EF4821 Group Project

European Swaption pricing under the Black-Karasinski and the Hull-White one-factor model

1 Introduction

The financial industry changed dramatically due to the collapse of the credit market in the 2008 financial crisis by the fall of Lehman-Brothers. The critical factor in the change is the interest rate. Before the crisis, conventionally, people believed that the interest rate would never go below zero. However, due to the drastic fall of the market, central banks worldwide start to lower the interest rate. In 2012, Denmark's National bank proposed a negative interest rate. Afterward, the situation has widened and spread around the world. Lopez and Röman (2016) suggest that term structure models assuming a log-normal distribution are no longer "work adjusted to reality" anymore because of below-zero interest rates. They also stated that Gaussian models such as the Hull-White model of Vasicek extension could properly price interest rate derivatives based on its normal distribution reliance. This project examines two models – Black-Karasinski and Hull-White 1-Factore model- on pricing European swaptions. The former model is based on the log-normal distribution, while the latter is on the normal distribution. We calibrated each model on market swaption data and compared the results for pricing and volatility calculations.

The remainder of the report is organized as follows. Section 2 reviews the two models. Section 3 states the data sources and formulates the calibration. Section 4 presents the calibration result and comparison. Section 5 concludes the whole project.

2 Model

2.1 Black-Karasinski model

The Black–Karasinski model is a mathematical model of the term structure of interest rates. It is a one-factor model as it describes interest rate movements as driven by a single source of randomness. The dependence of the drift on the node can be expressed as a linear dependence on the logarithm of the one-period rate:

$$\Delta \log r(t_i) = \kappa(t_i)[\theta(t_i) - \log r(t_{i-1})]\,\Delta t \pm \sigma(t_i)\sqrt{\Delta t}$$

where "+" and "-" with 1/2 (risk-neutral probability) and the mean-reversion parameter κ, mean θ, and volatility σ are all time-varying.
It allows the length of time steps to vary in order to remove the linkage between the mean-reversion parameter and the volatility. Denote the length of the time-step $t_i - t_{i-1}$ by T_i for i ≥ 2 according to the formula below. ($T_i$can be chosen arbitrarily when i = 1)

$$\tau_i = \frac{\left[\sqrt{\sigma(t_i)^2 + 4\kappa(t_i)\sigma(t_{i-1})^2\sqrt{\tau_{i-1}}} - \sigma(t_i)\right]^2}{4\kappa(t_i)^2\sigma(t_{i-1})^2\tau_{i-1}}$$

For swaption pricing, unlike the Hull-White model we will state in the next part, the Black-Karasinski model cannot yield analytical formulas for discount bonds and interest rate options since it does not have a closed-form solution. Therefore, pricing has to be performed through the numerical procedure. The following formula is a second-order expression for the swaption PV, which is obtained by applying the Green's function solution, according to Horvath (2017).

$$
\begin{aligned}
V_{\text{Payer}} = &\sum_{i=1}^{n}(D(0,t_{i-1}) - \kappa^{-1}D(0,t_i))N(-d_1(\xi^*,0,t_0)) \\
&+\sum_{i=1}^{n}\kappa^{-1}D(0,t_i)\int_{t_0}^{t_i}\overline{r}(u)\left(N\left(-d_2(\xi^*,0,u,t_0)\right) - N(-d_1(\xi^*,0,t_0))\right)du \\
&-\sum_{i=1}^{n}(D(0,t_{i-1}) - \kappa^{-1}D(0,t_i))\int_{0}^{t_0}\overline{r}(u)\left(N(-d_2(\xi^*,0,u,t_0)) - N(-d_1(\xi^*,0,t_0))\right)du \\
&-\sum_{i=1}^{n}\kappa^{-1}D(0,t_i)\int_{t_0}^{t_i}\overline{r}(v)\int_{0}^{v}\overline{r}(u)e^{\phi(u,v)I_r(u)}\left(N(-d_2^*(\xi^*,0,u,v,t_0)) - N(-d_2(\xi^*,0,v,t_0))\right)dudv \\
&+\sum_{i=1}^{n}\kappa^{-1}D(0,t_i)\int_{t_0}^{t_i}\overline{r}(v)\int_{0}^{v}\overline{r}(u)(N(-d_1^*(\xi^*,0,u,v,t_0)) - N(-d_2(\xi^*,0,v,t_0)) \\
&\qquad + N(-d_2(\xi^*,0,u,t_0)) - N(-d_1(\xi^*,0,t_0)))\,dudv + \mathcal{O}(\epsilon^3)
\end{aligned}
$$

2.2 Hull-White 1-Factor model

The Hull-White model is a single-factor interest model proposed by Hull and White (1990) as an extension of the Vasicek model. It assumes that short rates have a normal distribution and consistent with the current term structure of interest-rate volatilities. Consequently, forward interest rates can be negative with positive probability, just like the Vasicek model. The instantaneous short-rate process evolves under the risk-neutral measure as follows:

$$dr(t) = \theta(t)dt - a(t)r(t)dt + \sigma(t)dB(t)$$

where all parameters are time dependent.

For swaptions pricing, it can be calculated directly by replicating the payoffs of the swaption through a portfolio of zero-coupon bonds. The bond prices at time t in the Hull-White model are given by:

$$
\begin{aligned}
P(t,T) &= A(t,T)e^{-B(t,T)r(t)} \\
A(t,T) &= \frac{P(0,T)}{P(0,t)}\exp\left(B(t,T)F(0,t) - \frac{\sigma^2}{4a}B(t,T)^2(1-e^{-2at})\right) \\
B(t,T) &= \frac{1-e^{-a(T-t)}}{a}
\end{aligned}
$$

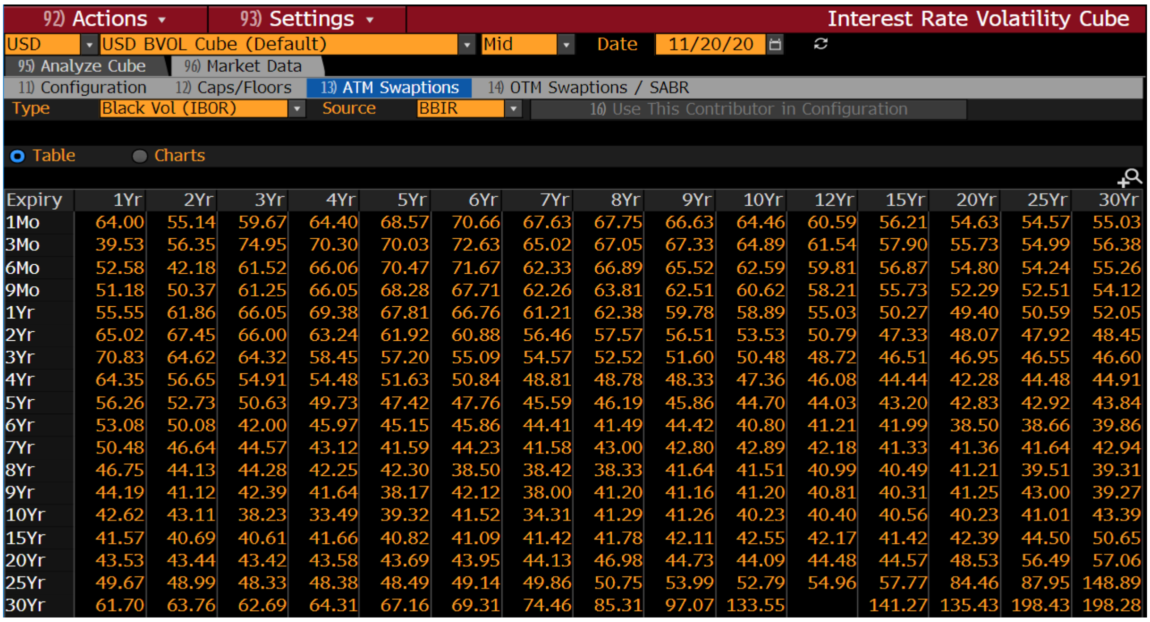
The following procedure can be used to price the swaption: 1) Determining the critical value of the yield r for which the price of the coupon-bearing bond equals the strike price of the option on the bond at option maturity; 2) Calculating the prices of options on the zero-coupon bonds that replicate the coupon-bearing bond. 3) Set the strike price of each option equal to the value the corresponding zero-coupon bond will have at time T at the critical value found before; Finally, the swaption can be obtained by summing all the option prices.

3 Calibration

Calibration is the process of determining specific model parameters to fit the market price of calibration instruments. For example, the α and σin the two models are stated in the previous section. In this section, we will walk through our chosen instruments and process for calibrating models.

3.1 Data

In this project, we used the European-type US swaption data quoted in Black Vol as our calibration instruments. The data is extracted from Bloomberg Interest Rate Volatility Cube on 20 November 2020, as shown in Figure 1. The underlying index or the floating rate for the swaption is the US 6-month LIBOR. The column of the swaption matrix represents different swaption expires, and the rows are of various maturities of the underlying swap. We chose the maturities and tenors that range from 1 year to 10 years for a more precise presentation of the result.



Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	6Yr	7Yr	8Yr	9Yr	10Yr	12Yr	15Yr	20Yr	25Yr	30Yr
1Mo	64.00	55.14	59.67	64.40	68.57	70.66	67.63	67.75	66.63	64.46	60.59	56.21	54.63	54.57	55.03
3Mo	39.53	56.35	74.95	70.30	70.03	72.63	65.02	67.05	67.33	64.89	61.54	57.90	55.73	54.99	56.38
6Mo	52.58	42.18	61.52	66.06	70.47	71.67	62.33	66.89	65.52	62.59	59.81	56.87	54.80	54.24	55.26
9Mo	51.18	50.37	61.25	66.05	68.28	67.71	62.26	63.81	62.51	60.62	58.21	55.73	52.29	52.51	54.12
1Yr	55.55	61.86	66.05	69.38	67.81	66.76	61.21	62.38	59.78	58.89	55.03	50.27	49.40	50.59	52.05
2Yr	65.02	67.45	66.00	63.24	61.92	60.88	56.46	57.57	56.51	53.53	50.79	47.33	48.07	47.92	48.45
3Yr	70.83	64.62	64.32	58.45	57.20	55.09	54.57	52.52	51.60	50.48	48.72	46.51	46.95	46.55	46.60
4Yr	64.35	56.65	54.91	54.48	51.63	50.84	48.81	48.78	48.33	47.36	46.08	44.44	42.28	44.48	44.91
5Yr	56.26	52.73	50.63	49.73	47.42	47.76	45.59	46.19	45.86	44.70	44.03	43.20	42.83	42.92	43.84
6Yr	53.08	50.08	42.00	45.97	45.15	45.86	44.41	41.49	44.42	40.80	41.21	41.99	38.50	38.66	39.86
7Yr	50.48	46.64	44.57	43.12	41.59	44.23	41.58	43.00	42.80	42.89	42.18	41.33	41.36	41.64	42.94
8Yr	46.75	44.13	44.28	42.25	42.30	38.50	38.42	38.33	41.64	41.51	40.99	40.49	41.21	39.51	39.31
9Yr	44.19	41.12	42.39	41.64	38.17	42.12	38.00	41.20	41.16	41.20	40.81	40.31	41.25	43.00	39.27
10Yr	42.62	43.11	38.23	33.49	39.32	41.52	34.31	41.29	41.26	40.23	40.40	40.56	40.23	41.01	43.39
15Yr	41.57	40.69	40.61	41.66	40.82	41.09	41.42	41.78	42.11	42.55	42.17	41.42	42.39	44.50	50.65
20Yr	43.53	43.44	43.42	43.58	43.69	43.95	44.13	46.98	44.73	44.09	44.48	44.57	48.53	56.49	57.06
25Yr	49.67	48.99	48.33	48.38	48.49	49.14	49.86	50.75	53.99	52.79	54.96	57.77	84.46	87.95	148.89
30Yr	61.70	63.76	62.69	64.31	67.16	69.31	74.46	85.31	97.07	133.55		141.27	135.43	198.43	198.28

Figure 1. The capture of Bloomberg Interest Rate Volatility Cube

Additional to the market swaption volatility data, the US instantaneous forward rate of different maturities has been extracted from Quandl. These rates are used to build the term structure of the model, and it is being used to calibrate the $\theta(t)$ parameter for both models.

3.2 Calibration process

We have utilized the QuantLib API for python to do our calibration, and the process mainly follows the flowchart in Figure 2 (Tilgenkamp, 2014). First, we use the extracted implied volatilities to obtain the market price of the swaption. Since the data is quoted in Black-Vol, it allows us to revert the price from Black's Formula. After setting up the market price, we start to utilize it as an input of the two model equations for swaption pricing. In this process, we try to find the best parameter values that optimize the objective functions we set up. Next, using the optimal parameter values as the model input to price the swaptions. Finally, the model prices of the swaptions are used to calculate the implied volatility of the model price.

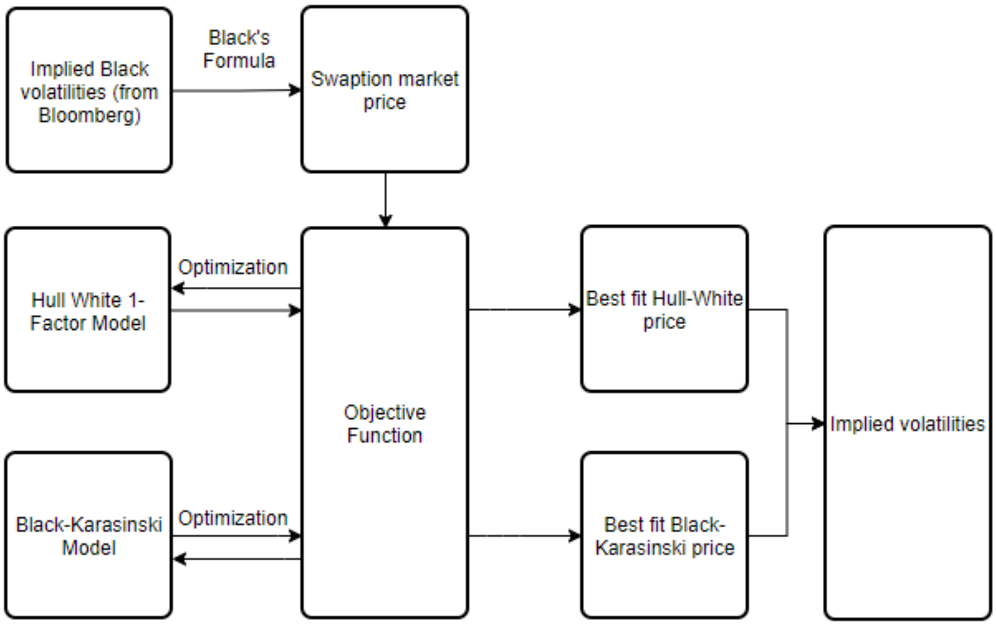


Figure 2. Calibration Process

3.2.1 Objective function and Optimization

The objective function is the way to access the goodness of fit for the model. The typical approach used in a lot of research is to minimize the sum of squared residuals. As we utilized the QuantLib API, the method we adapted is the Levenberg-Marquardt algorithm. The algorithm formulates the optimization for non-linear least squares and is used widely for curve fitting problems.

The general formula of the algorithm adjusted to our problem is given below:

$$\underset{a,\sigma}{\operatorname{argmin}} \sum_{i=1}^{n} \left[P_i^{mkt} - P_i^{model}(a,\sigma)\right]^2$$

where n suggests the number of swaption data we have, in our case it equals to 100; P_i^{mkt} represents the market price obtained from quotes; $P_i^{model}(a,\sigma)$ is the model price derived from a various set of α and σ, the derivation is based on the swaption pricing formula for each model stated in the previous section.

3.2.2 Calibration settings

In order to obtain a fair result, we set the maximum iterations of calibration to be 100,000. For different models, we used different pricing engines. As for Black-Karasinski, the classic tree pricing engine was implemented. On the other hand, we deployed the Finite-Difference pricing engine for the Hull-White 1-Factor model. The details of the method implementation can be found in Lee & Yang (2020).

4 Results and Comparison

4.1 Black Karasinski

For the Black-Karasinski model, the optimal parameters are shown below.

$\hat{a}$	$\hat{\sigma}$
0.04509	0.80301

Using the best-fit parameters, we plot two surfaces, 1) the surface for model price and market price; 2) the surface for implied volatilities derived from the model and market volatilities.

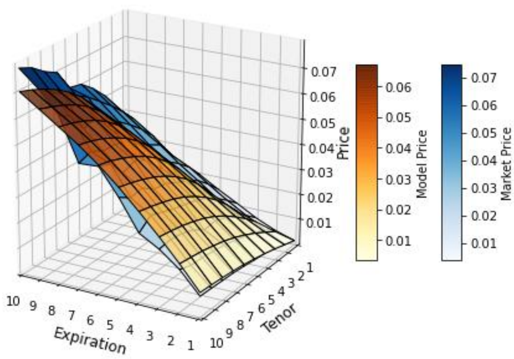


Figure 3. BK model pricing surface plot

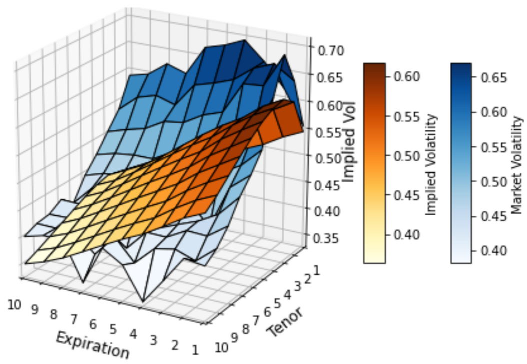


Figure 4. BK model volatility surface plot

As shown in the above two plots, the surface of model price and implied volatilities are smooth while that of market data are scratchy. The color range of both plots suggests that the model's overall behavior does not diverge significantly with market data. However, the volatility plot suggests an apparent deviation. For the pricing plot, when swaption maturities are small (e.g., expiration = 1), the difference between the model price and the market price seems insignificant. However, with the increasing maturities, the price surface deviates more from the market.

4.2 Hull-White 1-Factor

For the Hull-White 1-Factor model, the optimal parameters are shown below.

$\hat{a}$	$\hat{\sigma}$
0.01145	0.00763

We plotted the same two surfaces for the model, which are shown below.

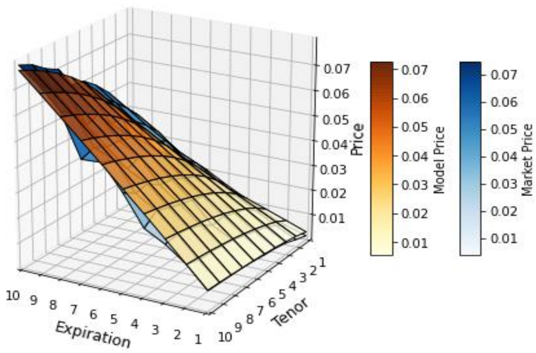


Figure 5. HW1F model pricing surface plot

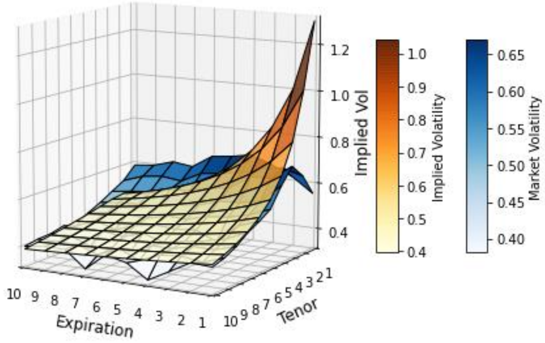


Figure 6. HW1F model volatility surface plot

The above plots have also shown the smooth surfaces of model price and implied volatilities while market data surfaces remain scratchy. Unlike the BK model comparison, both plots for the HW1F model suggest an insignificant deviation, especially for the pricing. For the implied volatility plot, when swaption maturities are large (e.g. expiration = 10), the difference between the model implied volatility, and that comes from the market seems indifferent. However, with the smaller maturities and tenor, the implied volatility surface deviates more from the market. For the pricing plot, a similar conclusion can be drawn as the one for the BK model. the price surface deviates more from the market as the expiration for the swaption increases.

4.3 Comparison

From the calibrated models, we calculated the sum of squared errors for pricing and volatility calculations. The result is shown in the below table.

Sum of Squared error

Model	Price error	Volatility error
Black Karasinski	0.00192	0.69470
Hull-White 1-Factor	0.00052	1.05672

We can conclude from the data that the Black Karasinski model performs better in modeling the volatility while the Hull-White 1-Factor model performs better in price modeling. In Figure 7 and 8, we plotted the relative differences between the model prices and market prices. The difference is calculated as $\frac{P^{model}-P^{mkt}}{P^{mkt}}$. As you can see, the BK model has an absolute deviation of around 20-30% when the underlying swap tenor is small or large. On the other hand, the HW 1-Factor model is relatively stable except when both swaption expiration and underlying swap maturities are small. The possible explanation for this behavior is that the prices for the short-term option are small. Therefore, a tiny difference between the prices can cause a comparatively sizeable relative price difference.

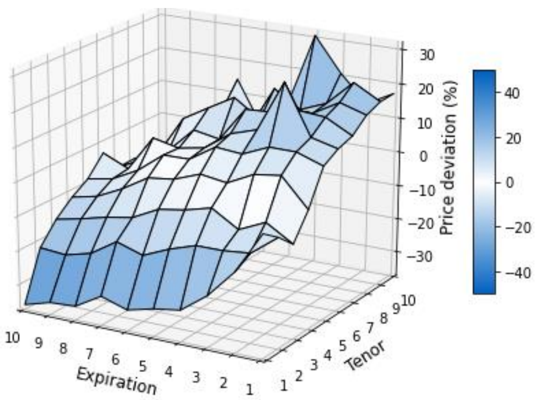


Figure 7. BK model relative pricing difference

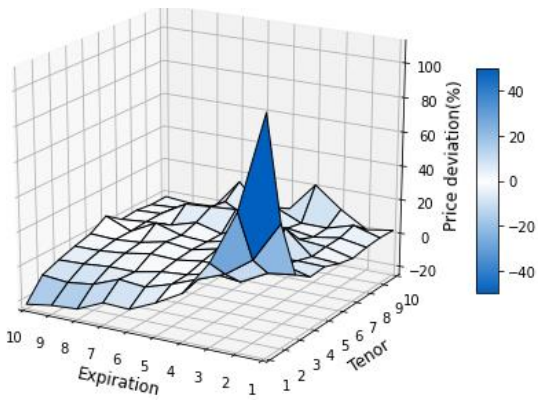


Figure 8. HW1F model relative pricing difference

5 Conclusion

We applied two term-structure models in this project - the Black-Karasinski model and the Hull-White 1-Factor model, on pricing the European swaption. We have calibrated the models by using market swaption volatilities data. From our calibration result, it can be concluded that the Hull-White 1-Factor model fits the market prices better than the Black-Karasinski model. Although there are no negative rates in our selected data, the difference between the two models may be explained by the contrast of the distribution assumption. Furthermore, both models' performance is not significantly different, and they all cannot price the swaption very accurately. Therefore, we suggest future works for adding more factors to explain the randomness of the model parameters, for example, the Hull-White 2-Factor model and G2 model.

References

Horvath, B., A. Jacquier, C. Turfus (2017) 'Analytic Option Prices for the Black-Karasinski Short Rate Model' https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3253833

Tilgenkamp, W. G. (2014). Swaption pricing under the hull-white one factor model. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.897.8509&rep=rep1&type=pdf

Lopez, V. L. (2016). Swaption pricing under the single Hull White model through the analytical formula and Finite Difference Methods (*Doctoral dissertation, Mälardalen University*). Retrieved from http://urn.kb.se/resolve?urn=urn:nbn:se:mdh:diva-32332

Yongwoong Lee, & Kisung Yang. (2020). Finite Difference Method for the Hull–White Partial Differential Equations. *Mathematics (Basel)*, *8*(1719), 1719. https://doi.org/10.3390/math8101719

Appendix

Appendix I. Market Price and Market Volatilities

Market Prices

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	0.00143	0.00389	0.00727	0.0115	0.0156	0.02007	0.02312	0.02847	0.03223	0.03673
2	0.00289	0.00685	0.0112	0.01566	0.02063	0.02584	0.02952	0.03579	0.04091	0.04448
3	0.00457	0.00922	0.01481	0.01931	0.02489	0.03009	0.03606	0.04095	0.04644	0.05162
4	0.00549	0.0105	0.01617	0.02238	0.02761	0.03364	0.03879	0.04524	0.05136	0.0569
5	0.006	0.01189	0.01791	0.0242	0.02973	0.0367	0.04181	0.04907	0.05559	0.06099
6	0.00669	0.01317	0.01734	0.0256	0.0321	0.03966	0.04555	0.04956	0.05979	0.06203
7	0.00721	0.01381	0.02036	0.02677	0.03284	0.04208	0.04693	0.05575	0.06286	0.07034
8	0.00745	0.01448	0.02205	0.02856	0.03609	0.04018	0.04717	0.05407	0.06574	0.07313
9	0.00773	0.01466	0.02281	0.03021	0.03524	0.04642	0.04971	0.0612	0.06901	0.07696
10	0.00799	0.01626	0.02216	0.02647	0.03829	0.04835	0.04789	0.06454	0.07278	0.07921

Market Volatilities

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	0.5555	0.6186	0.6605	0.6938	0.6781	0.6676	0.6121	0.6238	0.5978	0.5889
2	0.6502	0.6745	0.66	0.6324	0.6192	0.6088	0.5646	0.5757	0.5651	0.5353
3	0.7083	0.6462	0.6432	0.5845	0.572	0.5509	0.5457	0.5252	0.516	0.5048
4	0.6435	0.5665	0.5491	0.5448	0.5163	0.5084	0.4881	0.4878	0.4833	0.4736
5	0.5626	0.5273	0.5063	0.4973	0.4742	0.4776	0.4559	0.4619	0.4586	0.447
6	0.5308	0.5008	0.42	0.4597	0.4515	0.4586	0.4441	0.4149	0.4442	0.408
7	0.5048	0.4664	0.4457	0.4312	0.4159	0.4423	0.4158	0.43	0.428	0.4289
8	0.4675	0.4413	0.4428	0.4225	0.423	0.385	0.3842	0.3833	0.4164	0.4151
9	0.4419	0.4112	0.4239	0.4164	0.3817	0.4212	0.38	0.412	0.4116	0.412
10	0.4262	0.4311	0.3823	0.3349	0.3932	0.4152	0.3431	0.4129	0.4126	0.4023

Appendix II. Calibrated Black-Karasinski model

Calibrated Black-Karasinski model prices

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	0.0014	0.00335	0.00565	0.00829	0.01106	0.01383	0.0165	0.01905	0.0214	0.02358
2	0.00274	0.00608	0.00985	0.01377	0.01764	0.02124	0.02499	0.02856	0.03185	0.03492
3	0.00412	0.00875	0.01355	0.01859	0.02346	0.02799	0.03217	0.03599	0.03995	0.04359
4	0.00535	0.01111	0.01694	0.02261	0.02821	0.03342	0.03817	0.04245	0.04634	0.05042
5	0.00645	0.0131	0.01974	0.02589	0.03202	0.03775	0.04296	0.04758	0.05175	0.05594
6	0.00736	0.01476	0.02195	0.02861	0.03509	0.04121	0.0467	0.05165	0.05607	0.06028
7	0.00805	0.01598	0.02369	0.03069	0.03752	0.04386	0.04958	0.05479	0.0593	0.06372
8	0.00859	0.01705	0.02501	0.03229	0.03941	0.04594	0.05192	0.05711	0.0617	0.06646
9	0.00906	0.01781	0.02605	0.03356	0.0409	0.04766	0.05359	0.05884	0.06362	0.06863
10	0.00938	0.01842	0.02679	0.03464	0.0421	0.04882	0.05479	0.06007	0.06527	0.07039

Calibrated Black-Karasinski model prices VS Market prices (% difference)

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	-2.10%	-13.88%	-22.28%	-27.91%	-29.10%	-31.09%	-28.63%	-33.09%	-33.60%	-35.80%
2	-5.19%	-11.24%	-12.05%	-12.07%	-14.49%	-17.80%	-15.35%	-20.20%	-22.15%	-21.49%
3	-9.85%	-5.10%	-8.51%	-3.73%	-5.75%	-6.98%	-10.79%	-12.11%	-13.98%	-15.56%
4	-2.55%	5.81%	4.76%	1.03%	2.17%	-0.65%	-1.60%	-6.17%	-9.77%	-11.39%
5	7.50%	10.18%	10.22%	6.98%	7.70%	2.86%	2.75%	-3.04%	-6.91%	-8.28%
6	10.01%	12.07%	26.59%	11.76%	9.31%	3.91%	2.52%	4.22%	-6.22%	-2.82%
7	11.65%	15.71%	16.36%	14.64%	14.25%	4.23%	5.65%	-1.72%	-5.66%	-9.41%
8	15.30%	17.75%	13.42%	13.06%	9.20%	14.34%	10.07%	5.62%	-6.15%	-9.12%
9	17.21%	21.49%	14.20%	11.09%	16.06%	2.67%	7.81%	-3.86%	-7.81%	-10.82%
10	17.40%	13.28%	20.89%	30.87%	9.95%	0.97%	14.41%	-6.93%	-10.32%	-11.13%

Calibrated Black-Karasinski model implied volatilities

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	0.54496	0.52784	0.50617	0.49015	0.47125	0.45114	0.43013	0.4098	0.3903	0.37173
2	0.61204	0.5911	0.57326	0.54965	0.52215	0.49254	0.47244	0.45234	0.43293	0.41439
3	0.62834	0.60913	0.58113	0.55995	0.53557	0.50878	0.48167	0.45666	0.4386	0.4209
4	0.62439	0.60469	0.57926	0.55101	0.52892	0.50473	0.47946	0.45489	0.43202	0.41532
5	0.61312	0.59075	0.56683	0.53718	0.51566	0.49301	0.46993	0.44636	0.4238	0.40658
6	0.59634	0.57327	0.55049	0.52251	0.49997	0.47904	0.45676	0.4345	0.4135	0.39537
7	0.57695	0.55492	0.53265	0.50529	0.48448	0.46383	0.44245	0.42165	0.40091	0.38408
8	0.55662	0.53682	0.51443	0.4878	0.46859	0.44865	0.42834	0.40767	0.38755	0.37279
9	0.53618	0.51913	0.49684	0.47163	0.45362	0.43435	0.414	0.39381	0.37513	0.36179
10	0.5192	0.50225	0.47899	0.45696	0.43955	0.41996	0.39984	0.38	0.36406	0.35161

Calibrated Black-Karasinski model implied volatilities VS Market volatilities (% difference)

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	-1.90%	-14.67%	-23.37%	-29.35%	-30.50%	-32.42%	-29.73%	-34.31%	-34.71%	-36.88%
2	-5.87%	-12.36%	-13.14%	-13.09%	-15.67%	-19.10%	-16.32%	-21.43%	-23.39%	-22.59%
3	-11.29%	-5.74%	-9.65%	-4.20%	-6.37%	-7.65%	-11.73%	-13.05%	-15.00%	-16.62%
4	-2.97%	6.74%	5.49%	1.14%	2.44%	-0.72%	-1.77%	-6.75%	-10.61%	-12.31%
5	8.98%	12.03%	11.96%	8.02%	8.74%	3.23%	3.08%	-3.36%	-7.59%	-9.04%
6	12.35%	14.47%	31.07%	13.66%	10.74%	4.46%	2.85%	4.72%	-6.91%	-3.10%
7	14.29%	18.98%	19.51%	17.18%	16.49%	4.87%	6.41%	-1.94%	-6.33%	-10.45%
8	19.06%	21.65%	16.18%	15.46%	10.78%	16.53%	11.49%	6.36%	-6.93%	-10.19%
9	21.34%	26.25%	17.21%	13.26%	18.84%	3.12%	8.95%	-4.42%	-8.86%	-12.19%
10	21.82%	16.50%	25.29%	36.45%	11.79%	1.15%	16.54%	-7.97%	-11.76%	-12.60%

Appendix III. Calibrated Hull-White 1-Factor model

Calibrated Hull-White 1-Factor model prices

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	0.003	0.006	0.00893	0.01182	0.01469	0.01753	0.02034	0.02312	0.02585	0.02856
2	0.00421	0.00838	0.01248	0.01652	0.02054	0.0245	0.02842	0.03232	0.03613	0.03991
3	0.00511	0.01012	0.01507	0.01999	0.02484	0.02964	0.0344	0.03907	0.0437	0.04828
4	0.00578	0.01149	0.01716	0.02277	0.02827	0.03372	0.03912	0.04445	0.04971	0.05495
5	0.00636	0.01266	0.01891	0.02503	0.03108	0.0371	0.04303	0.04889	0.0547	0.06041
6	0.00686	0.01367	0.02033	0.02692	0.03348	0.03993	0.04631	0.05265	0.05887	0.06502
7	0.00725	0.01442	0.02154	0.02853	0.03545	0.04229	0.04905	0.05578	0.06236	0.06886
8	0.00759	0.01514	0.02254	0.02988	0.03713	0.04429	0.05143	0.0584	0.06529	0.07212
9	0.00793	0.01571	0.02341	0.03103	0.03856	0.04606	0.05339	0.06063	0.0678	0.07492
10	0.00815	0.01619	0.02414	0.03204	0.03977	0.04744	0.05502	0.06251	0.06997	0.07725

Calibrated Hull-White 1-Factor model prices VS Market prices (% difference)

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	109.79%	54.24%	22.83%	2.78%	-5.83%	-12.66%	-12.02%	-18.79%	-19.80%	-22.24%
2	45.67%	22.34%	11.43%	5.49%	-0.44%	-5.19%	-3.73%	-9.70%	-11.68%	-10.27%
3	11.82%	9.76%	1.76%	3.52%	-0.20%	-1.50%	-4.60%	-4.59%	-5.90%	-6.47%
4	5.28%	9.43%	6.12%	1.74%	2.39%	0.24%	0.85%	-1.75%	-3.21%	-3.43%
5	6.00%	6.48%	5.58%	3.43%	4.54%	1.09%	2.92%	-0.37%	-1.60%	-0.95%
6	2.54%	3.80%	17.24%	5.16%	4.30%	0.68%	1.67%	6.23%	-1.54%	4.82%
7	0.55%	4.42%	5.80%	6.57%	7.95%	0.50%	4.52%	0.05%	-0.80%	-2.10%
8	1.88%	4.56%	2.22%	4.62%	2.88%	10.23%	9.03%	8.01%	-0.68%	-1.38%
9	2.59%	7.16%	2.63%	2.71%	9.42%	-0.78%	7.40%	-0.93%	-1.75%	-2.65%
10	2.00%	-0.43%	8.94%	21.04%	3.87%	-1.88%	14.89%	-3.15%	-3.86%	-2.47%

Calibrated Hull-White 1-Factor model implied volatilities

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	1.3013	1.00103	0.82773	0.71446	0.63545	0.57787	0.53462	0.50089	0.47431	0.45269
2	1.01571	0.85202	0.74581	0.67116	0.61606	0.5744	0.54187	0.51572	0.49458	0.47695
3	0.81138	0.72075	0.65594	0.60762	0.57069	0.54161	0.51807	0.49901	0.48304	0.46949
4	0.68479	0.62948	0.58786	0.55555	0.53021	0.50971	0.49267	0.47841	0.46632	0.45582
5	0.60305	0.56704	0.53888	0.51667	0.49854	0.48342	0.47077	0.45993	0.45046	0.44229
6	0.5474	0.52271	0.50308	0.48697	0.47354	0.46219	0.45242	0.44393	0.4365	0.42985
7	0.50796	0.49033	0.47583	0.46383	0.45356	0.44478	0.43707	0.4302	0.42419	0.41874
8	0.47825	0.46518	0.45432	0.44506	0.43703	0.42998	0.42371	0.41814	0.41313	0.40858
9	0.45534	0.44552	0.43705	0.42965	0.42321	0.41738	0.41224	0.40759	0.4033	0.39944
10	0.43687	0.42906	0.42229	0.41626	0.41094	0.40615	0.40175	0.39781	0.39412	0.3908

Calibrated Hull-White 1-Factor model implied volatilities VS Market volatilities (% difference)

Expiration/Tenor	1	2	3	4	5	6	7	8	9	10
1	134.26%	61.82%	25.32%	2.98%	-6.29%	-13.44%	-12.66%	-19.70%	-20.66%	-23.13%
2	56.22%	26.32%	13.00%	6.13%	-0.51%	-5.65%	-4.03%	-10.42%	-12.48%	-10.90%
3	14.55%	11.54%	1.98%	3.96%	-0.23%	-1.69%	-5.06%	-4.99%	-6.39%	-6.99%
4	6.42%	11.12%	7.06%	1.97%	2.69%	0.26%	0.94%	-1.92%	-3.51%	-3.75%
5	7.19%	7.54%	6.43%	3.90%	5.13%	1.22%	3.26%	-0.43%	-1.77%	-1.05%
6	3.13%	4.38%	19.78%	5.93%	4.88%	0.78%	1.87%	7.00%	-1.73%	5.36%
7	0.63%	5.13%	6.76%	7.57%	9.06%	0.56%	5.12%	0.05%	-0.89%	-2.37%
8	2.30%	5.41%	2.60%	5.34%	3.32%	11.68%	10.28%	9.09%	-0.79%	-1.57%
9	3.04%	8.35%	3.10%	3.18%	10.88%	-0.91%	8.48%	-1.07%	-2.02%	-3.05%
10	2.50%	-0.47%	10.46%	24.29%	4.51%	-2.18%	17.09%	-3.65%	-4.48%	-2.86%