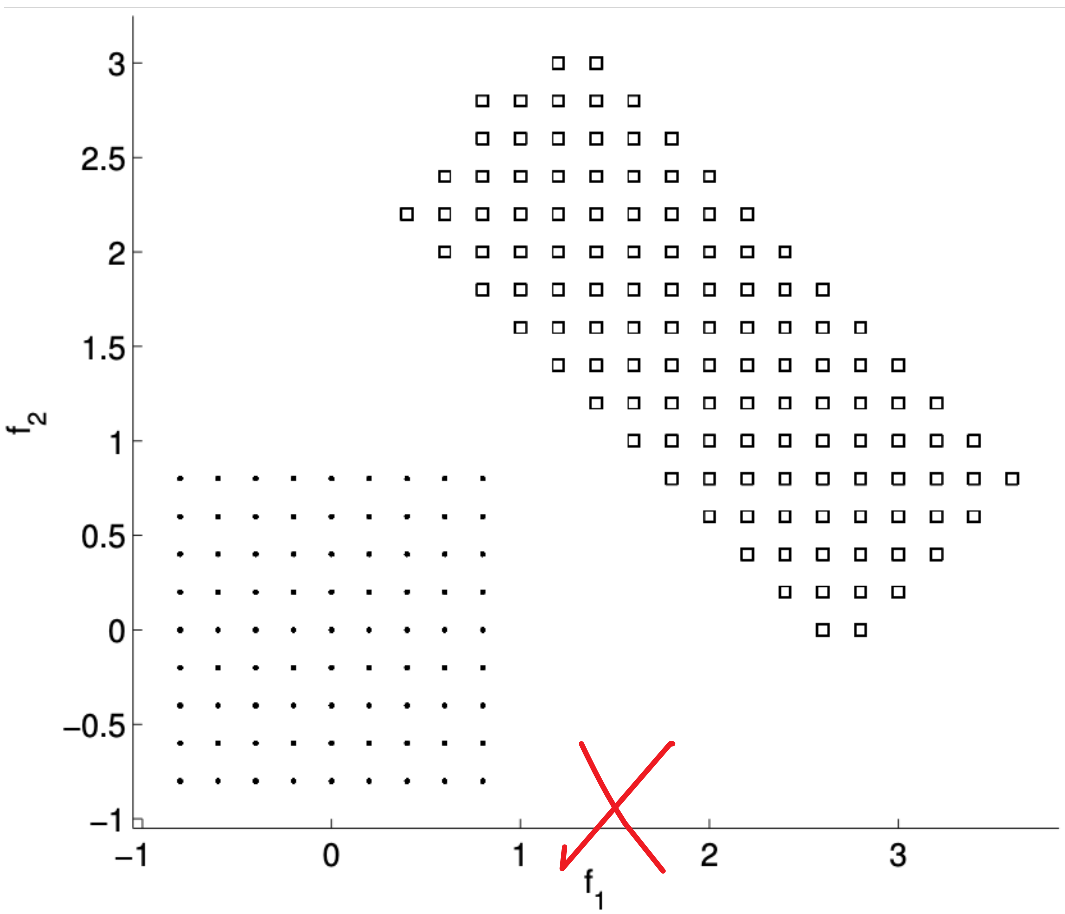


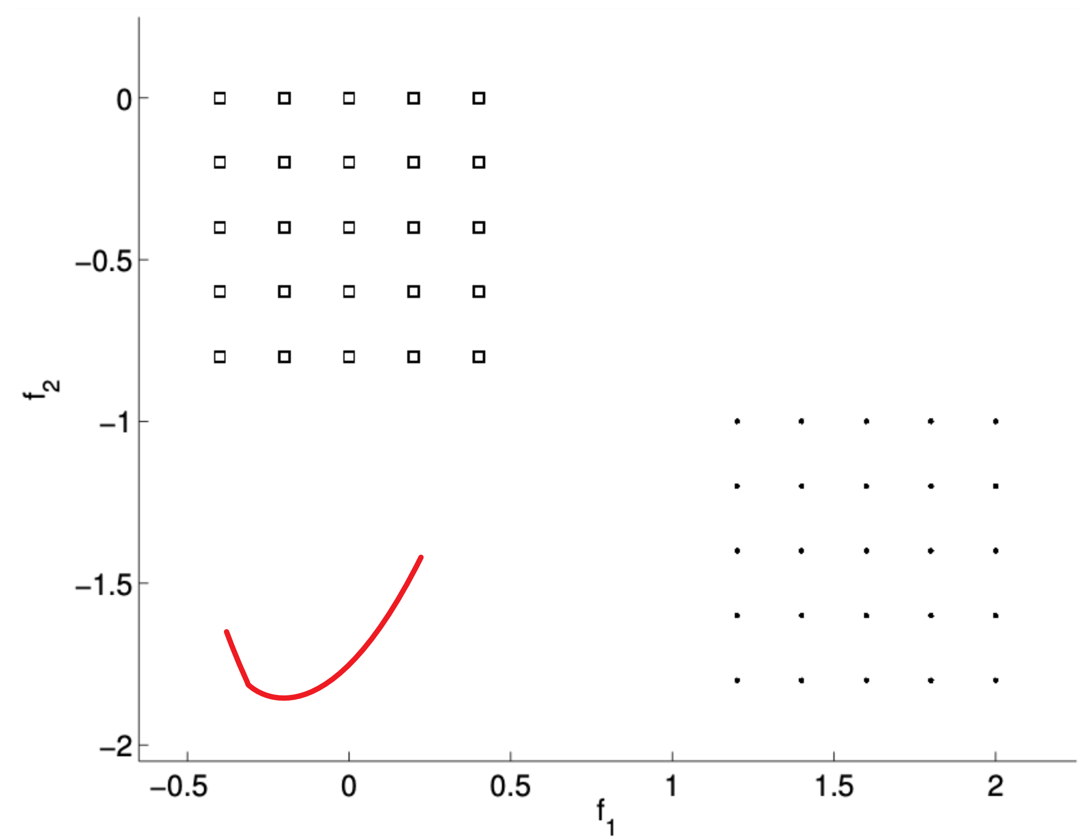
- Adapt the forward algorithm to this variant of HMM
  - Predict step
  - Update

Naïve Bayes

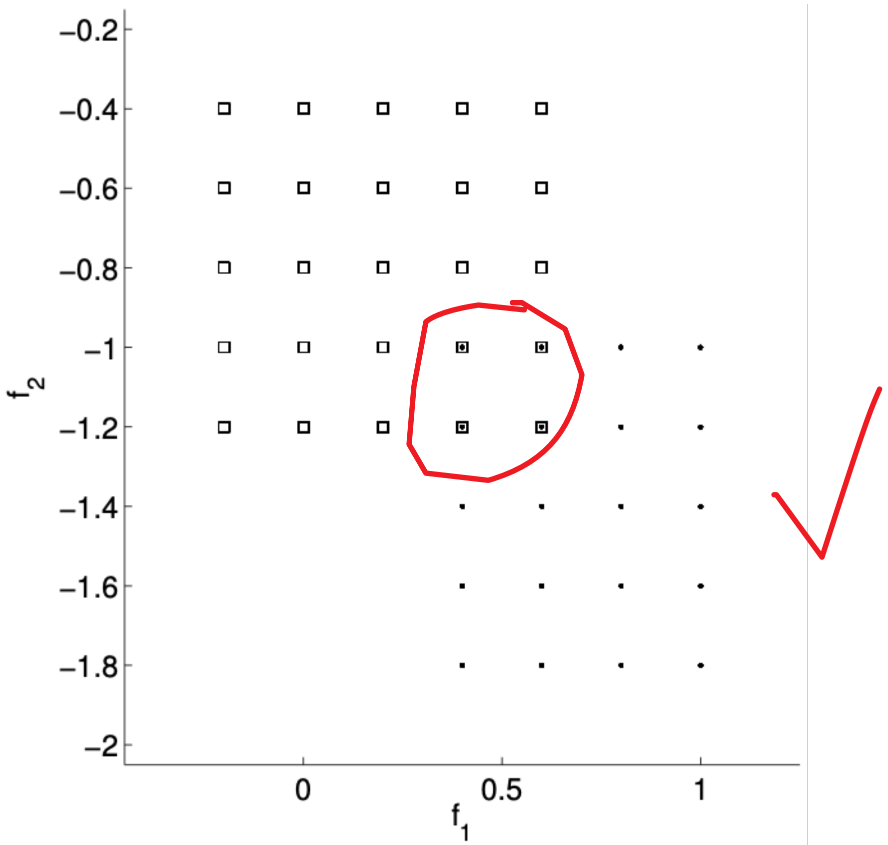
- Which of the following binary classification problems satisfy the assumption made in Naïve Bayes?

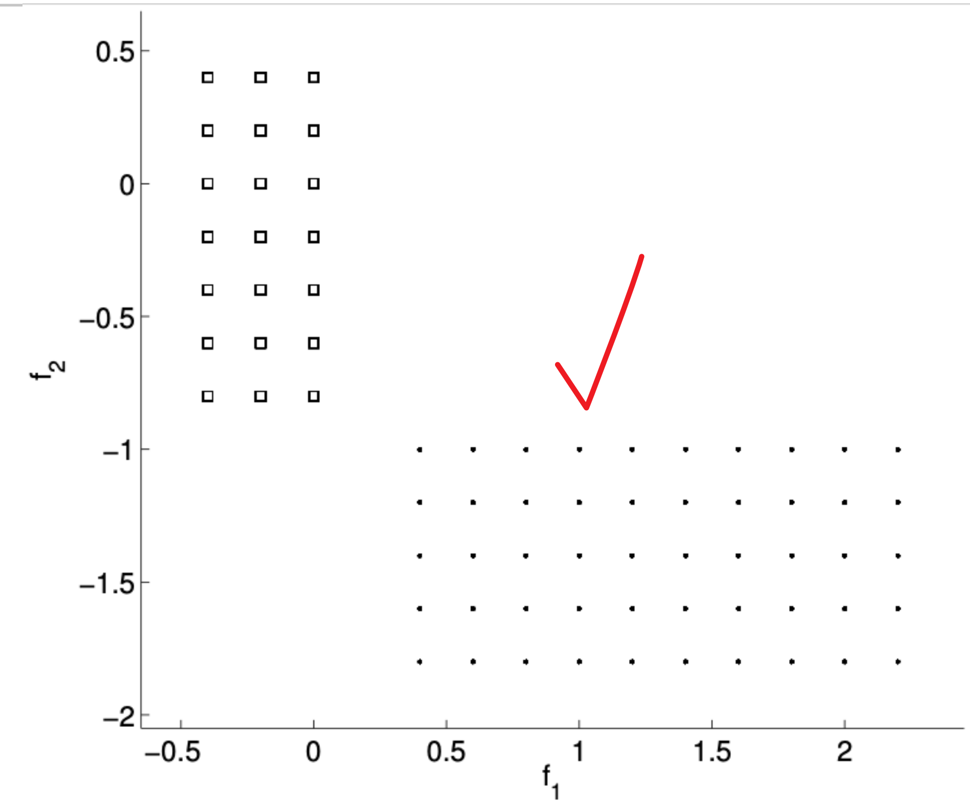


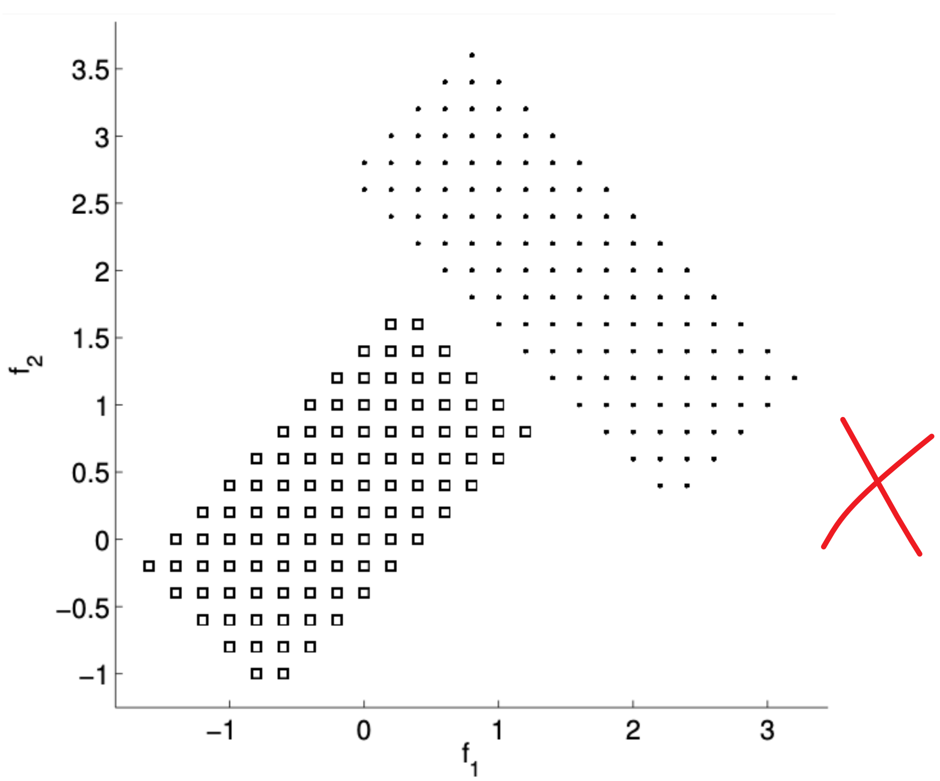


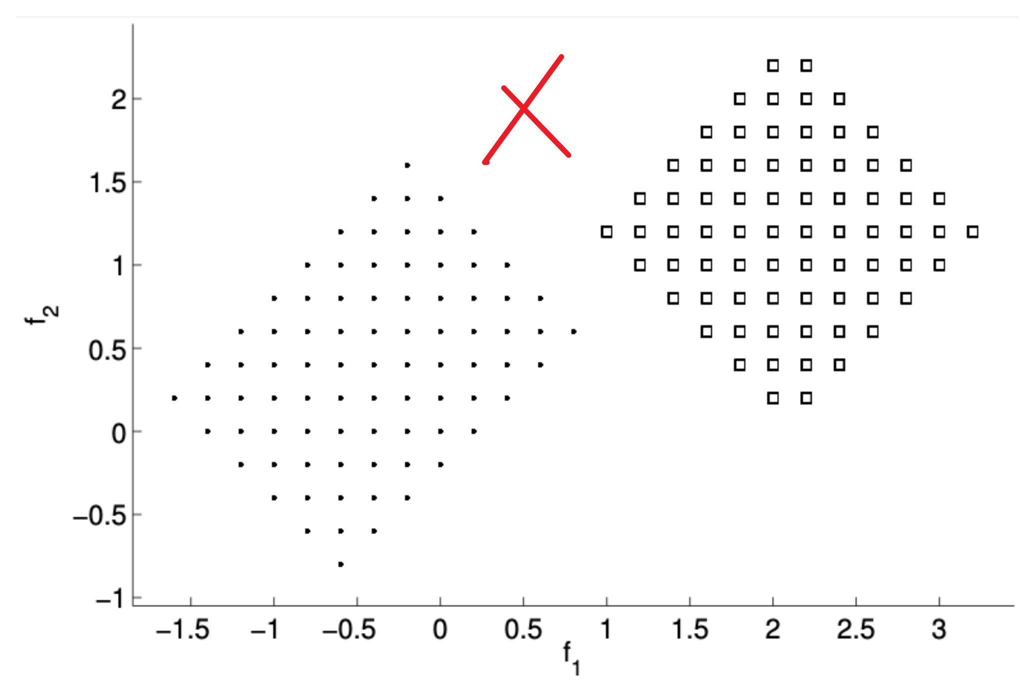


Naïve Bayes ctd.





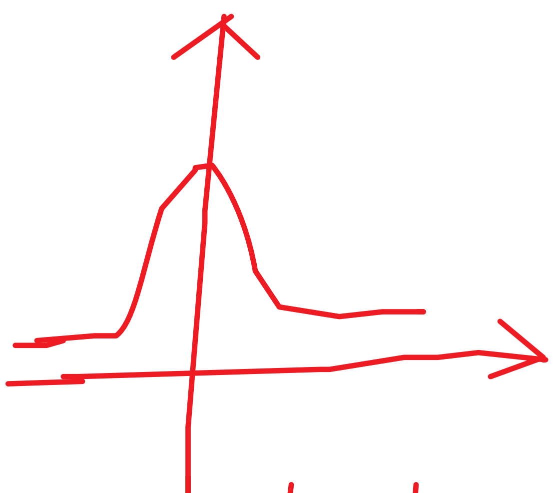




Discriminative Learning

For a binary classification problem, we choose the following model $\mathbb{P}(y = +1|x) = \Phi(w \cdot x)$ where Φ is the CDF of a standard normal distribution.

- What is the decision boundary? $P(y=+1|x) = \frac{1}{2}$
- Formulate the optimization problem to be solved to find w
- Formulate the stochastic gradient method to solve that problem



$\Phi(w \cdot x) = \frac{1}{2}$

$w \cdot x = \Phi^{-1}\left(\frac{1}{2}\right) = 0$

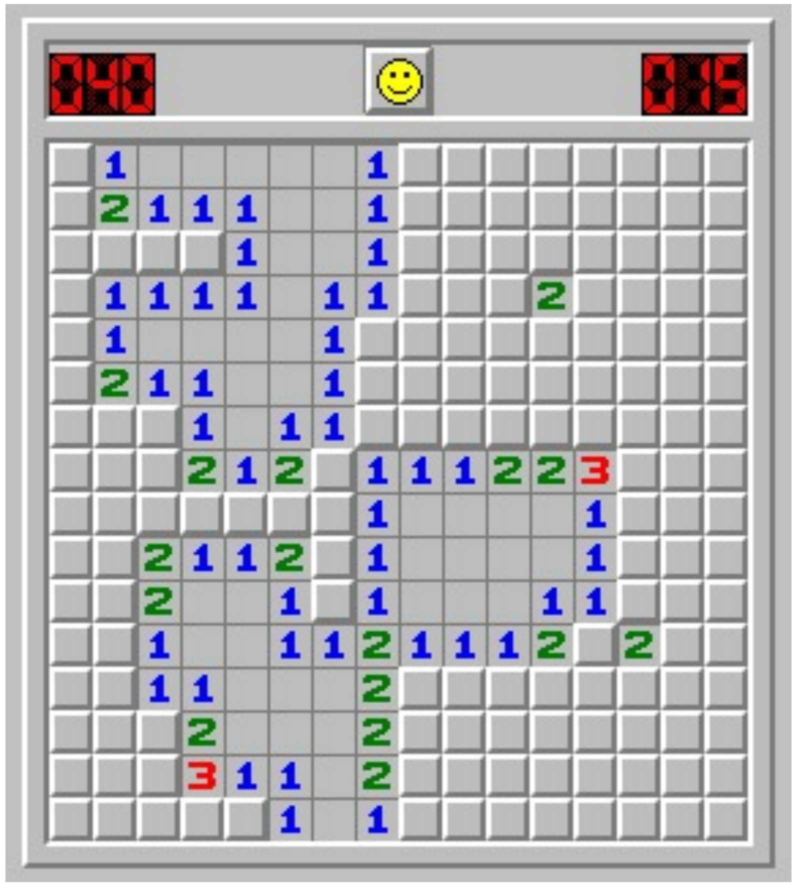
$X \sim N(0,1)$ $\rightarrow$ CDF $P(X \leq x)$

Propositional Logic

- Which of the following are correct? $\alpha \models \beta \Leftrightarrow \text{Model}(\alpha) \subseteq M(\beta)$
  - False $\models$ True ✓
  - True $\models$ False ✗
  - $(A \land B) \models (A \Leftrightarrow B)$
  - $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$
  - $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable
  - $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable
  - $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C

Application: Propositional Logic

- Minesweeper: Let $X_{i,j}$ be true iff square $[i, j]$ contains a mine.
  - Write down the assertion that exactly two mines are adjacent to [1,1] as a sentence involving some logical combination of $X_{i,j}$ propositions.
  - Generalize your assertion by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines
  - Explain precisely how an agent can use DPLL to prove that a given square does (or does not) contain a mine.



First-Order Logic

- For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it.
  - No two people have the same social security number.

    ✗ $\neg \exists\ x, y, n\ \text{Person}(x) \land \text{Person}(y) \Rightarrow [\text{HasSS\#}(x, n) \land \text{HasSS\#}(y, n)]$

    $\forall x, y,\ x \neq y \land \text{Person}(x) \land P(y) \Rightarrow \neg \exists n,\ \text{Has}(x, n) \land \text{Has}(y, n)$

  - John's social security number is the same as Mary's.

    ✓ $\exists\ n\ \text{HasSS\#}(\text{John}, n) \land \text{HasSS\#}(\text{Mary}, n)$

  - Everyone's social security number has nine digits.

    $\forall\ x, n\ \text{Person}(x) \Rightarrow [\text{HasSS\#}(x, n) \land \text{Digits}(n, 9)]$

    $\forall x\ \exists n\ \ldots$

Classic Planning

A monkey is in a room with some bananas hanging out of reach from the ceiling. A box is available that will enable the monkey to reach the bananas if he climbs on it. Initially, the monkey is at *A*, the bananas are at *B*, and the box is at *C*. The monkey and the box have height *Low*, but if the monkey climbs onto the box he will have height *High*, the same as the bananas. The actions available to the monkey include *EatBananas* if the monkey and the bananas are at the same location and height, *Go* from one place to another, *Push* an object from one place to another, and *ClimbUp* onto or *ClimbDown* from an object.

- Write down the initial state description
- Write down the STRIPS definitions of the five actions.