AM115 Workshop: The Age of the Earth

The basic idea, known before Lord Kevin to Fourier and Buffon, is the following. As Earth cools, the temperature gradient at the surface decreases. If, through theory, one can establish how the surface temperature gradient relates to the age, then measurements of the surface temperature gradient, in mines for example, will tell us the age of the earth. Lord Kevin was the first to make this calculation.

Kevin assumed that the earth originated as a molten sphere. Let's take the initial temperature to be uniformly 2000°C. After a short initial period, the surface temperature, as discussed earlier in the course, is set by a balance between the energy from the sun that Earth absorbs and the energy that Earth emits. For simplicity, we will take the surface temperature to be a constant of 0°C. Kevin assumed that Earth cools through diffusion. He had knowledge of the thermal diffusivity of rocks, which we will take to be $1.2 \times 10^{-6} m^2/s$, and the temperature gradient near the surface through measurements in the mines, which gives the value of about 25°C per kilometer. Instead of a sphere, we will further simplify the problem and consider the problem in one dimension, i.e. ignore the curvature associated with the sphere (After you have solved the problem, come back and revisit this simplification. Is it reasonable?).

Your task is to reproduce Lord Kevin's estimate of the age of the earth.

1. Solve the problem numerically.
    a. You should start by considering the domain over which you will solve the equation and the boundary conditions. One domain choice is $0<x<L$, where L is large enough (deep enough inside Earth) so that temperature is not changing significantly (which needs to be checked after the solution is obtained). What should the initial and boundary conditions be? Note that your computational domain shouldn't include the surface, as the temperature at the surface is known at all times. Instead, your computational domain should start one grid point below the surface. This is what is assumed in the code that is provided to you as a starting point (see part b).
    b. Decide how many grid points to use to represent your domain and solve the problem numerically (you can use the code heat_equation_for_student.m as a starting point).
    c. Pay special attention to making the numerical values and the units of space, time, and diffusivity consistent.
    d. Find the time when the temperature gradient drops below 25°C per kilometer. That's the age of Earth determined by Kelvin.
    e. Change the number of grid points that you use to see if your answer changes (i.e. check numerical convergence).
    f. Is the one-dimension approximation of the sphere reasonable?

2. Check the solution using the analytical solution that we shall present in class (also on Canvas, see "Analytical solution to the Age of Earth problem.pdf"). Note that the numerical solution, while approximate, is more versatile.
3. (Optional) We have assumed that the surface temperature is set by a balance between the energy from the sun that Earth absorbs and the energy that Earth emits. This only holds after a short initial period, during which Earth is losing much more heat than it is receiving from the Sun. One can model this by setting a flux boundary condition at the surface: the flux leaving Earth is the blackbody thermal radiation loss minus the absorbed solar energy. You could give that a try!