



City University of Hong Kong
2019/20 Semester B
EF4822 Financial Econometrics
Project Report – Group 6

Investigating Co-movements and Volatility Transmission
among the six main sectors return in America:
the effect in different economic regime

Abstract

This paper presents an empirical analysis of the time-varying volatility transmission among the six stock market sectors in the US in different regimes using the Markov-Chain Regime Switching Model and Dynamic Conditional Correlation Model. The interdependence and volatility transmission have been examined in the study. We found that the shock transmission effect with defensive stock markets has relatively small magnitude while large in sensitive sectors. The empirical finding also shows a significant correlation change between sectors when states switching. Providing insights for volatility forecasting and risk management, the results further suggest the usefulness in cross-market hedging and optimal portfolio decisions.

Keywords: US stock market, US sector indexes, Volatility, crisis, GARCH model, DCC-GARCH model, Markov-Chain Regime Switching Model, dynamic hedge, Log-likelihood, AIC.

I. Introduction

As the world's largest economy, America has the most international, active, and interactive stock market. Nowadays, with the increasing interaction in trade and stock markets, the volatility transmission between different sectors has been deepened. The degree of volatility transmission between markets is critical for investors as the investment decisions may be affected by this degree since investors have the incentives to include the stocks that are not highly correlated into their portfolios to minimize the portfolio risk. Therefore, the volatility correlation between different markets seems to be significant. In this paper, our objective is to investigate the dynamic volatility transmission between different stock market sectors in America throughout an arbitrarily long financial period and to figure out the volatility correlation patterns in both recession and prosperity periods, respectively.

According to Bradley T(2002), stock indexes are used as benchmarks for individual investors, mutual fund managers, and institutional investors to track the performance of publicly traded stocks in each sector. Therefore, in order to track the dynamic volatility transmission between different markets, using the data of corresponding sector indexes is a wise choice for us to model and test the volatility correlation.

In order to approach the objective of this paper, our project will be conducted and divided into two parts. Firstly, the Markov-Chain Regime Switching model will be used to determine the financial recession periods. The data selected to be used in this step is the quarterly return of the S&P 500 index from Q1 2002 to Q1 2020. Secondly, we will use the monthly return of the indexes of 6 main markets in America during the period between 2002-01-01 to 2020-03-31, which is S&P Financials index, S&P Energy index, S&P Consumer Discretionary index, S&P Health Care index, S&P Information Technology index and S&P Real Estate index, to fit into the Dynamic Conditional Correlation Model (DCC-GARCH model), which will generate the dynamic correlation graphs between every 2 of the above indexes. Finally, by comparing the results of the DCC model with the financial recession period determined by the Markov-Chain Regime Switching model, we will be able to conclude whether the volatility transmission will be increased during the financial recession period compared with the prosperity period. Additionally, we can observe that how sensitive that each sector reacts with the changing of the financial regime, which can offer some investing implications for the public.

This paper does pay some contributes to the previous research context. Some papers investigate the volatility spillover effect among neighboring countries during the financial crisis, say Jawadi, F. & Idi Cheffou, A. (2015) investigate whether the financial crisis has an impact on the volatility spillover effect among the international stock markets. Some papers took research on the volatility transmission among the stock market of different sectors, say Bradley T. (2002) investigated how volatility in one market explains fluctuations in other markets. This paper will combine the idea of the two research directions and investigate whether financial recessions affect the degree of volatility transmission between different stock markets in America. Moreover, the result of our research will be useful for calculating the dynamic hedge ratio and risk-minimizing portfolio weights for every two sectors.

II. Literature review

There are many pieces of research estimated the co-movements and volatility transmission between different sectors and different countries. Within the literature, models commonly used for examining the effect are VAR, GARCH, etc. Ewing (2002) analysed the interrelationship among five major sectors, i.e., capital goods, financials, industrials, transportation, and utilities, using the

technique of generalized forecast error variance decomposition within the vector auto-regression (VAR) framework. He uses S&P stock indexes monthly data and found that shocks in one sector had significant impacts on other sector returns. Hassan and Malik (2007) examined the volatility transmission effect among different US sector indexes by employing MGARCH model. They found a notable volatility transmission among different sectors.

Engle and Sheppard (2001) developed DCC-GARCH model, which simplifies the estimation process of the multivariate conditional variance. The model fits the DCC model with a list of univariate GARCH models. By doing so, Engle finds that the model gives a fair estimation of the changing correlation. Righi & Ceretta (2012) examined the sector volatility transmission effect of Brazil stock indexes by using GARCH-DCC model. They found a significant bilateral volatility transmission between the financial and consumer sectors. For better volatility forecasting, many studies have attempted to combine the Markov chain with the DCC-GARCH model, for instance, Cai (1994), Hamilton (1994), and Gary (1996).

In this paper, we examine the volatility transmission from two perspectives. First, the effect among major US sector indexes by employing Dynamic Conditional Correlation GARCH models on sector index daily returns. Second, the effect in different regimes using the Markov-Chain Regime Switching model of Hamilton (1989) to estimate the period of regime-switching. The experimental procedure for the first perspective is similar to that of Ewing (2002) and Hassan and Malik (2007), with the model changed to DCC-GARCH.

III. Methodology

Model specification

In this study, there are two main models being employed to conduct the proper research.

Markov-Chain Regime Switching Model

The widely applied Markov-Chain Regime Switching Model, illustrated by Hamilton (1994) can capture the various dynamic patterns by switching to different structures. It can characterize the time series behaviors in multiple regimes. Therefore, to identify the different periods of various states of an economy, we consider it as a suitable model. Here we use the simple two-stage Regime Switching Model. It can be defined as follows,

$$X_t = \begin{cases} a_0 + \sum_{j=1}^p a_j X_{t-j} + u_{1t} & \text{if } S_t = 1 \\ b_0 + \sum_{j=1}^p b_j X_{t-j} + u_{2t} & \text{if } S_t = 2 \end{cases} \quad \{u_{1t}\} \sim \text{i.i.d } (0, \sigma_1^2), \{u_{2t}\} \sim \text{i.i.d } (0, \sigma_2^2)$$

where unobservable state variable S_t assumes values in $\{1, 2\}$ and it is a first-order Markov chain with transition probabilities

$$P(S_t = 2 | S_{t-1} = 1) = P_{12}, \quad P(S_t = 1 | S_{t-1} = 2) = P_{21}$$

Dynamic Conditional Correlation Model (DCC-GARCH model)

To further illustrate the transmission effect among the six sectors in US stock market, the proper model should be employed to demonstrate the co-movement behaviors and relationships. In the financial

market, there are many evidences showing that the integration of stock market has been changing over the time (Graham, Kiviahio, & Nikkinen, 2012); thereby the alternative dynamic conditional correlation (DCC) models, where the conditional correlation is allowed to vary over time, was proposed by Tse, Tsui (2002) and Engle (2002) to address the unrealistic constant conditional correlation assumption problem. Also, in practice, the multivariate model helps make better decisions in many subjects such as hedging, Value-at-Risk forecasts as well as portfolio selection. The DCC model by Engle set up can be expressed in the following manner:

$$\begin{cases} r_t = \mu_t + a_t \\ \alpha_t = H_t^{1/2} z_t \\ H_t = D_t R_t D_t \end{cases}$$

$$z_t \sim i.i.d \ E(z_t) = 0, E(z_t z_t^T) = I$$

(r_t is $n \times 1$ vector of log-returns of n assets at time t)

The conditional covariance matrix is decomposed into conditional standard deviations and a correlation matrix as:

$$H_t = D_t R_t D_t$$

where $D_t = \begin{pmatrix} \sqrt{h_{1t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{h_{nt}} \end{pmatrix}$ is the conditional standard deviation, the elements in the diagonal matrix D_t are standard deviations from univariate GARCH models:

$$h_{it} = \alpha_{i0} + \sum_{j=1}^{q_i} \beta_j h_{t-j} + \sum_{j=1}^{p_i} \alpha_j a_{t-j}^2$$

Often the simplest model, GARCH (1,1), is adequate.

R_t is the correlation matrix $\begin{pmatrix} 1 & \cdots & \rho_{n1,t} \\ \vdots & 1 & \vdots \\ \rho_{1n,t} & \cdots & 1 \end{pmatrix}$. R_t is decomposed into:

$$R_t = (diag Q_t)^{-1/2} Q_t (diag Q_t)^{-1/2}$$

$$Q_t = (1 - a - b) \bar{Q} + a \epsilon_{t-1} \epsilon_{t-1}^T + b Q_{t-1}$$

where

1. $\bar{Q} = Cov [\epsilon_t \epsilon_t^T] = E [\epsilon_t \epsilon_t^T]$ is the unconditional covariance matrix of the standardized errors ϵ_t .
2. a and b are non-negative scalar parameters satisfying $a + b < 1$. The value of $a + b$ close to one indicates high persistence in the conditional variance.
 - a. a represents the impact of last shocks on a current conditional correlation
 - b. b captures the impact of the past correlation.

Thus, the elements of H_t is $[H_t]_{ij} = \sqrt{h_{it} h_{jt}} \rho_{ij,t}$, where $\rho_{ii,t} = 1$. Both D_t and R_t are designed to be time-varying. In this study, we only consider the DCC (1,1)-GARCH model.

Model fitting

Data pre-processing

Before we apply the model to our dataset, some pre-processing procedure needs to be conducted. S&P 500 index and six indices daily last price data are obtained from Bloomberg. The collection period is from 2002-01-01 to 2020-03-31. In our analysis, we specifically examine the following data: Financials (FI), Utilities (UI), Real Estate (RE), Consumer Discretionary (CD), Health Care (HC), and Information Technology (IT).

Statistics	Consumer Discretionary	Utilities	Financials	Health Care	Information Technology	Real Estate	S&P500
Minimum	-0.120815	-0.115430	-0.170050	-0.099931	-0.139149	-0.184723	-0.225582
Maximum	0.131033	0.135235	0.187693	0.124266	0.121436	0.207450	0.152218
Mean	0.000337	0.000212	0.000176	0.000267	0.000402	0.000333	0.014721
Median	0.000357	0.000430	0.000000	0.000271	0.000541	0.000387	0.024661
Variance	0.000175	0.000147	0.000363	0.000120	0.000221	0.000380	0.006567
Stdev	0.013226	0.012127	0.019061	0.010952	0.014859	0.019489	0.081035
Skewness	-0.072740	0.278015	0.351283	-0.014949	0.122543	0.431554	-0.873018
Kurtosis	9.727614	17.899596	18.543037	10.697888	8.513272	20.48354	0.660961
ADF(lag20)	15.417(0.01)	-15.261(0.01)	-15.385(0.01)	-15.99(0.01)	-15.531(0.01)	15.171(0.01)	2.7059(0.2879)

Table 1: Descriptive statistics for each sectors' daily return

Markov-Chain Regime Switching Model

We apply the model to the S&P500 index to estimate the period of appreciation regime and depreciation regime. From the table below, comparing the appropriateness of the various estimated two-state Markov switching models [AR(1)-AR(6)], the lagged-four MS model is selected as it has the lowest log-likelihood and highest Akaike information criteria.

Number of states	Number of lags for AR model	Log likelihood	AIC
2	1	95.39446	-182.7889
2	2	97.22831	-182.4566
2	3	98.73009	-181.4602
2	4	102.1832	-184.3663
2	5	101.1435	-178.287

Table 2: MS models

The model MS (2) – AR (4) was then tested for serial correlation by the Q-Q plot and ACF/PACF residuals charts (Appendix 5), which shows the model is somehow adequate.

DCC model

We start the DCC model procedure by testing for unit roots using Augmented Dickey Fuller (ADF) in each stock index (Table 1). The result suggests that all simple returns are stationary.

The Daily simple returns in chart 1 demonstrate the volatility clustering character (i.e. periods with high volatility and periods with low volatility), which indicates that a GARCH model can be used to fit the data. Higher volatility is shown in 2002, and 2007–2009 and especially for the Financials, Real estate, and IT sectors.

Then we further analyze the data by plotting the Auto Correlation Function (ACF) for return and the square of return (Chart 2, take the Financials sector as an example). Few significant linear dependencies can be observed from the first graph, but strong linear dependence can be seen from the

second graph, which indicates the strong volatility clustering effect. This can be confirmed by the Ljung-Box test, which rejects the null hypothesis of serial independence for each return as well. Hence, a GARCH model is a good choice of modeling.

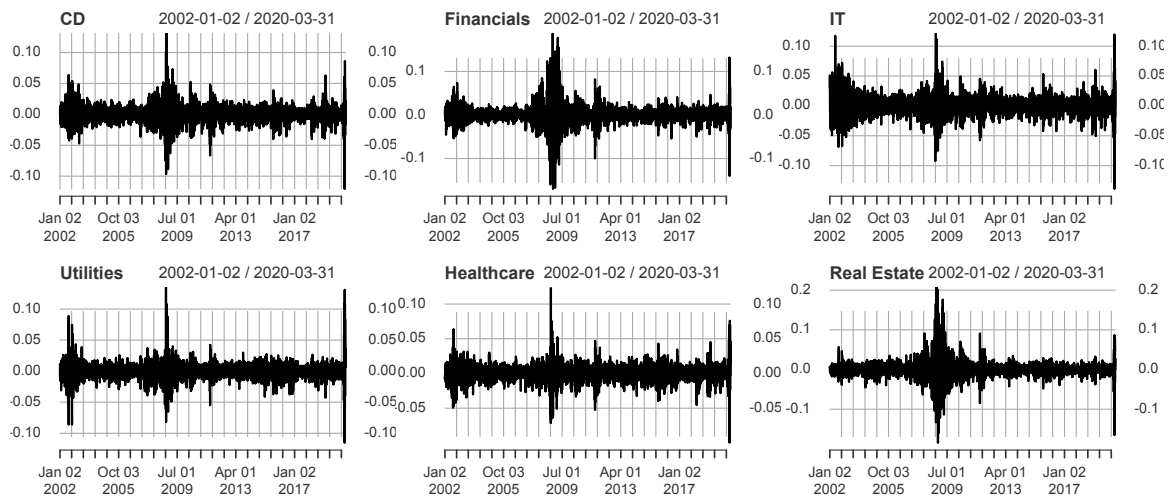


Chart 1: Daily simple returns plots

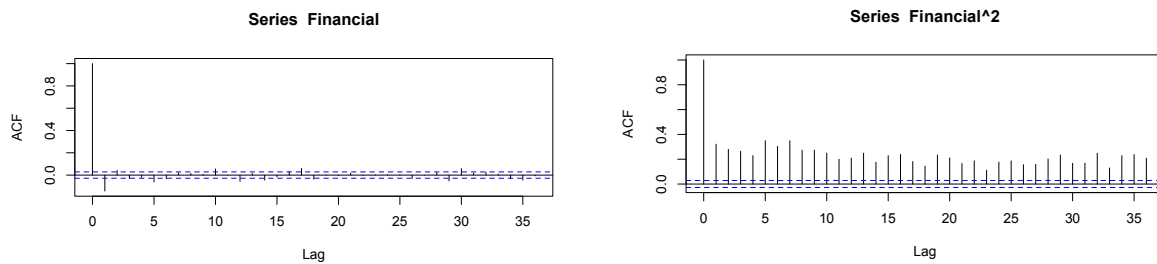


Chart 2: ACF for r and r^2 for the financials sector

To better estimate the return in different sectors, the mean evolution part parameters are selected based on the Schwarz-Bayesian information criterion (BIC) to reduce the model complicity. Take the Financials sector as an example; the ARMA (0,1) model was chosen as the Mean model, together with the univariate GARCH (1,1) model. The same model specification process is employed for all other indices. A DCC (1,1)-GARCH model with Gaussian distributed errors is then be specified for us further to identify the transmission or co-movement effect among the markets.

The optimal parameters of the fitted model are captures in appendix 15. α_1 and β_1 for each sector index data suggest the suitability of using GARCH (1, 1) for the particular series. As α_1 and β_1 for all series have a P-value less than 0.05 and an absolute T-value larger than 1.96, it can be concluded that GARCH (1, 1) is sufficient. By looking into the p-value and t-value of the joint DCC, the suitability of applying a DCC (1, 1) model can be estimated. As shown in the table that all values are significant for joint $dcca_1$ and joint $dccb_1$, the DCC (1, 1) model is sufficient to model the series.

IV. Result

Regime and correlation

Applying the MS (2) – AR (4) model, we generated the smoothed probabilities charts, which clearly show the likelihood in each period for each region. The mean in the two regimes are $\mu_1 = -0.1155$ and $\mu_2 = 0.0658$. Thus, Regime 1 in Chart 3 represents the depreciation state, while Regime 2 suggests the appreciation states.

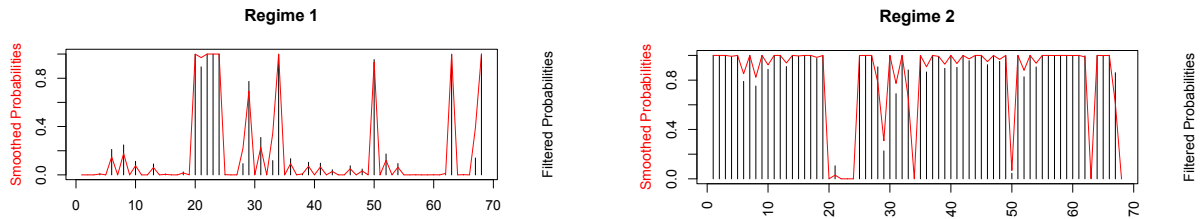


Chart 3: Graphical Representation of The Regime Probabilities

Regime 1 [Depreciation]		Regime 2 [Appreciation]	
Period	Average Probability	Period	Average Probability
2008 – 2009 (Q2)	0.829	2003 - 2007	0.975
2011 (Q3)	0.998	2009 (Q3) – 2011 (Q2)	0.815
2018 (Q4)	0.998	2011 (Q4) – 2018 (Q3)	0.947
2020 (Q1)	0.999	2019	0.902

Table 3: Smoothed Probabilities table

Applying the DCC-GARCH model, we obtained the conditional correlation and conditional covariance statistics – min, mean, and max, between each two-sector indexes. The result shows that, on average, Consumer Discretionary sector and Information Technology sector indexes have the highest conditional correlation, which means a higher chance of co-movements. In contrast, Utilities and Information Technology have the lowest conditional correlation, that is, less correlated or co-movements. In terms of conditional covariance, all conditional covariances are on a relatively small scale, among which, Financials and Real Estate, on average, has the most significant conditional covariance.

	Min	Mean	Max		Min	Mean	Max
Consumer & Info Tech	0.385	0.793	0.913	Financial & Real Estate	-4.27E-06	0.000256	0.00765
Financial & Consumer	0.384	0.789	0.897	Financial & Consumer	1.53E-05	0.000190	0.00396
Financial & Info Tech	0.096	0.711	0.879	Financial & Info Tech	9.97E-06	0.000184	0.00472
Consumer & Healthcare	0.316	0.699	0.883	Consumer & Real Estate	3.13E-06	0.000162	0.00446
Financial & Healthcare	0.237	0.674	0.855	Consumer & Info Tech	1.80E-05	0.000152	0.00366
Healthcare & Info Tech	0.273	0.652	0.857	Info Tech & Real Estate	7.05E-06	0.000147	0.00368
Consumer & Real Estate	0.042	0.592	0.831	Financial & Healthcare	9.84E-06	0.000127	0.00317
Financial & Real Estate	-0.054	0.575	0.873	Utilities & Real Estate	1.28E-05	0.000117	0.00398
Utilities & Real Estate	0.195	0.573	0.825	Healthcare & Info Tech	1.53E-05	0.000105	0.00283
Info Tech & Real Estate	0.111	0.516	0.792	Healthcare & Real Estate	6.07E-06	0.000103	0.00261
Healthcare & Real Estate	0.140	0.514	0.811	Financial & Utilities	-3.10E-05	0.000101	0.00327
Utilities & Healthcare	0.070	0.468	0.784	Consumer & Healthcare	1.18E-05	0.000101	0.00236
Consumer & Utilities	-0.081	0.446	0.773	Utilities & Info Tech	-1.20E-05	8.17E-05	0.00281
Financial & Utilities	-0.242	0.425	0.810	Consumer & Utilities	-1.44E-05	7.87E-05	0.00233
Utilities & Info Tech	-0.075	0.407	0.746	Utilities & Healthcare	3.99E-06	6.65E-05	0.00224

Table 4: statistics of the conditional correlation and covariance between each two sector indexes

Analyzing the Regime Switching and DCC model in parallel, we obtained the following charts for mean conditional correlation and mean conditional covariance of every pair of sectors. The mean conditional correlation/covariance is calculated as the mean of the mean conditional

correlation/covariance between every two sectors in a specified period. The comparison aims to find the changing trend of average conditional correlation/covariance of all the six main sectors.

From Chart 4 and Chart 5, we can see the tendency of conditional correlation and covariance when economic states change. If the regime switch from appreciation to depreciation, both data tend to increase; if the states change from downturn to uptrend, both data tend to decrease. This implies that if the economic change from expansion to contraction, the co-movements and covariance between every two sectors on average will increase; on the contrary, if the economic change from recession to expansion, the co-movements and covariance between every two sectors in average will decrease. Therefore, the empirical result conforms that regime-switching does affect the interrelationship among sector stock indexes, and the effect on average is more evident in conditional covariance than conditional correlation.

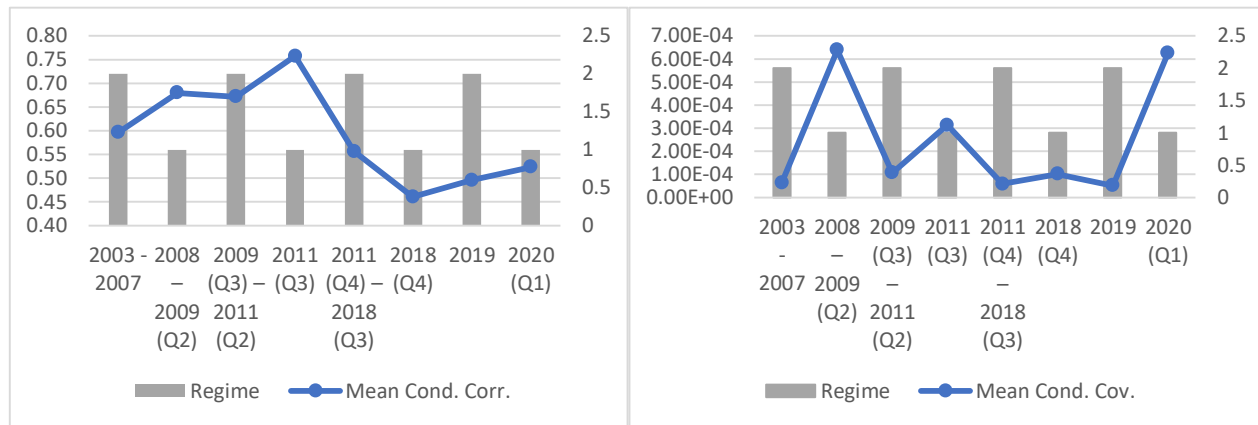


Chart 4: Regime Switching and Mean Conditional Correlations Change

Chart 5: Regime Switching and Mean Conditional Covariance Change

Application in real-world hedging

In the real world stock market, investors not just invest in a specific one stock but try means to make their portfolio or take hedge to minimize the total risk of the investment. However, the above risk-minimizing methods require an accurate estimation of the time-varying covariance matrix. Next, we will follow the applications outlined by Kroner and Ng (1998) to illustrate the applications of the results of our project.

risk-minimizing portfolio weight of two-sector indexes

We would like to use the results of the dynamic conditional covariance matrix to construct an optimal, fully invested portfolio holdings any two of the sector indexes subject to a no-shorting constraint. In this case, we focus on the risk-minimizing instead of return performance. Therefore, the expected returns are assumed to be zero.

According to the knowledge we acquired in the previous portfolio theory course, as for the risk-minimizing portfolio constructed by the assets a and b, the optimal weight for the asset a is:

$$X = \frac{\sigma_b^2 - \rho \sigma_a \sigma_b}{\sigma_a^2 + \sigma_b^2 - 2\rho \sigma_a \sigma_b} \quad (1)$$

Therefore, at time t, as for the portfolio constructed by two sector indexes, the optimal weight for the index of sector 1 is:

$$W_t^* = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}} \quad (2)$$

According to the no short-selling assumption,

$$W_t^* = \begin{cases} 0 & \text{if } W_t < 0 \\ W_t & \text{if } 0 \leq W_t \leq 1 \\ 1 & \text{if } W_t > 1 \end{cases} \quad (3)$$

In the discrete case, we need to rebalance our portfolio weight at different time point to make the portfolio effective. However, it is also possible to use the average covariance value to construct the portfolio. We will show the average covariance matrix and further calculate the average optimal weighted.

	CD	FI	UI	HC	IT	RE
CD	0.0001685811					
FI	0.0001902456	0.0003455609				
UI	7.866499e-05	0.0001013221	0.00013836			
HC	0.0001006394	0.0001275171	6.646552e-05	0.0001135018		
IT	0.0001515501	0.0001843313	8.168555e-05	0.0001051435	0.0002114228	
RE	0.0001619921	0.0002560391	0.0001169411	0.0001033829	0.0001466899	0.0003666417

Table 5: average covariance matrix for the six indexes

For example, we would like to construct a portfolio that consists of two stocks in the Customer Discretionary sector and the Health Care sector. Then the average optimal weight for the stock in CD is calculated to be 0.15918004. This indicates that in this portfolio, the value of CD stock should be accounted for 15.92% while the value of HC stock should be accounted for 84.08%.

risk-minimizing hedge ratio

Except for constructing a portfolio, people also use hedging strategies to minimize their risk. In this case, we would like to use two stocks in different sectors to hedge our investment. We would like to long \$1 in the first sector and short \$β in the second sector.

Let X_0 denotes the original price of stock 1, X_1 denotes the price of stock 1 at the end of the period, Y_0 denotes the original price of stock 2, Y_1 denotes the price of stock 2 at the end of the period, Q denotes the quantity long for stock 1, K denotes the quantity short for stock 2. Then, the income at the end of the period is:

$$QX_1 + K(Y_0 - Y_1) \quad (4)$$

The variance of the income is:

$$Q^2 * Var(X_1) + K^2 * Var(Y_1) - 2Q * K * cov(X_1, Y_1) \quad (5)$$

In order to minimize the variance of income, we can minimize the following equation:

$$\left(\frac{K}{Q}\right)^2 * Var(Y_1) - 2 * \left(\frac{K}{Q}\right) cov(X_1, Y_1) + Var(X_1) \quad (6)$$

Therefore, the optimal K/Q , which is the optimal hedge ratio β is:

$$\frac{cov(X_1, Y_1)}{Var(Y_1)} \quad (7)$$

In our case, it is equivalent to:

$$\beta_t = \frac{h_{12,t}}{h_{22,t}} \quad (8)$$

This ratio is the optimal hedge ratio at time t , in real life, we need to keep rebalancing the portfolio in order to make our hedge strategy effective.

	CD	FI	UI	HC	IT	RE
CD	0.001990668					
FI	0.002457294	0.003988799				
UI	0.001691045	0.002463788	0.003628799			
HC	0.001605865	0.002364310	0.001871704	0.001988844		
IT	0.002192716	0.003061761	0.002131047	0.002042225	0.003079991	
RE	0.001841437	0.002641316	0.002916561	0.001878142	0.002274644	0.003500266

Table 6: The last day covariance matrix of the six indexes

For example, we want to use the stock in CD sector to hedge the investment in IT sector at the last day of our study. The β is calculated to be 1.10149759, which indicates that a \$1.10149759 short position in the CD stock should be taken to hedge the \$1 long position in the IT stock. However, this portfolio should be rebalanced in the next day.

V. Limitation and future development

The preferred volatility models identified above, and the conclusion can be used to manage risks better and improve the pricing of the derivative product in US market. However, there are many extensions of our model could be further considered.

Firstly, our model is based on the univariate GARCH (1,1) model. However, better results might be yield by the other marginal models such as the EGARCH, QGARCH, and GJR GARCH, which capture the asymmetry in the conditional variances. Secondly, we assume the error distribution follows a Gaussian distribution but, other error distributions, such as Student's t -distributions, skew Student's t -distributions, and Normal Inverse Gaussian (NIG) might give a better fit. Besides, it is too restrictive to deploy the same error distributions for all the series; different choices might allow a better fit as well. Finally, to further forecast volatility, other factors that may contribute to the dynamics of US stock market volatility should be considered as extended input. The factors may be represented by macroeconomics variables such as the interest rate, Federal Funds Rate, employment rate and so on. Meanwhile, monetary policies and major events could also have significant impacts on the volatility transformation among market sectors.

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VII. Appendix

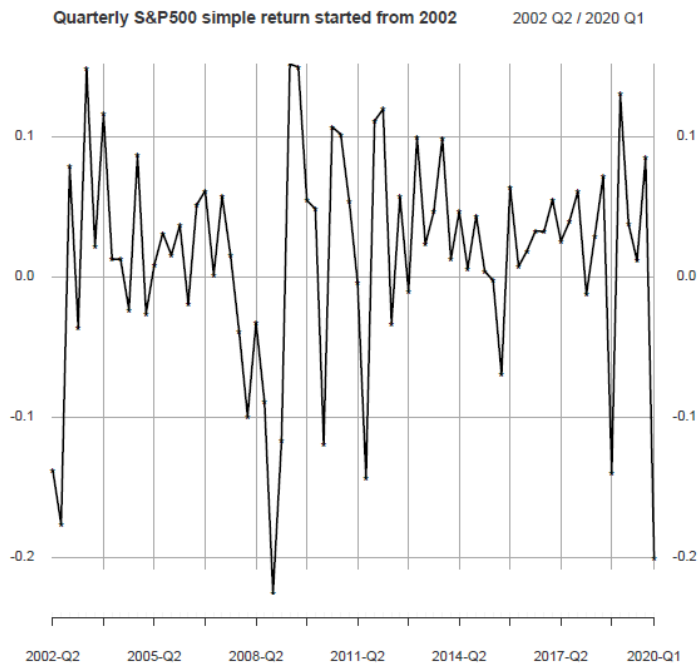
Appendix 1: S&P 500 raw data

```
> SPX=read.table("0.csv",header=T,fill = TRUE,sep=',',fileEncoding="UTF-8-BOM")
>
> head(SPX)
      Date   Price
1 2002-01-01 1154.67
2 2002-01-03 1165.27
3 2002-01-04 1172.51
4 2002-01-07 1164.89
5 2002-01-08 1160.71
6 2002-01-09 1155.14
```

Appendix 2: S&P 500 data transformation (from daily to quarterly)

```
> SPX_P <- as.numeric(SPX$Price)
> S_date <- as.Date(as.character(SPX$Date),"%Y-%m-%d")
> SPX <- xts(SPX_P, S_date)
>
> SPX <- to.quarterly(SPX)
> SPX = SPX$SPX.Close
> SPX = diff(SPX[,1])/lag(SPX[,1])
> SPX<-SPX[-1,]
> names(SPX)[1]<-"SPX"
> basicStats(SPX)
      SPX
nobs      72.000000
NAs        0.000000
Minimum    -0.225582
Maximum     0.152218
1. Quartile -0.013940
3. Quartile  0.058849
Mean        0.014721
Median      0.024661
Sum         1.059945
SE Mean     0.009550
LCL Mean    -0.004321
UCL Mean     0.033764
Variance    0.006567
Stdev       0.081035
Skewness    -0.873018
Kurtosis    0.660961
```

Appendix 3: S&P 500 quarterly simple return



Appendix 4: Fit the Markov Chain Regime Switching Model (2)- AR(4)

```
> mod<-lm(SPX ~ 1)
> SPX_ar4 <- msmFit(mod, k=2, sw=c(TRUE,TRUE,TRUE,TRUE,TRUE,TRUE), p=4)
> summary(SPX_ar4)
Markov Switching Model

Call: msmFit(object = mod, k = 2, sw = c(TRUE, TRUE, TRUE, TRUE, TRUE,
TRUE), p = 4)

      AIC      BIC    logLik
-184.3663 -119.9762 102.1832

Coefficients:

Regime 1
-----
      Estimate Std. Error t value Pr(>|t|)
(Intercept) (S) -0.1155      0.0181  -6.3812 1.757e-10 ***
SPX_1(S)        0.4250      0.2630   1.6160 0.106094
SPX_2(S)       -1.1189      0.4824  -2.3194 0.020373 *
SPX_3(S)        1.1505      0.2829   4.0668 4.766e-05 ***
SPX_4(S)       -0.6922      0.2224  -3.1124 0.001856 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.04238054
Multiple R-squared: 0.7373

Standardized Residuals:
      Min      Q1      Med      Q3      Max
-6.064674e-02  3.347731e-15  6.162148e-05  5.563046e-03  1.032715e-01

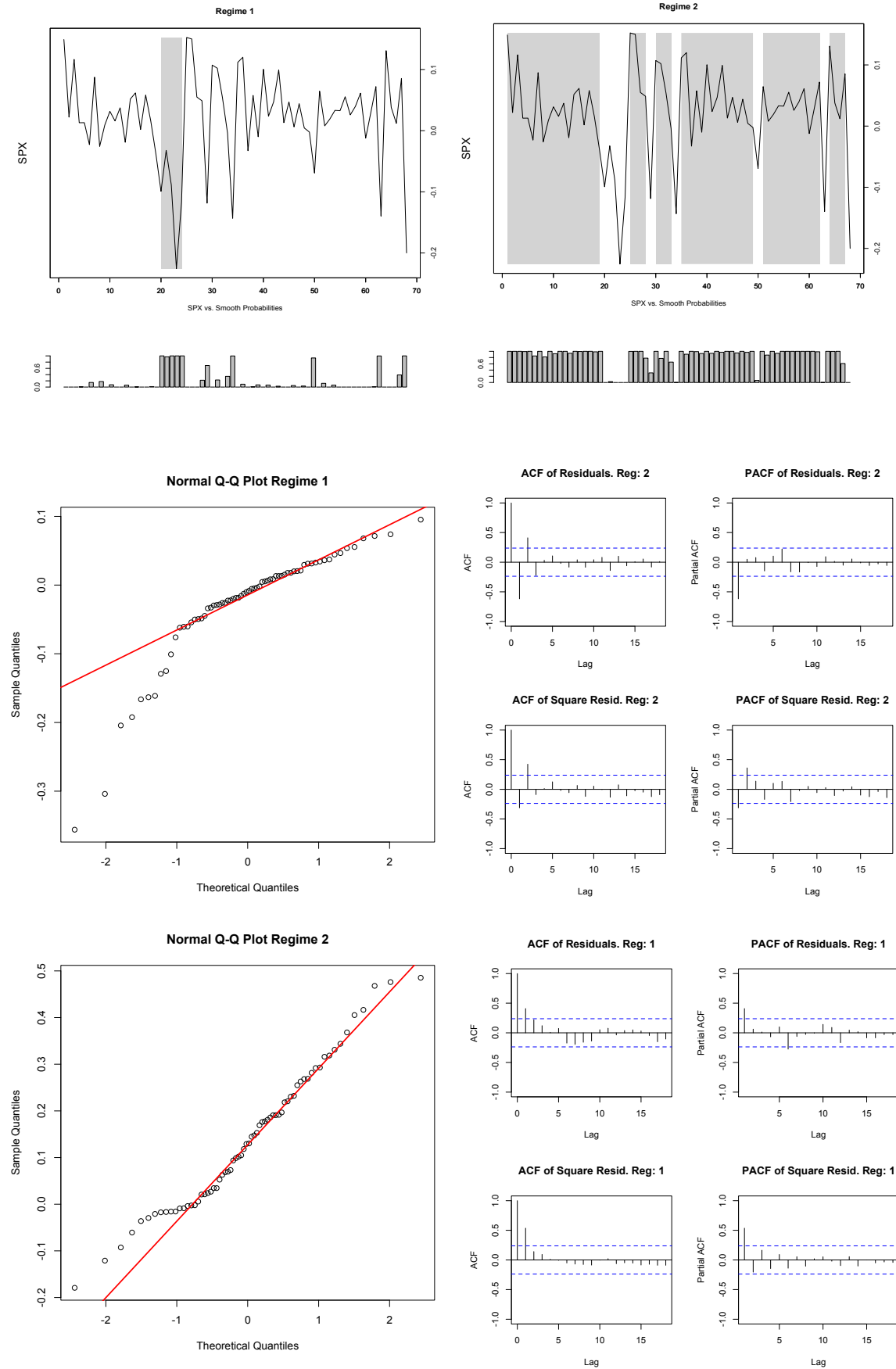
Regime 2
-----
      Estimate Std. Error t value Pr(>|t|)
(Intercept) (S)  0.0658      0.0067   9.8209 < 2.2e-16 ***
SPX_1(S)       -0.3565      0.0868  -4.1071 4.007e-05 ***
SPX_2(S)       -0.2116      0.0783  -2.7024 0.006884 **
SPX_3(S)       -0.2031      0.0730  -2.7822 0.005399 **
SPX_4(S)       -0.1589      0.0778  -2.0424 0.041112 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03847935
Multiple R-squared: 0.3739

Standardized Residuals:
      Min      Q1      Med      Q3      Max
-0.076022993 -0.023108833 -0.002547463  0.020105725  0.095252256

Transition probabilities:
      Regime 1 Regime 2
Regime 1 0.4141219 0.1053417
Regime 2 0.5858781 0.8946583
```

Appendix 5: Regimes plot



Appendix 6: Consumer Discretionary data extraction and conversion

```
> #Consumer Discretionary
> Consumer=read.table("1.csv",header=T,fill = TRUE,sep=',',fileEncoding="UTF-8-BOM")
> Consumer_P <- as.numeric(Consumer$Price)
> C_date <- as.Date(as.character(Consumer$Date),"%Y-%m-%d")
> Consumer <- xts(Consumer_P, C_date)
> Consumer=diff(Consumer[,1])/lag(Consumer[,1])
> Consumer<-Consumer[-1,]
> names(Consumer)[1]<-"Consumer"
> basicStats(Consumer)
```

	Consumer
nobs	4760.000000
NAs	0.000000
Minimum	-0.120815
Maximum	0.131033
1. Quartile	-0.005017
3. Quartile	0.006160
Mean	0.000337
Median	0.000357
Sum	1.604499
SE Mean	0.000192
LCL Mean	-0.000039
UCL Mean	0.000713
Variance	0.000175
Stdev	0.013226
Skewness	-0.072740
Kurtosis	9.727614

Appendix 7: Utilities data extraction and conversion

```
> #Utilities
> Utilities=read.table("2.csv",header=T,fill = TRUE,sep=',',fileEncoding="UTF-8-BOM")
> Utilities_P <- as.numeric(Utilities$Price)
> U_date <- as.Date(as.character(Utilities$Date),"%Y-%m-%d")
> Utilities <- xts(Utilities_P, U_date)
> Utilities=diff(Utilities[,1])/lag(Utilities[,1])
> Utilities<-Utilities[-1,]
> basicStats(Utilities)
```

	x
nobs	4760.000000
NAs	0.000000
Minimum	-0.115430
Maximum	0.135235
1. Quartile	-0.004846
3. Quartile	0.005962
Mean	0.000212
Median	0.000430
Sum	1.007920
SE Mean	0.000176
LCL Mean	-0.000133
UCL Mean	0.000556
Variance	0.000147
Stdev	0.012127
Skewness	0.278015
Kurtosis	17.899596

Appendix 8: Financial Services data extraction and conversion

```
> #Financial Services
> Financial=read.table("3.csv",header=T,fill = TRUE,sep=',',fileEncoding="UTF-8-BOM")
> Financial_P <- as.numeric(Financial$Price)
> F_date <- as.Date(as.character(Financial$Date), "%Y-%m-%d")
> Financial <- xts(Financial_P, F_date)
> Financial=diff(Financial[,1])/lag(Financial[,1])
> Financial<-Financial[-1,]
> names(Financial)[1]<-"Financial"
> basicStats(Financial)
      Financial
nobs      4760.000000
NAs         0.000000
Minimum    -0.170050
Maximum     0.187693
1. Quartile -0.006127
3. Quartile  0.006636
Mean         0.000176
Median       0.000000
Sum          0.837401
SE Mean      0.000276
LCL Mean     -0.000366
UCL Mean      0.000718
Variance     0.000363
Stdev        0.019061
Skewness     0.351283
Kurtosis     18.543037
```

Appendix 9: Health care data extraction and conversion

```
> #Health Care
> Healthcare = read.table("4.csv",header=T,fill = TRUE,sep=',',fileEncoding="UTF-8-BOM")
> Healthcare_P <- as.numeric(Healthcare$Price)
> H_date <- as.Date(as.character(Healthcare$Date), "%Y-%m-%d")
> Healthcare <- xts(Healthcare_P, H_date)
> Healthcare = diff(Healthcare[,1])/lag(Healthcare[,1])
> Healthcare<-Healthcare[-1,]
> names(Healthcare)[1]<-"Healthcare"
> basicStats(Healthcare)
      Healthcare
nobs      4760.000000
NAs         0.000000
Minimum    -0.099931
Maximum     0.124266
1. Quartile -0.004404
3. Quartile  0.005548
Mean         0.000267
Median       0.000271
Sum          1.271024
SE Mean      0.000159
LCL Mean     -0.000044
UCL Mean      0.000578
Variance     0.000120
Stdev        0.010952
Skewness     -0.014949
Kurtosis     10.697888
```


Appendix 10: Information Technology data extraction and conversion

```
> #Information Technology
> Info = read.table("5.csv",header=T,fill = TRUE,sep=',',fileEncoding="UTF-8-BOM")
> Info_P <- as.numeric(Info$Price)
> I_date <- as.Date(as.character(Info$Date), "%Y-%m-%d")
> Info<- xts(Info_P, I_date)
> Info = diff(Info[,1])/lag(Info[,1])
> Info<-Info[-1,]
> names(Info)[1]<-"Info tech"
> basicStats(Info)
```

	Info.tech
nobs	4760.000000
NAs	0.000000
Minimum	-0.139149
Maximum	0.121436
1. Quartile	-0.005734
3. Quartile	0.006900
Mean	0.000402
Median	0.000541
Sum	1.914151
SE Mean	0.000215
LCL Mean	-0.000020
UCL Mean	0.000824
Variance	0.000221
Stdev	0.014859
Skewness	0.122543
Kurtosis	8.513272

Appendix 11: Real Estate data extraction and conversion

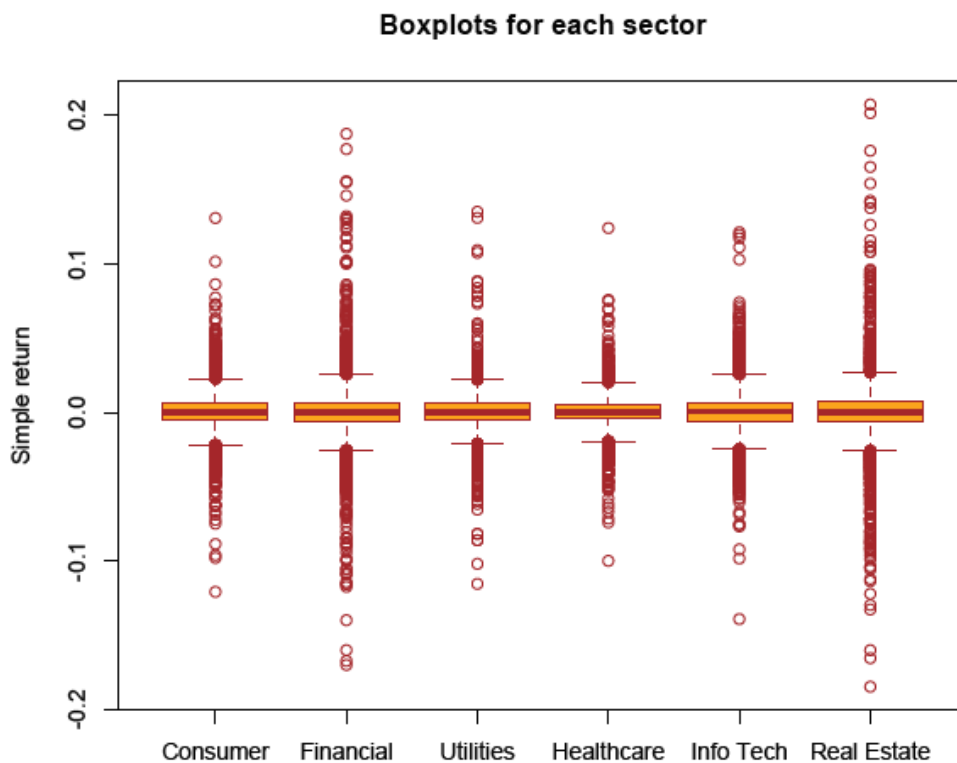
```
> #Real Estate
> Real=read.table("6.csv",header=T,fill = TRUE,sep=',',fileEncoding="UTF-8-BOM")
> Real_P <- as.numeric(Real$Price)
> R_date <- as.Date(as.character(Real$Date), "%Y-%m-%d")
> Real<- xts(Real_P, R_date)
> Real=diff(Real[,1])/lag(Real[,1])
> Real<-Real[-1,]
> names(Real)[1]<-"Real Estate"
> basicStats(Real)
```

	Real.Estate
nobs	4760.000000
NAs	0.000000
Minimum	-0.184723
Maximum	0.207450
1. Quartile	-0.006026
3. Quartile	0.007113
Mean	0.000333
Median	0.000387
Sum	1.586860
SE Mean	0.000282
LCL Mean	-0.000220
UCL Mean	0.000887
Variance	0.000380
Stdev	0.019489
Skewness	0.431554
Kurtosis	20.483541

Appendix 12: Create data frame and box plot of each sector's return series

```
> data <- data.frame(Consumer, Financial,Utilities,Healthcare,Info,Real)
> names(data)[1] <- "Consumer"
> names(data)[2] <- "Financial"
> names(data)[3] <- "Utilities"
> names(data)[4] <- "Healthcare"
> names(data)[5] <- "Info Tech"
> names(data)[6] <- "Real Estate"
```

```
boxplot(data$Consumer,data$Financial,data$Utilities,data$Healthcare,data$Info,data$Real,main="Boxplots
for each sector",names=c("Consumer","Financial","Utilities","Healthcare","Info Tech","Real
Estate"),ylab="Simple return",col="orange",border="brown")
```



Appendix 13: Fit ARIMA model for each sector return series based on “BIC”

```
> auto.arima(Financial,ic=c('bic'))
Series: Financial
ARIMA(1,0,0) with zero mean

Coefficients:
      ar1
    -0.1439
s.e.    0.0143

sigma^2 estimated as 0.0003558:  log likelihood=12146.18
AIC=-24288.35  AICc=-24288.35  BIC=-24275.42
> #ARIMA(1,0,0)
>
> auto.arima(Utilities,ic=c('bic'))
Series: Utilities
ARIMA(1,0,0) with zero mean

Coefficients:
      ar1
    -0.0865
s.e.    0.0145

sigma^2 estimated as 0.000146:  log likelihood=14266.26
AIC=-28528.52  AICc=-28528.52  BIC=-28515.58
> # ARIMA(1,0,0)
>
> auto.arima(Healthcare,ic=c('bic'))
Series: Healthcare
ARIMA(0,0,1) with zero mean

Coefficients:
      mal
    -0.0825
s.e.    0.0148

sigma^2 estimated as 0.0001192:  log likelihood=14748.33
AIC=-29492.65  AICc=-29492.65  BIC=-29479.72
> #ARIMA(0,0,1)
>
> auto.arima(Info,ic=c('bic'))
Series: Info
ARIMA(2,0,0) with zero mean

Coefficients:
      ar1      ar2
    -0.1077   -0.0025
s.e.    0.0145    0.0145

sigma^2 estimated as 0.0002184:  log likelihood=13307.86
AIC=-26609.72  AICc=-26609.72  BIC=-26590.32
> #ARIMA(2,0,0)
>
> auto.arima(Real,ic=c('bic'))
Series: Real
ARIMA(0,0,1) with zero mean

Coefficients:
      mal
    -0.2013
s.e.    0.0142

sigma^2 estimated as 0.0003649:  log likelihood=12086.23
AIC=-24168.45  AICc=-24168.45  BIC=-24155.52
> #ARIMA(0,0,1)
```

Appendix 14: Fit univariate GARCH model

```
> C_spec <- ugarchspec(mean.model=list(armaOrder=c(0,1)),distribution.model = "norm")
> F_spec <- ugarchspec(mean.model=list(armaOrder=c(1,0)),distribution.model = "norm")
> U_spec <- ugarchspec(mean.model=list(armaOrder=c(1,0)),distribution.model = "norm")
> H_spec <- ugarchspec(mean.model=list(armaOrder=c(0,1)),distribution.model = "norm")
> I_spec <- ugarchspec(mean.model=list(armaOrder=c(2,0)),distribution.model = "norm")
> R_spec <- ugarchspec(mean.model=list(armaOrder=c(0,1)),distribution.model = "norm")
```

Appendix 15: Fit DCC GARCH model

```
> uspec.n = multispec(c(C_spec,F_spec,U_spec,H_spec,I_spec,R_spec))
> multf = multifit(uspec.n, data)
> multf
```

```
*-----*
*      GARCH Multi-Fit      *
*-----*
No. Assets :6
GARCH Model Fit
-----
Optimal Parameters:
[[1]]
      mu      mal      omega      alphas      betas
6.5150e-04 -1.3578e-02 2.1644e-06 1.0434e-01 8.8103e-01

[[2]]
      mu      ar1      omega      alphas      betas
6.0106e-04 -5.8568e-02 2.3337e-06 1.0454e-01 8.8677e-01

[[3]]
      mu      ar1      omega      alphas      betas
4.8514e-04 -1.3723e-02 1.9382e-06 9.9141e-02 8.8548e-01

[[4]]
      mu      mal      omega      alphas      betas
5.2620e-04 -4.2685e-02 2.3881e-06 1.0110e-01 8.7558e-01

[[5]]
      mu      ar1      ar2      omega      alphas      betas
8.8193e-04 -3.7398e-02 -1.2100e-02 2.9366e-06 9.8296e-02 8.8663e-01

[[6]]
      mu      mal      omega      alphas      betas
6.1483e-04 -3.3502e-02 2.4110e-06 1.2534e-01 8.6676e-01
```

```
> dccl_1 = dccspec(uspec = uspec.n, dccOrder = c(1, 1), distribution = 'mvnrm')
> dccl_1
```

```
*-----*
*          DCC GARCH Spec          *
*-----*
Model           : DCC(1,1)
Estimation      : 2-step
Distribution     : mvnrm
No. Parameters  : 48
No. Series      : 6
```

```
>
> fit = dccfit(dccl_1, data = data, fit.control = list(eval.se = TRUE), fit = multf)
> fit
```

```
*-----*
*          DCC GARCH Fit          *
*-----*
```

```
Distribution     : mvnrm
Model            : DCC(1,1)
No. Parameters   : 48
[VAR GARCH DCC UncQ] : [0+31+2+15]
No. Series       : 6
No. Obs.         : 4760
Log-Likelihood   : 99448.02
Av.Log-Likelihood : 20.89
```

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
[Consumer].mu	0.000651	0.000124	5.24009	0.000000
[Consumer].mal	-0.013578	0.015893	-0.85435	0.392912
[Consumer].omega	0.000002	0.000001	2.18724	0.028725
[Consumer].alphal	0.104336	0.013949	7.47973	0.000000
[Consumer].betal	0.881027	0.014510	60.72027	0.000000
[Financial].mu	0.000601	0.000137	4.39613	0.000011
[Financial].arl	-0.058568	0.015980	-3.66507	0.000247
[Financial].omega	0.000002	0.000002	1.36377	0.172641
[Financial].alphal	0.104538	0.022456	4.65518	0.000003
[Financial].betal	0.886772	0.023032	38.50124	0.000000
[Utilities].mu	0.000485	0.000118	4.11754	0.000038
[Utilities].arl	-0.013723	0.015902	-0.86301	0.388130
[Utilities].omega	0.000002	0.000001	1.46879	0.141889
[Utilities].alphal	0.099141	0.021300	4.65454	0.000003
[Utilities].betal	0.885480	0.022386	39.55594	0.000000
[Healthcare].mu	0.000526	0.000110	4.77508	0.000002
[Healthcare].mal	-0.042685	0.016684	-2.55836	0.010517
[Healthcare].omega	0.000002	0.000001	2.24569	0.024724
[Healthcare].alphal	0.101100	0.010683	9.46327	0.000000
[Healthcare].betal	0.875577	0.012740	68.72520	0.000000
[Info Tech].mu	0.000882	0.000151	5.85318	0.000000
[Info Tech].arl	-0.037398	0.015698	-2.38238	0.017201
[Info Tech].ar2	-0.012100	0.016297	-0.74243	0.457829
[Info Tech].omega	0.000003	0.000002	1.85395	0.063747
[Info Tech].alphal	0.098296	0.013183	7.45627	0.000000
[Info Tech].betal	0.886628	0.015126	58.61436	0.000000
[Real Estate].mu	0.000615	0.000134	4.59587	0.000004
[Real Estate].mal	-0.033502	0.016113	-2.07918	0.037601
[Real Estate].omega	0.000002	0.000001	1.66455	0.096002
[Real Estate].alphal	0.125342	0.022633	5.53797	0.000000
[Real Estate].betal	0.866756	0.022158	39.11622	0.000000
[Joint]dccal	0.022716	0.002219	10.23524	0.000000
[Joint]dccbl	0.968484	0.003762	257.46018	0.000000

Information Criteria

```
-----
Akaike      -41.765
Bayes       -41.699
Shibata     -41.765
Hannan-Quinn -41.742
```

Elapsed time : 21.52373

Appendix 16: Conditional Correlation between each two sectors in different regimes

Regime	2	1	2	1	2	1	2	1
Period	2003 - 2007	2008 – 2009 (Q2)	2009 (Q3) – 2011 (Q2)	2011 (Q3)	2011 (Q4) – 2018 (Q3)	2018 (Q4)	2019	2020 (Q1)
(1)	0.81	0.84	0.78	0.83	0.76	0.65	0.75	0.78
(2)	0.53	0.51	0.56	0.73	0.31	0.11	0.18	0.27
(3)	0.7	0.67	0.65	0.78	0.67	0.61	0.61	0.69
(4)	0.72	0.74	0.75	0.78	0.69	0.62	0.71	0.79
(5)	0.6	0.81	0.74	0.78	0.51	0.38	0.3	0.35
(6)	0.5	0.57	0.61	0.69	0.37	0.07	0.24	0.28
(7)	0.69	0.74	0.71	0.84	0.7	0.73	0.64	0.63
(8)	0.78	0.84	0.83	0.88	0.78	0.84	0.83	0.78
(9)	0.58	0.78	0.73	0.77	0.56	0.4	0.43	0.4
(10)	0.48	0.63	0.63	0.71	0.4	0.26	0.39	0.39
(11)	0.43	0.56	0.58	0.63	0.33	0.07	0.24	0.32
(12)	0.54	0.53	0.61	0.72	0.6	0.61	0.61	0.66
(13)	0.62	0.69	0.69	0.76	0.66	0.76	0.66	0.67
(14)	0.48	0.61	0.57	0.74	0.51	0.47	0.46	0.43
(15)	0.48	0.68	0.64	0.72	0.5	0.33	0.39	0.41
Mean	0.60	0.68	0.67	0.76	0.56	0.46	0.50	0.52

*(1) Financial & Consumer

(2) Financial & Utilities

(3) Financial & Healthcare

(4) Financial & Info Tech

(5) Financial & Real Estate

(6) Consumer & Utilities

(7) Consumer & Healthcare

(8) Consumer & Info Tech

(9) Consumer & Real Estate

(10) Utilities & Healthcare

(11) Utilities & Info Tech

(12) Utilities & Real Estate

(13) Healthcare & Info Tech

(14) Healthcare & Real Estate

(15) Info Tech & Real Estate

Appendix 17: Conditional Covariance between each two sectors in different regimes

Regime	2	1	2	1	2	1	2	1
Period	2003 - 2007	2008 - 2009 (Q2)	2009 (Q3) - 2011 (Q2)	2011 (Q3)	2011 (Q4) - 2018 (Q3)	2018 (Q4)	2019	2020 (Q1)
(1)	8.61E-05	0.00098	0.00015	0.00046	8.45E-05	0.00016	8.12E-05	0.00077
(2)	5.46E-05	0.00047	8.33E-05	0.00029	3.53E-05	1.91E-05	1.73E-05	0.00062
(3)	6.22E-05	0.00053	8.87E-05	0.00035	7.27E-05	0.00011	6.01E-05	0.00067
(4)	9.01E-05	0.00081	0.00014	0.00042	8.76E-05	0.00017	9.28E-05	0.00093
(5)	8.10E-05	0.0019	0.00022	0.00052	6.94E-05	6.79E-05	3.08E-05	0.00074
(6)	4.75E-05	0.00034	6.82E-05	0.0002	3.22E-05	1.54E-05	2.01E-05	0.00044
(7)	5.78E-05	0.00034	7.14E-05	0.00027	6.10E-05	0.00016	6.20E-05	0.00047
(8)	9.22E-05	0.00051	0.00012	0.00034	7.85E-05	0.00027	0.000105	0.0007
(9)	6.88E-05	0.00099	0.00016	0.00038	5.76E-05	8.78E-05	3.98E-05	0.00055
(10)	3.84E-05	0.00024	5.10E-05	0.00016	3.27E-05	3.56E-05	2.93E-05	0.00044
(11)	4.98E-05	0.0003	6.40E-05	0.00018	3.31E-05	1.57E-05	2.45E-05	0.00054
(12)	6.15E-05	0.00053	0.0001	0.00024	5.58E-05	8.05E-05	4.55E-05	0.00077
(13)	6.24E-05	0.0003	6.93E-05	0.00024	6.54E-05	0.00018	7.58E-05	0.00058
(14)	4.89E-05	0.00052	8.76E-05	0.00029	5.00E-05	7.41E-05	3.92E-05	0.0005
(15)	6.91E-05	0.00081	0.00014	0.00035	5.75E-05	7.82E-05	4.37E-05	0.00066
Mean	6.47E-05	6.38E-04	1.08E-04	3.13E-04	5.82E-05	1.02E-04	5.11E-05	6.25E-04

*(1) Financial & Consumer

(2) Financial & Utilities

(3) Financial & Healthcare

(4) Financial & Info Tech

(5) Financial & Real Estate

(6) Consumer & Utilities

(7) Consumer & Healthcare

(8) Consumer & Info Tech

(9) Consumer & Real Estate

(10) Utilities & Healthcare

(11) Utilities & Info Tech

(12) Utilities & Real Estate

(13) Healthcare & Info Tech

(14) Healthcare & Real Estate

(15) Info Tech & Real Estate

Appendix 18: Conditional Correlation between two sectors

