

# Controlling the Spread of Two Secrets in Diverse Social Networks (Student Abstract)

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## Abstract

Information diffusion in social networks is a well-studied concept in social choice theory. We propose the study of the diffusion of two secrets in a heterogeneous environment from the complexity perspective, that is, there are two different networks with the same set of agents (e.g., the structure of the set of followers might be different in two distinct social networks).

Formally, our model combines two group identification processes for which we do have independent desiderata—either constructive, where we would like a given group of agents to be exposed to a secret, or destructive, where a given group of agents should not be exposed to a secret. To be able to reach these targets, we can either delete an agent or introduce a previously latent agent.

Our results are mostly negative—all of the problems are NP-hard. Therefore, we propose a parameterized study with respect to the natural parameters, the number of influenced agents, the size of the required/protected agent sets, and the duration of the diffusion process. Most of the studied problems remain W[1]-hard even for a combination of these parameters. We complement these results with nearly optimal XP algorithms.

## Introduction

In the summer, Danny met an astonishing woman named Sandy, who was on vacation in Danny's town. They spent pretty romantic times at the beach and fell in love immediately. Unfortunately, all good things come to an end, which is also applicable to these summer holidays. Luckily, Sandy's parents decided not to return to Australia, and she enrolled at the same high school as Danny. None of them were aware of the presence of each other at the school.

Apart from the holiday bubble, Danny is a completely different person than Sandy knew. He is a leader of a greaser gang. Of course, such a person cannot tell his friends how he actually spent the holidays, so he spiced up his love story a little. On the other hand, Sandy is a decent girl, as she has always been, so her variant of the story emphasizes the romance of the rapport.

Once Sandy and Danny finally met, the situation gets much more complicated. Danny really wants to get back together with Sandy, but on the other hand, he does not want

to lose his face in front of the other gang members. Therefore, he decided to date Sandy in secret. However, things are not always as ideal as one would imagine, which is especially true when attending high school. After a short while, their potential relationship began to be rumored among their classmates.

When Danny found out, he freaked out. Fortunately, Sandy is calmer in nature and, moreover, is an honored member of a science club. With the knowledge she has about computational social choice, she decided to approach the problem using a scientific method. It must be said that even her motivation is not completely crystal clear, as she became more acquainted with the chairman of the science club at a recent argumentation competition.

Sandy's goal is obvious. There are, in terms of relationships, two mostly disjoint networks of classmates, and the goal is to convince the right people of the falsity of the rumor so that the information does not reach members of the science club in the first network and members of Danny's greaser gang in the second network.

Motivated by this scenario, we derive the formal setting of the **DELETE AGENT LSR DESTRUCTIVE-DESTRUCTIVE GROUP IDENTIFICATION** problem (DLDD for short) of opinion spreading in diverse social networks. We have a group of agents  $A$  and two social networks (this is formally captured by two directed graphs with identical vertex set  $A$ ). We use the liberal starting rule to determine the agents which know the respective secrets—the ones with self-loop. Then the secrets spread through the network along directed edges. Each agent knowing a secret will share it in the appropriate network. Finally, we are given two groups of agents  $D_1$  and  $D_2$  and a positive integer  $k$ . We want to decide if there is a group of at most  $k$  agents  $X$  such that if we remove these agents, then the agents in  $D_1$  will not learn the first secret and the agents in  $D_2$  will not learn the second secret.

This problem has several natural modifications: We may consider a variant where the operation is adding an agent (instead of removing it), where the goal is to ensure that some agents learn the secret, or variants with different starting rules. We investigate DLDD (and other settings) from a parameterized complexity view, showing hardness of the problem even when many natural values are restricted (given as parameters or known to be constant). The hardness results are complemented with XP algorithms.

**Previous work.** Our work is at the intersection of network dynamics and group identification. There is vast work on opinion spread in a social network; one of the most discussed models leads to the so-called target set selection, where one chooses the smallest set of initial agents so that an opinion spreads in a weighted undirected graph. The target set selection is a notoriously hard problem from the perspective of both classical and parameterized complexity, however, in our work the activation sets are given and our task is to secure certain targets.

In the group identification line of work, the most important for us are the recent works on manipulation of the outcome. Erdélyi, Reger, and Yang (2019) study the complexity of control destructive or constructive; both these goals are solvable in polynomial time. Boehmer et al. (2020) then introduced the combination of the two goals in the same social network (with the same identification process) and studied it from both a classical and parameterized perspective.

In our work, we assume two different social networks with the same set of agents. This setting was already proposed in the group identification by Cho and Ju (2017), where the problem studied is to find a partition of the agents into disjoint groups of socially qualified agents. However, we do have different goals for different opinions.

## Our Contribution

We study a new variant of the group identification problem with more opinions. In our work, we start with analysis of the easy settings of the problem (which belong to the complexity class P). For most of the remaining variants, we show the hardness of such settings in terms of both classical computational complexity and parameterized complexity.

Among other things, we prove that both DLDD and DLCD (variant with one constructive target) are NP-complete. If we parameterize these variants using different combinations of the most natural parameters, then we show that for some settings, such as DLCD where the only parameter is the number  $k$  of agents we are allowed to affect, the problem is W[1]-hard, and we provide XP algorithms for these settings. For other variants, we introduce XP algorithms without proving W[1]-hardness or the existence of FPT algorithms, which we leave as an open problem for future research. The results are put into a comprehensive format, showing our results, their impact on other settings, and the list of settings with undetermined complexity.

**Sketch of results.** The DLDD can be proved to be NP-hard by reduction from 3-SAT as follows (sketch): The graph contains vertices which represent literal occurrences. The rough idea is to encode the two conditions of the 3-SAT into the edges of the two opinions. The first condition is that each clause contains at least one true valuation of a literal occurrence—this is tackled by joining an auxiliary source and sink by a path over the respective three literals in the first opinion  $\varphi_1$ . To disconnect the path, one of the three vertices will need to be removed. Second condition is that all literal occurrences of a variable are consistent—only positive literals or negative literals evaluate to true. This can be tackled by joining all positive and all negative variable occurrences

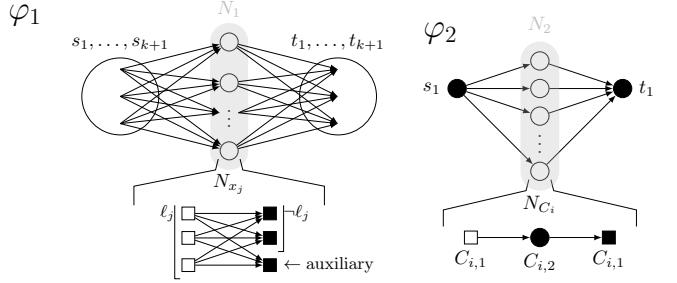


Figure 1: Representation of the graph construction for the edges of  $\varphi_1$  on the left, and  $\varphi_2$  on the right over the same set of agent-vertices, showing NP-hardness for DLDD setting.

into a complete bipartite (oriented) graph, which is entered from the source and left to the sink using edges of the second opinion  $\varphi_2$ . See 1 for diagram on how to connect the edges of the first and second opinion.

This construction gives us that the problem is NP-hard for cases when the opinion spread takes a constant number of steps and when one of the target sets has a constant size. Complementing this result are XP algorithms. When the number of removed vertices is a parameter  $k$ , then we may simply check each possible set in  $n^k$  which is in XP. On the other hand, we are able to prove that the problem is in XP for parameters being the sizes of both target sets.

The three results partition all settings of DLDD into ones which are XP and ones which are NP-hard. We provide similar results in other settings of the problem.

## Conclusion

This work investigates many settings of the liberal starting rule (LSR) of the DLDD, DLCD, and other settings of the problem, getting many results in its parameterized complexity analysis. Next, we plan to investigate the range between LSR and so-called consent starting rule (CSR) – here, the agent starts to spread the secret only if some number of other agents would tell him his secret. We conjecture that the settings which are easy on LSR and are hard on CSR become hard when this threshold is constant.

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