PRELIMINARY PREPRINT VERSION: DO NOT CITE
The AAAI Digital Library will contain the published
version some time after the conference.

Amortized Generation of Sequential Algorithmic Recourses for Black-box Models

Sahil Verma,[1,2] Keegan Hines,[1] John P. Dickerson[1,3]
[1] Arthur AI
[2] University of Washington
[3] University of Maryland
vsahil@cs.washington.edu, keegan@arthur.ai, john@arthur.ai

Abstract

Explainable machine learning (ML) has gained traction in recent years due to the increasing adoption of ML-based systems in many sectors. *Algorithmic Recourses* (ARs) provide "what if" feedback of the form "if an input datapoint were x' instead of x, then an ML-based system's output would be y' instead of y." ARs are attractive due to their actionable feedback, amenability to existing legal frameworks, and fidelity to the underlying ML model. Yet, current AR approaches are single shot—that is, they assume x can change to x' in a single time period. We propose a novel stochastic-control-based approach that generates *sequential* ARs, that is, ARs that allow x to move stochastically and sequentially across intermediate states to a final state x'. Our approach is model agnostic and black box. Furthermore, the calculation of ARs is amortized such that once trained, it applies to multiple datapoints without the need for re-optimization. In addition to these primary characteristics, our approach admits optional desiderata such as adherence to the data manifold, respect for causal relations, and sparsity—identified by past research as desirable properties of ARs. We evaluate our approach using three real-world datasets and show successful generation of sequential ARs that respect other recourse desiderata.

1 Introduction

Machine learning (ML) models are increasingly used to make predictions in systems that directly or indirectly impact humans. This includes critical applications like healthcare (Faggella 2020), finance (Singla 2020), hiring (Sennaar 2019), and parole (Tashea 2017). To understand ML models better and to promote their equitable impact in society, it is necessary to assess stakeholders'—both expert (Holstein et al. 2019) and layperson (Saha et al. 2020)—comprehension of and needs for general observability into their systems (Poursabzi et al. 2021; Ehsan et al. 2021a). The nascent Fairness, Accountability, Transparency, and Ethics in machine learning (aka "FATE ML") community conducts research to develop methods to detect (and counteract) bias in ML models, develop techniques that make complex models explainable, and propose policies to advise and adhere to the regulations of algorithmic decision-making (see Appendix A). Here, we focus on ML model explainability.

Research in explainable ML is bifurcated. One high-level approach aims to develop inherently interpretable models such as decision trees and linear models (Rudin 2019). Another high-level approach aims to utilize existing complex classification techniques (such as deep neural networks) but to bolster them with surrogate models that can render their predictions and/or internal processes understandable (Adadi and Berrada 2018). This is achieved through explaining models holistically (global explanation) or single predictions from the model (local explanation).

Algorithmic Recourses (ARs). ARs find the minimal change in a datapoint such that the ML model ends up classifying the new datapoint in the desired class. Such new datapoint(s) is termed as a counterfactual. (We provide an in-depth discussion of terminology in Appendix D.) For example, if an individual were denied a loan request, a recourse might tell them that their request would be approved if they were to increase their income by $2000. ARs provide a precise recommendation and are therefore more actionable than other forms of local explainability like feature importance. Recent research in this area has aimed to ensure ARs are actionable and useful by incorporating additional desiderata into the recourse generation problem. As described in Section 2, these include notions of sparsity, causality, and realism of ARs, among others. What is needed (see, e.g., Verma, Dickerson, and Hines 2020; Chou et al. 2021; Karimi et al. 2020b) is a generalized approach that can accommodate such varied constraints and can also be computed efficiently.

Operationalizing ARs. We propose a novel approach (FastAR) for generating ARs by translating a given recourse generation problem into a Markov Decision Process (MDP). FastAR aims to learn a policy that can generate ARs for given data distribution. Upon learning that policy once, it can generate ARs for multiple datapoints (from that distribution) without the need to re-optimize (which is required by most previous approaches; see Appendix B). Thus, FastAR *amortizes* the cost of repeatedly computing ARs. FastAR also allows enforcing desirable properties of ARs, such as closeness to the training data distribution (data manifold), respect of causal relations between the features, and mutability and actionability of different features. FastAR works for *black-box* models and is therefore *model agnostic*.

Via the learned policy, FastAR outputs ARs as a sequence

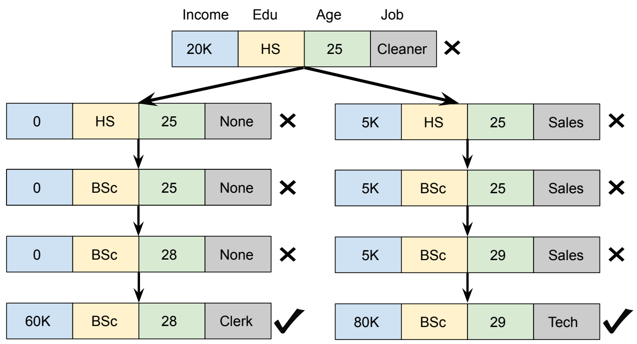


Figure 1: Example of Stochastic Algorithmic Recourses. Starting with a datapoint (✘ denotes undesired class prediction), FASTAR *can stochastically generate ARs that lead to different counterfactual states (✔ denotes desired class prediction).*

of steps that lead an individual to a counterfactual state. To our knowledge, we are the first to leverage techniques from stochastic control to provide such *sequential ARs* (Ramakrishnan, Lee, and Albarghouthi 2020; Naumann and Ntoutsi 2021). That sequence can also adhere to particular *sparsity* constraints (e.g., only one feature changing per step).

Sequential and "rolled out" ARs have several advantages, directly addressing gaps identified by recent survey papers (Verma, Dickerson, and Hines 2020; Chou et al. 2021; Karimi et al. 2020b) and workshops (Ehsan et al. 2021b): 1) action sparsity allows an individual to focus their effort on changing a small number of features at a time; and 2) presentation of ARs as a set of discrete and sequential steps is closer to real-world actions, rather than one-step continuous change, which most previous approaches do. Singh et al. (2021) recently conducted a user-study with 54 participants, wherein each of them was presented with 15 scenarios and asked if they preferred one-shot or directed sequential AR in that scenario. When overall results were pulled, the study concluded a preference for sequential ARs with high confidence.

Figure 1 shows an example of sequential ARs which are generated for an individual whose loan request was denied (shown by ✘). Instead of a one-shot solution, FASTAR delineates all intermediate steps to reach a counterfactual state (shown by ✔). FASTAR also models the stochastic factors like the duration to complete a BSc degree, no or part-time job during the course, and the salary variance in the new job after graduation, which lead to different recourse paths and hence different counterfactual states (as shown in Figure 1).

In summary, our contributions are:

1. A novel algorithm that translates an AR problem into a Markov decision process (MDP). To the best of our knowledge, our stochastic-control-based approach is the first to address several roadblocks to using ARs in practice that have been identified by the community (Verma, Dickerson, and Hines 2020; Chou et al. 2021; Karimi et al. 2020b).
2. The first approach that generates sequential and amortized ARs, and also works for black-box models.
3. An extensive evaluation with three real-world datasets and nine baselines.

2 Desiderata of *Practical* ARs

The overarching goal of an AR is to provide practical guidance to an individual seeking to change their treatment (e.g., class label) by a deployed ML model. Apart from the necessary property of a AR having a desired class label, other desiderata have been identified in the literature, enumerated here:

- *Actionability*: ARs should only recommend changes to the features that are actionable (Ustun, Spangher, and Liu 2019; Kanamori et al. 2020; Dandl et al. 2020). Actionable features are dataset and preference dependent.
- *Sparsity*: Social studies have argued that smaller explanations are more comprehensible to humans (Miller 2019). Therefore ARs should make changes to a small set of features (Van Looveren and Klaise 2020; Karimi et al. 2020a).
- *Data manifold*: To obey the correlations between features, their input domain, and to be realistic, ARs should adhere to the training data manifold (Dhurandhar et al. 2019; Kanamori et al. 2020; Dandl et al. 2020).
- *Causal constraints*: In order to adhere to real-world constraints in ARs, causal constraints between features must be respected. They can encode facts like age cannot decrease or increase in education level increases age (Mahajan, Tan, and Sharma 2020).
- *Model-agnostic*: For wide-spread applicability, AR generating approaches should be model-agnostic (Laugel et al. 2018; Guidotti et al. 2018a).
- *Black-box models*: For applicability to proprietary ML models, AR generating approaches should work for black-box models (Sharma, Henderson, and Ghosh 2019).
- *Amortized*: An *amortized* approach can generate ARs for several datapoints without optimizing separately for each of them. Such an approach is effective for deployment (Mahajan, Tan, and Sharma 2020).

FASTAR satisfies all the above desiderata. To the best of our knowledge, it is the first approach to do so (see Table 1). The choice of action space helps produce ARs that consider actionability among features and are sparse. It only modifies the actionable features. Its ARs are realistic as they adhere to the training data manifold and respect causal relations between features. FASTAR works for black-box models and, therefore, is model-agnostic. It learns a policy that can produce ARs for several input datapoints without the need to optimize again; and, therefore, generates amortized ARs.

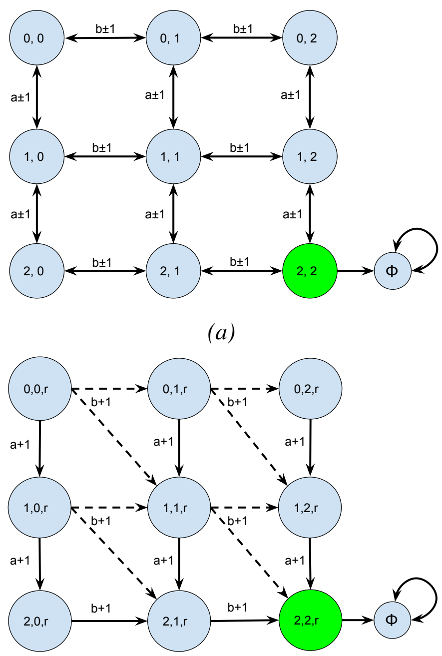
3 Examples of Translating ARs to MDPs

We now give two examples of translating an AR problem into an MDP. Once modeled as an MDP, we can use various off-the-shelf algorithms (from planning or RL) to learn a policy to generate ARs.

Example 1: Consider two categorical features $\mathsf{a}, \mathsf{b} \in \{0, 1, 2\}$. The combinations of possible values for a and b form the state space for the MDP (represented by $\mathcal{S}$). The directed edges in Figure 2a show that upon taking a specific action, an agent can move from one state to another, e.g., it transits from state (0, 1) to (0, 2) by taking the action b+1, which increments the value of feature b by 1. Actions a+1 and a-1 respectively increase and decrease the value of fea-

Table 1: Desiderata comparison of various AR generating approaches. FASTAR *is the **first and only one** which satisfies all desiderata.*

Approach	Actionability	Sparsity	Agnostic	Black-box	Amortized	Manifold	Constraints
CFE Expl. (Wachter, Mittelstadt, and Russell 2017)	✗	✓	✗	✗	✗	✗	✗
Recourse (Ustun, Spangher, and Liu 2019)	✓	✓	✗	✗	✗	✗	✗
CEM (Dhurandhar et al. 2019)	✗	✓	✗	✗	✗	✓	✗
MACE (Karimi et al. 2020a)	✓	✓	✗	✗	✗	✗	✗
DACE (Kanamori et al. 2020)	✓	✗	✗	✗	✗	✓	✗
DiCE (Mothilal, Sharma, and Tan 2020)	✓	✓	✗	✗	✗	✗	✗
DiCE VAE (Mahajan, Tan, and Sharma 2020)	✓	✗	✗	✗	✓	✓	✓
Spheres (Laugel et al. 2018)	✗	✓	✓	✓	✗	✗	✗
LORE (Guidotti et al. 2018a)	✗	✓	✓	✓	✗	✗	✗
Weighted (Grath et al. 2018)	✗	✗	✓	✓	✗	✗	✗
CERTIFAI (Sharma, Henderson, and Ghosh 2019)	✓	✗	✓	✓	✗	✗	✗
Prototypes (Van Looveren and Klaise 2020)	✗	✓	✓	✓	✗	✓	✗
MOC (Dandl et al. 2020)	✓	✓	✓	✓	✗	✓	✗
FASTAR	✓	✓	✓	✓	✓	✓	✓



(a)

(b)

Figure 2: Transition function for the two examples. Circles show all the states, and edges show possible transitions. 1) Left-hand-side shows the transition function for a dataset with two features a *and* b*, with no restrictions on the values both of them can take within the input domain. The transition edges are therefore bidirectional. 2) Right-hand-side shows the transition function for a dataset with three features: age (*a*), education-level (*b*), and race (*r*). The transition edges are unidirectional as both age and education cannot decrease. Since race is immutable, there are no actions for* r*. Since an increase in education stochastically affects age, the dashed edges represent a 50% probability of transition.*

ture a by 1 (similarly for feature b. These actions constitute the action space for the MDP (represented by $\mathcal{A}$). The third component of the MDP is the transition function which is represented by $T : s \times a \rightarrow s'$. This denotes that if an agent takes action a in state s then it will move to state s'. This transition function is deterministic because taking the action a in state s will always land the agent in the state s'.

The final component of the MDP is the reward function. Taking action costs something (negative reward), and reaching desirable states generate a positive reward. In this MDP taking any action costs a constant amount of 1 and reaching the terminal state (ϕ) gives a reward of +10. The terminal state (ϕ) can only be reached via (2,2) (using any action), the state in green color. All actions in the terminal state lead to itself with 0 cost. This represents the situation in which a ML model classifies only (2,2) in the desired class.

The aim is to learn a policy that reaches a terminal state from any state at the lowest cost (e.g., taking the fewest number of steps). Cost (or reward) can be discounted in the traditional way using a discount factor $\gamma \in [0, 1)$. Formally, for this example with a discrete state space and discrete action space, our MDP is:

- States = $\{s \in \mathcal{S} : \{0, 0\}, \{0, 1\}, \{0, 2\}, \{1, 0\}, \dots\}$.
- Actions = $\{a \in \mathcal{A} : \text{a+1}, \text{a-1}, \text{b+1}, \text{b-1}\}$.
- Transition function $T : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{S}$
- Reward function $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$.
- Discount factor $\gamma \in [0, 1)$, capturing the tradeoff between current and future reward.

Our goal is finding a policy $\pi : \mathcal{S} \rightarrow \mathcal{A}$ that, given a state $s \in \mathcal{S}$ (an input datapoint), returns an action $a \in \mathcal{A}$ that represents the best first step to take to reach a new state, hopefully closer to the ML model's decision boundary. FASTAR would then call this precomputed policy repeatedly to find an optimal path to a counterfactual state.

Example 2: Now, consider a more realistic dataset having 3 features: age (denoted by a), education-level (denoted by b), and race (denoted by r). This is accompanied by real-world constraints like age and education-level cannot decrease, education-level affects age, and race is immutable. When we increase the education-level (b) by 1, there is a 50% chance that age group (a) will remain the same and a 50% chance that it will increase by 1. These interactions between features can be captured by a structural causal model (SCM), as we discuss in Section 4. The transition function for the MDP representing the AR problem for this dataset is, therefore, stochastic.

Defined formally, here are the components for this MDP:

- States = $\{s \in \mathcal{S} : \{0, 0, 0\}, \{0, 1, 0\}, \{0, 2, 1\}, \dots\}$.

- Actions = $\{a \in \mathcal{A} : \mathsf{a+1}, \mathsf{a-1}, \mathsf{b+1}, \mathsf{b-1}\}$.
- Transition function $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S}' \rightarrow \{0,1\}$ s.t. $\forall s \in \mathcal{S}, \forall a \in \mathcal{A}, \ \sum_{s' \in \mathcal{S}} T(S, A, S') = 1$.
- Reward function $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$.
- Discount factor $\gamma \in [0, 1)$.

Figure 2b shows the transition function for this problem. The action that increases the education-level (b) now has a probabilistic transition to two destination states, represented by dashed unidirectional edges. Each transition edge has a 50% probability of occurrence. Unidirectionality comes from the fact that education-level cannot decrease. The edges change feature a are also unidirectional as age cannot decrease. The reward function is identical to the previous example; optionally, it can be changed to accommodate adherence to the data manifold (Section 2) or having different costs for changing different features, which we describe in Section 4. Additional examples can be found in Appendix C.

4 An Algorithmic Approach for Generating MDPs from AR Problems

We now present a general approach for translating an AR problem setup into an MDP. Algorithm 1 generates all components of an MDP: state space, action space, transition function, reward function, and additional parameters such as discount factor. We detail this process below.

State space. Features can be broadly categorized into numerical (Num) and categorical (Cat) kinds. Numerical features can take real number values within a specified domain, while categorical features are mapped to a set of integers. Consequently, the state space $\mathcal{S}$ of our MDP (line 1) consists of the product of the continuous domains for numerical features (a subset of $\mathbb{R}^{|\mathsf{Num}|}$) and product of the integer domains for categorical features (a subset of $\mathbb{Z}^{|\mathsf{Cat}|}$).

Action space. To facilitate capturing actionability (Ustun, Spangher, and Liu 2019) and causal relationships between features (Karimi, Schölkopf, and Valera 2020), we further categorize features as follows:

- *Actionable* features can be directly changed by an individual, e.g., income, education level, age.
- *Mutable but not actionable* features are mutable but cannot be modified directly by an individual, e.g., credit score cannot be directly changed by a person, it changes due to change in features like income and credit history.
- *Immutable* features cannot change, e.g., race, birthplace.

The agent is permitted to change only the actionable numerical and categorical features (denoted by NumA and CatA). Consequently, the action space $\mathcal{A}$ is a subset of $\mathbb{R}^{|\mathsf{NumA}|} \times \mathbb{Z}^{|\mathsf{CatA}|}$ (line 2). Categorical features are changed within their discrete domain, while numerical features are changed within their continuous domain. Line 13 further enforces the infeasibility of out-of-domain actions.

Transition function. The transition function (line 12) finds the modified state when an action is taken. This function is influenced by the structural causal model (SCM), which is an *optional* input to Algorithm 1. An SCM consists of a triplet M = $\langle \mathsf{U}, \mathsf{V}, \mathsf{F} \rangle$. U is the set of *exogenous* features and V is the set of *endogenous* features. In terms of a causal graph, the exogenous features U consist of features that have no parents, i.e., they can change independently. The endogenous features V consists of features that have parents in U and/or other features in V. They change as an effect of change in their parents. F is the set of functions that determine the relationship between exogenous and endogenous features. They are termed as structural equations.

Since knowing the exact SCM is often infeasible, Mahajan, Tan, and Sharma (2020) overcome this limitation by utlizing constraints from domain knowledge. Algorithm 1 also accepts such contraints in unary (Un) and binary (Bin) forms.

ALGORITHM 1: Generate MDP from an Algorithmic Recourse Problem

Input : Training Dataset (D), ML model (f), Structural Causal Model (SCM), Numerical actionable features (NumA), Categorical actionable feature (CatA), Data Manifold distance function (`DistD`), Data Manifold adherence (λ), Desired Label (L), Distance Function (`DistF`), Discount Factor (γ)

Output : MDP

// States consist of all numerical (Num) and categorical (Cat) features.

1 State space $\mathcal{S} \subseteq \mathbb{R}^{|\mathsf{Num}|} \times \mathbb{Z}^{|\mathsf{Cat}|}$

// Actions change the actionable numerical and categorical features.

2 Action space $\mathcal{A} \subseteq \mathbb{R}^{|\mathsf{NumA}|} \times \mathbb{Z}^{|\mathsf{CatA}|}$; denote actions $A \in \mathcal{A}$

3 **Function** `Reward`(f, L, CurrState, A, D, λ, `DistD`, SCM)

4   NextState ← `Transition`(CurrState, A, SCM)

5   **if** `argmax`(f(NextState)) = L **then**

6   CFReward ← Pos // High positive reward

7   **else**

8   CFReward ← f(NextState)[L] // Probability of classification in the desired class

9   **return** `DistF`(CurrState, A, D) // action cost

10   $+\lambda *$ `DistD`(NextState, D) // Manifold distance cost

11   +CFReward // Counterfactual label reward

12 **Function** `Transition`(CurrState, A, SCM)

  // Action does not violate feature domain and unary constraints

13   **if** `Allowed` *(A)* & `InDomain` *(A)* **then**

14   NextState ← CurrState + A // Modify features

15   **else**

16   **return** CurrState

  // Modify the endogenous features

17   **for** V ∈ SCM **do**

18   **if** $A \in$ `Parent`(V) **then**

19   NextState[V] ← F(U) // Stochastic or deterministic update of endogenous features

20   **return** NextState

21 MDP ← $\{\mathcal{S}, \mathcal{A},$ `Transition`, `Reward`, $\gamma\}$

Even if this does not provide us with the precise functional form of the constraint, its nature can help the FASTAR's recourses to be realistic. Unary constraints are derived from the property of one feature, e.g., age and education level cannot decrease. Binary constraints are derived from the relation between two features, e.g., if the education level increases, age increases. If an action does not violate the domain of the feature it is changing, nor the constraints in the SCM, then the feature is modified in NextState (line 14). If the modified feature is an exogenous feature, we update its children using the F functions (line 17-19).

Note that if no SCM is input to the algorithm, that will allow transitions from any state to any state (with intermediate states), and FASTAR would generate ARs using this unconstrained transition function.

Reward function. Line 3 defines a reward function that, given a state and an action, returns a reward based on three components derived from the initial AR problem:

- Given the current state (CurrState), action (A), training dataset (D), and distance function `DistF`, the first part returns the appropriate cost to take that action (line 9). The distance function can either be the ℓ_p norm of the change produced by the action or a more complex function.
- The second part adds a cost if a datapoint is far from the training data manifold (line 10) (which is computed using the `DistD` function) A λ factor is used to control the strictness of data manifold adherence.
- The third part rewards the agent with a large positive value if a counterfactual state is reached (CFReward in line 11). To avoid sparse rewards, we partially reward the agent with a small reward equal to the probability of NextState being classified in the desired class (line 8). However, the sparse rewards can only be used if the underlying ML model provides the class label probabilities instead of only the class label, e.g., a neural network or random forest.

Other parameters. MDPs require additional parameters such as the discount factor $\gamma \in [0, 1]$. At a high level, setting $\gamma < 1$ penalizes longer paths; for additional intuition, see Sutton and Barto (2018). We note that λ, DistD, and DistF are user-specified and domain-specific parameters that directly impact the reward function for the MDP. We instantiate them in the evaluation section (see section 5).

5 Evaluation

We provide experimental validation of FASTAR using three real-world datasets and comparison using nine baselines. Our research questions (RQ) are motivated by the recourse desiderata discussed in Section 2, and are enumerated here:

RQ1 Does FASTAR successfully generate ARs for various input datapoints (validity)?

RQ2 How much change is required to reach a counterfactual state (proximity)?

RQ3 How many features are changed to reach a counterfactual state (sparsity)?

RQ4 Do the generated ARs adhere to the data manifold (realisticness)?

RQ5 Do the generated ARs respect causal and feature immutability constraints (feasibility)?

RQ6 How much time does FASTAR take to generate ARs (amortizability)?

Datasets. Motivated by most previous AR generating approaches (Verma, Dickerson, and Hines 2020), we use three datasets in our experiments: German Credit, Adult Income, and Credit Default (Dua and Graff 2017). These datasets have 20, 13 (omitted education-num as it has one to one mapping with education), and 23 features respectively. We split the datasets into 80%-10%-10% for training, validation, and testing, respectively. Each dataset has two labels, '1' and '0', where '1' is the desired label. We trained a simple classifier: a neural network with two hidden layers (5 and 3 neurons) with ReLU activations. The test accuracy of the classifier was 83.0% for German Credit, 83.7% for Adult Income, and 83.2% for Credit Default. Note that the classifier's accuracy is relatively less important for FASTAR's validation.

Implementation Algorithm. Any appropriate method for computing an optimal policy $\pi^* : \mathcal{S} \to \mathcal{A}$, or any approximately optimal policy, to the MDP output of Algorithm 1 can be used. Our MDPhas a continuous state and action space, and therefore we use a policy gradient algorithm. Specifically, we use proximal policy optimization (PPO) with generalized advantage estimate (GAE) (Schulman et al. 2017; Mnih et al. 2016; Schulman et al. 2018) to train the agent. We justify this choice and answer several other related questions in Appendix E. The features in all datasets are scaled between -1 and 1 before training both the ML model and the RL agent.

5.1 Baselines

Since, to our knowledge, FASTAR is the first approach to generate amortized ARs for black-box models, there exist no previous approaches which we can directly compare against. Nevertheless, we compare FASTAR to several previous popular AR generating approaches.

Baselines we developed. To compare FASTAR to approaches that generated ARs in an amortized manner for black-box models, we developed two baselines:

- **Random:** This approach tries to reach a counterfactual state by executing random actions from the action space.
- **Greedy:** At each step, this approach greedily chooses the action (among all actions) which gives the highest reward.

Previous AR generating approaches. Based on the level of required model access, AR generating approaches can be categorized as: 1) access to complete model internals, i.e., weights of neurons or nodes of decision trees, 2) access to model gradients (restricted to differentiable models like neural networks), and 3) access to only the `predict` function (black-box). We choose popular methods from all categories:

- **Complete model internal access.** We chose MACE (Karimi et al. 2020a) from this category.
- **Gradients access.** Here we chose DiCE-Gradient (Mothilal, Sharma, and Tan 2020) and DiCE-VAE (Mahajan, Tan, and Sharma 2020). Notably, DiCE-VAE is the *only other* amortized AR generation method, however, it requires gradients and is restricted to differentiable models.
- **Black-box.** Open-source repository of the aforementioned DiCE method also had three black-box and model-agnostic approaches, namely: DiCE-Genetic, DiCE-KD-Tree, and

Table 2: Causal constraints and immutable features for the datasets. We assume FASTAR *is provided with them.*

Dataset	Causal constraints	Immutable features
German Credit	Age and Job cannot decrease	Foreign worker, Number of liable people, Personal status, Purpose
Adult Income	Age and Education cannot decrease, increasing Education increases Age	Marital-status, Race, Native-country, Sex
Credit Default	Age and Education cannot decrease, increasing Education increases Age	Sex, Marital status

DiCE-Random. We choose these three and Prototypes (Van Looveren and Klaise 2020) for this category. We did not compare with MOC (Dandl et al. 2020) as DiCE-Genetic it also a genetic algorithm based approach and has uses Python code.

5.2 Experimental Methodology

Here we describe the specific details of some approaches:

FASTAR specifics. As stated in Section 4, the recourses generated by FASTAR are realistic if provided with the actionability of features and causal constraints. These constraints can be provided using the complete/partial SCM of the data generating process or using domain knowledge. We assume these constraints are provided to FASTAR and are shown in Table 2. As described in Algorithm 1, this directly impacts the transition function. We use a particular instantiation of Algorithm 1 in the experiments:

- Action space: To produce sequential ARs, actions modify only one feature at a time. However, endogenous features may simultaneously change due to change in their parent.
- Cost of action: We treat DistF function as a hyperparameter and use several values for it in the experiments.
- Data manifold distance: Following previous work (Dandl et al. 2020; Kanamori et al. 2020), we train k-Nearest Neighbor (KNN) algorithm on the training dataset and use it to find the ℓ_1 distance of a given datapoint from its nearest neighbor ($k = 1$) in the dataset (DistD). We use several values of the adherence factor λ in the experiments.
- Counterfactual state reward ($\mathsf{CFReward}$): The agent receives a reward equal to the probability of its state belonging to the desired class (this ranges between 0 and 1). However, when a counterfactual state is reached, the agent is rewarded with 100 points.
- Discount Factor: We use a discount factor $\gamma = 0.99$. This value encourages the agent to learn a policy that takes a few steps to reach a counterfactual state.

We explore the impact of λ hyperparameter in Appendix G, and give more implementation details in Appendix F.

MACE specifics. MACE requires as input the type of ML classifier to be used. We could not use a neural network because of the MACE's long runtime (see section 5.3), and and therefore choose logistic regression (LR) and random forest (RF), which had a reasonable runtime.

All approaches are requested to generate ARs for the test datapoints that are predicted as '0' by the classifier. Due to the small size of the German Credit dataset, we generate ARs for datapoints that are predicted as '0' both in the training and test sets. Thus we request ARs for 257 datapoints in the German credit, 7229 datapoints in the Adult Income, and 5363 datapoints in the Credit Default datasets. Since MACE uses a different classifier, the number of datapoints predicted as '0' were slightly different. More details are provided in section 5.3. FASTAR, random, and greedy approaches stop when they reach a counterfactual state (predicted as '1') or exhaust 50 actions. Other baselines have no such timeout.

5.3 Results

Table 3 shows the performance of FASTAR and all the baselines on the recourse desiderata. We report the average validity, average proximity (separately for the numerical and categorical features), average sparsity, average data manifold distance, average causal constraints adherence, and the average time to generate the ARs per datapoint.

Answer to RQ1: As shown in Table 3, FASTAR has very high validity for all datasets. For Adult Income, FASTAR gets the highest validity at 100%, while for Credit Default and German Credit, it achieves the second and third highest validity, respectively. Random and greedy approaches have low validity in general. DiCE-Genetic has validity in the high range, but this comes at the cost of proximity, sparsity, and data manifold distance. DiCE-KDTree is unable to generate AR even for a single datapoint in all three datasets. DiCE-Random achieves 100% validity for all datasets, and just like DiCE-Genetic, this comes at the cost of proximity, sparsity, and data manifold distance. The conclusion is similar for DiCE-Gradient's and Prototypes' validity. DiCE-VAE's validity is lower than 80% for all datasets. MACE also achieves 100% validity but is very expensive to run. Due to this, it was impractical to run MACE for the larger datasets, Adult Income and Credit Default (we show MACE run only for the German Credit dataset). MACE was even more expensive when the underlying classifier was a neural network, and we had to abandon that experiment. For the classifiers used for MACE, '0' was predicted for 210 datapoints by logistic regression (LR) and 287 datapoints by random forest (RF). MACE was supposed to generate ARs for these datapoints.

Answer to RQ2: We measure proximity for numerical and categorical features separately (Prox-Num and Prox-Cat, respectively). For numerical features, the distance is the sum of the ℓ_1 norm respectively divided by the median average deviation for each numerical feature. For categorical features, the distance is the number of categorical features changed divided by the total number of categorical features. These metrics were proposed and used in previous works (Mahajan, Tan, and Sharma 2020). FASTAR's ARs are most proximal for Adult Income and Credit Default datasets, and second best for German Credit. The random approach, Prototypes, and the five variants of DiCE have large proximity values. The greedy approach performs well on this metric, but its validity is very low. MACE's performance is about average.

Answer to RQ3: FASTAR achieves the lowest sparsity among all approaches. Following previous works (Mothilal, Sharma, and Tan 2020), we measure sparsity at the start and endpoint of a recourse. Random, Prototypes, DiCE-VAE, DiCE-Genetic, and DiCE-Gradient's performance is abysmal.

Table 3: Comparison of FASTAR *to all baselines for various AR evaluation metrics.* `Validity` *is the percentage an AR is actually classified in the desired class.* `Prox-Num` *and* `Prox-Cat` *refers to the L1 distance of the datapoint and its AR for the numerical and categorical features respectively.* `Sparsity` *is the number of features that were changed to produce the AR.* `Manifold dist.` *is the distance of the AR as returned by the trained kNN algorithm.* `Constraints` *refer to the causal constraints adherence by the generated AR.* `Time` *is the average time to generate ARs. For* `Validity` *and* `Constraints`*, a higher value is better, and for all other columns, a lower value is better. MACE and DiCE-Gradient could not be run for larger datasets owing to their large computation time.*

Dataset	Approach	#DataPts.	Validity	Prox-Num	Prox-Cat	Sparsity	Manifold dist.	Constraints	Time (s)
German Credit	Random	257	23.7	0.17	0.57	11.33	1.08	41.0	0.31
	Greedy	257	49.8	**0.07**	0.087	1.81	0.48	**100.0**	4.59
	DiCE-Genetic	257	98.1	0.67	0.26	6.52	2.39	45.6	1.71
	DiCE-KDTree	257	0.0	N/A	N/A	N/A	N/A	N/A	0.17
	DiCE-Random	257	**100.0**	0.33	0.10	1.93	2.40	93.4	0.17
	Prototypes	100	**100.0**	0.26	0.56	12.8	0.97	14.0	13.7
	DiCE-Gradient	257	**100.0**	0.27	0.29	6.33	2.19	82.9	7.10
	DiCE-VAE	257	77.8	0.80	0.42	10.12	0.97	5.0	0.15
	MACE (LR)	210	**100.0**	0.36	**0.017**	1.99	0.60	97.1	38.45
	MACE (RF)	287	**100.0**	0.22	0.02	2.64	**0.38**	74.2	101.29
	FASTAR	257	97.3	0.10	0.063	**1.22**	0.72	**100.0**	**0.07**
Adult Income	Random	7229	80.9	0.56	0.77	10.07	1.00	29.0	0.25
	Greedy	7229	97.7	**0.04**	0.02	1.18	**0.17**	95.0	0.27
	DiCE-Genetic	7229	89.5	0.71	0.27	4.43	0.46	23.0	3.43
	DiCE-KDTree	7229	0.0	N/A	N/A	N/A	N/A	N/A	0.59
	DiCE-Random	7229	**100.0**	0.82	0.04	1.64	1.24	90.0	0.22
	Prototypes	100	**100.0**	0.28	0.57	9.0	0.68	27.0	25.4
	DiCE-Gradient	500	84.0	0.18	0.012	2.78	0.51	82.4	59.75
	DiCE-VAE	7229	77.1	0.75	0.65	9.99	0.30	0.13	0.12
	FASTAR	7229	**100.0**	**0.04**	**0.0**	**1.00**	0.18	**100.0**	**0.015**
Credit Default	Random	5363	12.8	4.85	0.68	14.54	1.30	41.5	0.63
	Greedy	5363	65.1	0.15	**0.072**	1.25	**0.22**	99.9	4.67
	DiCE-Genetic	5363	92.6	3.93	0.49	16.67	2.75	27.9	3.58
	DiCE-KDTree	5363	0.0	N/A	N/A	N/A	N/A	N/A	0.45
	DiCE-Random	5363	**100.0**	5.80	0.20	2.33	3.09	97.7	0.39
	Prototypes	100	**100.0**	4.8	0.87	21.0	1.25	0.0	25.6
	DiCE-Gradient	100	81.0	0.77	0.40	15.98	1.35	85.2	479.17
	DiCE-VAE	5363	76.4	1.6	0.68	20.1	0.31	8.9	0.18
	FASTAR	5363	99.9	**0.01**	0.11	**1.008**	0.32	**100.0**	**0.051**

This is surprising because DiCE-Gradient has a post-hoc step specifically for reducing sparsity. Greedy, MACE, and DiCE-Random's performance is about average.

Answer to RQ4: FASTAR achieves low average manifold distance. It performs second best for Adult Income and Credit Default and is in the middle for German Credit. The greedy approach, MACE, and DiCE-VAE also perform well on this metric. The random approach, Prototypes, and all variants of DiCE (except DiCE-VAE) perform poorly on this metric.

Answer to RQ5: By construction, FASTAR always respects causal constraints encoded in its transition function: it has 100% adherence in all datasets. The DiCE based approaches (except DiCE-VAE), MACE, and random approach take as input the immutable features, but not the other causal constraints and hence do not perform well. DiCE-VAE and Prototypes do not accept immutable features and hence perform the worst for this metric. The greedy approach performs well on this metric, even though it does not have a knowledge of the causal constraints.

Answer to RQ6: The final column in Table 3 reports the average computation time per AR. Owing to amortization, FASTAR can generate ARs very quickly and takes the lowest time among all approaches. The next best performers are DiCE-VAE and DICE-Random. FASTAR is **2**× faster than DiCE-VAE on average (up to **8**× faster), **8**× faster than DiCE-random on average (up to **15**× faster). DiCE-random and random approach perform similarly. The difference even more staggering for DiCE-Genetic, Prototypes, and greedy approach. MACE and Dice-Gradient were the slowest. FASTAR is about **1000**× faster than MACE on average (up to **1447**× faster) and **4500**× faster than DiCE-Gradient on average (up to **9400**× faster). While amortization allows for the rapid generation of new ARs, there exists a one-time training cost. We give details about it in Appendix F.

6 Conclusion

We propose a novel RL-based approach, FASTAR, that generates amortized and sequential recourses for black-box ML models. To the best of our knowledge, we are the first to propose such an approach. The ARs generated by FASTAR possess desirable properties and when evaluated on the recourse metrics, they perform better than several popular baselines.

References

Adadi, A.; and Berrada, M. 2018. Peeking inside the black-box: A survey on Explainable Artificial Intelligence (XAI). *IEEE Access*, PP.

Andrews, R.; Diederich, J.; and Tickle, A. B. 1995. Survey and Critique of Techniques for Extracting Rules from Trained Artificial Neural Networks. *Know.-Based Syst.*, 8.

Brockman, G.; Cheung, V.; Pettersson, L.; Schneider, J.; Schulman, J.; Tang, J.; and Zaremba, W. 2016. OpenAI Gym. arXiv:arXiv:1606.01540.

Carvalho, D. V.; Pereira, E. M.; and Cardoso, J. S. 2019. Machine learning interpretability: A survey on methods and metrics. *Electronics*, 8.

Chipman, H. A.; George, E. I.; and Mcculloch, R. E. 1998. Making Sense of a Forest of Trees. In *Proceedings of the 30th Symposium on the Interface*.

Chou, Y.-L.; Moreira, C.; Bruza, P.; Ouyang, C.; and Jorge, J. 2021. Counterfactuals and Causability in Explainable Artificial Intelligence: Theory, Algorithms, and Applications. *arXiv preprint arXiv:2103.04244*.

Craven, M. W.; and Shavlik, J. W. 1995. Extracting Tree-Structured Representations of Trained Networks. In *Conference on Neural Information Processing Systems (NeurIPS)*. Cambridge, MA, USA: MIT Press.

Dandl, S.; Molnar, C.; Binder, M.; and Bischl, B. 2020. Multi-Objective Counterfactual Explanations. *arXiv:2004.11165 [cs, stat]*.

Deng, H. 2014. Interpreting Tree Ensembles with inTrees. *arXiv:1408.5456*.

Dhurandhar, A.; Chen, P.-Y.; Luss, R.; Tu, C.-C.; Ting, P.; Shanmugam, K.; and Das, P. 2018. Explanations Based on the Missing: Towards Contrastive Explanations with Pertinent Negatives. In *Conference on Neural Information Processing Systems (NeurIPS)*.

Dhurandhar, A.; Pedapati, T.; Balakrishnan, A.; Chen, P.-Y.; Shanmugam, K.; and Puri, R. 2019. Model Agnostic Contrastive Explanations for Structured Data. *arXiv:1906.00117 [cs, stat]*.

Dhurandhar, A.; and Shanmugam, K. 2020. Counterfactual vs Contrastive Explanations in Artificial Intelligence. https://towardsdatascience.com/counterfactual-vs-contrastive-explanations-in-artificial-intelligence-e67a9cfc7e4e. Accessed: 2021-05-15.

Domingos, P. 1998. Knowledge Discovery Via Multiple Models. *Intell. Data Anal.*, 2.

Dua, D.; and Graff, C. 2017. UCI Machine Learning Repository.

Dunkelau, J.; and Leuschel, M. 2019. Fairness-Aware Machine Learning.

Ehsan, U.; Liao, Q. V.; Muller, M.; Riedl, M. O.; and Weisz, J. D. 2021a. Expanding Explainability: Towards Social Transparency in AI systems. In *CHI*.

Ehsan, U.; Wintersberger, P.; Liao, Q. V.; Mara, M.; Streit, M.; Wachter, S.; Riener, A.; and Riedl, M. O. 2021b. Operationalizing Human-Centered Perspectives in Explainable AI. In *CHI*.

Faggella, D. 2020. Machine Learning for Medical Diagnostics – 4 Current Applications. https://emerj.com/ai-sector-overviews/machine-learning-medical-diagnostics-4-current-applications/. Accessed: 2020-10-15.

Ghallab, M.; Nau, D.; and Traverso, P. 2016. *Automated Planning and Acting*. USA: Cambridge University Press, 1st edition. ISBN 1107037271.

Grath, R. M.; Costabello, L.; Van, C. L.; Sweeney, P.; Kamiab, F.; Shen, Z.; and Lecue, F. 2018. Interpretable Credit Application Predictions With Counterfactual Explanations. *arXiv:1811.05245 [cs]*.

Guidotti, R.; Monreale, A.; Ruggieri, S.; Pedreschi, D.; Turini, F.; and Giannotti, F. 2018a. Local Rule-Based Explanations of Black Box Decision Systems.

Guidotti, R.; Monreale, A.; Ruggieri, S.; Turini, F.; Giannotti, F.; and Pedreschi, D. 2018b. A Survey of Methods for Explaining Black Box Models. *ACM Comput. Surv.*, 51.

Henelius, A.; Puolamäki, K.; Boström, H.; Asker, L.; and Papapetrou, P. 2014. A Peek into the Black Box: Exploring Classifiers by Randomization. *Data Min. Knowl. Discov.*, 28.

Holstein, K.; Wortman Vaughan, J.; Daumé III, H.; Dudik, M.; and Wallach, H. 2019. Improving fairness in machine learning systems: What do industry practitioners need? In *Conference on Human Factors in Computing Systems (CHI)*, 1–16.

Joshi, S.; Koyejo, O.; Vijitbenjaronk, W.; Kim, B.; and Ghosh, J. 2019. Towards Realistic Individual Recourse and Actionable Explanations in Black-Box Decision Making Systems.

Kanamori, K.; Takagi, T.; Kobayashi, K.; and Arimura, H. 2020. DACE: Distribution-Aware Counterfactual Explanation by Mixed-Integer Linear Optimization. In *International Joint Conference on Artificial Intelligence (IJCAI)*.

Karimi, A.-H.; Barthe, G.; Balle, B.; and Valera, I. 2020a. Model-Agnostic Counterfactual Explanations for Consequential Decisions. In *Proceedings of the 23rd International Conference on Artificial Intelligence and Statistics*.

Karimi, A.-H.; Barthe, G.; Schölkopf, B.; and Valera, I. 2020b. A survey of algorithmic recourse: definitions, formulations, solutions, and prospects. arXiv:2010.04050.

Karimi, A.-H.; Schölkopf, B.; and Valera, I. 2020. Algorithmic Recourse: from Counterfactual Explanations to Interventions. *arXiv:2002.06278 [cs, stat]*.

Karimi, A.-H.; von Kügelgen, J.; Schölkopf, B.; and Valera, I. 2020c. Algorithmic recourse under imperfect causal knowledge: a probabilistic approach.

Keane, M. T.; and Smyth, B. 2020. Good Counterfactuals and Where to Find Them: A Case-Based Technique for Generating Counterfactuals for Explainable AI (XAI). arXiv:2005.13997.

Kostrikov, I. 2018. PyTorch Implementations of Reinforcement Learning Algorithms. https://github.com/ikostrikov/pytorch-a2c-ppo-acktr-gail.

Krishnan, R.; Sivakumar, G.; and Bhattacharya, P. 1999. Extracting decision trees from trained neural networks. *Pattern Recognition*, 32.

Krishnan, S.; and Wu, E. 2017. PALM: Machine Learning Explanations For Iterative Debugging. In *Proceedings of the 2nd Workshop on Human-In-the-Loop Data Analytics*.

Lash, M. T.; Lin, Q.; Street, W. N.; Robinson, J. G.; and Ohlmann, J. W. 2017. Generalized Inverse Classification. In *SDM*.

Laugel, T.; Lesot, M.-J.; Marsala, C.; Renard, X.; and Detyniecki, M. 2018. Comparison-Based Inverse Classification for Interpretability in Machine Learning. In *Information Processing and Management of Uncertainty in Knowledge-Based Systems, Theory and Foundations (IPMU)*. Springer International Publishing.

Le, T.; Wang, S.; and Lee, D. 2019. GRACE: Generating Concise and Informative Contrastive Sample to Explain Neural Network Model's Prediction. arXiv:1911.02042.

Liang, E.; Liaw, R.; Nishihara, R.; Moritz, P.; Fox, R.; Goldberg, K.; Gonzalez, J. E.; Jordan, M. I.; and Stoica, I. 2018. RLlib: Abstractions for Distributed Reinforcement Learning. In *ICML*.

Liang, E.; Nishihara, R.; Liaw, R.; and et al. 2020. Ray. https://github.com/ray-project/ray.

Mahajan, D.; Tan, C.; and Sharma, A. 2020. Preserving Causal Constraints in Counterfactual Explanations for Machine Learning Classifiers. *arXiv:1912.03277 [cs, stat]*.

Miller, T. 2019. Explanation in artificial intelligence: Insights from the social sciences. *Artificial Intelligence*, 267.

Mnih, V.; Badia, A. P.; Mirza, M.; Graves, A.; Lillicrap, T.; Harley, T.; Silver, D.; and Kavukcuoglu, K. 2016. Asynchronous Methods for Deep Reinforcement Learning. In *Proceedings of The 33rd International Conference on Machine Learning*. PMLR.

Mothilal, R. K.; Sharma, A.; and Tan, C. 2020. Explaining Machine Learning Classifiers through Diverse Counterfactual Explanations. In *Proceedings of the Conference on Fairness, Accountability, and Transparency (FAccT)*, FAT* '20.

Naumann, P.; and Ntoutsi, E. 2021. Consequence-aware Sequential Counterfactual Generation. arXiv:2104.05592.

Pawelczyk, M.; Broelemann, K.; and Kasneci, G. 2020. Learning Model-Agnostic Counterfactual Explanations for Tabular Data. *Proceedings of The Web Conference (WWW)*.

Pearl, J. 2000. *Causality: Models, Reasoning, and Inference*. USA: Cambridge University Press. ISBN 0521773628.

Poulin, B.; Eisner, R.; Szafron, D.; Lu, P.; Greiner, R.; Wishart, D. S.; Fyshe, A.; Pearcy, B.; MacDonell, C.; and Anvik, J. 2006. Visual Explanation of Evidence in Additive Classifiers. In *Conference on Innovative Applications of Artificial Intelligence (IAAI)*.

Poursabzi, F. S.; Goldstein, D. G.; Hofman, J. M.; Wortman Vaughan, J. W.; and Wallach, H. 2021. Manipulating and measuring model interpretability. In *CHI*.

Poyiadzi, R.; Sokol, K.; Santos-Rodriguez, R.; De Bie, T.; and Flach, P. 2020. FACE: Feasible and Actionable Counterfactual Explanations. *Conference on Artificial Intelligence (AAAI)*.

Ramakrishnan, G.; Lee, Y. C.; and Albarghouthi, A. 2020. Synthesizing Action Sequences for Modifying Model Decisions. *Proceedings of the AAAI Conference on Artificial Intelligence*, 34.

Rathi, S. 2019. Generating Counterfactual and Contrastive Explanations using SHAP.

Ribeiro, M. T.; Singh, S.; and Guestrin, C. 2016. "Why Should I Trust You?": Explaining the Predictions of Any Classifier. In *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, KDD '16. New York, NY, USA: Association for Computing Machinery.

Rudin, C. 2019. Stop explaining black box machine learning models for high stakes decisions and use interpretable models instead. *Nature Machine Intelligence*, 1(5).

Russell, C. 2019. Efficient Search for Diverse Coherent Explanations. In *Proceedings of the Conference on Fairness, Accountability, and Transparency (FAccT)*, FAT* '19.

Saha, D.; Schumann, C.; Mcelfresh, D.; Dickerson, J.; Mazurek, M.; and Tschantz, M. 2020. Measuring non-expert comprehension of machine learning fairness metrics. In *International Conference on Machine Learning (ICML)*.

Schulman, J.; Moritz, P.; Levine, S.; Jordan, M.; and Abbeel, P. 2018. High-Dimensional Continuous Control Using Generalized Advantage Estimation. arXiv:1506.02438.

Schulman, J.; Wolski, F.; Dhariwal, P.; Radford, A.; and Klimov, O. 2017. Proximal Policy Optimization Algorithms. arXiv:1707.06347.

Selvaraju, R. R.; Cogswell, M.; Das, A.; Vedantam, R.; Parikh, D.; and Batra, D. 2017. Grad-CAM: Visual Explanations from Deep Networks via Gradient-Based Localization. In *IEEE International Conference on Computer Vision*.

Sennaar, K. 2019. Machine Learning for Recruiting and Hiring – 6 Current Applications. https://emerj.com/ai-sector-overviews/machine-learning-for-recruiting-and-hiring/. Accessed: 2020-10-15.

Sharma, S.; Henderson, J.; and Ghosh, J. 2019. CERTIFAI: Counterfactual Explanations for Robustness, Transparency, Interpretability, and Fairness of Artificial Intelligence models.

Singh, R.; Dourish, P.; Howe, P.; Miller, T.; Sonenberg, L.; Velloso, E.; and Vetere, F. 2021. Directive Explanations for Actionable Explainability in Machine Learning Applications. arXiv:arXiv:2102.02671.

Singla, S. 2020. Machine Learning to Predict Credit Risk in Lending Industry. https://www.aitimejournal.com/@saurav.singla/machine-learning-to-predict-credit-risk-in-lending-industry. Accessed: 2020-10-15.

Sutton, R. S.; and Barto, A. G. 2018. *Reinforcement Learning: An Introduction*. Cambridge, MA, USA: A Bradford Book. ISBN 0262039249.

Tashea, J. 2017. Courts Are Using AI to Sentence Criminals. That Must Stop Now. https://www.wired.com/2017/04/courts-using-ai-sentence-criminals-must-stop-now/. Accessed: 2020-10-15.

Tjoa, E.; and Guan, C. 2019. A Survey on Explainable Artificial Intelligence (XAI): Towards Medical XAI. arXiv:1907.07374.

Turner, R. 2016. A Model Explanation System: Latest Updates and Extensions.

Ustun, B.; Spangher, A.; and Liu, Y. 2019. Actionable recourse in linear classification. In *Proceedings of the Conference on Fairness, Accountability, and Transparency (FAccT)*.

Van Looveren, A.; and Klaise, J. 2020. Interpretable Counterfactual Explanations Guided by Prototypes.

Verma, S.; Dickerson, J.; and Hines, K. 2020. Counterfactual Explanations for Machine Learning: A Review. arXiv:2010.10596.

Verma, S.; and Rubin, J. 2018. Fairness Definitions Explained. In *Proceedings of the International Workshop on Software Fairness*, FairWare '18. New York, NY, USA: Association for Computing Machinery.

Wachter, S.; Mittelstadt, B.; and Russell, C. 2017. Counterfactual Explanations Without Opening the Black Box: Automated Decisions and the GDPR. *SSRN Electronic Journal*.

White, A.; and Garcez, A. d. 2019. Measurable Counterfactual Local Explanations for Any Classifier. *arXiv:1908.03020 [cs]*.

Zhou, B.; Khosla, A.; Lapedriza, A.; Oliva, A.; and Torralba, A. 2016. Learning Deep Features for Discriminative Localization. In *CVPR*.

Zien, A.; Krämer, N.; Sonnenburg, S.; and Rätsch, G. 2009. The Feature Importance Ranking Measure. In *Machine Learning and Knowledge Discovery in Databases*. Berlin, Heidelberg: Springer Berlin Heidelberg.