Equivalence in Argumentation Frameworks with a Claim-centric View – Classical Results with Novel Ingredients

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Abstract

A common feature of non-monotonic logics is that the classical notion of equivalence does not preserve the intended meaning in light of additional information. Consequently, the term strong equivalence was coined in the literature and thoroughly investigated. In the present paper, the knowledge representation formalism under consideration is claim-augmented argumentation frameworks (CAFs) which provide a formal basis to analyze conclusion-oriented problems in argumentation by adapting a claim-focused perspective. CAFs extend Dung AFs by associating a claim to each argument representing its conclusion. In this paper, we investigate both ordinary and strong equivalence in CAFs. Thereby, we take the fact into account that one might either be interested in the actual arguments or their claims only. The former point of view naturally yields an extension of strong equivalence for AFs to the claim-based setting while the latter gives rise to a novel equivalence notion which is genuine for CAFs. We tailor, examine and compare these notions and obtain a comprehensive study of this matter for CAFs. We conclude by investigating the computational complexity of naturally arising decision problems.

1 Introduction

Equivalence is an important subject of research in knowledge representation and reasoning. Given a knowledge base $\mathcal{K}$, finding an equivalent one, say $\mathcal{K}'$, helps to obtain a better understanding or more concise representation of $\mathcal{K}$. From a computational point of view, equivalence is particularly interesting whenever a certain subset of a collection of information can be replaced without changing the intended meaning. In propositional logics, for example, replacing a subformula ϕ of Φ with an equivalent one, say ϕ', yields a formula $\Phi[\phi/\phi']$ equivalent to Φ. That is, we may view ϕ as an independent module of Φ. Within the KR community it is folklore that this is usually not the case for non-monotonic logics (apart from folklore, we refer the reader to (Baumann and Strass 2016) for a rigorous study of this matter).

Motivated by this observation, the notion of strong equivalence was introduced in the literature. In a nutshell, strong equivalence requires the aforementioned property by design: $\mathcal{K}$ and $\mathcal{K}'$ are strongly equivalent if for any $\mathcal{H}$, the knowledge bases $\mathcal{K} \cup \mathcal{H}$ and $\mathcal{K}' \cup \mathcal{H}$ are equivalent. Although a naive implementation would require to iterate over an infinite number of possible $\mathcal{H}$, researchers discovered techniques to decide strong equivalence of two knowledge bases efficiently, most notably for logic programming (Lifschitz, Pearce, and Valverde 2001) and argumentation frameworks (AFs) (Oikarinen and Woltran 2011). The possibility to replace parts of a framework in a semantical neutral way is particularly important whenever dynamics in argumentation are considered. The latter topic is rightly one of the most active research areas within the community at the moment (cf. (Gabbay et al. 2021). In this paper, we extend this line of research to a recent extension of AFs, called Claim-augmented argumentation frameworks (CAFs).

Dung (Dung 1995) boosted the research in abstract argumentation frameworks which can by now be considered a classical area in knowledge representation and reasoning. AFs have been thoroughly investigated since then and various extensions have been proposed; e.g., the addition of supports (Cayrol and Lagasquie-Schiex 2005), recursive (Baroni et al. 2011) and collective (Nielsen and Parsons 2006) attacks, or probabilities (Thimm 2012) to mention a few. In recent years, the focus on conclusion-oriented reasoning (Baroni and Riveret 2019; Dvorák and Woltran 2020) became increasingly popular. While traditional argumentation formalisms focus on the identification of acceptable arguments, the emphasis in claim-focused argumentation lies on the argument's conclusions (*claims*). Building on the observation that a claim can be supported by different arguments, it becomes evident that the traditional argument-focused perspective is often insufficient to capture claim-based reasoning. *Claim-augmented argumentation frameworks* as introduced by (Dvorák and Woltran 2020) address this issue by extending AFs with a function that assigns a claim to each argument. They are in particular well-suited to analyze instantiation-based approaches, e.g., instantiations of logic programs (Caminada et al. 2015b), rule-based formalisms, e.g., ABA+ (Bondarenko, Toni, and Kowalski 1993; Caminada et al. 2015a), or logic-based instantiations (Besnard and Hunter 2001; Gorogiannis and Hunter 2011), where the focus lies on the claims of the constructed arguments.

The goal of this paper is to investigate equivalence notions for reasoning with a claim-centered point of view. Due to their generality, CAFs form an ideal basis to obtain a comprehensive study of this matter. Our main contributions are:

- We provide characterization results of strong equivalence between CAFs via semantics-dependent kernels for each CAF semantics which has been considered in the literature so far. Moreover, we discuss ordinary equivalence for CAFs and present dependencies between semantics for this weaker equivalence notion.
- We introduce novel equivalence concepts based on argument renaming which are genuine for CAFs. We show that ordinary equivalence up to renaming coincides with ordinary equivalence while strong equivalence up to renaming can be characterized via kernel isomorphism.
- We present a rigorous complexity analysis of deciding equivalence between two CAFs for all of the aforementioned equivalence notions. We show that deciding ordinary equivalence can be computationally hard, up to the third level of the polynomial hierarchy.

2 Motivation

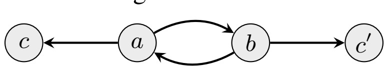
Abstract argumentation frameworks have proved to be a powerful and expressive tool in that they are capable of capturing the behavior of various knowledge representation formalisms by so-called *instantiations*, see e.g. (Modgil and Prakken 2014; Toni 2014; Wu, Caminada, and Gabbay 2009; Caminada et al. 2015b).

Such intertranslations possess various advantages. First, since AFs are graphical and accessible by design, representing a knowledge base as an AF may yield a more user-friendly representation. Second, they facilitate the investigation of theoretical results since this way different research areas can benefit from each other. Moreover, some formalisms have semantics which are solely based on the evaluation of an constructed AF. Our main motivation to investigate the behavior of CAFs is that they can help streamlining such instantiations. To illustrate this, we consider the following simple example where we translate a logic program.

Example 2.1. Let P be the following program:

$$a \leftarrow \text{not } b. \quad b \leftarrow \text{not } a. \quad c \leftarrow \text{not } a. \quad c \leftarrow \text{not } b.$$

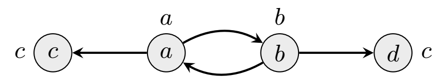
The corresponding AF (Wu, Caminada, and Gabbay 2009) would have the following structure:



Thereby, both arguments c and c' are associated with the conclusion c. Since Dung-style AFs are not tailored to capture such a relationship between two arguments, some more technical machinery is required. First, the extensions of the AF are translated into labelings as e.g. done in (Caminada 2006a). Second, given a labeling of the constructed AF, two more mappings are required to translate labelings into models and vice versa ((Wu, Caminada, and Gabbay 2009, Definitions 21 and 22)) to then obtain the desired correspondence between LPs and AFs.

The recently introduced claim-centered AFs (CAFs) (Dvorák and Woltran 2020) provide a natural solution to this problem by extending Dung-style AFs in a way that to each argument an associated claim (or: conclusion) is assigned as well. As pointed out by (Dvorák and Woltran 2020), using CAFs to instantiate knowledge representation formalisms streamlines the evaluation. Let us return to the previous example, this time utilizing a CAF.

Example 2.2. The LP from above yields the following CAF, including claims depicted next to the respective argument.



As the reader may verify, from a structural point of view the CAF is comparable to the AF from above, however it captures the fact that two occurring arguments represent the same conclusion. CAF semantics are equipped to deal with scenarios of this kind out of the box, thus no labelings and no further mappings are required, thus establishing a closer relation between the base formalism (in this case: our LP) and the instantiated instance (in this case: our CAF).

CAFs thus provide a more robust translation and better preserve properties of the instantiated knowledge base. They therefore provide a simple, yet promising extension of Dung-AFs and this paper contributes to their investigation by examining one of the basic problems in knowledge representation and reasoning, namely (strong) equivalence.

3 Background

Abstract Argumentation. We fix a non-finite background set $\mathcal{U}$. An argumentation framework (AF) (Dung 1995) is a directed graph $F = (A, R)$ where $A \subseteq \mathcal{U}$ is a finite set of arguments and $R \subseteq A \times A$ models *attacks* between them. We use $\mathcal{AF}$ to denote the set of all AFs.

For two arguments $a, b \in A$, if $(a, b) \in R$ we say that a *attacks* b as well as the set $E \subseteq A$ *attacks* b if $a \in E$. The *range* of a set $E \subseteq A$ is defined as $E_F^{\oplus} = E \cup E_F^{+}$ where $E_F^{+} = \{a \in A \mid E \text{ attacks } a\}$. E is conflict-free in F (for short, $E \in cf(F)$) iff for no $a, b \in E$, $(a, b) \in R$. E *defends* an argument a if any attacker of a is attacked by E. A *semantics* is a function $\sigma : \mathcal{AF} \to 2^{2^{\mathcal{U}}}$ with $F \mapsto \sigma(F) \subseteq 2^A$. This means, given an AF $F = (A, R)$ a semantics returns a set of subsets of A. These subsets are called *σ-extensions*.

In this paper we consider so-called *naive*, *admissible*, *complete*, *grounded*, *preferred*, *stable*, *semi-stable* and *stage* semantics (abbr. na, ad, co, gr, pr, stb, ss, stg). Apart from naive, semi-stable and stage semantics (Verheij 1996; Caminada 2006b), all mentioned semantics were already introduced in (Dung 1995).

Definition 3.1. Let $F = (A, R)$ be an AF and $E \in cf(F)$.

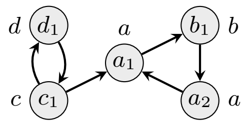
1. $E \in na(F)$ iff E is $\subseteq$-maximal in $cf(F)$,
2. $E \in ad(F)$ iff E defends all its elements,
3. $E \in co(F)$ iff $E \in ad(F)$ and for any a defended by E we have, $a \in E$,
4. $E \in gr(F)$ iff E is $\subseteq$-minimal in $co(F)$, and
5. $E \in pr(F)$ iff E is $\subseteq$-maximal in $ad(F)$,
6. $E \in stb(F)$ iff $E \in cf(A)$ and E attacks any $a \in A \setminus E$,
7. $E \in ss(F)$ iff $E \in ad(F)$ and there is no $D \in ad(F)$ with $E_F^{\oplus} \subsetneq D_F^{\oplus}$,
8. $E \in stg(F)$ iff $E \in cf(F)$ and there is no $D \in cf(F)$ with $E_F^{\oplus} \subsetneq D_F^{\oplus}$.

Claim-based Argumentation. A *claim-augmented argumentation framework (CAF)* (Dvorák and Woltran 2020) is a triple $\mathcal{F} = (A, R, cl)$ where $F = (A, R)$ is an AF and $cl : A \to \mathcal{C}$ is a function which assigns a claim to each argument in A; $\mathcal{C}$ is a set of (countable infinite) possible claims. The claim-function is extended to sets in the natural way, i.e. for a set $E \subseteq A$, we let $cl(E) = \{cl(a) \mid a \in E\}$.

There are several ways in which semantics for AFs extend to CAFs. The most basic one is to choose an appropriate AF semantics and consider the claims of the induced extensions.

Definition 3.2. For a CAF $\mathcal{F} = (A, R, cl)$, $F = (A, R)$, and a semantics σ, we define the inherited variant of σ *(i-σ)* as $\sigma_c(\mathcal{F}) = \{cl(E) \mid E \in \sigma(F)\}$. We call $E \in \sigma(F)$ with $cl(E) = S$ a σ_c-realization of S in $\mathcal{F}$.

Example 3.3. Consider the following CAF $\mathcal{F}$:



Let us focus on stable semantics. For the underlying AF F we have the unique stable extension $E = \{c_1, b_1\}$. It is thus easy to see that $stb_c(\mathcal{F}) = \{\{c, b\}\}$. Moreover, $\{c_1, b_1\}$ is a stb_c-realization of E.

Let us now turn to the semantics which actually operate on the level of the claims instead of focusing on the underlying arguments. For this, we need to generalize the notion of defeat to claims. A set of arguments $E \subseteq A$ *defeats* a claim $c \in cl(A)$ in $\mathcal{F}$ if E attacks every $a \in A$ with $cl(a) = c$ (in F); we write

$$E^{\ddagger}_{\mathcal{F}} = \{c \in cl(A) \mid E \text{ defeats } c \text{ in } \mathcal{F}\}$$

to denote the set of all claims which are defeated by E in $\mathcal{F}$. The claim-range of a set of claims $S = cl(E)$ is denoted by $E^{\oplus}_{\mathcal{F}} = cl(E) \cup E^{\ddagger}_{\mathcal{F}}$.

Example 3.4. Consider again the CAF $\mathcal{F}$ from the previous example. Although c_1 defeats a_1, it does not defeat the claim a. However, $E = \{c_1, b_1\}$ defeats a, i.e. $a \in E^{\ddagger}_{\mathcal{F}}$. The claim-range of E is thus $E^{\oplus}_{\mathcal{F}} = \{a, b, c, d\}$.

Observe that the range of a set of claims is not a well-defined concept: In our example CAF $\mathcal{F}$, the claim-range of $\{a\}$ could either be $\{a, b\}$ induced by the realization $\{a_1\}$ or it could be $\{a\}$, which is induced by the realization $\{a_2\}$. Nonetheless, we can define semantics based on the claim-range by focusing on the underlying set E of arguments. We consider *cl-preferred*, *cl-naive*, *cl-cf-stable*, *cl-ad-stable*, *cl-semi-stable* and *cl-stage* semantics (abbr. *cl-pr*, *cl-na*, *cl-stb$_{cf}$*, *cl-stb$_{ad}$*, *cl-ss*, *cl-stg*) as introduced in (Rapberger 2020; Dvorák, Rapberger, and Woltran 2020a).

Definition 3.5. Let $\mathcal{F} = (A, R, cl)$ be a CAF with underlying AF $F = (A, R)$. For a set of claims $S \subseteq cl(A)$,

- $S \in$ *cl-pr*$(\mathcal{F})$ if S is $\subseteq$-maximal in $ad_c(\mathcal{F})$;
- $S \in$ *cl-na*$(\mathcal{F})$ if S is $\subseteq$-maximal in $cf_c(\mathcal{F})$;
- $S \in$ *cl-stb$_\tau$*$(\mathcal{F})$, $\tau \in \{cf, ad\}$, if there is a τ_c-realization E of S which defeats any $c \in cl(A) \setminus S$ (i.e., $E^{\oplus}_{\mathcal{F}} = cl(A)$);
- $S \in$ *cl-ss*$(\mathcal{F})$ if there is an ad_c-realization E of S in $\mathcal{F}$ such that there is no $D \in ad(F)$ with $E^{\oplus}_{\mathcal{F}} \subsetneq D^{\oplus}_{\mathcal{F}}$;
- $S \in$ *cl-stg*$(\mathcal{F})$ if there is an cf_c-realization E of S in $\mathcal{F}$ such that there is no $D \in cf(F)$ with $E^{\oplus}_{\mathcal{F}} \subsetneq D^{\oplus}_{\mathcal{F}}$.

A set $E \subseteq A$ *cl-σ-realizes* the claim-set S in $\mathcal{F}$ if $cl(E) = S$ and E satisfies the respective requirements; e.g., $E \in cf(F)$ and $E^{\oplus}_{\mathcal{F}} = cl(A)$ for cl-cf-stable semantics. We call E a *cl-σ*-realization of S in $\mathcal{F}$.

Example 3.6. Consider the semantics *cl-stb$_{cf}$*. We have that $S = \{c, b\} \in$ *cl-stb$_{cf}$*$(\mathcal{F})$ since the realization $E = \{c_1, b_1\}$ for S has full claim-range as we already observed before. Moreover, $S' = \{d, a\} \in$ *cl-stb$_{cf}$*$(\mathcal{F})$ as well: We consider the realization $E' = \{d_1, a_1\}$. The claims c and b are defeated by E' and hence, $E'^{\oplus}_{\mathcal{F}} = \{a, b, c, d\}$. Note that E' is not a stable extension of the underlying AF.

Basic relations between i-semantics carry over from AF semantics, e.g., $stb_c(\mathcal{F}) \subseteq ss_c(\mathcal{F}) \subseteq pr_c(CF) \subseteq co_c(CF) \subseteq ad_c(\mathcal{F}) \subseteq cf_c(\mathcal{F})$ and $stb_c(\mathcal{F}) \subseteq stg_c(\mathcal{F}) \subseteq na_c(\mathcal{F}) \subseteq cf_c(\mathcal{F})$. As shown in (Dvorák, Rapberger, and Woltran 2020a), we have $stb_c(\mathcal{F}) \subseteq$ *cl-stb$_{ad}$*$(\mathcal{F}) \subseteq$ *cl-stb$_{cf}$*$(\mathcal{F}) \subseteq$ *cl-stg*$(\mathcal{F}) \subseteq na_c(\mathcal{F})$ and *cl-stb$_{ad}$*$(\mathcal{F}) \subseteq$ *cl-ss*$(\mathcal{F}) \subseteq pr_c(\mathcal{F})$. Moreover, each *cl-$\sigma$*-claim-set of $\mathcal{F}$ is $\subseteq$-maximal in $\sigma_c(\mathcal{F})$ for $\sigma \in \{pr, na\}$.

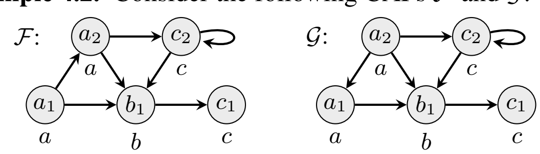
Notation. We write $\mathcal{F} = (F, cl)$ as an abbreviation for $\mathcal{F} = (A, R, cl)$ with AF $F = (A, R)$ (similar for CAFs $\mathcal{G}$ or $\mathcal{H}$ for which we denote the corresponding AFs by G and H, respectively). Also, we use the subscript-notation $A_{\mathcal{F}}$, $R_{\mathcal{F}}$, $cl_{\mathcal{F}}$, and $F_{\mathcal{F}}$ to indicate the affiliations.

4 Ordinary Equivalence

We start our analysis by investigating ordinary equivalence.

Definition 4.1. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *ordinary equivalent* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv^{\rho}_{o} \mathcal{G}$, if we have $\rho(\mathcal{F}) = \rho(\mathcal{G})$.

Example 4.2. Consider the following CAFs $\mathcal{F}$ and $\mathcal{G}$:



Although $\mathcal{F}$ and $\mathcal{G}$ disagree only on the direction of the attack between the arguments a_1 and a_2, we observe that $\mathcal{F}$ and $\mathcal{G}$ are not ordinary equivalent under i-stable semantics: $stb_c(\mathcal{F}) = \emptyset$ while $\mathcal{G}$ has the unique i-stable claim-set $\{a, c\}$ witnessed by the stable extension $\{a_2, c_1\}$ of G.

If we consider instead cl-stable semantics, we observe that the two CAFs agree on their outcome: First notice that $\{a, c\}$ is also cl-ad-stable (cl-cf-stable) in $\mathcal{G}$ (every stb_c-realization is admissible and has full claim-range). Moreover, we have that $\{a, c\}$ is also cl-ad-stable (cl-cf-stable) in $\mathcal{F}$ since the set $\{a_1, c_1\}$ is admissible and defeats every remaining claim. As a side remark, we mention that the claim-set $\{a, c\}$ has two realizations in $\mathcal{F}$ and $\mathcal{G}$ since both of the sets $\{a_1, c_1\}$, $\{a_2, c_1\}$ are conflict-free and have full claim-range. We obtain that the CAFs $\mathcal{F}$ and $\mathcal{G}$ are ordinary equivalent with respect to *cl-stb$_{ad}$* and *cl-stb$_{cf}$* semantics.

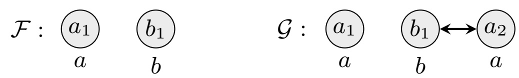
There are only few relations that hold in general between the semantics for ordinary equivalence. We summarize them as follows:

Proposition 4.3. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$,*

- $\mathcal{F} \equiv_o^{\rho} \mathcal{G} \Rightarrow \mathcal{F} \equiv_o^{cl\text{-}pr} \mathcal{G}$, $\rho \in \{ad_c, pr_c\}$;
- $\mathcal{F} \equiv_o^{co_c} \mathcal{G} \Rightarrow \mathcal{F} \equiv_o^{\rho} \mathcal{G}$, $\rho \in \{gr_c, cl\text{-}pr\}$;
- $\mathcal{F} \equiv_o^{cf_c} \mathcal{G} \Leftrightarrow \mathcal{F} \equiv_o^{cl\text{-}na} \mathcal{G}$;
- $\mathcal{F} \equiv_o^{na_c} \mathcal{G} \Rightarrow \mathcal{F} \equiv_o^{\rho} \mathcal{G}$, $\rho \in \{cf_c, cl\text{-}na\}$.

Interestingly, we observe that the relations for AF semantics presented in (Oikarinen and Woltran 2011) do not carry over to inherited semantics. This is due to the fact that i-preferred (i-naive) semantics are not necessarily $\subseteq$-maximal i-admissible (i-conflict-free) claim-sets; for CAFs, this role is instead taken over by cl-preferred (cl-naive) semantics.

Example 4.4. Assume we are given two CAFs as follows:



Clearly, $ad_c(\mathcal{F}) = ad_c(\mathcal{G}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. On the other hand, $\{a, b\}$ is the unique i-preferred claim-set of $\mathcal{F}$ while $pr_c(\mathcal{G}) = \{\{a\}, \{a, b\}\}$ witnessed by the extensions $\{a_1, a_2\}$ and $\{a_1, b_1\}$. Thus $\mathcal{F} \equiv_o^{ad_c} \mathcal{G} \not\Rightarrow \mathcal{F} \equiv_o^{pr_c} \mathcal{G}$. The example furthermore shows $\mathcal{F} \equiv_o^{cf_c} \mathcal{G} \not\Rightarrow \mathcal{F} \equiv_o^{na_c} \mathcal{G}$ since cf_c and ad_c as well as the respective variants of naive and preferred semantics coincide in $\mathcal{F}$ and $\mathcal{G}$.

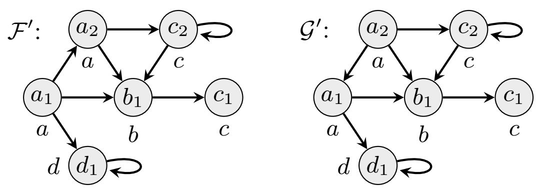
The relations presented in Proposition 4.3 follow since cl-preferred claim-sets are $\subseteq$-maximal in $ad_c(\mathcal{F})$, $co_c(\mathcal{F})$ and $pr_c(\mathcal{F})$ for any CAF $\mathcal{F}$; moreover, the i-grounded claim-set is the $\subseteq$-minimal i-complete extension. Similar observations hold for conflict-free and naive semantics; additionally, we observe that $\mathcal{F} \equiv_o^{\rho} \mathcal{G}$, $\rho \in \{cl\text{-}na, na_c\}$, implies $\mathcal{F} \equiv_o^{cf_c} \mathcal{G}$ since cf_c semantics satisfies downward closure (every subset of a conflict-free set is conflict-free). We can construct counter-examples for the remaining cases.

5 Strong Equivalence

In this section, we discuss strong equivalence for CAFs. We introduce a novel kernel which characterizes strong equivalence for cl-cf-stable and cl-stage semantics moreover, we show that the remaining semantics can be characterized via known kernels for AFs.

A crucial observation is that ordinary equivalence is not robust when it comes to expansions of the frameworks, e.g., if an update in the knowledge base induces new arguments or attacks. Let us illustrate this at the following example:

Example 5.1. Assume we are given an updated version of $\mathcal{F}$ and $\mathcal{G}$ from Example 4.2 where an additional argument has been introduced. Let $\mathcal{F}'$ and $\mathcal{G}'$ be given as follows:



$\mathcal{F}'$ and $\mathcal{G}'$ no longer agree on their cl-ad-stable claim-sets: In $\mathcal{G}'$, the set $\{a_2, c_1\}$ does not defeat claim d, thus $cl\text{-}stb_{ad}(\mathcal{G}') = \emptyset$ while $\{a, c\}$ remains cl-ad-stable in $\mathcal{F}'$.

Let us introduce a stronger notion of equivalence which addresses such situations. We say that two CAFs are *strongly equivalent* to each other if they possess the same extensions independently of any such (simultaneous) expansions of the frameworks. Before we can define this notion formally, we require an additional concept which ensures that the expansion of the frameworks is well-defined.

Definition 5.2. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *compatible* to each other if $cl_{\mathcal{F}}(a) = cl_{\mathcal{G}}(a)$ for all $a \in A_{\mathcal{F}} \cap A_{\mathcal{G}}$. The union $\mathcal{F} \cup \mathcal{G}$ of two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$ is defined componentwise, i.e., $\mathcal{F} \cup \mathcal{G} = (A_{\mathcal{F}} \cup A_{\mathcal{G}}, R_{\mathcal{F}} \cup R_{\mathcal{G}}, cl_{\mathcal{F}} \cup cl_{\mathcal{G}})$.

We are ready to introduce strong equivalence for CAFs.

Definition 5.3. Two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$ are *strongly equivalent* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv_s^{\rho} \mathcal{G}$, iff $\rho(\mathcal{F} \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$ for each CAF $\mathcal{H}$ which is compatible with $\mathcal{F}$ and $\mathcal{G}$.

The definition extends strong equivalence for AFs. We write $F \equiv_s^{\sigma} G$ to denote strong equivalence of two AFs F and G w.r.t. the semantics σ.

Strong equivalence for AFs has been characterized via syntactic equivalence of so-called (semantics-dependent) kernels. Let us recall the definitions of the stable and the naive kernel (Oikarinen and Woltran 2011; Baumann, Linsbichler, and Woltran 2016) as they exhibit interesting overlaps with our novel kernel for cl-cf-stable semantics.

Definition 5.4. For an AF $F = (A, R)$, we define the *stable kernel* $F^{sk} = (A, R^{sk})$ with

$$R^{sk} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\};$$

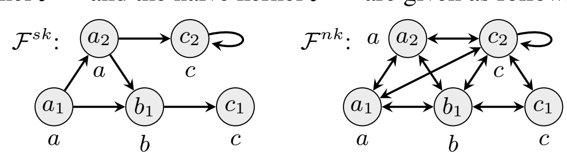
and the *naive kernel* $F^{nk} = (A, R^{nk})$ with

$$R^{nk} = R \cup \{(a, b) \mid a \neq b, \{(a, a), (b, b), (b, a)\} \cap R \neq \emptyset\}.$$

For a CAF $\mathcal{F} = (F, cl)$, we write $\mathcal{F}^{sk}$ ($\mathcal{F}^{nk}$) to denote (F^{sk}, cl) ((F^{nk}, cl), respectively).

The stable kernel characterizes strong equivalence for stable and stage semantics, i.e., $F \equiv_s^{\sigma} G$ iff $F^{sk} = G^{sk}$ for $\sigma \in \{stb, stg\}$ (Oikarinen and Woltran 2011); similarly, $F \equiv_s^{\sigma} G$ iff $F^{nk} = G^{nk}$ for $\sigma \in \{cf, na\}$ (Baumann, Linsbichler, and Woltran 2016).

Example 5.5. For the CAF $\mathcal{F}$ from Example 4.2, the stable kernel $\mathcal{F}^{sk}$ and the naive kernel $\mathcal{F}^{nk}$ are given as follows:



In the remaining part of this section, we characterize strong equivalence for all semantics under consideration by identifying appropriate kernels. Let us start with cl-cf-stable semantics. An interesting observation is that the CAFs $\mathcal{F}'$ and $\mathcal{G}'$ from Example 5.1 yield the same cl-cf-stable claim-sets even after the argument d_1 has been added. In fact, it can be shown that $\mathcal{F}$ and $\mathcal{G}$ yield the same cl-cf-stable claim-sets under any possible expansion. The reason is that the direction of the attack between a_1 and a_2 is irrelevant since both arguments possess the same claim a. Thus it suffices to include one of them in a cl-cf-stable claim-set in case not both of them are attacked.

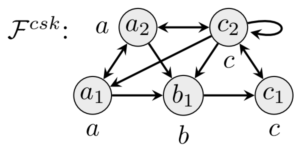
Let us now introduce the cf-stable kernel for CAFs.

Definition 5.6. For a CAF $\mathcal{F} = (A, R, cl)$, we define the cf-stable kernel as $\mathcal{F}^{csk} = (A, R^{csk}, cl)$ with

$$\begin{aligned} R^{csk} = R \cup \{(a,b) \mid a \neq b, \\ (a,a) \in R \vee (cl(a) = cl(b) \wedge \{(b,a),(b,b)\} \cap R \neq \emptyset)\}. \end{aligned}$$

We denote the underlying AF (A, R^{csk}) by F^{csk}.

Example 5.7. Consider again our previous CAF $\mathcal{F}$. We construct the cf-stable kernel $\mathcal{F}^{csk}$ of $\mathcal{F}$ as follows:



Remark 5.8. The cf-stable kernel consists of a combination of the stable and the naive kernel for AFs, where the claim-independent part stems from the stable kernel while the case where two arguments have the same claim relates to the naive kernel. In a nutshell, it is save to introduce attacks (a, b), $a \neq b$ where a is self-attacking without changing stable semantics because attacks of this form neither interfere with the conflict-free extensions of an AF nor change the range of a conflict-free set. In case two arguments have the same claim, it is irrelevant which of these arguments is included in an extension. It is thus save to introduce attacks between two arguments in case their union is conflicting.

The following main theorem formalizes that the cf-kernel characterizes strong equivalence for claim-level cf-stable and stage semantics.

Theorem 5.9. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F}^{csk} = \mathcal{G}^{csk}$ iff $\mathcal{F} \equiv_s^\rho \mathcal{G}$ for $\rho \in \{cl\text{-}stb_{cf}, cl\text{-}stg\}$.*

The remaining semantics under consideration can be characterized via known AF kernels. We recall the AF kernels from the literature (Oikarinen and Woltran 2011).

Definition 5.10. For an AF $F = (A, R)$, we define the *admissible kernel* $F^{ak} = (A, R^{ak})$ with

$$R^{ak} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R, \{(b,a),(b,b)\} \cap R \neq \emptyset\};$$

the *complete kernel* $F^{gk} = (A, R^{gk})$ with

$$R^{ck} = R \setminus \{(a,b) \mid a \neq b, (a,a), (b,b) \in R\};$$

and the *grounded kernel* $F^{gk} = (A, R^{gk})$ with

$$R^{gk} = R \setminus \{(a,b) \mid a \neq b, (b,b) \in R, \{(b,a),(a,a)\} \cap R \neq \emptyset\}.$$

It has been shown that the grounded (complete) kernel characterizes strong equivalence for grounded (complete) semantics; moreover, for any two AFs F and G we have $F \equiv_s^\sigma G$ iff $F^{ak} = G^{ak}$ for $\sigma \in \{ad, pr, ss\}$ (Oikarinen and Woltran 2011). We write $F^{k(\rho)}$ to denote the kernel which characterizes strong equivalence for the semantics ρ.

It can be shown that two CAFs are strongly equivalent under cl-ad-stable and cl-semi-stable semantics iff their admissible kernels coincide.

Theorem 5.11. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv_s^\rho \mathcal{G}$ iff $F^{ak} = G^{ak}$ for $\rho \in \{cl\text{-}stb_{ad}, cl\text{-}ss\}$.*

Moreover, each each inherited semantics σ_c can be characterized by the respective kernel for σ.

Theorem 5.12. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv_s^{\sigma_c} \mathcal{G}$ iff $F \equiv_s^\sigma G$ for any considered AF semantics σ.*

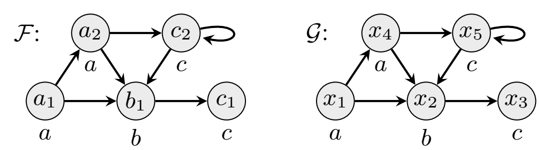
For cl-naive and cl-preferred semantics, it can be shown that strong equivalence w.r.t. cl-naive and cl-preferred semantics coincides with strong equivalence w.r.t. their inherited counterparts. This implies that two CAFs are strongly equivalent w.r.t. cl-preferred semantics iff their admissible kernels coincide; likewise, two CAFs are strongly equivalent w.r.t. cl-naive semantics iff their naive kernels coincide.

Theorem 5.13. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv_s^{cl\text{-}\sigma} \mathcal{G}$ iff $\mathcal{F} \equiv_s^{\sigma_c} \mathcal{G}$ for $\sigma \in \{na, pr\}$.*

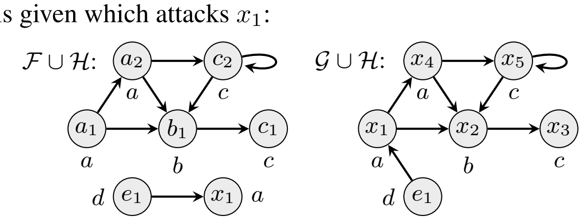
6 Renaming and Equivalence

In the previous section we were assuming that we are interested in the actual arguments and not just the claims and their interactions. In this section, we will also provide another point of view which entirely abstracts from the underlying arguments and thus treats a CAF as a collection of claims and their relationships. To illustrate this, let us consider the following example.

Example 6.1. Assume we are given two CAFs $\mathcal{F}$ (cf. Example 4.2) and $\mathcal{G}$ which both stem from instantiating the same knowledge base using different argument naming schemes – the CAF $\mathcal{F}$ relates argument names with the corresponding claim (e.g., arguments with claim a are named a_i) while $\mathcal{G}$ uses a consecutive numbering for all arguments:



It is evident that $\mathcal{F}$ and $\mathcal{G}$ are ordinary equivalent w.r.t. all considered semantics despite the mismatch in argument names because they represent the same knowledge base. However, when we consider equivalence in a dynamic setting, we observe that different argument naming patterns can cause unwanted effects. To illustrate this let us suppose we are given $\mathcal{H}$ in a way that a novel argument e_1 with claim e is given which attacks x_1:

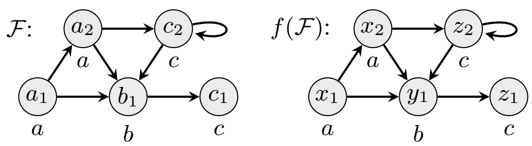


This is fine when insisting on the arguments, but on a claim-level one could of course argue that $\mathcal{H}$ did not yield the same modification on both sides and thus disrupts the similarity between $\mathcal{F}$ and $\mathcal{G}$ in an unintended way.

The example suggests that the usual notion of strong equivalence does not handle situations where we are interested in claims only very well. Our goal is hence to develop notions of equivalence which handle such scenarios in a more intuitive way. The first step to formalize the underlying idea is the following notion of a renaming.

Definition 6.2. For a CAF $\mathcal{F}$ and a set A' of arguments we call a mapping $f : A_{\mathcal{F}} \to A'$ a *renaming for $\mathcal{F}$*. By $f(\mathcal{F})$ we denote the CAF $(f(F), cl_f) := (f(A), R_f, cl_f)$ where $(a, b) \in R_f$ iff $(f(a), f(b)) \in R_{\mathcal{F}}$ and $cl_f(f(a)) = cl_{\mathcal{F}}(a)$.

Example 6.3. Consider again our previous CAF $\mathcal{F}$. Let us assume we are given $A' = \{x_1, x_2, y_1, z_1, z_2\}$. The renaming f with $a_i \mapsto x_i$, $b_1 \mapsto y_1$ and $c_i \mapsto z_i$ induces the following CAF $f(\mathcal{F})$:



We observe that f does not change the structure of $\mathcal{F}$ on claim-level. In particular, we observe that $\rho(\mathcal{F}) = \rho(f(\mathcal{F}))$ for all considered semantics ρ.

The last observation we made was no coincidence in the specific situation. More precisely, for the semantics considered in this paper, renaming does not change the meaning of our CAF.

Proposition 6.4. *For a CAF $\mathcal{F}$, an arbitrary set A' of arguments and a renaming f we have $\rho(\mathcal{F}) = \rho(f(\mathcal{F}))$ for any semantics ρ considered in this paper.*

Having formally established that names of arguments do not change the given semantics, let us proceed with defining notions of equivalence that build upon this insight.

Definition 6.5. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *ordinary equivalent up to renaming* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv^{\rho}_{or} \mathcal{G}$, if there is some set A of arguments and some renaming $f : A_{\mathcal{F}} \to A$ for $\mathcal{F}$ s.t. $\rho(f(\mathcal{F})) = \rho(\mathcal{G})$.

So, informally speaking, Definition 6.5 requires that $\mathcal{F}$ and $\mathcal{G}$ are equivalent, at least after the underlying arguments are relabeled in a suitable way. However, in Proposition 6.4 we have actually already established that this adjustment is superfluous for our semantics. More formally, we infer the following result.

Proposition 6.6. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, $\mathcal{F} \equiv^{\rho}_{or} \mathcal{G}$ iff $\mathcal{F} \equiv^{\rho}_{o} \mathcal{G}$ for any semantics ρ under consideration.*

Considering this result, it becomes apparent that we could also require that $\rho(f(\mathcal{F})) = \rho(\mathcal{G})$ holds for any renaming, not just for one in particular.

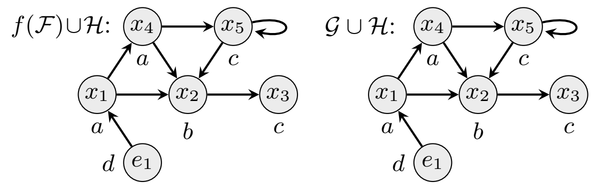
Proposition 6.7. *For two CAFs $\mathcal{F}$ and $\mathcal{G}$ we have that for all semantics considered in this paper $\mathcal{F} \equiv^{\rho}_{or} \mathcal{G}$ implies $\rho(f(\mathcal{F})) = \rho(\mathcal{G})$ for any renaming f for $\mathcal{F}$.*

Now we utilize the notion of a renaming in order to define a strong equivalence-like relation which is more suitable than strong equivalence for situations like the one described in Example 6.1.

Definition 6.8. Two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$ are *strongly equivalent up to renaming* to each other w.r.t. a semantics ρ, in symbols $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$, if there is a renaming $f : A_{\mathcal{F}} \to A_{\mathcal{G}}$ for $\mathcal{F}\rho(f(\mathcal{F}) \cup \mathcal{H}) = \rho(\mathcal{G} \cup \mathcal{H})$ for each CAF $\mathcal{H}$ which is compatible with $\mathcal{F}$ and $\mathcal{G}$.

Let us reconsider our motivating Example 6.1.

Example 6.9. Recall the CAFs $\mathcal{F}$, $\mathcal{G}$ from before and consider a renaming f with $f(a_1)=x_1$, $f(b_1)=x_2$, $f(c_1)=x_3$, $f(a_2)=x_4$, and $f(c_2)=x_5$. Augmenting both $f(\mathcal{F})$ and $\mathcal{G}$ with the CAF $\mathcal{H}$, we obtain the following desired situation:



Notice that Proposition 6.4 ensures that our renaming for $\mathcal{F}$ only prevents $\mathcal{H}$ from introducing a novel argument, while preserving the semantics of $\mathcal{F}$.

Strong equivalence up to renaming implies the usual strong equivalence. This can be obtained by setting $f = id$.

Proposition 6.10. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, if $\mathcal{F} \equiv^{\rho}_{s} \mathcal{G}$, then $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$ for all considered semantics ρ.*

Even without using Proposition 6.4 explicitly we can infer that strong equivalence survives moving to a renamed version of f as well.

Proposition 6.11. *For any two compatible CAFs $\mathcal{F}$ and $\mathcal{G}$, if $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$, then $f(\mathcal{F}) \equiv^{\rho}_{sr} \mathcal{G}$ for any renaming f for $\mathcal{F}$, for all semantics ρ under consideration.*

Let us now come to the kernels. Since our notion of strong equivalence up to renaming allows for changing the names of the arguments, we expect our kernels to behave similarly. More specifically, we also need to consider renamed versions of the CAFs before evaluating the kernels. However, checking strong equivalence up to renaming will surely require to take the structure of the CAFs into consideration. We thus define what we mean by a CAF isomorphism.

Definition 6.12. Two CAFs $\mathcal{F}$ and $\mathcal{G}$ are *isomorphic* to each other iff there is a renaming $f : A_{\mathcal{F}} \to A_{\mathcal{G}}$ such that for all $a, b \in A_{\mathcal{F}}$, $cl_{\mathcal{F}}(a) = cl_{\mathcal{G}}(f(a))$ and $(f(a), f(b)) \in R_{\mathcal{G}}$ iff $(a, b) \in R_{\mathcal{F}}$; f is called *isomorphism* between $\mathcal{F}$ and $\mathcal{G}$.

CAFs $\mathcal{F}$ and $f(\mathcal{F})$ from Example 6.3 are isomorphic. The given renaming f naturally is a CAF-isomorphism between $\mathcal{F}$ and $f(\mathcal{F})$. The following proposition collects basic properties of CAF isomorphisms.

Proposition 6.13. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, (a) if $\mathcal{F}$ and $\mathcal{G}$ are isomorphic, then $\rho(\mathcal{F}) = \rho(\mathcal{G})$ for any considered semantics ρ; and (b) if f is a renaming for $\mathcal{F}$, then $\mathcal{F}$ and $f(\mathcal{F})$ are isomorphic.*

As it turns out, we obtain *exactly* the result we desire to: We check strong equivalence up to renaming by choosing the appropriate kernel for ρ, computing the kernels of $\mathcal{F}$ and $\mathcal{G}$ and then checking whether those are isomorphic to each other. Informally speaking, our tailored notion of equivalence which does not take the names of arguments into account yields the exact same kernels after relabeling the arguments in a suitable way.

Theorem 6.14. *For any two CAFs $\mathcal{F}$ and $\mathcal{G}$, for any semantics ρ under consideration, $\mathcal{F} \equiv^{\rho}_{sr} \mathcal{G}$ iff $\mathcal{F}^{k(\rho)}$ and $\mathcal{G}^{k(\rho)}$ are isomorphic.*

Example 6.15. For our CAFs $\mathcal{F}$ and $\mathcal{G}$ from Example 6.1 we see that their kernels are isomorphic. Hence $\mathcal{F}$ and $\mathcal{G}$ are strongly equivalent up to renaming w.r.t. all semantics considered in this paper.

7 Computational Complexity

In this section we examine the computational complexity of deciding equivalence between two CAFs $\mathcal{F}$ and $\mathcal{G}$ for every equivalence notion which has been established in this paper. We assume the reader to be familiar with the polynomial hierarchy. Moreover, by $\mathrm{QSAT}^{\exists}_n$ ($\mathrm{QSAT}^{\forall}_n$) we denote the generic Σ^{P}_n-complete (Π^{P}_n-complete) problem, i.e. checking validity of a corresponding QBF. Our results reveal that ordinary equivalence can be computationally hard, up to the third level of the polynomial hierarchy for both variants of semi-stable and stage semantics as well as for i-preferred semantics. For the remaining semantics under consideration, the problem is Π^{P}_2-complete; the only exception is i-grounded semantics for which deciding ordinary equivalence is P-complete. Moreover, we show that deciding strong equivalence up to renaming extends the list of problems which lie in NP but are not known to be NP-complete.

First we present our complexity results for ordinary equivalence. We formulate the following decision problem:

$\textsc{Ver-OE}_\rho$
Input: Two CAFs $\mathcal{F}$, $\mathcal{G}$.
Output: TRUE iff $\mathcal{F}$, $\mathcal{G}$ are ordinary equivalent w.r.t. ρ.

We obtain the following computational complexity results for deciding ordinary equivalence:

Theorem 7.1. $\textsc{Ver-OE}_\rho$ *is*

- P*-complete for* $\rho = gr_c$*;*
- Π^{P}_2*-complete for* $\rho \in \{cf_c, ad_c, co_c, na_c, cl\text{-}pr, cl\text{-}na, stb_c, cl\text{-}stb_{cf}, cl\text{-}stb_{ad}, \}$*; and*
- Π^{P}_3*-complete for* $\rho \in \{pr_c, ss_c, stg_c, cl\text{-}stg, cl\text{-}ss\}$*.*

Observe that the computational complexity results from Theorem 7.1 extend to ordinary equivalence up to renaming by Proposition 6.6 for any semantics under consideration. Having established complexity results for ordinary equivalence it remains to discuss the computational complexity of strong equivalence and strong equivalence up to renaming.

$\textsc{Ver-SE}_\rho$
Input: Two CAFs $\mathcal{F}$, $\mathcal{G}$.
Output: TRUE iff $\mathcal{F}$, $\mathcal{G}$ are strongly equivalent w.r.t. ρ.

Recall that in Section 5, we have shown that strong equivalence of two CAFs $\mathcal{F}$ and $\mathcal{G}$ can be characterized via syntactic equivalence of their kernels. Since the computation and comparison of the kernels of $\mathcal{F}$ and $\mathcal{G}$ can be done in polynomial time, we obtain tractability of strong equivalence for every semantics under consideration.

Theorem 7.2. *The problem* $\textsc{Ver-SE}_\rho$ *can be solved in polynomial time for any semantics ρ considered in this paper.*

Finally, we consider strong equivalence up to renaming. An analogous decision problem be formulated as follows:

$\textsc{Ver-SER}_\rho$
Input: Two CAFs $\mathcal{F}$, $\mathcal{G}$.
Output: TRUE iff $\mathcal{F}$, $\mathcal{G}$ are strongly equivalent up to renaming w.r.t. ρ.

As outlined above, the computation of the kernels lies in P and is therefore negligible; the complexity of verifying strong equivalence up to renaming thus stems entirely from deciding whether two labelled graphs (i.e., the kernels of the given CAFs) are isomorphic. As a consequence we obtain that the complexity of $\textsc{Ver-SER}_\rho$ coincides with the complexity of the well-known graph isomorphism problem.

Theorem 7.3. *The problem* $\textsc{Ver-SER}_\rho$ *is exactly as hard as the graph isomorphism problem for any semantics ρ considered in this paper.*

8 Conclusion and Future Work

In this paper, we considered ordinary and strong equivalence as well as novel equivalence notions based on argument renaming for CAFs w.r.t. all semantics for CAFs which have been considered in the literature so far and provided a complexity analysis of all considered equivalence notions.

Our characterization results for strong equivalence are in line with existing studies for related argumentation formalisms (Oikarinen and Woltran 2011; Dvorák, Rapberger, and Woltran 2020b); in addition, we adapt an argument-independent view by considering equivalence under renaming. Equivalence of logic-based argumentation has been studied in (Amgoud, Besnard, and Vesic 2014); they show that under certain conditions on the underlying logic, unnecessary arguments can be removed while retaining (strong) equivalence. In contrast to their work, our studies are independent of the underlying formalism of the instantiated argumentation system as we do not impose any further constraints on the arguments or their claims; in this way, it is even possible to test equivalence between argumentation systems stemming from entirely different base formalisms.

For future work, we want to extend our strong equivalence studies by considering certain constraints of the framework modifications. What has been commonly investigated in the literature are *normal expansions* where attacks can only be introduced if they involve newly added arguments. We also mention that our investigation was focusing on CAFs with unrestricted attacks. While this reflects the behavior of certain instantiations (Cyras and Toni 2016; Modgil and Prakken 2014), oftentimes out-going attacks are characterized by the conclusions of arguments. It would be interesting to extend our results to this class of CAFs.

Acknowledgements

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