

Combating Collusion Rings is Hard but Possible

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Abstract

A recent report of Littmann [Commun. ACM '21] outlines the existence and the fatal impact of collusion rings in academic peer reviewing. We introduce and analyze the problem CYCLE-FREE REVIEWING that aims at finding a review assignment without the following kind of collusion ring: A sequence of reviewers each reviewing a paper authored by the next reviewer in the sequence (with the last reviewer reviewing a paper of the first), thus creating a *review cycle* where each reviewer gives favorable reviews. As a result, all papers in that cycle have a high chance of acceptance independent of their respective scientific merit.

We observe that review assignments computed using a standard Linear Programming approach typically admit many short review cycles. On the negative side, we show that CYCLE-FREE REVIEWING is NP-hard in various restricted cases (i.e., when every author is qualified to review all papers and one wants to prevent that authors review each other's or their own papers or when every author has only one paper and is only qualified to review few papers). On the positive side, among others, we show that, in some realistic settings, an assignment without any review cycles of small length always exists. This result also gives rise to an efficient heuristic for computing (weighted) cycle-free review assignments, which we show to be of excellent quality in practice.

1 Introduction

As recently pointed out by Littman (2021), the integrity and legitimacy of scientific conference publications (particularly important in the context of computer science) is threatened by so-called “collusion rings”, which are sets of authors that unethically review and support each other while breaking anonymity and hiding conflicts of interest. Despite the fact that details are usually not disclosed for various reasons, it is inevitable that the process of assigning papers to reviewers is the key point to engineer technical barriers against such incidents. Whereas assignments at very small venues could be performed manually, support by (semi-)automatic systems becomes necessary already for medium-size conferences. Today computational support for finding review assignments is well-established and has improved the quality of the reviewing and paper assignment process in many

ways (see the surveys of Shah (2021) and Price and Flach (2017) for details). Still there is huge potential for improving processes and further computational support is urgently requested (Price and Flach 2017; Shah 2021).

When aiming to prevent collusion rings, one of the most basic properties one can request from a review assignment is that the assignment does not contain any *review cycle* of length z , that is, a sequence of z agents each reviewing a paper authored by the next agent in the sequence (with the last agent reviewing a paper authored by the first). This property is of high practical relevance: For example, in the AAAI'21 review assignment the non-existence of review cycles of length at most $z = 2$ was a soft constraint (Leyton-Brown and Mausam 2021). Yet, there is a lack of systematic studies concerning the computation of such assignments. Motivated by this, we propose and analyze CYCLE-FREE REVIEWING, the problem of computing an assignment of papers to agents that is free of review cycles of length at most z , both from a theoretical and practical perspective.

Related Work. The literature is rich in the general context of peer reviewing (see, e.g., the works of Goldsmith and Sloan (2007); Taylor (2008); Garg et al. (2010); Long et al. (2013); Lian et al. (2018); Kobren, Saha, and McCallum (2019); Stelmakh, Shah, and Singh (2021) on computational aspects of finding a “good” review assignment, and the survey of Shah (2021)). Closest to our work are Barrot et al. (2020) and Guo et al. (2018). In the context of product reviewing, among others, Barrot et al. (2020) propose and analyze a restricted case which translates to our setting as follows: Given a set of single-author papers and a set of agents each writing a single paper and each having some conflicts of interest over papers, find a review assignment of papers to agents, where each agent serves as a reviewer providing one review and each paper must receive one review. They show that in this setting finding an assignment without review cycles of length at most z corresponds to finding a 2-factor without cycles of length at most z , which is known to be NP-hard for $z \geq 5$ but polynomial-time solvable for $z \leq 3$ (Hell et al. 1988). Closer to our setting is that of Guo et al. (2018), who also consider the computation of cycle-free review assignments. They propose two simple heuristics and conduct experiments measuring the quality of their heuristics and the number of review cycles in a weight-

maximizing solution on two instances, mostly focusing on the influence of the number of reviews per paper and per reviewer.

Outline and Contributions. Our contribution is three-fold. First, in Section 3, we show the intractability of CYCLE-FREE REVIEWING in various restricted settings: We show NP-hardness even when just forbidding review cycles of length at most two in “sparse” and “dense” settings (e.g., if each reviewer can review only “few” or can review “almost all” papers, see Theorems 1 to 3). Furthermore, solving a question left open by Barrot et al. (2020), we show NP-hardness if each agent writes just one single-author paper and can review only few papers (Theorem 4).

Second, in Section 4, we develop greedy heuristics. In contrast to Guo et al. (2018) we provide a theoretical analysis for the heuristics. In particular, we prove that, if the considered instance satisfies certain near-realistic conditions (such as that each paper has few authors and that for each paper there are many possible reviewers), then these heuristics are guaranteed to output a z -cycle-free review assignments in polynomial time.

Third, in Section 5, we present and discuss the results of our experiments. Our core results are:

1. Existing linear-programming-based methods for computing maximum-weight review assignments (as often used in practice) produce assignments where a high fraction (20% or more) of agents and papers belong to some review cycles of length two.
2. For $z \in \{2, 3, 4\}$ maximum-weight z -cycle-free assignments computed by one of our heuristics (see Section 4) or computed via Integer Linear Programming are almost as good as the maximum-weight review assignments with cycles (solution quality loss less than 4% resp. 1%).
3. Somewhat surprisingly, we show that adding additional reviewers that are authors of some papers to the reviewer pool increases the number of papers that belong to review cycles in maximum-weight (non cycle-free) assignments.

2 Preliminaries

For $n \in \mathbb{N}$, we set $[n] := \{1, \dots, n\}$. In an instance of CYCLE-FREE REVIEWING, we are given a set P of papers and set A of agents, where each paper $p \in P$ is authored by a subset $\text{aut}(p) \subseteq A$ of agents. Moreover, we are given for each agent $a \in A$ a subset $\text{rev}(a) \subseteq P$ of papers the agent is *qualified to review*¹. We capture this information in a bipartite graph $(A \cup P, E_A \cup E_P)$ with $E_A = \{(a, p) \mid a \in A, p \in \text{rev}(a)\}$ and $E_P = \{(p, a) \mid p \in P, a \in \text{aut}(p)\}$ (see also Table 1 for an overview). A (*peer*) *review assignment* $E' \subseteq E_A$ is a subset of edges from agents to papers, where we say that a *reviews* p in E' if $(a, p) \in E'$. Given a review assignment $E' \subseteq E_A$, for an agent $a \in A$, let $N^+(a, E') = \{p \in P \mid (a, p) \in E'\}$ be the subset of papers agent a reviews in E' and, for a paper $p \in P$, let $N^-(p, E') = \{a \in A \mid (a, p) \in E'\}$ be the subset of agents that review

¹Being “qualified to review” can encode that the agent is capable of reviewing the paper or that the agent does not have a conflict of interest with one of the co-authors or both.

Table 1: Notation overview

Variable	Explanation
$V = A \cup P$	vertex set consisting of agents A and papers P with $n_A = A $ and $n_P = P $
E_A	$(a, p) \in E_A \subseteq A \times P$ shows a can review p
E_P	$(p, a) \in E_P \subseteq P \times A$ shows a authors p
$N^-(v, E)$	in-neighbors of $v \in V$ wrt. $E \subseteq \binom{V}{2}$, i.e., $N^-(v, E) := \{u \in V \mid (u, v) \in E\}$
$N^+(v, E)$	out-neighbors of $v \in V$ wrt. $E \subseteq \binom{V}{2}$, i.e., $N^+(v, E) := \{u \in V \mid (v, u) \in E\}$
Δ_U^-, Δ_U^+	maximum in- and out-degree in U resp., e.g., $\Delta_U^- := \max_{u \in U} N^-(u, E_A \cup E_P) $
δ_U^-, δ_U^+	minimum in- and out-degree in U resp., e.g., $\delta_U^+ := \min_{u \in U} N^+(u, E_A \cup E_P) $
Δ_A^-, δ_A^-	maximum resp. minimum number of papers per author
Δ_P^+, δ_P^+	maximum resp. minimum number of authors per paper
Δ_A^+, δ_A^+	maximum resp. minimum number of papers any author is qualified to review
Δ_P^-, δ_P^-	maximum resp. minimum number of potential reviewers for any paper

p in E' . For $c, d \in \mathbb{N}$ a review assignment $E' \subseteq E_A$ is called *c-d-valid* if each agent reviews at most c papers and each paper is reviewed by d agents, that is, $|N^+(a, E')| \leq c$ for all $a \in A$ and $|N^-(p, E')| = d$ for all $p \in P$. In a review assignment $E' \subseteq E_A$, we say that papers p_1, \dots, p_z and agents a_1, \dots, a_z form a *review cycle (of length z)* if a_i is an author of p_i ($(p_i, a_i) \in E_P$) for all $i \in [z]$, a_i reviews p_{i+1} in E' ($(a_i, p_{i+1}) \in E'$) for $i \in [z-1]$ and a_z reviews p_1 in E' ($(a_z, p_1) \in E'$). Notably, a review cycle of length z in E' corresponds to a directed cycle of length $2z$ in $(A \cup P, E' \cup E_P)$ and a review cycle of length one corresponds to an author reviewing one of its own papers. We say that a review assignment E' is *z-cycle free* if there is no review cycle of length $i \in [z]$ in E' .

Using this notation, we define our central problem and refer to Table 1 for further necessary variable definitions:

[WEIGHTED] CYCLE-FREE REVIEWING

Input: A directed bipartite graph $(A \cup P, E_A \cup E_P)$ and non-negative integers c_{reviewer} , d_{paper} , and z [and a weight function $w : E_A \mapsto \mathbb{Z}$ and an integer W].

Question: Is there a $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid and z -cycle-free review assignment $E' \subseteq E_A$ [of weight at least W , i.e., $\sum_{e \in E'} w(e) \geq W$]?

We refer to $\sum_{e \in E'} w(e)$ as the weight of the review assignment E' . The proofs (or their completions) for results marked by (\star) can be found in the full version (Boehmer, Bredereck, and Nichterlein 2021).

3 NP-Hardness in Various Restricted Cases

From the work of Barrot et al. (2020, Theorem 4.12) it follows that CYCLE-FREE REVIEWING is NP-hard in the

single-author-single-paper setting ($\Delta_A^- = \Delta_P^+ = 1$) even if $c_{\text{reviewer}} = d_{\text{paper}} = 1$ and $z = 2$. However, as in reality instances of CYCLE-FREE REVIEWING are hardly arbitrary but have a quite strong structure, in this section we prove that the NP-hardness of CYCLE-FREE REVIEWING upholds even if the given instance fulfills further quite restrictive conditions, e.g., each agent is qualified to review all papers or our problem specific parameters ($\Delta_A^-, \Delta_P^+, \Delta_A^+, \Delta_P^-, c_{\text{reviewer}}, d_{\text{paper}}, z$) are small constants.

Sparse Review Graph and Small Weights. We start by considering the case where all our parameters are small. Specifically, we show the NP-hardness of CYCLE-FREE REVIEWING for arbitrarily $z \geq 2$ even if each paper is only authored by at most two agents, each agent authors at most two papers, each agent is only qualified to review at most three papers, and for each paper only at most three agents are qualified to review it (see Table 1 for definitions).

Theorem 1 (★). *For any $z \geq 2$, CYCLE-FREE REVIEWING is NP-hard, even if $\Delta_A^+ = \Delta_P^- = 3$, $\Delta_A^- = \Delta_P^+ = 2$, $n_A = n_P$, and $c_{\text{reviewer}} = d_{\text{paper}} = 1$. For any $z \geq 2$, $\delta_P^- \geq 1 \leq \delta_A^+$, CYCLE-FREE REVIEWING is NP-hard, even if $\Delta_A^- = \Delta_P^+ = 2$, $n_A = n_P$, and $c_{\text{reviewer}} = d_{\text{paper}} = 1$. Both hardness results still hold if agents are not allowed to review papers of co-authors.*

Proof sketch. To show the first statement, we reduce from the NP-hard variant of 3-SAT where each literal appears in two clauses (Berman, Karpinski, and Scott 2003). We set $c_{\text{reviewer}} = d_{\text{paper}} = 1$ and $z \geq 2$. Given a propositional formula φ , for each variable x , we introduce three agents $a_x, a_{\bar{x}}, b_x$ and three papers $p_x, p_{\bar{x}}$, and q_x . Agents a_x and b_x are qualified to review p_x , agents $a_{\bar{x}}$ and b_x are qualified to review $p_{\bar{x}}$ and agents a_x and $a_{\bar{x}}$ are qualified to review q_x . Intuitively, either does a_x review p_x (which corresponds to setting x to false) or $a_{\bar{x}}$ review $p_{\bar{x}}$ (which corresponds to setting x to true).

For each clause $c = \ell_1 \vee \ell_2 \vee \ell_3$, we introduce an agent a_c , two dummy agents, and three papers p_c^1, p_c^2 , and p_c^3 . Agent a_c and both dummy agents are qualified to review all three papers. Notably, for one $i \in [3]$, a_c needs to review p_c^i (which corresponds to c being fulfilled because of ℓ_i). Concerning the authors of each paper, for each clause $c = \ell_1 \vee \ell_2 \vee \ell_3$ and $i \in [3]$, a_c is an author of p_{ℓ_i} and a_{ℓ_i} is an author of p_c^i .

Note that letting a_c review p_c^i for some $c \in C$ and $i \in [3]$ results in a 2-cycle if and only if a_{ℓ_i} reviews p_{ℓ_i} , which corresponds to setting ℓ_i to false. Thus, the assignments of variables encoded in a 1-1-valid 2-cycle-free review assignment needs to satisfy φ , as for each $c \in C$, a_c reviews p_c^i for some $i \in [3]$. This results in a cycle except if a_{ℓ_i} reviews p_{ℓ_i} which corresponds to setting ℓ_i to true. Conversely, it is possible to show that each satisfying assignment for φ results in a 1-1-valid cycle-free review assignment. \square

While we prove hardness for arbitrary δ_A^+ and δ_P^- , in our construction (presented in our full version (Boehmer, Bredereck, and Nichterlein 2021)), there are always agents that are not qualified to review “many” papers (around $\frac{2}{3}$) and always papers that cannot be reviewed by “many” agents

(around $\frac{2}{3}$). Thus, interpreting a qualification as the absence of a conflict of interest, for our NP-hardness agents need to have many conflicts. In Section 4, we prove that this does not happen by accident, as if the number of conflicts per agent/paper (and $\Delta_A^-, \Delta_P^+, c_{\text{reviewer}}$, and d_{paper}) are “small”, then CYCLE-FREE REVIEWING always admits a solution.

In WEIGHTED CYCLE-FREE REVIEWING it is possible to encode the “qualifications” of agents into weights: If we modify the reduction from above and give an agent-reviewer pair weight one if the agent is qualified to review the paper and weight zero otherwise, we get that WEIGHTED CYCLE-FREE REVIEWING is NP-hard even if each agent is qualified to review all papers and we have few non-zero weights.

Corollary 1. *For any $z \geq 2$, WEIGHTED CYCLE-FREE PEER REVIEWING is NP-hard, even if each agent is qualified to review all papers, each agent gives only at most three papers a non-zero weight, for each paper at most three agents give it a non-zero weight, $\Delta_A = \Delta_P^+ = 2$, $n_A = n_P$, and $c_{\text{reviewer}} = d_{\text{paper}} = 1$.*

No Conflicts of Interest. We now extend the hardness from Corollary 1 for the case where each agent is qualified to review all papers (no conflicts) to the unweighted case. However, our new reduction relies on the existence of papers with many authors and agents authoring many papers.

Theorem 2 (★). *CYCLE-FREE REVIEWING is NP-hard even if each agent is qualified to review all papers, $n_A = n_P$, $c_{\text{reviewer}} = d_{\text{paper}} = 1$, and $z = 2$.*

The reduction from Theorem 2 heavily relies on the possibility that an agent reviews a paper written by an agent with whom she has a joint paper. As some conferences might declare an automatic conflict of interest for co-authors, we now consider the case where an agent is qualified to review all papers that are not authored by one of her co-authors:

Theorem 3 (★). *CYCLE-FREE REVIEWING is NP-hard even if each agent is qualified to review all papers that are not written by one of her co-authors, $c_{\text{reviewer}} = d_{\text{paper}} = 1$, and $z = 2$.*

Single-Author-Single-Paper Setting. In their theoretical analysis, Barrot et al. (2020) focus on CYCLE-FREE REVIEWING where each agent writes a single-author paper (we speak of an agent and its paper interchangeably) and qualifications are symmetric, i.e., if an agent a is qualified to review agent b , then b is qualified to review a . They prove that this problem is NP-hard for $c_{\text{reviewer}} = d_{\text{paper}} = 1$ and $z = 5$ (without bounds on Δ_A^+ or Δ_P^-) but polynomial-time solvable for arbitrary $c_{\text{reviewer}} = d_{\text{paper}}$ for $z = 2$. We close the gap between these two results and extend their general picture by proving that for $c_{\text{reviewer}} = d_{\text{paper}} = 2$, CYCLE-FREE REVIEWING is NP-hard for $z = 3$ even if qualifications are symmetric and each agent is only qualified to review four agents, i.e., we need to decide for each agent a which two of these four agents review a and which two of these agents will get a review from a .

Theorem 4 (★). *CYCLE-FREE REVIEWING is NP-hard, even if $z = 3$, $c_{\text{reviewer}} = d_{\text{paper}} = 2$, $\Delta_A^- = \Delta_P^+ = 1$,*

$n_A = n_P$, each agent is qualified to review exactly four papers and if an agent a can review the paper written by agent b , then b can review the paper of a .

4 Polynomial-Time Solvable Special Cases

In this section, we identify conditions under which a short-cycle-free review assignment provably exists and can be computed in polynomial time. As we will see in our experiments, the subsequently presented algorithms provide short-cycle-free review assignments even beyond the theoretical limitations we discuss below. As we are interested in computing z -cycle-free review assignments for $z \geq 1$, no author is allowed to review one of its own papers. That is why throughout this section we assume that we do not have $(a, p) \in E_A$ and $(p, a) \in E_P$ at the same time.

Our algorithms in this section are based on the following simple observation: Given a partial z -cycle-free review assignment E' and a paper $p \in P$ that requires more assigned reviewers, the number of potential reviewers that would create a z -cycle—if assigned to review p —is bounded by a function in z , the maximum number Δ_P^+ of authors per paper, and the maximum number c_{reviewer} of reviews per agent; the precise function is given in the subsequent proofs. Thus, assuming that the minimum number δ_P^- of potential reviewers for each paper is large compared to z , Δ_P^+ , d_{paper} , and c_{reviewer} , for each paper p there are always reviewers that can be assigned to review p without creating a z -cycle. Note that in practice we can expect that z , d_{paper} , and c_{reviewer} are quite small. Moreover, while the minimum number of fitting reviewers might be not very large, it is not uncommon to assign papers to reviewers that are not “perfect”. Thus, interpreting δ_P^- as the number of community members that do not have a conflict of interest actually yields relative large values for δ_P^- in practice.

We start with a very restrictive setting and then, step by step, generalize the approach and the results. First, each paper is written by exactly one author, each agent has at most one paper and we want a completely cycle-free review assignment (i.e., z -cycle-free for every $z \in \mathbb{N}$). This of course implies that some agents cannot be authors of papers and so the number n_P of papers is smaller than the number n_A of agents. However, it allows Algorithm 1 to work (implicitly) with the topological ordering of the (acyclic) review assignment while constructing it.

Proposition 1. *If $\Delta_A^- \leq 1 = \delta_P^+ = \Delta_P^+$, $d_{\text{paper}} \leq c_{\text{reviewer}}$, and $\delta_P^- \geq n_P + d_{\text{paper}}$, then Algorithm 1 computes a d_{paper} - d_{paper} -valid and completely cycle-free review assignment in linear time.*

Proof. We first show the correctness of Algorithm 1. Clearly, if in each iteration of the loop in Line 3 the set of eligible reviewers R (see Line 6) is of size at least d_{paper} , then a completely cycle-free review assignment is created as each agent only reviews papers from agents “occurring” later during the algorithm. Observe that if $|S_i| \geq n_A - \delta_P^- + d_{\text{paper}}$ for $i \in \{0, \dots, n_P - 1\}$, then in iteration i we have $|R| \geq d_{\text{paper}}$: There are at most $n_A - \delta_P^-$ agents in S_i that cannot review p (the corresponding edge is not in E_A) and, thus, at

Algorithm 1: A greedy algorithm computing a d_{paper} - d_{paper} -valid completely cycle-free assignment E' .

```

1  $E' \leftarrow \emptyset; S_0 \leftarrow$  agents without papers
   /*  $\phi_i(a)$  is the free reviewing capacity
      of  $a$  before iteration  $i$  of the
      for-loop from Line 3; each agent
      reviews at most  $d_{\text{paper}}$  papers */
2 foreach  $a \in A$  do  $\phi_0(a) \leftarrow d_{\text{paper}}$ 
   /* Assign reviewers to one paper per
      iteration: */
3 for  $i \leftarrow 0$  to  $n_P - 1$  do
4   foreach  $a \in A$  do  $\phi_{i+1}(a) \leftarrow \phi_i(a)$ 
5   select some  $(p, a) \in E_P$  where  $p$  has no reviews yet
   /* collect qualified reviewers and
      assign  $d_{\text{paper}}$  of them to  $p$  */
6    $R \leftarrow \{b \in S_i \mid (b, p) \in E_A\}$ 
7   for  $j \leftarrow 1$  to  $d_{\text{paper}}$  do
8     arbitrary  $b \in R$  reviews  $p$ :  $E' \leftarrow E' \cup \{(b, p)\}$ 
9      $\phi_{i+1}(b) \leftarrow \phi_i(b) - 1; R \leftarrow R \setminus \{b\}$ 
   /* collect possible reviewers for
      next paper */
10   $S_{i+1} := \{a\} \cup \{b \in S_i \mid \phi_{i+1}(b) > 0\}$ 
11 return  $E'$ 

```

least d_{paper} agents in S_i are eligible to review p . It remains to show that $|S_i| \geq n_A - \delta_P^- + d_{\text{paper}}$ for all $i \in \{0, \dots, n_P - 1\}$ follows from our assumptions. By assumption of the lemma we have $n_P \leq \delta_P^- - d_{\text{paper}}$. Hence, $|S_0| = n_A - n_P \geq n_A - \delta_P^- + d_{\text{paper}}$. We next show that $|S_i| \geq |S_0|$ for all $i \in [n_P - 1]$. Observe that at the start we have $\sum_{a \in S_0} \phi_0(a) = |S_0| \cdot d_{\text{paper}}$. Moreover, after the i th iteration of the loop in Line 3 we have $\sum_{a \in S_{i+1}} \phi_{i+1}(a) = \sum_{a \in S_i} \phi_i(a)$ as each paper gets d_{paper} reviews and the reviewer a in $S_{i+1} \setminus S_i$ starts with $\phi_{i+1}(a) = d_{\text{paper}}$. Observe that $\phi_i(a) \leq d_{\text{paper}}$ for all $a \in A$ and $i \in \{0, \dots, n_P - 1\}$. Thus, we have $|S_i|d_{\text{paper}} \geq \sum_{a \in S_i} \phi_i(a) = \sum_{a \in S_0} \phi_0(a) = |S_0|d_{\text{paper}}$ and, hence, $|S_i| \geq |S_0|$. This completes the correctness proof.

As to the running time, everything outside the loop starting in Line 3 clearly runs in linear time. As to the part inside the loop, note that by keeping just one array of length n_A we can store the values of ϕ in linear time. Moreover, the reviewers for p are selected arbitrarily from R , which is doable in $|N^-(p, E_A)|$ time. Hence, the loop in Line 3 can be processed in $O(n_P + |E_A|)$ time. Thus, the overall algorithm runs in $O(n_A + n_P + |E_A|)$, that is, linear time. \square

For our next result we replace the completely cycle-free property of the resulting review assignment with z -cycle freeness. This implies that the idea of constructing the review assignment along its topological ordering (as done by Algorithm 1) cannot be employed. Instead, Algorithm 2 constructs greedily a maximal z -cycle-free assignment and then extends the assignment by replacing one review assignment by two other assignments. The argument behind the replace-

Algorithm 2: Greedy algorithm to compute a $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid z -cycle-free review assignment E' .

```

1  $E' \leftarrow \emptyset$ 
2 while  $\exists p \in P : |N^-(p, E')| < d_{\text{paper}}$  do
3   if  $\exists (a, p) \in E_A \setminus E' : E' \cup \{(a, p)\}$  is  $z$ -cycle free
    and  $|N^+(a, E')| < c_{\text{reviewer}}$  then
      /* Case 1: greedy assignment of
       reviews as long as no  $z$ -cycles
       are created: */
      4    $E' \leftarrow E' \cup \{(a, p)\}$ 
5   else
     /* Case 2: replace one review
      assignment by two: */
     6   pick  $(a', p') \in E'$  and  $a \in A$  so that
        $|N^+(a, E')| < c_{\text{reviewer}}$ ,  $(a', p), (a, p') \in E_A$ ,
       and  $(E' \setminus \{(a', p')\}) \cup \{(a', p), (a, p')\}$  is
        $z$ -cycle free
     7    $E' \leftarrow (E' \setminus \{(a', p')\}) \cup \{(a', p), (a, p')\}$ 
8 return  $E'$ 
```

ment strategy is an extension of the argument in Algorithm 1 that there are always enough reviewers to assign in Lines 7 to 9.

To keep our arguments simple we first consider the case that each agent reviews at most one paper and each paper requires one review. Moreover, as before, we are in the setting that each paper has one author and each agent authors at most one paper. Formally, we have the following.

Proposition 2. *If $\Delta_A^- \leq 1 = \delta_P^+ = \Delta_P^+ = c_{\text{reviewer}} = d_{\text{paper}}$, $n_A \geq n_P$, $\delta_A^+ > z$, $\delta_P^- > z$, and $n_P \leq \delta_A^+ + \delta_P^- - 2z$, then Algorithm 2 computes a $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid z -cycle-free review assignment in polynomial time.*

Proof. Obviously, Algorithm 2 terminates after at most n_P iterations of the while loop as in each iteration the number of assigned reviews increases. Moreover, a $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid z -cycle-free review assignment is returned if a, a', p' as described in case 2 (Line 6) always exist. To prove their existence, we introduce some notation. For some $v \in A \cup P$ let $N_z^+(v, E' \cup E_P)$ be the z -out-neighborhood of v , that is, the set of vertices that can be reached from v in the review graph $(A \cup P, E' \cup E_P)$ via a path of length at least one and at most $2z$. Similarly, let $N_z^-(v, E' \cup E_P)$ be the $2z$ -in-neighborhood of v , that is, the set of vertices that can reach v in the review graph $(A \cup P, E' \cup E_P)$ via a path of length at least one and at most $2z$. Note that if $v \in N_z^-(v, E' \cup E_P)$, then also $v \in N_z^+(v, E' \cup E_P)$ and v is contained in a review cycle of length z (that is a directed cycle of length $2z$ in $(A \cup P, E' \cup E_P)$). Subsequently, we present upper bounds on the size of $N_z^-(v, E' \cup E_P)$ and $N_z^+(v, E' \cup E_P)$ for $v \in A \cup P$ thereby proving the existence of a, a', p' .

Let $p \in P$ be the paper without reviewer selected in Line 2 when the algorithm enters case 2. Let $A_p \subseteq A$ be

the set of agents that could review p without creating a z -cycle, that is, $A_p := \{a \in A \mid (a, p) \in E_A \wedge E' \cup \{(a, p)\}$ is z -cycle free}. Since $d_{\text{paper}} = c_{\text{reviewer}} = \Delta_P^+ = 1$, there are at most z agents whose assignment to review p would create a review cycle, that is, $|N_z^+(p, E' \cup E_P) \cap A| \leq z$, and thus $|A_p| \geq \delta_P^- - z$. Since we are in case 2, no more review assignments could be added without creating a z -cycle. Hence, the algorithm assigned the at least $\delta_P^- - z$ potential reviewers in A_p to different papers. Let P_p be the set of these papers. Since $d_{\text{paper}} = c_{\text{reviewer}} = 1$ we have $|P_p| = |A_p| \geq \delta_P^- - z$.

Let $a \in A$ be an arbitrary agent without assigned review, that is, $\#p'' : (a, p'') \in E'$. Since $d_{\text{paper}} = c_{\text{reviewer}} = \Delta_A^- = 1$, we have $|N_z^-(a, E' \cup E_P) \cap P| \leq z$. Thus, there are $\delta_A^+ - z$ papers that a could review without creating a z -cycle; let P_a denote the set of these papers. Since we assume that $n_P \leq \delta_A^+ + \delta_P^- - 2z$, it follows that there is a $p' \in P_a \cap P_p$. By definition of P_p there is an agent a' with $(a', p') \in E'$ and $a' \in A_p$. Thus, a, a', p' exist and E' can be updated to $(E' \setminus \{(a', p')\}) \cup \{(a', p), (a, p')\}$ in Line 7. \square

We now turn our attention to our general case where agents can author and review many papers and papers can have multiple authors and can require several reviews. While the conditions that guarantee the existence of a z -cycle-free review assignment need adjustments, we can still use Algorithm 2 together with a correctness proof that follows a similar pattern as the proof of Proposition 2.

Theorem 5 (★). *If $n_A \cdot c_{\text{reviewer}} \geq n_P \cdot d_{\text{paper}}$, $\delta_A^+ > 2(\Delta_A^- \cdot d_{\text{paper}})^z + c_{\text{reviewer}}$, $\delta_P^- > 2(\Delta_P^+ \cdot c_{\text{reviewer}})^z + d_{\text{paper}}$, and $n_P \leq \delta_A^+ - 2(\Delta_A^- \cdot d_{\text{paper}})^z - c_{\text{reviewer}} + (c_{\text{reviewer}}/d_{\text{paper}})(\delta_P^- - 2(\Delta_P^+ \cdot c_{\text{reviewer}})^z - d_{\text{paper}})$, then Algorithm 2 computes a $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid z -cycle-free review assignment in polynomial time.*

To simplify the statement of Theorem 5 consider a “symmetric” case where $n_A \geq n_P$, $\delta_P^- = \delta_A^+$, and $\Delta_A^+ = \Delta_P^-$. For brevity, set $n := n_P$, $\delta := \delta_P^-$, and $\Delta := \Delta_A^+$. Let coi be the maximum number of papers any agent is not qualified to review/has a conflict of interest with, that is, $\text{coi} = n - \delta$. Setting $c_{\text{reviewer}} = 6$ and $d_{\text{paper}} = 3$ as in our experiments we get:

Corollary 2. *If $n - 6 \geq 1.5 \cdot \text{coi} + \Delta^z(6^z \cdot 2 + 3^z)$, then there always exists a 6-3-valid z -cycle-free review assignment that can be found in polynomial time.*

Considering that AAAI’22 had 9,251 submissions and that there was a submission limit of 10 papers per author and assuming that each paper has at most ten authors (implying that $\Delta = 10$) and that each author has at most 700 conflict of interests, it follows that there is a 6-3-valid 2-cycle-free review assignment computable with Algorithm 2.

As we see in the experiments in the next section, our algorithm returns 2/3/4-cycle-free review assignments even well beyond the theoretical guarantees given above. We also remark that Algorithm 2 allows for an easy extension to the weighted case which we use in our experiments in the next section. To this end, in the first case (Line 4) we do not pick an arbitrary edge (a, p) but a eligible edge of maximum weight to be added to the assignment E' .

5 Experiments

In this section, we compare the weight of review assignments computed using different methods and analyze the occurrences of review cycles.² For this, we use a dataset from the 2018 International Conference on Learning Representations (ICLR ’18) prepared by Xu et al. (2019). Xu et al. (2019) collected all 911 papers submitted to ICLR ’18 and the identity of all 2428 authors. As reviewers identities are unknown, they considered all authors to be reviewers and computed for each author-paper pair a similarity score.³

From the dataset of Xu et al. (2019), we created multiple instances of WEIGHTED CYCLE-FREE REVIEWING as follows. Given a number n_P of papers and a ratio r_{AP} of the numbers of agents and papers, we sample a subset of n_P of the 911 ICLR ’18 papers and set this as our set of papers. Subsequently, we compute the set of all authors of one of these papers and sample a subset of $r_{AP} \cdot n_P$ authors and set this as our set of agents. Notably, the created instances can be seen as particularly challenging when it comes to avoiding review cycles, as in reality also “uncritical” reviewers, i.e., reviewers that do not author any paper, exist.

As done in other papers using the same dataset, we focus on the case with $d_{\text{paper}} = 3$ and $c_{\text{reviewer}} = 6$, i.e., every paper needs exactly three reviews and each agent can review at most six papers (Xu et al. 2019; Jecmen et al. 2020). We consider three different types of review assignments: As “optimal” we denote a maximum-weight $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid review assignment. Such an assignment can be computed using a simple Linear Program (LP) as, for instance, described by Taylor (2008). As “optimal z -cycle free” we denote a maximum-weight $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid z -cycle-free review assignment. This solution can be computed by treating the LP of Taylor (2008) as an Integer Linear Program (ILP) and adding for each possible i -cycle for $i \in [z]$ a separate constraint which imposes that at least one of the agent-paper pairs from the cycle is not assigned. We solved all (I)LPs using Gurobi Optimization, LLC (2021). Lastly, as “heuristic z -cycle free”, we denote a $c_{\text{reviewer}}\text{-}d_{\text{paper}}$ -valid z -cycle-free review assignment computed by the weighted variant of Algorithm 2 as described at the end of Section 4.⁴ In all experiments conducted in this section, the heuristic always returned a solution despite the fact that most of the time we are beyond the setting in which Theorem 5 guarantees this behavior of the heuristic. In experiment I presented in the following subsection, for $z = 2/3/4$, an unoptimized implementation of our heuristic was always able to find a z -cycle-free review assignment in less than 30 seconds, being on average around 2 times faster than the “optimal” LP, on average around 3.7 times faster than the “optimal 2-cycle free” ILP, and on average more than 100 times faster than

²The code for our experiments is available at github.com/nboehmer/Combating-Collusion-Rings-is-Hard-but-Possible.

³To the best of our knowledge, in all other publicly available datasets, there are similarity scores for reviewer-paper pairs but the link between the identities of authors and reviewers is missing (as this is considered sensitive information).

⁴We could not use the heuristics of Guo et al. (2018) as these are not available and their algorithm details are ambiguous.

the “optimal 3-cycle free” ILP.

5.1 Experiment I

In this experiment, we focus on the case where the total number of needed reviews is the same as the total number of reviews that can be written, which is in some sense the most “challenging” but probably also one of the more realistic scenarios. Specifically, for $n_P \in \{150, 175, \dots, 900\}$, we prepared 100 instances with $r_{AP} = 0.5$ as described above and computed for each of these instances the optimal, heuristic 2/3/4-cycle-free, and optimal 2-cycle-free review assignment. Moreover, for all instances with $n_P \leq 225$, we also computed the optimal 3-cycle-free review assignment (for larger instances the ILP solver run out of memory.)

To measure the “price of z -cycle freeness”, in Figure 1, we display the weights of different cycle-free review assignments divided by the weight of an optimal review assignment. What stands out here is that forbidding the existence of 2-cycles only comes at the cost of decreasing the assignment’s weight by on average at most 0.8% (if the optimal 2-cycle-free assignment is used). Turning to the results produced by our heuristic, the quality decrease for 2/3/4-cycle-free assignments lies, on average, around 3.1%, 3.2%, and 3.3%. The weight of assignments computed using our heuristic is thus clearly worse than the weight of the optimal cycle-free assignment, yet still not far away from the weight of an optimal assignment. What is particularly surprising here is that for both our heuristic and the optimal cycle-free assignment, whether 2, 3 or 4 cycles are forbidden seems to be rather irrelevant for the quality decrease. All in all, it is encouraging that 2/3/4-cycle freeness can be realized at a low cost independent of whether our heuristic or an ILP is used.

The necessity of dealing with review cycles is underlined by the data displayed in Figure 2. Here, we show the fraction of agents that are contained in at least one review cycle of some length in an optimal assignment and in a heuristic 2/3-cycle-free assignment. Overall, as the number of papers increases the fraction of agents contained in review cycles constantly decreases, yet for all considered values of n_P the results are worrisome. In the optimal assignment for 150 papers, the fraction of agents contained in a review cycle of length at most 2/3/4 is, on average, 40%/58%/76%, while even for 900 papers, still 32%/41%/55% of agents are contained in a review cycle. Considering heuristic z -cycle-free assignments, the fraction of agents contained in a cycle of length $z + 1$ is considerably lower than for the optimal solution but still non-negligible (the results for optimal 2/3-cycle-free assignments are similar to the displayed results for our heuristic).

We also computed the fraction of papers that are contained in at least one review cycle. The results are as in Figure 2 with all values roughly halved, e.g., even in the optimal assignment for 900 papers, 15%/20%/27% of papers are contained in a review cycle of length at most 2/3/4. An intuitive explanation for this difference between agents and papers is that the number of papers is twice the number of agents and that there exist some papers without reviewing authors. Overall, it is striking that even for a high number

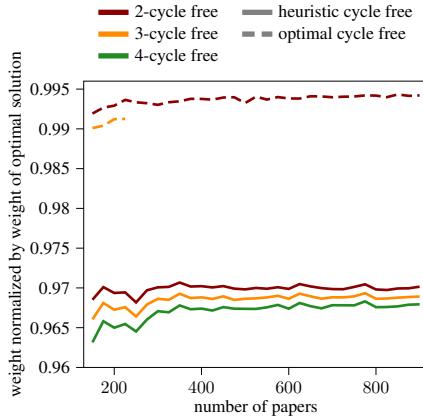


Figure 1: For different values of z , weight of an optimal/heuristic z -cycle-free assignment divided by the weight of an optimal assignment.

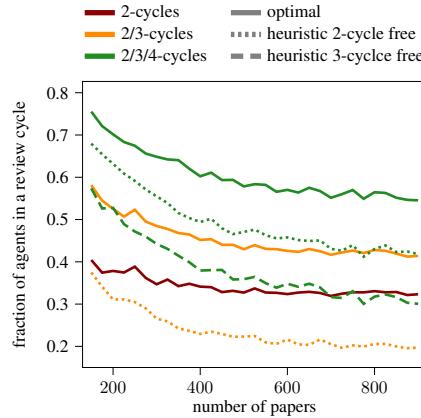


Figure 2: Fraction of agents that are part of a review cycle of at most some length for different types of assignment.

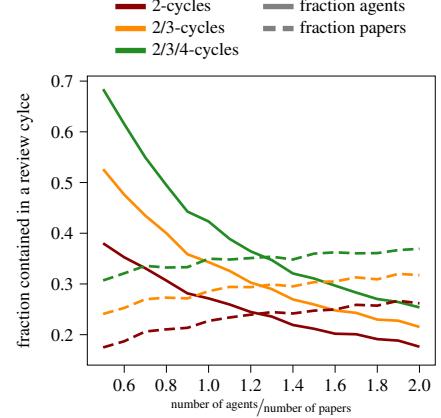


Figure 3: Fraction of agents/papers that are part of a review cycle of at most some length in an optimal assignment for 200 papers and between 100 and 400 agents.

of papers, in an optimal assignment around 15% of papers could have a considerably higher chance of getting accepted if two agents coordinate to give each others paper better reviews and 32% of reviewers would have an opportunity to participate in such a collusion.

5.2 Experiment II

In this experiment, we analyze how the results from experiment I depend on the assumption that the supply and demand of reviews exactly matches. In particular, as described before, for $r_{AP} \in \{0.5, 0.6, \dots, 1.9, 2\}$ we prepared 100 instances with 200 papers and $r_{AP} \cdot 200$ agents (we also repeated this experiment for 400 and 600 papers producing similar results) and computed the different types of review assignments. Considering the assignment weights, increasing r_{AP} from 0.5 to 2, the normalized weight of an optimal 2/3-cycle-free assignment decreases by 0.005 to 0.987/0.985, while the normalized weight of a heuristic 2/3/4-cycle-free assignment increases by 0.01 to 0.982/0.979/0.976; our heuristic performs particularly well if there are (considerably) more reviews available then needed; this supports our theoretical statements for our heuristic in Section 4.

Turning to the possible impact of review cycles, we visualize the fraction of agents/papers contained in a review cycle in an optimal assignment in Figure 3.⁵ While the fraction of agents contained in a review cycle constantly and significantly decreases if more and more agents are added, the fraction of papers contained in a cycle constantly increases. The former observation is quite intuitive, as when more and more agents are added, the average review load decreases and even if the number of review cycles remains the same, it is likely that the fraction of agents contained in one gets smaller. The latter observation is less intuitive but probably a consequence of the fact that, starting with $r_{AP} = 0.5$, for

some papers none of the authors is part of the agent set, implying that these papers cannot be part of a review cycle; however, if we start to add more and more agents, more and more papers can potentially be part of a review cycle. Overall, it might be quite counter intuitive that adding more and more reviewers (that are also authors) to the reviewer pool does not decrease the number of papers contained in a review cycle but increases them.

6 Conclusion

Our work provides a first systematic analysis of CYCLE-FREE REVIEWING. On the theoretical side, we show that CYCLE-FREE REVIEWING is a computationally hard problem even in very restricted settings, yet practically relevant polynomial-time solvable special cases exist. In our practical analysis, we could show that in assignments that do not care for review cycles a high fraction of authors and papers will likely be part of a short review cycle. While collusion rings can certainly also emerge without the existence of review cycles, for example, when authors coordinate over multiple conferences (Littman 2021; Shah 2021), allowing so many easy opportunities means to leave a huge door unlocked without good reason: Our heuristic significantly improves the situation, since it seems to always find a cycle-free review assignment at a very low quality loss.

For future work, it would be valuable to further investigate the limits of our heuristic. While our current bounds are certainly not tight, there are also clear limitations for possible extensions imposed by our NP-hardness results in quite restrictive settings from Section 3. However, a concrete and practically very relevant open question is whether the minimum degree in our analysis can be replaced by the average degree; this would make the results much more robust against outliers. Finally, due to the lack of data, we tested our model on just one dataset. Obtaining more data to test our and other models on would be extremely valuable.

⁵For readability, we do not display the values for the optimal/heuristic cycle-free assignment, as their relationship to the optimal assignment is again similar as in Figure 2.

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References

- Barrot, N.; Lemeilleur, S.; Paget, N.; and Saffidine, A. 2020. Peer Reviewing in Participatory Guarantee Systems: Modelling and Algorithmic Aspects. In *Proceedings of the 19th International Conference on Autonomous Agents and Multiagent Systems (AAMAS '20)*, 114–122. IFAAMAS.
- Berman, P.; Karpinski, M.; and Scott, A. D. 2003. Approximation Hardness of Short Symmetric Instances of MAX-3SAT. *ECCC*, (49).
- Boehmer, N.; Bredereck, R.; and Nichterlein, A. 2021. Combating Collusion Rings is Hard but Possible. fpt.akt.tu-berlin.de/publications/Combating-Collusion-Rings-is-Hard-but-Possible-AAAI22.pdf.
- Garg, N.; Kavitha, T.; Kumar, A.; Mehlhorn, K.; and Mestre, J. 2010. Assigning Papers to Referees. *Algorithmica*, 58(1): 119–136.
- Goldsmith, J.; and Sloan, R. H. 2007. The AI Conference Paper Assignment Problem. In *Proceedings of the 22nd AAAI Conference Workshop on Preference Handling for Artificial Intelligence (MPREF '07)*, 53–57. AAAI Press.
- Guo, L.; Wu, J.; Chang, W.; Wu, J.; and Li, J. 2018. K-Loop Free Assignment in Conference Review Systems. In *Proceedings of the 2018 International Conference on Computing, Networking and Communications (ICNC '18)*, 542–547. IEEE Computer Society.
- Gurobi Optimization, LLC. 2021. Gurobi Optimizer Reference Manual.
- Hell, P.; Kirkpatrick, D. G.; Kratochvíl, J.; and Kríz, I. 1988. On Restricted Two-Factors. *SIAM J. Discret. Math.*, 1(4): 472–484.
- Jecmen, S.; Zhang, H.; Liu, R.; Shah, N. B.; Conitzer, V.; and Fang, F. 2020. Mitigating Manipulation in Peer Review via Randomized Reviewer Assignments. In *Advances in Neural Information Processing Systems 33 (NeurIPS '20)*.
- Kobren, A.; Saha, B.; and McCallum, A. 2019. Paper Matching with Local Fairness Constraints. In *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining (KDD '19)*, 1247–1257. ACM.
- Leyton-Brown, K.; and Mausam. 2021. AAAI 2021 - introduction. <https://slideslive.com/38952457/aaai-2021-introduction?ref=account-folder-79533-folders>. Minute 8 onwards in the video.
- Lian, J. W.; Mattei, N.; Noble, R.; and Walsh, T. 2018. The Conference Paper Assignment Problem: Using Order Weighted Averages to Assign Indivisible Goods. In *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence (AAAI '18)*, 1138–1145. AAAI Press.
- Littman, M. L. 2021. Collusion rings threaten the integrity of computer science research. *Commun. ACM*, 64(6): 43–44.
- Long, C.; Wong, R. C.; Peng, Y.; and Ye, L. 2013. On Good and Fair Paper-Reviewer Assignment. In *Proceedings of the 2013 IEEE 13th International Conference on Data Mining (ICDM '13)*, 1145–1150. IEEE Computer Society.
- Price, S.; and Flach, P. A. 2017. Computational support for academic peer review: a perspective from artificial intelligence. *Commun. ACM*, 60(3): 70–79.
- Shah, N. B. 2021. Systemic Challenges and Solutions on Bias and Unfairness in Peer Review. <http://www.cs.cmu.edu/~nihars/preprints/SurveyPeerReview.pdf>.
- Stelmakh, I.; Shah, N.; and Singh, A. 2021. PeerReview4All: Fair and Accurate Reviewer Assignment in Peer Review. *J. Mach. Learn. Res.*, 22(163): 1–66.
- Taylor, C. J. 2008. On the optimal assignment of conference papers to reviewers. https://repository.upenn.edu/cis_reports/889.
- Xu, Y.; Zhao, H.; Shi, X.; and Shah, N. B. 2019. On Strategyproof Conference Peer Review. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence (IJCAI '19)*, 616–622. ijcai.org.