# An Algorithmic Introduction to Savings Circles

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## Abstract

Rotating savings and credit associations (roscas) are informal financial organizations common in settings where communities have reduced access to formal institutions. In a rosca, a fixed group of participants regularly contribute sums of money to a pot. This pot is then allocated periodically using lottery, aftermarket, or auction mechanisms. Roscas are empirically well-studied in the economics literature. Due to their dynamic nature, however, roscas have proven challenging to study theoretically, and typical economic economic analyses stop at coarse ordinal welfare comparisons to other credit allocation mechanisms and leave much of roscas' ubiquity unexplained. This work takes an algorithmic perspective on the study of roscas. We present worst-case welfare approximation guarantees, building on tools from the price of anarchy. We further use simulations to compare the welfare of outcomes as key features of the environment vary. These cardinal welfare analyses help rationalize the prevalence of roscas. We conclude by discussing several other promising avenues.

## 1 Introduction

Rotating saving and credit associations (roscas) are financial institutions common in low- and middle-income nations, as well as immigrant and refugee populations around the world. In a rosca, a group of individuals meet regularly for a defined period of time. At each meeting, members contribute a sum of money into a pot, which is then allocated via some mechanism, such as a lottery or an auction. Recipients often use this sum of money to purchase of durable goods (e.g., farming equipment, appliances, and vehicles), to buffer shocks (e.g. an unexpected medical expense), to pay of loans, and to meet other financial needs. These roscas often exist outside of legal frameworks and do not typically have a central authority to resolve disputes or enforce compliance. Instead, they provide a decentralized mechanism for peer-to-peer lending, where members who receive the pot earlier borrow from those who receive it later, and a structure for mutual support and community empowerment.

Roscas are in use in over 85 countries and are especially prevalent in communities with reduced access to formal financial institutions (Aredo 2004; Bouman 1995a; Klonner 2002; La Ferrara 2002; Raccanello and Anand 2009). Roscas account for about one-half of Cameroon's national savings; likewise, over one in six households in Ethiopia's highlands participate in *ekub*, the region's variant of roscas Bouman (1995a). Roscas and other mutual aid organizations also play a special role after shocks and disasters, such as the covid-19 pandemic, due to their ability to provide quick, targeted support within communities (Chevée 2021; Mesch et al. 2020; Travlou 2020).

Roscas are well-studied within the economics literature, with over three decades of research, starting with Besley, Coate, and Loury (1993); Kovsted and Lyk-Jensen (1999); Kuo (1993) (see the supplementary materials for further related works). Economic theory, in particular, has sought to explain how roscas act as insurance, savings, and lending among members. While such studies have deepened our understanding of roscas, they are typically constrained in two main ways. First, the standard economic approach solves exactly for equilibria, which can be especially difficult when agents are asymmetric, and more so because of roscas' dynamic nature. Second, much of the existing theory focuses on coarse-grained comparisons between the welfare of roscas and other mechanisms for allocating credit, and often concludes that roscas allocate credit suboptimally.

The present work initiates an algorithmic study of roscas. Viewing roscas through the lens of approximation and using tools from price of anarchy, we study the welfare properties of rosca outcomes without directly solving for them, and obtain cardinal welfare guarantees. We specifically quantify the allocative efficacy of roscas: *how well do roscas coordinate saving and lending among participants with heterogeneous investment opportunities*? We show roscas enable a group's lending and borrowing in a way that approximately maximizes the groups' total utility. We do so under a wide range of assumptions on both agents' values for investment and the mechanisms used for allocating the pots. This robustness may provide one explanation for their prevalence.

Our work generalizes the saving and lending formulation of Besley, Coate, and Loury (1994). We assume each participant seeks to purchase an investment, such as a durable good, but can only do so upon winning the rosca's pot. We analyze the welfare properties of typical pot allocation mechanisms, such as swap roscas, where the agents are given an initial random allocation and then swap positions in an after-market through bilateral trade agreements. We also study the price of anarchy in auction-based roscas where, during each meeting, participants bid to decide a winner among those who have not yet received a pot. Each round, participants must weigh the value of investing earlier against the utility-loss from spending to win that round.

Our technical contributions are as follows. For swap roscas, we prove that all outcomes guarantee at most a factor 2 loss. For auction-based roscas, we give full-information price of anarchy results: we study second-price sequential roscas and give a price of anarchy of 3 under a standard no-overbidding assumption. For first-price sequential roscas, we provide a ratio of $(2e+1)e/(e-1)$. Our work provides new applications of and extensions to the *smoothness* framework of Syrgkanis and Tardos (2013). However, due to the round-robin nature of rosca allocations and the fact that all payments are redistributed to members, standard smoothness arguments do not immediately yield bounds. Via a new sequential composition argument, we show that roscas based on smooth mechanisms are themselves smooth, and we go beyond smoothness to bound the distortionary impact of redistributed payments. Our above results hold under the well-studied assumption of *quasilinear* utility for money. We also simulate several natural families of swap rosca instances, and consider the impact of nonlinear utilities for money on swap rosca outcomes.

Overall, this work aims to provide greater exposure to mutual aid organizations more generally and roscas specifically to the algorithmic community. In doing so, we present a case study that techniques within algorithmic game theory can present an important toolkit to further understand and examine fundamental questions related to these financial organizations. We view this as an important research direction since, as new technologies are being introduced in low-access contexts, the need to understand these prevalent financial organizations is even more pressing. We close with a discussion on promising algorithmic research directions.

# 2 Model and Preliminaries

A rosca consisting of $n$ agents takes place over $n$ discrete and fixed time periods, or *rounds*. During each round three things occur: (1) each agent contributes an amount $p_0$ into the rosca common pot, (2) a winner for the pot is decided among those who have not yet won, and (3) the winning agent is allocated the entire pot worth $np_0$. Typically, the contributions $p_0$ are decided ex ante during the formation of the rosca. As is common in previous literature, we will not model the selection process for $p_0$, but instead take it as given (c.f. Klonner 2001).

With the rosca contribution $p_0$ fixed, we can cast roscas as an abstract multi-round allocation problem where every agent is allocated exactly one pot, and each pot is allocated to exactly one agent, illustrated in Alg 1. Each agent's value for the allocations is described by a real-valued vector $\mathbf{v}_i = (v_i^1, \ldots, v_i^n)$, with $v_i^t$ representing agent $i$'s value for winning the pot in round $t$ and having access to that money at that time. We denote allocations by $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$, where $\mathbf{x}_i = (x_i^1, \ldots, x_i^n)$ is an indicator vector, and $x_i^t = 1$ if and only if agent $i$ receives the pot in round $t$.

Based on common observations from the literature on roscas, we can make further assumptions on our model parameters. First, we assume values for allocation are non-increasing over time: i.e., for $t < t'$ and any $i$, $v_i^t \geq v_i^{t'}$. This is would follow if rosca funds are used to make lumpy investments, e.g. in a durable good, as is common in practice (Besley, Coate, and Loury 1993, 1994; Kovsted and Lyk-Jensen 1999; Klonner 2008). Agents prefer to own the good earlier rather than later, though different agents' values for owning the good earlier may vary.

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Algorithm 1: Rosca Multi-Round Allocation

**Constants**: $n$: the number of agents in the rosca, and hence also the number of rounds. $p_0$: amount contributed by each agent to the pot in each round of the rosca.

**Inputs**: Valuations $\mathbf{v}$ where $v_i^t$ indicates the value to agent $i$ of winning the pot in round $t$. $Alloc$ an allocation mechanism.

For each round $t \in \{1, 2, \ldots, n\}$

1. Each agent contributes $p_0$ into the pot
2. $Alloc$ selects the winning agent (who has not yet won a round) based on valuations $(v_1^t, v_2^t, \ldots, v_n^t)$
3. The winning agent receives the pot worth $np_0$
4. *Optional*: Some agents make payments based on $Alloc$, which are redistributed to the others as rebates.

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## 2.1 Roscas with Payments

A variety of different pot allocation mechanisms are common in practice (see Ardener 1964; Bouman 1995b). This work considers roscas where agents make payments to influence their allocations, and assumes as a first-order approximation that agents are rational. Payments in roscas take the form $\mathbf{p} = (\mathbf{p}_1, \ldots, \mathbf{p}_n)$, where $\mathbf{p}_i = (p_i^1 \ldots, p_i^n)$. As agents' abilities to save money over time are typically limited, we assume agents' utilities are additively separable across rounds, but possibly nonlinear in money. That is, agent $i$ with value vector $\mathbf{v}_i$ has utility for allocation $\mathbf{x}_i$ and payments $\mathbf{p}_i$ given by

$$u_i^{\mathbf{v}_i}(\mathbf{x}_i, \mathbf{p}_i) = \mathbf{x}_i \cdot \mathbf{v}_i - \sum\nolimits_t C(p_i^t),$$

for some disutility function $C$ that is both increasing and satisfies $C(0) = 0$. In a given round, $p_i^t$ could be positive, if the allocation mechanism requires $i$ to make payments, or negative, if a different agent's payments are redistributed to $i$. We refer to the latter as *rebates*, and assume all payments are redistributed each round, i.e. $\sum_i p_i^t = 0$ for all $t$.

An agent who makes positive payments in round $t$ has less money to spend in round $t$, and one who receives rebates in the form of negative payments has more to spend. The function $C(\cdot)$ describes agents' preferences for these changes in wealth. A more precise interpretation of $C(p_i^t)$ is as follows: assume that each agent $i$ has a per-round income of $w$. Without participating in the rosca, they would receive a utility $U(w)$ from consumption of that income, for some increasing consumption utility function $U$. Upon contributing $p_0$ into the rosca pot each round, the agent's baseline consumption utility is $U(w - p_0)$. If the rosca's allocation procedure requires additional payments (or distributes rebates) of $p_i^t$, an agent's utility from consumption becomes $U(w - p_0 - p_i^t)$. The disutility function $C$ then represents the agent's difference in utility for consumption,

$$C(p) = U(w - p_0) - U(w - p_0 - p),$$

which is increasing.

A large body of anthropological and empirical work on roscas shows that participants in the same rosca tend to have similar economic circumstances (see Ardener 1964; Aredo 2004; Mequanent 1996), so we follow the theory literature and assume $U$ and $w$, and hence $C$, are identical across agents, even if the value for receiving the pot differ between agents (Besley, Coate, and Loury 1994; Kovsted and Lyk-Jensen 1999; Klonner 2001). It is typical to assume consumption utility $U$ is weakly concave, and hence $C$ is weakly convex (Anderson and Baland 2002; Klonner 2003b, 2001). The special case of *quasilinear utilities*, where $C(p) = p$, is especially well-studied in the algorithmic game theory literature.

To measure allocative performance of a rosca, we study the total utility of participants:

$$\text{WEL}^{\mathbf{v}}(\mathbf{x}, \mathbf{p}) = \sum\nolimits_i u_i^{\mathbf{v}_i}(\mathbf{x}_i, \mathbf{p}_i).$$

Following the interpretation of $C$ in terms of consumption utility $U$, $\text{WEL}^{\mathbf{v}}(\mathbf{x}, \mathbf{p})$ represents the gain in utility to all agents for a given allocation $\mathbf{x}$ and payments $\mathbf{p}$, above the baseline total utility of $n^2 U(w - p_0)$, obtained by each of the $n$ agents obtaining utility $U(w - p_0)$ for $n$ rounds. Among all possible matchings $\mathbf{x}$ and payment profiles $\mathbf{p} \in \mathbb{R}^{n \times n}$, the optimal welfare-outcome is given by the maximum-weight matching $\mathbf{x}^* = \text{argmax}_{\mathbf{x}} \sum_i \mathbf{x}_i \cdot \mathbf{v}_i$ and payments $\mathbf{p}^* = (0, \ldots, 0)$, whose welfare is denoted $\text{OPT}(\mathbf{v}) = \text{WEL}^{\mathbf{v}}(\mathbf{x}^*, \mathbf{p}^*)$. To quantify the inefficiency of a rosca outcome $(\mathbf{x}, \mathbf{p})$, we study the approximation ratio $\text{OPT}(\mathbf{v})/\text{WEL}^{\mathbf{v}}(\mathbf{x}, \mathbf{p})$. When rosca outcomes are equilibria of auctions, as in Section 3, this ratio is also known as the price of anarchy (PoA).

**Roadmap.** In Section 3, we prove a constant-approximation for auction roscas, and in Section 4, we do the same for swap roscas. Both sets of results focus on the case of quasilinear utilities, where $C(p) = p$, and hence all welfare loss comes from allocative inefficiency. In Section 4, we further conduct simulations to study the impact of nonlinear utility on swap rosca welfare. Section 5 gives directions for future work.

## 3 Auction Roscas

Auctions are a common mechanism for allocating pots in roscas (Ardener 1964; Bouman 1995a; Klonner 2003a). Two major sources of variety in auction roscas are (1) when bids are solicited from participants and (2) the type of auction run for the bidding process. The bidding may occur either at the beginning, in which case a single (up-front) auction determines the full schedule of pot allocations, or sequentially, in which case a separate auction is held at each period to determine the allocation for the corresponding pot. We will consider first- and second-price (equivalently, ascending- and descending-price) auctions for this purpose. Payments in such roscas are typically redistributed as rebates among all of the non-winning participants.

Complicating our analysis is the fact that outcomes depend on agents' bidding behavior. We assume agents play a *Nash equilibrium (NE)* of the rosca's auction game. That is, their bidding strategy maximizes their utility given the bidding strategies of other participants. Our analysis will use the *smoothness* framework of Syrgkanis and Tardos (2013), along with new arguments to handle rosca-specific obstacles. Throughout, we assume quasilinear utilities.

### 3.1 Proof Template: Up-Front Roscas

We begin our analysis by considering roscas with up-front bidding. In an up-front rosca, each agent $i$ submits a bid $b_i$ at the beginning of the rosca. Agents pay their bids, and are then assigned pots in decreasing order of their bids, with the highest bidder receiving the pot in round 1, and so on. Each agent $i$'s payments are redistributed evenly among the other participants in the form of reduced per-period payments into the rosca. Under quasilinear utilities, it is not relevant to the agents' utilities what round payments are made; the only relevant outcome is total payments, which we write as $p_i = \sum_t p_i^t$ when context allows. We can further assume per-period payments remain fixed and that the agents receive the redistributed payments up-front in the form of a rebate.

We decompose the agents' total payments into their *gross payments* $\hat{p}_i$ and *rebates* $\hat{r}_i$, with $p_i = \hat{p}_i - \hat{r}_i$. Formally:

**Definition 1.** *In an* up-front rosca *with quasilinear agents, each agent $i$ submits a bid $b_i$, with $\mathbf{b} = (b_1, \ldots, b_n)$. Let $r_i$ denote the rank of agent $i$'s bid. Allocations are $x_i^t(\mathbf{b}) = 1$ if $t = r_i$ and $0$ otherwise. Agent $i$'s gross payment is $\hat{p}_i(\mathbf{b}) = b_i$, and their rebate is $\hat{r}_i(\mathbf{b}) = \sum_{i' \neq i} b_{i'}/(n-1)$.*

In this literature, results typically characterize equilibrium exactly, and typically make ordinal welfare comparisons to other credit market structures. By studying the price of anarchy, we obtain approximate, cardinal comparisons which help rationalize roscas even when they allocate funds suboptimally.

Our auction rosca analyses all follow from a two-step argument. First, we use or modify the smoothness framework of Syrgkanis and Tardos (2013) to obtain a tradeoff between bidders' utilities and their gross payments. Without rebates, typical auction analyses conclude by noting that high payments imply high welfare. However, because gross payments in roscas are redistributed, it could happen that both gross payments and rebates are high, but welfare is low. Our second step is to rule out this problem. For up-front roscas, we can demonstrate both steps simply. The first step follows from Lemma A.20 of Syrgkanis and Tardos (2013):

**Lemma 1.** *With quasilinear agents, any Nash equilibrium $\mathbf{b}$ of any up-front rosca with values $\mathbf{v}$ satisfies*

$$\sum\nolimits_i u_i^{\mathbf{v}_i}(\mathbf{b}) \geq \tfrac{1}{2}\text{OPT}(\mathbf{v}) - \sum\nolimits_i \hat{p}_i(\mathbf{b}). \qquad (1)$$

The lefthand side of (1) is the equilibrium welfare. It therefore suffices for the second step to upper bound the gross payments on the righthand side.

**Lemma 2.** *Let* $\mathbf{b}$ *be a Nash equilibrium of an up-front rosca with quasilinear agents and values* $\mathbf{v}$*. The for any agent* $i$*,* $\hat{p}_i(\mathbf{b}) \leq \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b})$*.*

*Proof.* Assume for some $i$ that $\hat{p}_i(\mathbf{b}) > \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b})$. Then $i$ must be overbidding. They could improve their utility by bidding 0, which, in an up-front rosca, does not change their rebates: $\hat{r}_i(0, \mathbf{b}_{-i}) = \hat{r}_i(\mathbf{b}) > \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b}) - \hat{p}_i(\mathbf{b}) + \hat{r}_i(\mathbf{b})$. □

Since $\sum_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{b})$ is equal to the equilibrium welfare, Lemmas 1 and 2 together imply the following.

**Theorem 1.** *With quasilinear agents, every Nash equilibrium of an up-front rosca has PoA at most* 4*.*

### 3.2 Sequential Roscas

We now consider roscas with separate sequentially-held first- or second-price auctions for each pot as opposed to the single-auction format from the previous section.

**Definition 2.** *A* first-price rosca *runs a first-price auction in each round. That is, if the highest-bidding agent in round* $t$ *among those who have not yet won is agent* $i^*$*, with bid* $b^t_{i^*}$*, then* $x^t_{i^*} = 1$*,* $x^t_i = 0$ *for all other bidders* $i$*. The gross payments are* $\hat{p}^t_{i^*} = b^t_{i^*}$ *and* $\hat{p}^t_i = 0$ *otherwise. The rebates are* $\hat{r}^t_i = b^t_{i^*}/(n-1)$ *for all* $i \neq i^*$*.*

**Definition 3.** *A* second-price rosca *runs a second-price auction in each round. That is, if the highest-bidding agent in round* $t$ *among those who have not yet won is agent* $i^*$*, with second-highest bid* $b^t_{(2)}$*, then* $x^t_{i^*} = 1$*,* $x^t_i = 0$ *for all other bidders* $i$*. The gross payments are* $\hat{p}^t_{i^*} = b^t_{(2)}$ *and* $\hat{p}^t_i = 0$ *otherwise. The rebates are* $\hat{r}^t_i = b^t_{(2)}/(n-1)$ *for all* $i \neq i^*$*.*

Sequential auctions further require a choice of monitoring scheme, whereby the auctioneer discloses information about agents' bids after each round. Our results will hold for any deterministic monitoring scheme. A key subtlety is that agents' actions are now *behavioral strategies*: that is, at each stage, agent observes the disclosed history of play so far, and can condition their future bids on this history. We denote by $\mathbf{a} = (a_1, \ldots, a_n)$ a vector of behavioral strategies, and reserve the notation $b^t_i$ for the bid of agent $i$ in a round $t$.

As with up-front roscas, we first derive a tradeoff between utility and gross payments, and second consider the impact of rebates. The sequential format complicates both steps. Our first step will follow from a novel composition argument, where we show that both first- and second-price roscas retain the smoothness properties of their single-item analogs. For second-price roscas, a standard no-overbidding assumption then bounds the auction's rebates and implies a welfare bound. For first-price roscas, we give a more involved analysis that bounds overbidding and yields an unconditional guarantee. Overbidding can both occur and harm welfare, so such an analysis is necessary.

To understand utilities and gross payments in sequential roscas, we use a composition argument. Notice that both first- and second-price roscas can be thought of as the sequential composition $n$ single-item auctions, with a rule excluding past winners. Formally:

**Definition 4** (Round-Robin Composition)**.** *Given a single-item auction* $M$*, the* $n$-item round-robin composition *of* $M$ *is a multi-round allocation mechanism for* $n$ *items using the following procedure: during each round* $t$*, each agent* $i$ *who has not yet been allocated an item submits a bid* $b^t_i$*. The mechanism then runs* $M$ *among these remaining agents to determine the allocation and payments for that round.*

The following definition of *smoothness*, adapted from Syrgkanis and Tardos (2013), lets us characterize both first- and second-price roscas with the same framework. In fact, it applies to any auction where in round $t$, each agent who has not yet won submits a real-valued bid $b^t_i$, which we term *sequential single-bid auctions*. Note that this includes single-item auctions; will show that smoothness of single-item auctions implies smoothness of their round-robin composition.

**Definition 5.** *Let* $M$ *be a sequential single-bid auction. We say* $M$ *is* $(\lambda, \mu_1, \mu_2)$*-smooth if for every value profile* $\mathbf{v}$ *and action profile* $\mathbf{a}$*, there exists a randomized action* $a^*_i(a_i, \mathbf{v})$ *for each* $i$ *such that:*

$$\sum\nolimits_i (\mathbf{v}_i \cdot \mathbf{x}_i(a^*_i(a_i, \mathbf{v})) - p_i(a^*_i(a_i, \mathbf{v})) \geq \lambda \mathrm{OPT}(\mathbf{v}) - \mu_1 \sum\nolimits_i \hat{p}_i(\mathbf{a}) - \mu_2 \sum\nolimits_i B_i(\mathbf{a}),$$

*where* $B_i(\mathbf{a})$ *is* $i$*'s bid in the round where they win, or* 0 *if no such round exists.*

Syrgkanis and Tardos (2013) show that single-item first-price and second-price auctions are $(1 - 1/e, 1, 0)$-smooth, and $(1, 0, 1)$-smooth, respectively. However, the smoothness result they prove for a form of sequential composition fails to hold for round-robin composition, due to the restriction that each agent can only win once. We instead give a new composition argument tailored specifically to the rosca setting, which uses the fact that values are decreasing over time. Our composition result follows a useful definition.

**Definition 6.** *A single-item mechanism* $M$ *with allocation rule* $\mathbf{x}$ *and payments* $\mathbf{p}$ *is* strongly individually rational (IR) *if (1) for every profile of actions* $\mathbf{a}$*,* $x_i(\mathbf{a}) = 0$ *only if* $\hat{p}_i(\mathbf{a}) = 0$*, and (2) there exists an action* $\perp$ *such that for all* $i$ *and* $\mathbf{a}_{-i}$*,* $\hat{p}_i(\perp, \mathbf{a}_{-i}) = 0$*.*

**Lemma 3.** *Let* $M$ *be a strongly individually-rational single-item mechanism. If* $M$ *is* $(\lambda, \mu_1, \mu_2)$*-smooth for* $\lambda \leq 1$ *and* $\mu_1, \mu_2 \geq 0$*, then its round-robin composition is* $(\lambda, \mu_1 + 1, \mu_2)$*-smooth as long as* $v^t_i \geq v^{t+1}_i$ *for all* $i$ *and* $t$*.*

We prove the lemma in the supplement. Our proof augments the main idea from the Syrgkanis and Tardos (2013) composition result with ideas from Kesselheim, Kleinberg, and Tardos (2015), who consider smoothness of non-sequential mechanisms for cardinality-constrained allocation environments. As a corollary of Lemma 3, we obtain that first- and second-price roscas are respectively $(1 - 1/e, 2, 0)$ and $(1, 1, 1)$-smooth.

The next task is analyzing the impact of rebates. If we assume no agent overbids, then payments (and hence rebates) are necessarily bounded by values, and we obtain a similar conclusion to Lemma 2. Moreover, we show in the supplement that an overbidding assumption is necessary for second-price roscas, as is often the case for auctions with second-price payments. The overbidding assumption we require is as follows:

**Definition 7.** *Action profile* $\mathbf{a}$ *satisfies* no-overbidding *if* $B_i(\mathbf{a}) \leq \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})$ *for every agent* $i$.

**Theorem 2.** *Let* $M$ *be a strongly IR, single-item auction that is* $(\lambda, \mu_1, \mu_2)$*-smooth, with* $\lambda \leq 1$. *With quasilinear agents, every no-overbidding Nash equilibrium of the corresponding auction rosca with rebates has PoA at most* $(2 + \mu_1 + \mu_2)/\lambda$.

*Proof.* Lemma 3 implies that the rosca is $(\lambda, 1 + \mu_1, \mu_2)$-smooth before rebates. We can therefore write:

$$\begin{aligned}
\sum\nolimits_i u_i^{\mathbf{v}_i}(\mathbf{a}) &\geq \sum\nolimits_i u_i^{\mathbf{v}_i}(a_i^*, \mathbf{a}_{-i}) \\
&\geq \sum\nolimits_i (\mathbf{v}_i \cdot \mathbf{x}_i(a_i^*, \mathbf{a}_{-i}) - \hat{p}_i(a_i^*, \mathbf{a}_{-i})) \\
&\geq \lambda \mathrm{OPT}(\mathbf{v}) - (1 + \mu_1) \sum\nolimits_i \hat{p}_i(\mathbf{a}) \\
&\qquad - \mu_2 \sum\nolimits_i B_i(\mathbf{a}) \\
&\geq \lambda \mathrm{OPT}(\mathbf{v}) - (1 + \mu_1 + \mu_2) \sum\nolimits_i B_i(\mathbf{a}) \\
&\geq \lambda \mathrm{OPT}(\mathbf{v}) - (1 + \mu_1 + \mu_2) \sum\nolimits_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})
\end{aligned}$$

Since both $\sum_i u_i^{\mathbf{v}_i}(\mathbf{a})$ and $\sum_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})$ are equal to equilibrium welfare, the result follows. $\square$

**Corollary 1.** *For quasilinear agents, any Nash equilibrium of the first-price rosca satisfying no-overbidding has PoA at most* $3e/(e-1)$.

**Corollary 2.** *For quasilinear agents, any Nash equilibrium of the second-price rosca satisfying no-overbidding has PoA at most* 3.

### 3.3 Relaxing No-Overbidding

The no-overbidding assumption in the previous section rules out behavior where agents overbid in early rounds to induce others to bid high in later rounds, thereby resulting in high rebates. When this behavior is extreme, agents' payments could conceivably far exceed their values, which in turn complicates the smoothness-based approach. The following example gives a Nash equilibrium of a first-price rosca where overbidding leads to welfare loss.

**Example 1.** *Consider three agents, with* $\mathbf{v}_1 = (1, 0, 0)$, $\mathbf{v}_2 = (2, 2, 0)$, *and* $\mathbf{v}_3 = (2, 2, 0)$. *The following behavioral strategies form a Nash equilibrium. Agent 1 bids 2 in round 1. Agents 2 and 3 bid 1 in round 1. If agent 1 bids less than 2 in round 1, agents 2 and 3 bid 0 in round 2. Otherwise, they bid 2. The optimal welfare in this example is* 4, *but the equilibrium welfare is* 3.[1]

Despite the loss exhibited in Example 1, we can obtain a constant price of anarchy for first-price roscas without an overbidding assumption. Lemma 4 below shows that overbidding cannot drive payments much higher than equilibrium welfare. The lemma extends the following logic: in equilibrium, the agent who wins in the final round has no competition, and is therefore making zero payments. Consequently, the agent who wins in the second-to-last round cannot expect any rebates from round $n$, and therefore has no incentive to overbid. This, in turn, limits the rebates due the agent who wins the round before that, and so on. These limits on rebates in turn limit the extent of overbidding that might occur. Throughout this section, index agents such that in round $t$, the winner is agent $t$.

**Lemma 4.** *Fix a Nash equilibrium of a first-price rosca. Then:*

$$\hat{p}_t^t \leq v_t^t + \tfrac{1}{n-1} \sum_{t'=t+1}^{n} v_{t'}^{t'} (\tfrac{n}{n-1})^{t'-t-1}.$$

We provide the proof in the supplementary materials.

**Theorem 3.** *In any Nash equilibrium of the first-price rosca, the PoA is at most* $(2e+1)e/(e-1)$.

The result follows from summing the bounds on $\hat{p}_i(\mathbf{a})$ from Lemma 4, which can be arranged to obtain an upper bound of $e \sum_i \mathbf{v}_i \cdot \mathbf{x}_i(\mathbf{a})$ of the total gross payments. The theorem then follows from applying smoothness as before.

## 4 Swap Roscas

Several common rosca formats make allocations which do not depend on agents' values. Examples include roscas based on random lottery allocations or those based on seniority or social status (Anderson, Baland, and Moene 2009; Kovsted and Lyk-Jensen 1999). To improve total welfare, it is common practice in such roscas for the participants to engage in an *aftermarket* by buying or selling their assigned allocations when it is mutually beneficial, i.e. to swap rounds in which they are allocated the pot (Mequanent 1996). In this section, we formally define these swap roscas and show that, for agents with quasilinear cost function $C(p) = p$, this aftermarket is guaranteed to converge to an outcome that yields at least half of the optimal welfare. We then give simulations which show that often the guarantee is even better, even for strictly convex $C$.

### 4.1 Theoretical Analysis

We assume that the aftermarket occurs via a series of two-agent swaps, which is consistent with both the literature and practice (Mequanent 1996; Bouman 1995b; Ardener 1964). We assume these swaps can occur at any round $t$. We denote by $\mathbf{p}^t$ the vector of payments for round $t$, which are initialized to 0 for each round and updated as swaps occur. A swap occurs if and only if it is utility-improving for two agents under some set of payments. Formally:

**Definition 8.** *Given initial allocation* $\mathbf{x}$ *and payments* $\mathbf{p}^t$ *at round* $t$, *a* swap *is given by a pair of agents* $i$, $i'$ *assigned to rounds* $j, j' \geq t$, *respectively, and a payment* $\hat{p}$. *A swap is valid if* $v_i^{j'} - C(p_i^t + \hat{p}) > v_i^j - C(p_i^t)$ *and* $v_{i'}^j - C(p_{i'}^t - \hat{p}) > v_{i'}^{j'} - C(p_{i'}^t)$. *Upon executing a swap, set* $x_i^j, x_{i'}^{j'} \leftarrow 0$, $x_i^{j'}, x_{i'}^j \leftarrow 1$, $p_i^t \leftarrow p_i^t + \hat{p}$, *and* $p_{i'}^t \leftarrow p_i^t - \hat{p}$.

[1] This example does not satisfy the refinement of subgame perfection. Note, however, that our welfare guarantees do not require such a restriction.

Note that with quasilinear agents, all valid swaps must strictly improve allocative efficiency since $C(p) = p$. That is $v_i^{j'} + v_{i'}^{j} > v_i^{j} + v_{i'}^{j'}$, and the validity of a swap does not depend on the initial payments $\mathbf{p}^t$. We then study roscas of the following form:

**Definition 9.** *A* swap rosca *starts from an initial allocation* $\mathbf{x}$ *and initial payments* $\mathbf{p} = \{\mathbf{p}^t\}_{t=1}^n$ *of* $0$ *for each agent and round. At each round* $t = 1, \ldots, n$*, participants execute valid swaps and we update the allocation and payment accordingly. We do so until there are no valid swaps.*

Note that for non-linear $C$, new swaps may become valid moving from round $t$ to $t+1$, as each new round's payments reset to $0$. For quasilinear agents, however, Definition 9 executes all swaps in round 1. In this case, the resulting allocation is guaranteed to be stable to pairwise swaps.

**Definition 10.** *An allocation* $\mathbf{x}$ *is* swap-stable *if for all agents* $i, i'$ *assigned to* $j, j'$*, we have that* $v_i^j + v_{i'}^{j'} \geq v_i^{j'} + v_{i'}^{j}$.

For quasilinear agents, swap-stability is guaranteed regardless of the initial allocation. Convergence of the swap process follows from the fact that the total allocated value $\sum_i \mathbf{v}_i \cdot \mathbf{x}_i$ strictly increases each swap and that the number of allocations is finite.

**Theorem 4.** *For quasilinear agents, the welfare approximation for every swap rosca is at most* $2$.

*Proof.* Without loss of generality, assume that the welfare-optimal allocation assigns each agent $i$ to be allocated the pot in round $i$, so the optimal welfare is $\sum_i v_i^i$. Now let $\pi(i)$ denote the *round* when agent $i$ is allocated the pot in the swap rosca's final allocation, and $\pi^{-1}(i)$ the *agent* allocated the pot in round $i$. Note that $\pi$ and $\pi^{-1}$ are bijections. Furthermore, under quasilinear utilities, all payments between agents are welfare-neutral, and hence the rosca welfare is given by $\sum_i v_i^{\pi(i)}$.

For any agent $i$, note that swap-stability implies

$$v_i^{\pi(i)} + v_{\pi^{-1}(i)}^i \geq v_i^i + v_{\pi^{-1}(i)}^{\pi(i)} \geq v_i^i.$$

Summing over all agents $i$, we get

$$\sum\nolimits_i v_i^{\pi(i)} + \sum\nolimits_i v_{\pi^{-1}(i)}^i \geq \sum\nolimits_i v_i^i.$$

Since $\pi$ and $\pi^{-1}$ are bijections, both sums on the lefthand side are equal to the rosca welfare, and the righthand side is the optimal welfare, giving us a 2-approximation. $\square$

Example 1 in the appendix shows that this bound is tight.

## 4.2 Experimental Results

So far, we presented purely theoretical results that partly rationalize the prevalence of auction and swap roscas. Two limitations yet prevent a comprehensive view of roscas' allocative efficiency. First, the worst-case nature of the results give little detail about outcomes in *typical* instances. Second, our results hold only under quasilinear utilities, which is likely less realistic for extremely vulnerable participants.

This section complements our theoretical results with computational experiments that shed light on these latter questions for swap roscas. We simulate swap roscas under natural instantiations of agents' values, and with agents' costs for payments taking a well-studied but non-linear form. We find that the approximation ratio of these roscas in more typical scenarios is significantly better than the worst-case ratio we prove, even after relaxing quasilinearity. Our experiments also allow us to study the way rosca performance changes as agents' values for their payments become more convex. In particular, we use *constant relative risk aversion (CRRA)* utilities, given by the formula:

$$C(p; W, a) = (1 - a)^{-1}(W^{1-a} - (W - p)^{1-a}),$$

where the parameter $W$ represents the agent's starting wealth, and $a$ governs the convexity of the function, with $a = 0$ being quasilinear. For $a > 0$, CRRA utilities have a vertical asymptote at $p = W$, as agents are unable to spend beyond their means. We choose $W$ to be less than many of agents' maximum values for the rosca pot, to capture that most agents cannot afford the durable good without a rosca's help (Anderson and Baland 2002). Note that as $a \to 1$, $C(p; W, a) \to \ln(W) - \ln(W - p)$. We choose CRRA utilities because they are standard for modeling preferences for wealth in economics (see, e.g. Romer 1996).

We give two sets of simulations. In each, we run 9- and 30-person roscas (typical sizes for small- and medium-sized roscas), and compare three quantities: the optimal welfare under our selected value profile, the expected approximation ratio of a random allocation before any swaps, and the approximation ratio for a swap rosca run from a random allocation. Our swap roscas are simulated according to the description in Section 4. For a pair of agents $i$ and $j$ for whom there exists a valid swap, there are generally many payments which will incentivize a swap. Our simulations choose the smallest such payment.

## 4.3 Experiment: CRRA Utilites

Our first experiment fixes a profile of agent values, and studies the performance of swap roscas as the covexity parameter $a$ and starting wealth $W$ vary. The value profile, comprised of 9 agents, features 6 with *cutoff values* of the form $v_i^t = \overline{v}$ for all $i \leq \hat{t}$ for some $\hat{t}$, and three agents with values which are roughly linearly decreasing in time. The average maximum value among cutoff agents is 5, which matched the average value for linearly decreasing values. We give all value profiles explicitly in the supplement. We consider values of $a$ ranging from $0$ (quasilinear) to $2$ (very convex), focusing on smaller values, as larger values of $a$ tend to represent very similar, extreme functions. We take $W$ in the range $\{1, \ldots, 5\}$, as this puts agents' wealth levels generally below their values for the rosca pot. Rosca welfares are averaged over $10,000$ simulation runs, each starting with a random initial allocation that agents can pay to improve through swaps. Results for this simulation can be found in Table 1.

Across all values of $W$, the approximation ratio of swap roscas generally worsens (increases) as the level of convexity $a$ increases. Intuitively, this is likely due to the fact that since $C$ is convex, an agent receiving payments for a swap values them less than the agent offering the payments. Consequently, swaps are less likely to occur, even

Table 1: Swap Rosca Performance Under Different CRRA Parameters (OPT = 45, random baseline ratio = 1.601)

| | | W | | | | |
|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** |
| **a** | **0** | 1.035 | 1.034 | 1.035 | 1.034 | 1.035 |
| | **0.1** | 1.121 | 1.119 | 1.070 | 1.067 | 1.063 |
| | **0.2** | 1.122 | 1.121 | 1.080 | 1.074 | 1.074 |
| | **0.3** | 1.122 | 1.119 | 1.118 | 1.086 | 1.081 |
| | **0.5** | 1.122 | 1.124 | 1.121 | 1.119 | 1.120 |
| | **0.75** | 1.122 | 1.122 | 1.123 | 1.121 | 1.121 |
| | **1** | 1.124 | 1.122 | 1.121 | 1.123 | 1.122 |
| | **1.5** | 1.123 | 1.123 | 1.122 | 1.122 | 1.123 |
| | **2** | 1.122 | 1.125 | 1.123 | 1.122 | 1.123 |

if they would lead to improved allocative efficiency. Meanwhile, the effect of $W$ depends on the level of convexity $a$. When $0 < a < 0.5$, agents with higher wealth $W$ have more money to spend on swaps, making swaps more likely to occur and hence improve allocative efficiency. Thus, approximation ratios improve (decrease) with higher $W$. However, as convexity increases, the disincentive to swap caused by convexity overcomes the benefit of having greater wealth with which to pay for swaps, and the approximation ratios no longer change with $W$. For all parameter values chosen, however, swap roscas led to a marked improvement over the approximation ratio from random allocation alone, suggesting that even under extreme convexity, agents are able to identify local improvements to social welfare. We also repeated this experiment with a 30-agent rosca using similar value profiles and observed the same trends. We present the results in the supplement.

### 4.4 Experiment: Distributional Diversity

Our second set of experiments, discussed in more detail in the supplement, varies the distribution of values across the population of agents, again for 9- and 30-person roscas. This allows us to study the way the distribution of need across a population impacts rosca welfare. We find that performance is surprisingly insensitive to wide inequality in values of agents in the population.

## 5 Discussion and Conclusion

In this work, we study roscas, which are organizations that are especially highly-prevalent in communities where individuals have reduced access to formal financial institutions. We focus specifically on the allocative efficacy of roscas as lending and saving mechanisms. We derive welfare guarantees for roscas under a variety of general settings and allocation protocols and show that many commonly-observed roscas provide a constant factor welfare approximation to the optimal allocation. This guarantee, we believe, gives partial evidence for the ubiquity of roscas.

In addition to these specific results, our work also serves as a first case-study for the potential for algorithmic game theory to further our understanding of roscas and, more generally, building a broader theory for how communities self-organize to create opportunity. We highlight opportunities for further exploration below. First, in our work, we model the savings aspects of roscas. However, roscas are also frequently used as insurance mechanisms where participants experiencing unanticipanted needs may bid to obtain the pot earlier than they may have otherwise planned (Calomiris and Rajaraman 1998; Klonner 2003b, 2001). There remain many gaps in our understanding of roscas when participants' values and incomes evolve stochastically over time.

Another challenge is understanding the tension between allocative efficiency and wealth inequality. Participants with valuable investment opportunities might not bid as aggressively, if their low wealth causes them to value cash highly. This is exacerbated when participants experience income shocks, which is often experienced by economically vulnerable individuals (Abebe, Kleinberg, and Weinberg 2020). Ethnographic work shows that altruism plays a significant role in alleviating this tension (Klonner 2008; Sedai, Vasudevan, and Pena 2021). Roscas often serve a dual role of community-building institutions. Consequently, participants tend to observe signals about each others shocks, and act with mutual aid in mind (Klonner 2008; Mequanent 1996).

Despite often working outside of formal institutions and without any legal enforcement, studies show that "rosca enforcement" is not often an issue. For instance, studies show that early recipients of the pot rarely default (Smets 2000; Van den Brink and Chavas 1997). This is partially explained by community-based norms and standards. We believe modeling these phenomena could help explain the dynamics of this enforcement mechanism and the robustness of roscas.

Finally, there are many questions on how aspects of the population and environment govern the performance of roscas: i.e., under what conditions would one prefer one type of rosca over another? Similarly, how do roscas perform when their members are evolve over time, e.g., with some participants joining part way through the rosca and potentially holding more leverage? Likewise, rosca formation is known to be crucial, with many roscas preferring individuals with similar socio-economic backgrounds. Modeling and examining the rosca formation process can improve our understanding of the interaction between the rosca formation process and their functionality, efficacy, and robustness.

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