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Advanced Artificial Intelligence

Lecture 2b: Solving Search Problems

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Problem Solving using Search

Presentation Structure:

- Problem Solving Methods
- Problem Formulation
- State Space
- Blind/Uninformed Search:
  - Breadth First, Depth First, Uniform Cost, Iterative Deepening, Bidirectional Search
- Intelligent/Informed Search:
  - Greedy Search, A* Algorithm
- Practical Application Examples

Solution Search

Methodology to carry out the Solution search:

1. **Start with the initial state**
2. **Execute the goal test**
3. **If the solution was not found, use the operators to expand the current state generating new successor states (expansion)**
4. **Execute the objective test**
5. **If we have not found the solution, choose which state to expand next (search strategy) and carry out this expansion**
6. **Return to 4)**

Solution Search - Search Tree

- **Search tree composed by nodes. Leaf nodes either have no successors or have not yet been expanded!**
- **Important to distinguish between the search tree and the state space!**

- **Tree Node (five components):**
  - **Corresponding state**
  - **Node that gave rise to it (father)**
  - **Operator applied to generate it**
  - **Node depth**
  - **Path cost from the starting node**

- **datatype** NODE

  components: STATE, PARENT-NODE,
  OPERATOR, DEPTH, PATH-COST
- **Border:**
  - **Set of nodes waiting to be expanded**
  - **Represented as a queue**

Search Strategies

- A search strategy is defined by the order of node expansion

- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: How long does it takes (total number of nodes generated)?
  - space complexity: How much memory it needs (maximum number of nodes in memory)?
  - optimality: does it always find the best (least-cost) solution?

- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the least-cost solution
  - m: maximum depth of the state space (may be ∞)

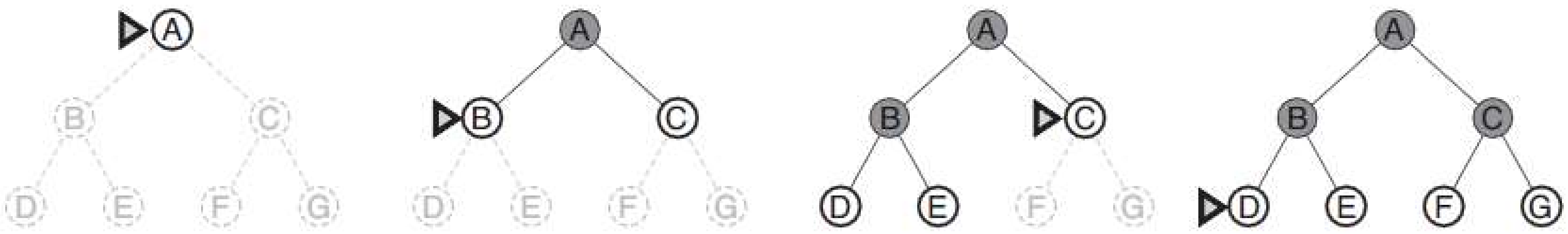
- Types of search Strategies:
  - Uninformed Research (blind)
  - Informed Search (heuristic/intelligent)

Breadth-first Search

Breadth-first search

- **Strategy: Nodes at lowest depth are expanded first**
- **Good: Very systematic search**
- **Bad: Usually it takes a long time and above all it takes up a lot of space**
- **Assuming branching factor b then $n=1+b+b^2+b^3+ \ldots +b^n$**
- **Exponential complexity in space and time: $O(b^d)$**
- **In general, only small problems can be solved like this!function**
  BREADTH-FIRST-SEARCH(problem) returns a solution or failure

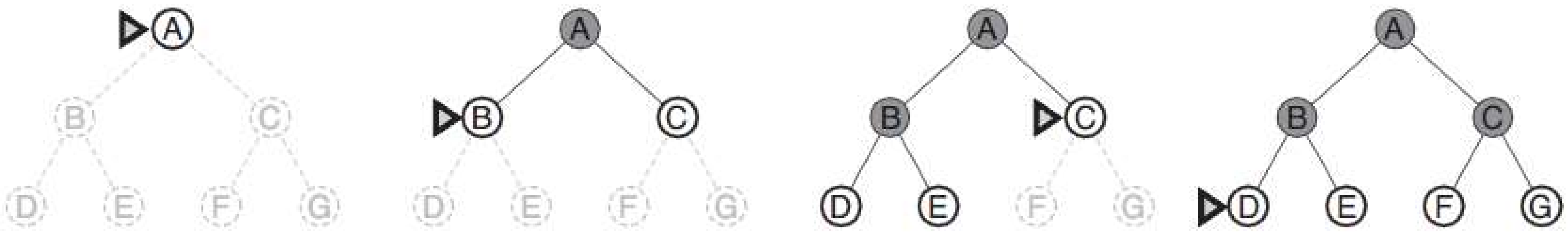
  GENERAL-SEARCH(problem,ENQUEUE-AT-END)



Uniform Cost Search

Uniform Cost Search

- **Strategy: Always expand the border node with the lowest cost (measured by the solution cost function)**
- **Breadth first Search is equal to Uniform Cost Search if g (n) = Depth (n)**



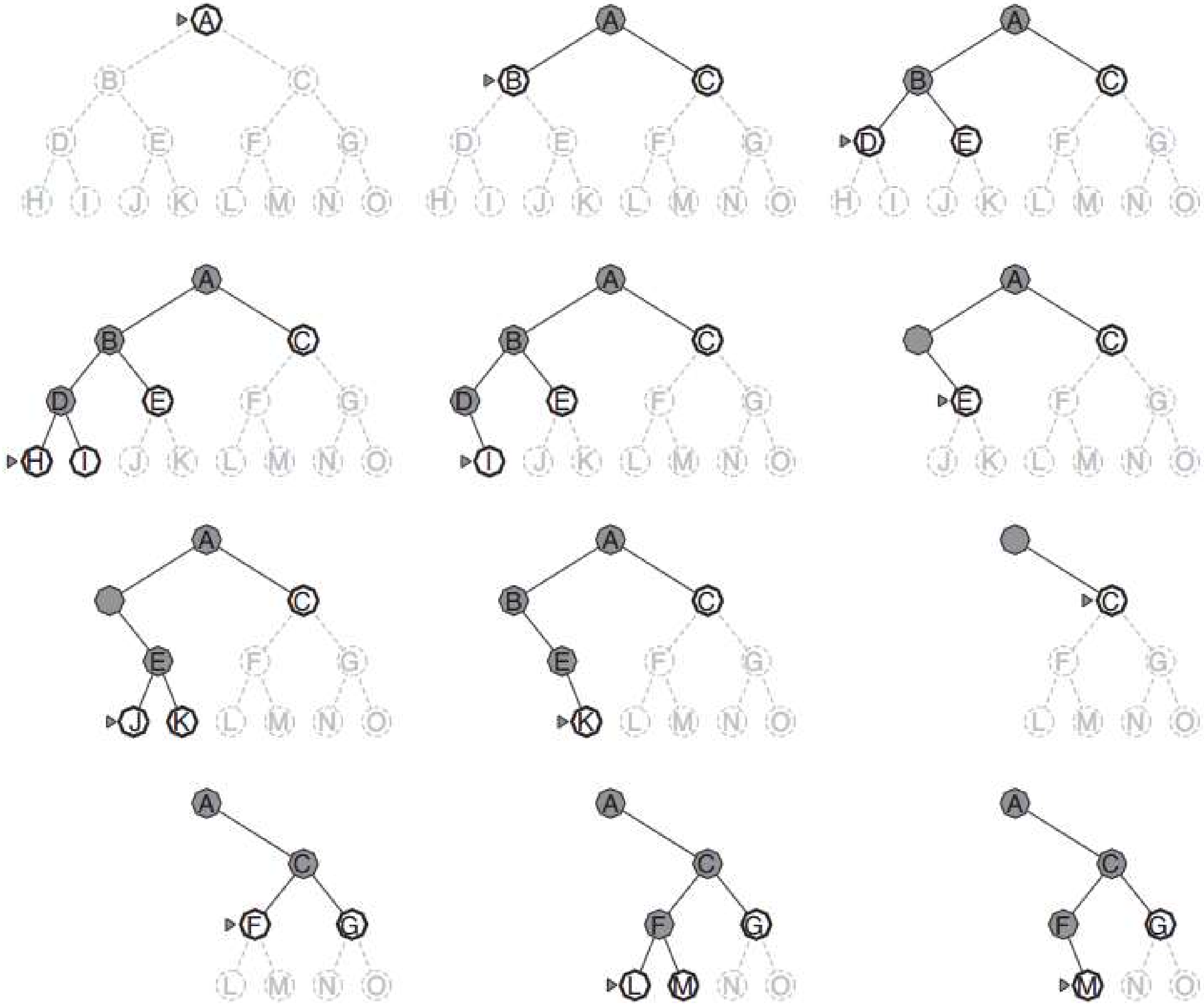
Depth-First Search

Depth-First Search

- **Strategy: Always expand one of the deepest nodes in the tree**
- **Good: Very little memory required, good for problems with lots of solutions**
- **Bad: Cannot be used for trees with infinite depth, can get stuck in wrong branches**
- **Complexity in time O (b^m) and space O (bm).**
- **Sometimes a limit depth is defined and it becomes a Search with Limited Depth**
- **function DEPTH-FIRST-SEARCH(problem) returns a solution or failure**

  GENERAL-SEARCH(problem,ENQUEUE-AT-FRONT)

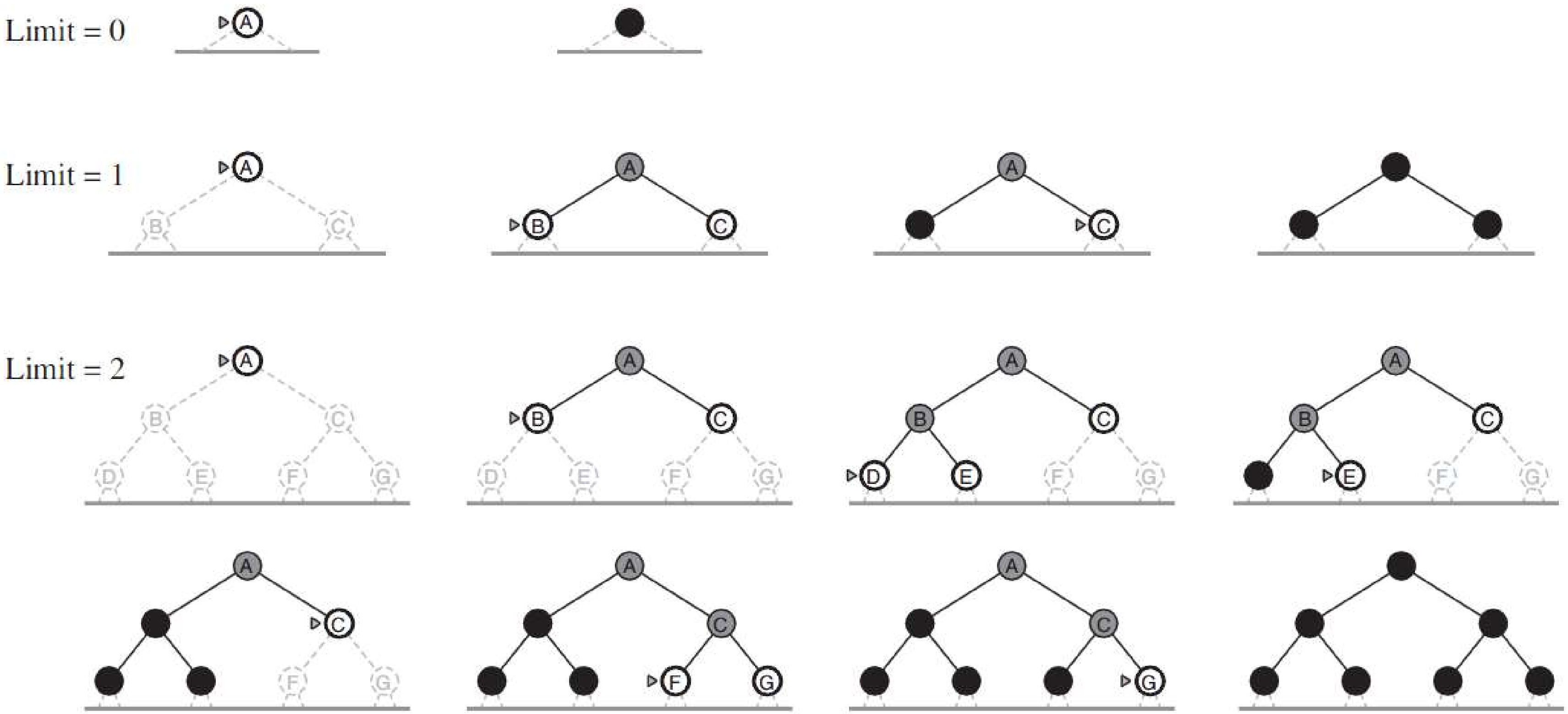
Depth-First Search (2)



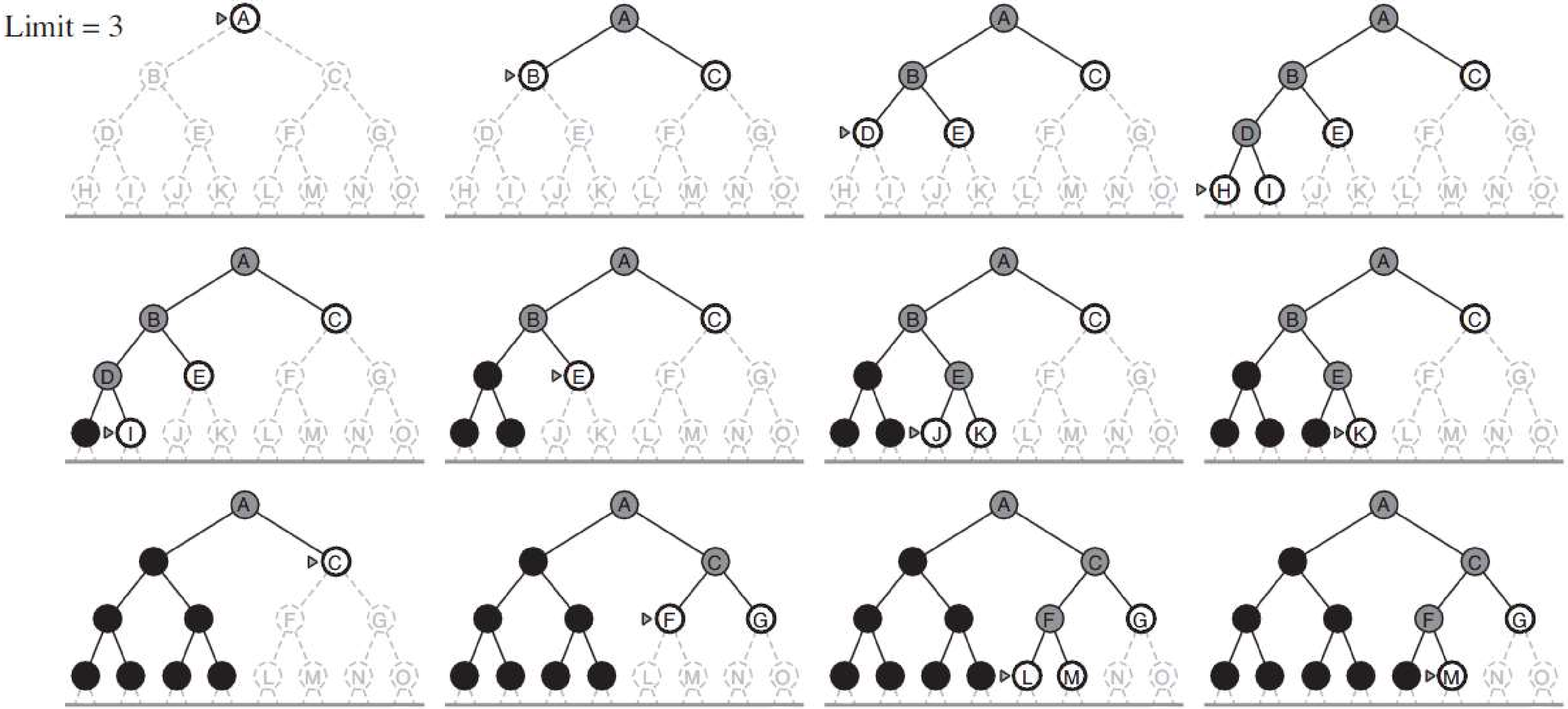
Iterative Deepening Search

- Strategy: Perform limited depth search, iteratively, always increasing the depth limit
- Complexity in time $O(b^d)$ and in space $O(bd)$.
- In general it is the best strategy for problems with a large search space and where the depth of the solution is not known

Iterative Deepening Search (2)

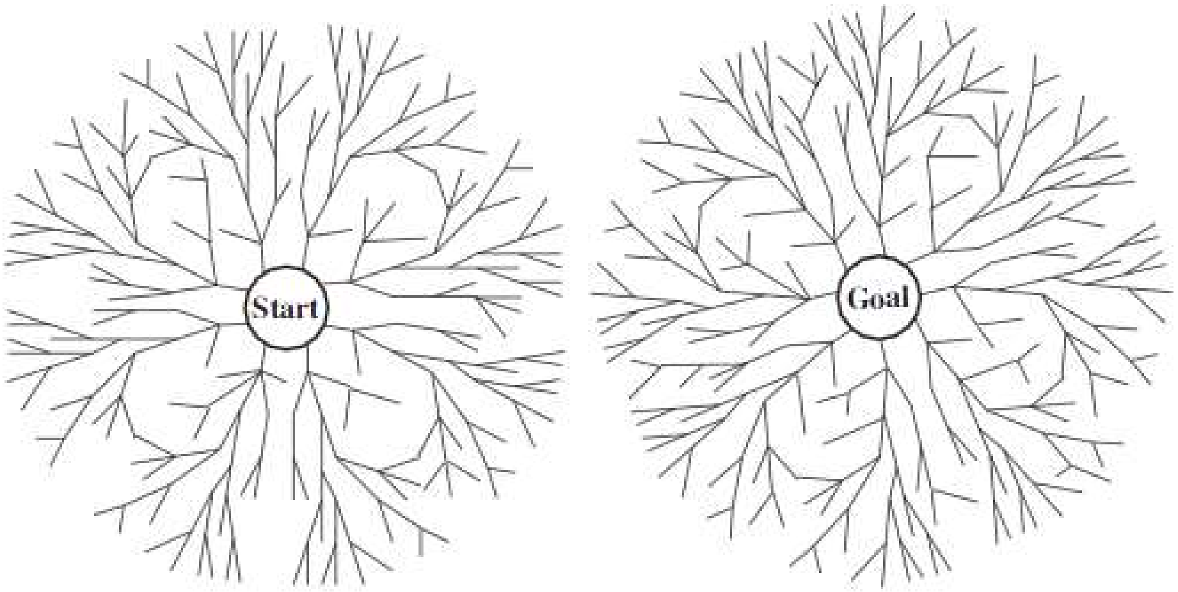


Iterative Deepening Search (3)



Bidirecional Search

- Strategy: Run forward search from the initial state and backward search from the target, simultaneously
- Good: Can greatly reduce complexity over time
- Problems:
  - Is it possible to generate predecessors?
  - What if there are many objective states?
  - How to do the "matching" between the two searches?
  - What kind of research to do in the two halves?



Comparison between Search Strategies

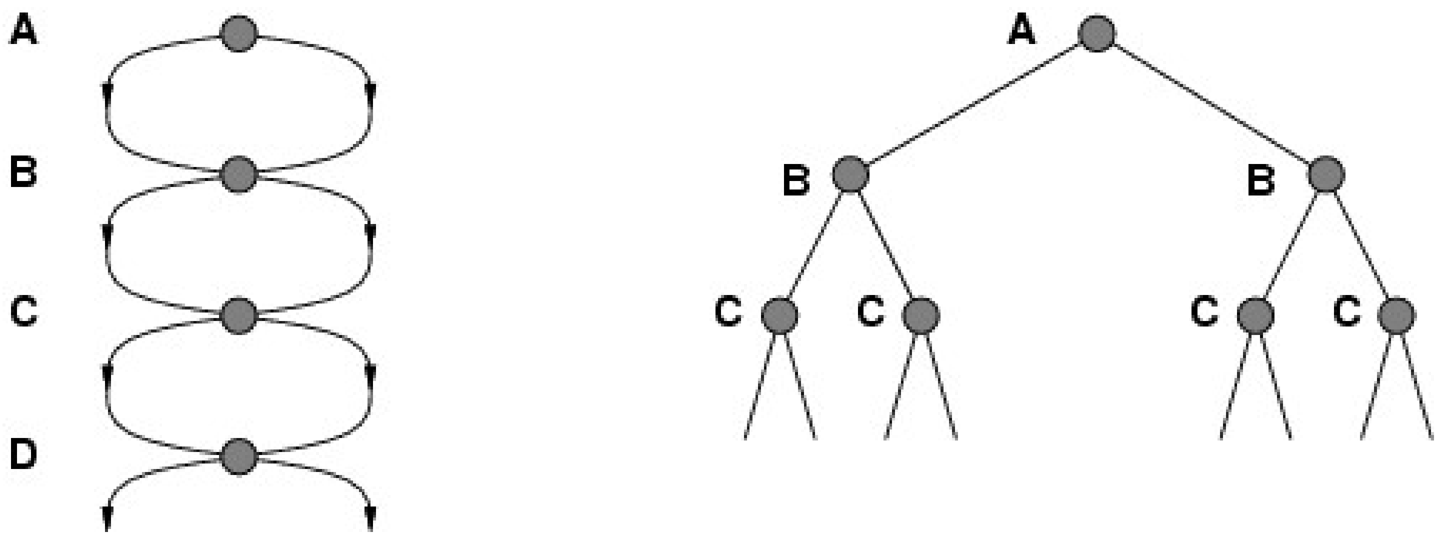
Evaluation of search strategies:

- **b is the branching factor**
- **d is the depth of the solution**
- **m is the maximum depth of the tree**
- **l is the search limit depth**

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Repeated States

- **Failure to detect repeated states can make a linear problem an exponential problem!**
- **Avoid Repeated States**
  - Do not return to the previous state
  - Do not create cycles (do not return to states where you passed in the sequence)
  - Do not use any repeated states (is it possible? What is the computational cost?)



Exercises – Search

Formulate the following problems (analysed in the previous class) as search problems and solve them using the various search strategies studied:

- Missionaries and Cannibals
- Bucket Filling problem
- Towers of Hanoi
- Cryptograms
- 8-Queens and N-Queens
- 8-Puzzle and N-Puzzle

Note: For all problems try to make the solution as generic as possible in order to allow solving versions with different data

Informed/Intelligent/Heuristic Search

Informed/Intelligent/Heuristic Search:

- **Use problem information to prevent the search algorithm from being "lost wandering in the dark"!**

Search Strategy:

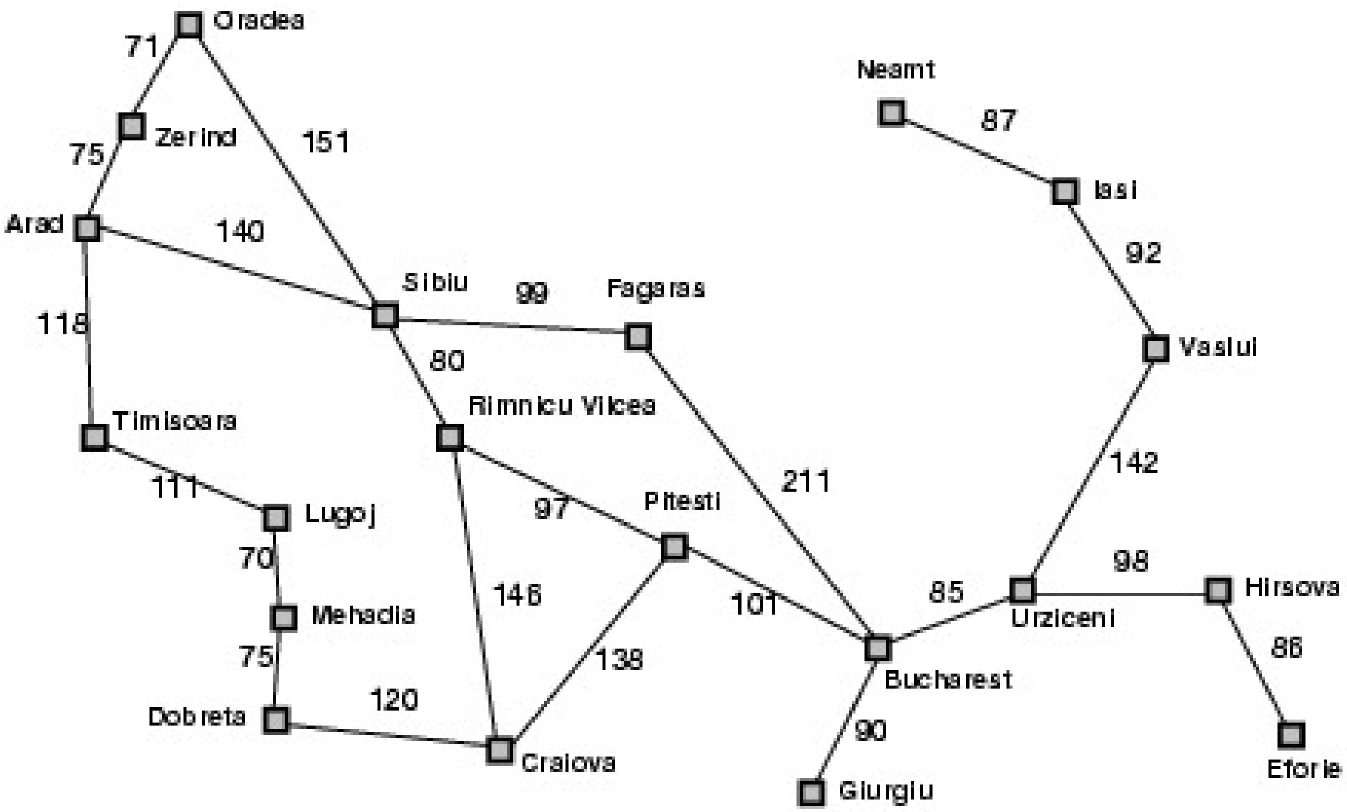
- **Defined by choosing the order of expansion of the nodes!**

Best-First Search

- **Uses an evaluation function that returns a number indicating the interest to expand a node**
- **Greedy-Search - $f(n) = h(n)$ - estimates distance to solution**
- **A* Algorithm - $f(n) = g(n) + h(n)$ - estimates cost of the best solution that passes through n**

Informed Search Example

- **Initial State: Arad; Objective: Bucharest; h(n)= straight line distance**



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
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Greedy-Search

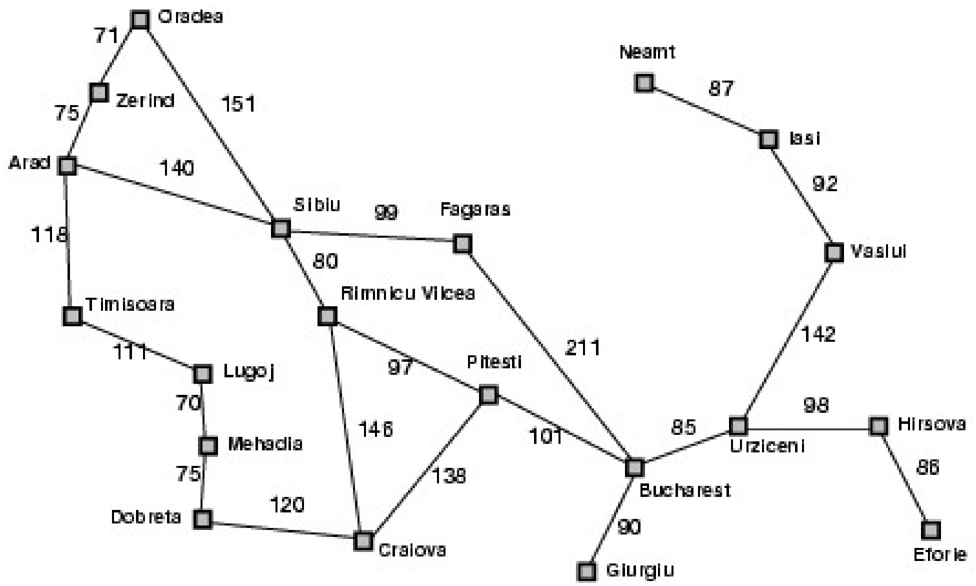
Strategy:

- **Expand the node that appears to be closest to the solution**
- **h (n) = estimated cost of the shortest path from state n to the objective (heuristic function)**
- **function GREEDY-SEARCH(problem) returns a solution or failure**

  return BEST-FIRST-SEARCH(problem,h)
- **Example:**
  - $h_{SLD}(n)$ = straight line distance between n and the objective

Greedy-Search

- **Initial State: Arad; Objective: Bucharest; h(n)= straight line distance**



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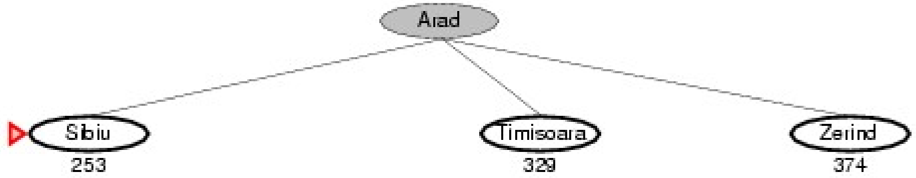
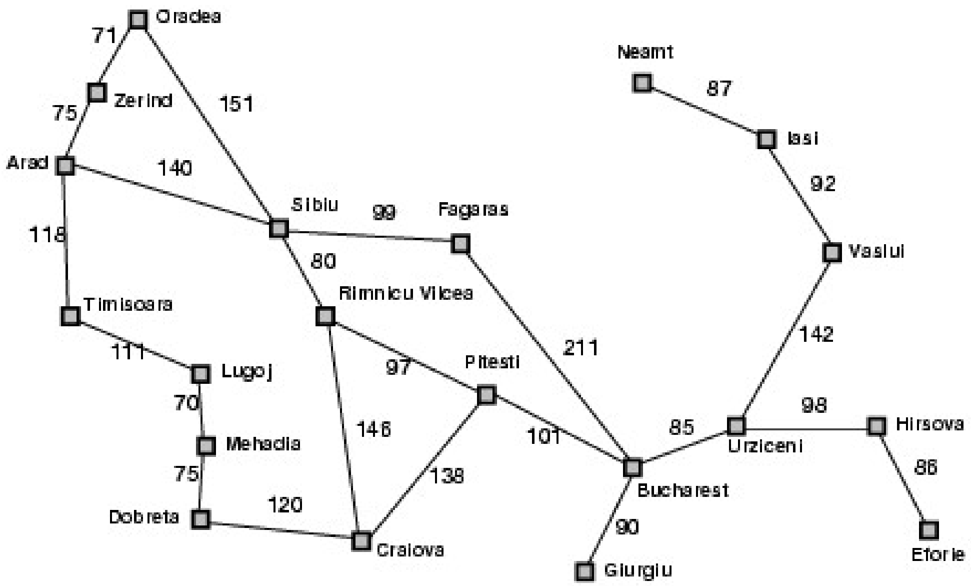
Greedy-Search

- **Initial State: Arad; Objective: Bucharest; h(n)= straight line distance**





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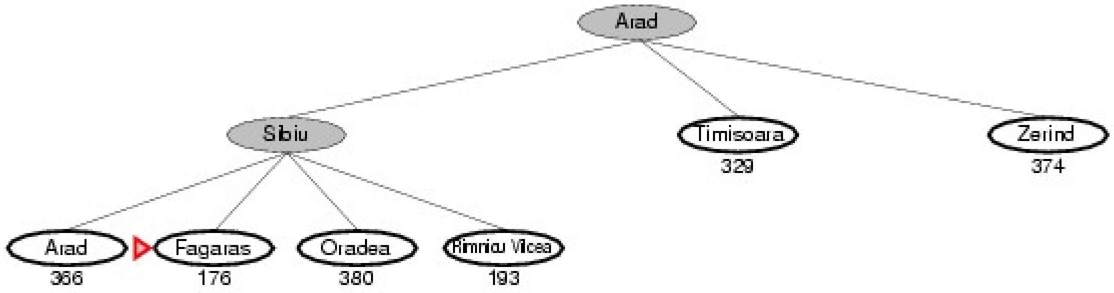
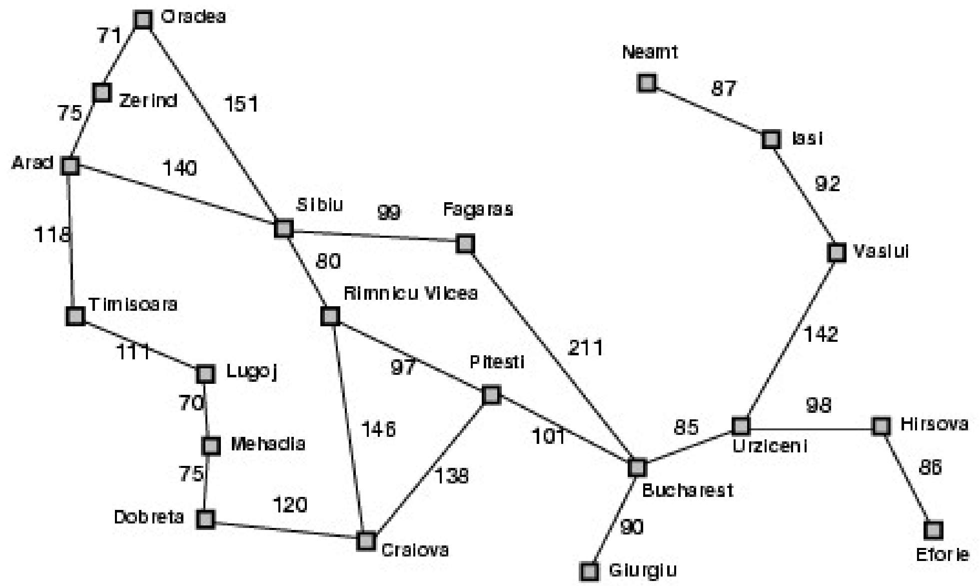
Greedy-Search

- **Initial State: Arad; Objective: Bucharest; h(n)= straight line distance**





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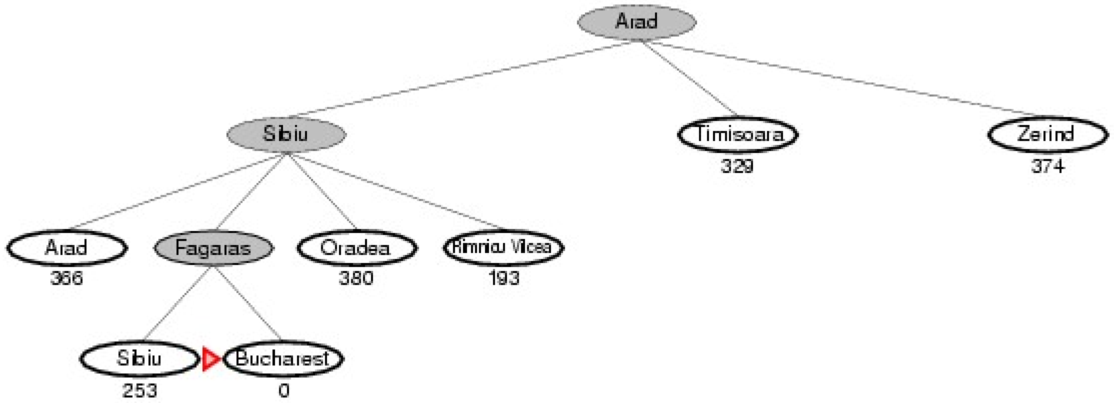
Greedy-Search

- **Estado Inicial: Arad; Objetivo: Bucharest; h(n)= distância em linha reta**



Greedy-Search

Greedy Search Properties:

- **Complete? No! Cycles! (ex:, Iasi → Neamt → Iasi → Neamt → Iasi …)**
- **Susceptible to false starts**
- **Time complexity? O(b^m)**
  - but with a good heuristic function it can decrease considerably
- **Complexity in space? O(b^m)**
  - Keeps all nodes in memory
- **Optimal? No! You don't always find the optimal solution!**
- **It is necessary to detect repeated states!**

A* Algorithm

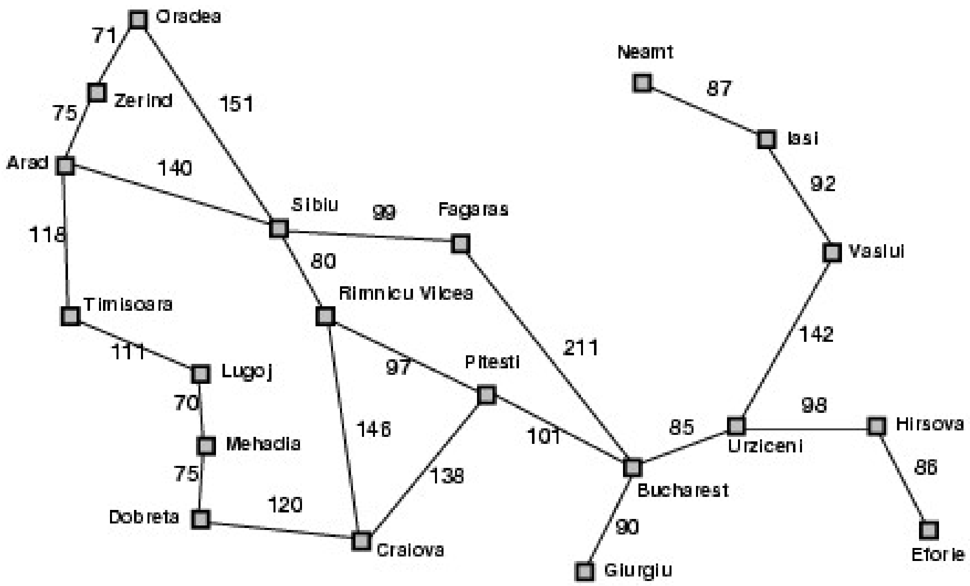
A* Algorithm Strategy:

- **The A * algorithm combines the greedy with the uniform search minimizing the sum of the path already carried out with the minimum expected until the solution.**
- **It Uses the function: f (n) = g (n) + h (n)**
  - g (n) = total cost, so far, to reach state n
  - h (n) = estimated cost to reach the objective (you cannot overestimate the cost to reach the solution! You have to be optimistic!)
- **f (n) = estimated cost of the cheapest solution that passes through node n**
- **function A*-SEARCH(problem) returns a solution or failure**

  return BEST-FIRST-SEARCH(problem,g+h)
- **Algorithm A * is optimal and complete!**
- **Exponential time complexity (but depends on the quality of the heuristic)**
- **Complexity in space: Keeps all nodes in memory!**

A* Algorithm

- **Initial State: Arad; Objective: Bucharest; f(n) = g(n) + h(n);**
- **g(n) = cost from start to node n: h(n)= straight line distance to objective**



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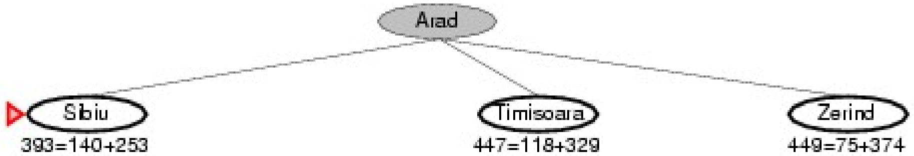

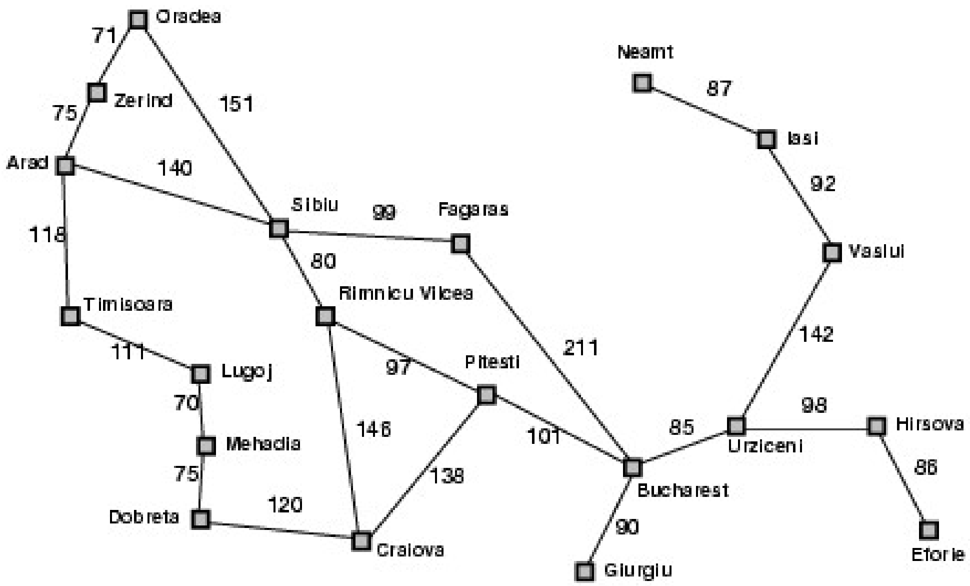

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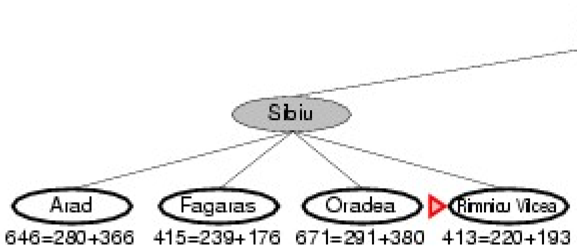

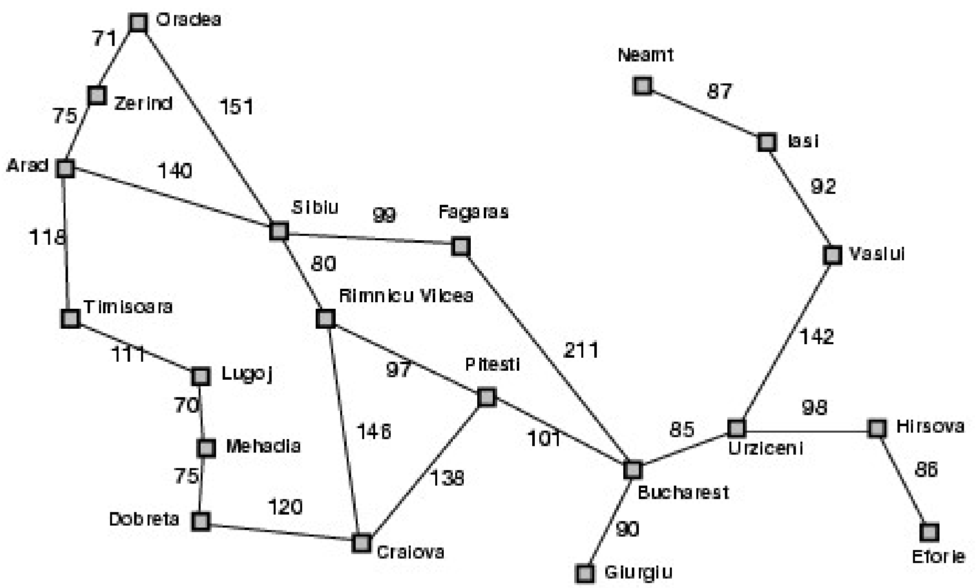

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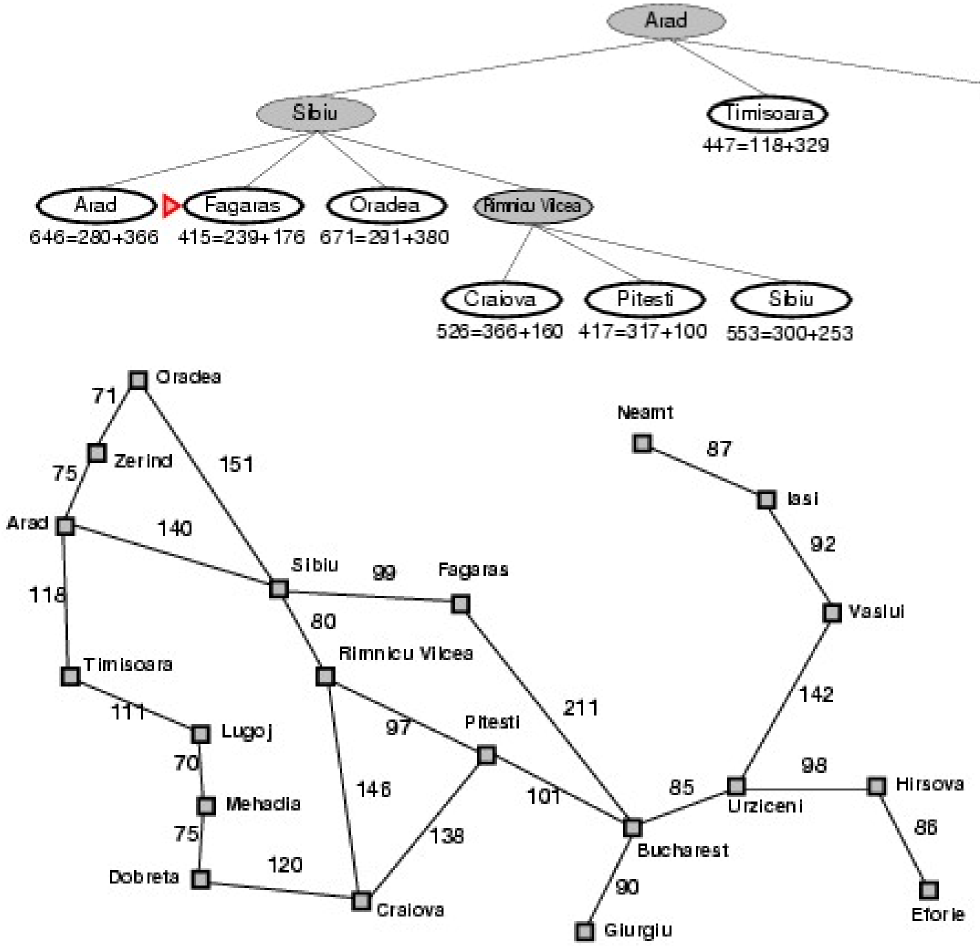

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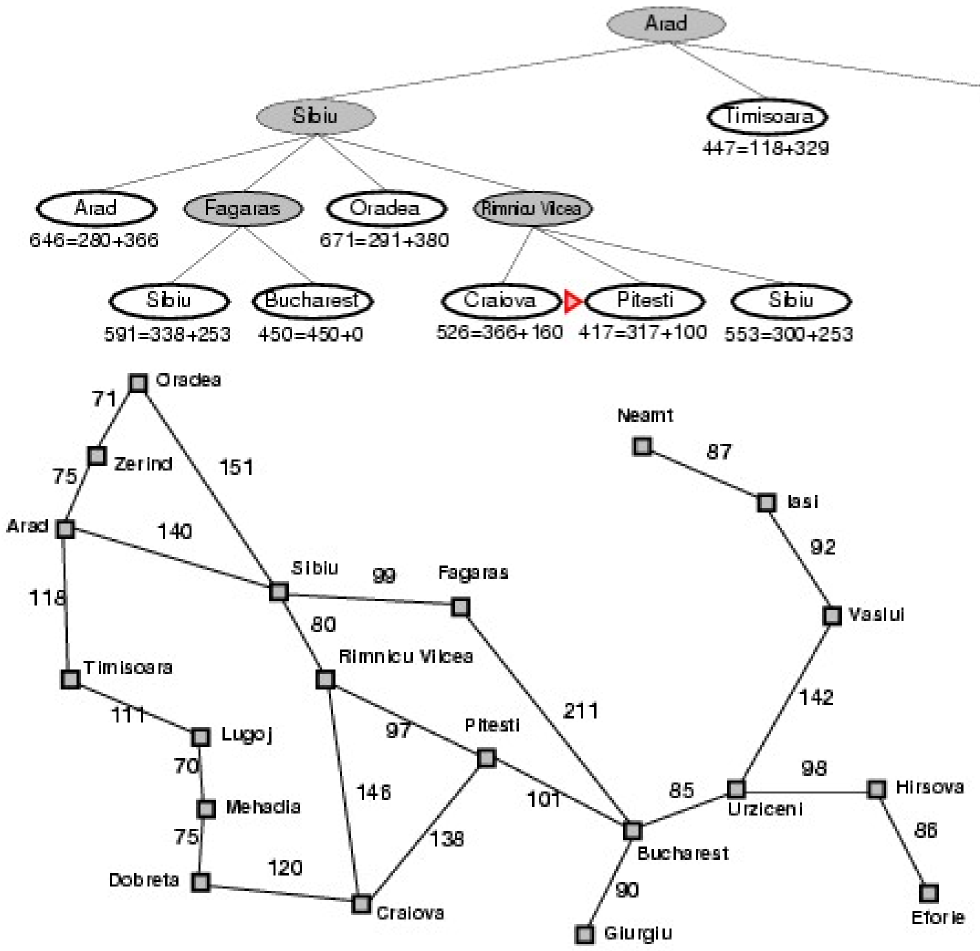

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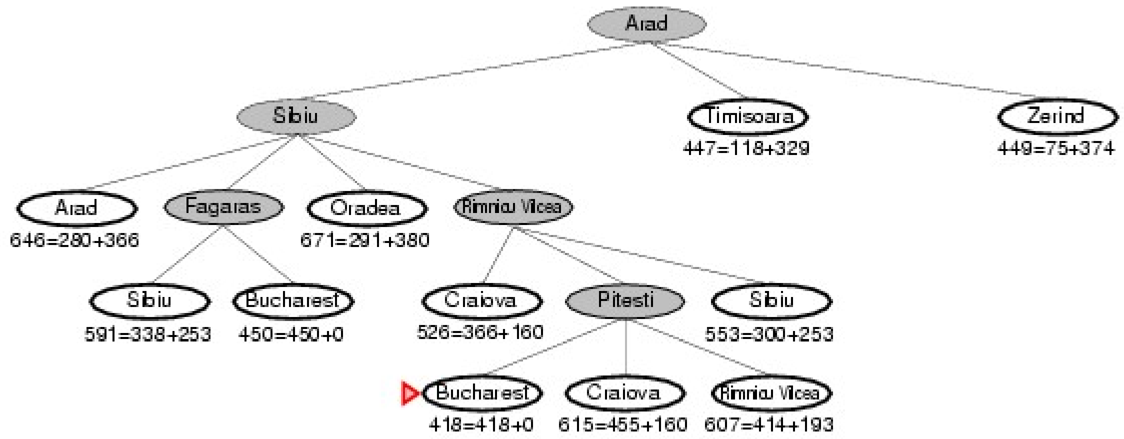
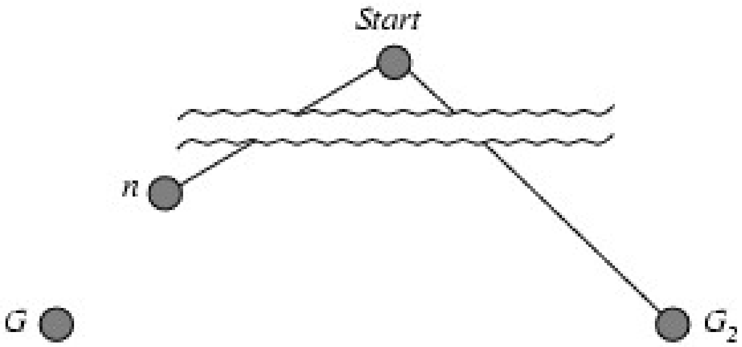
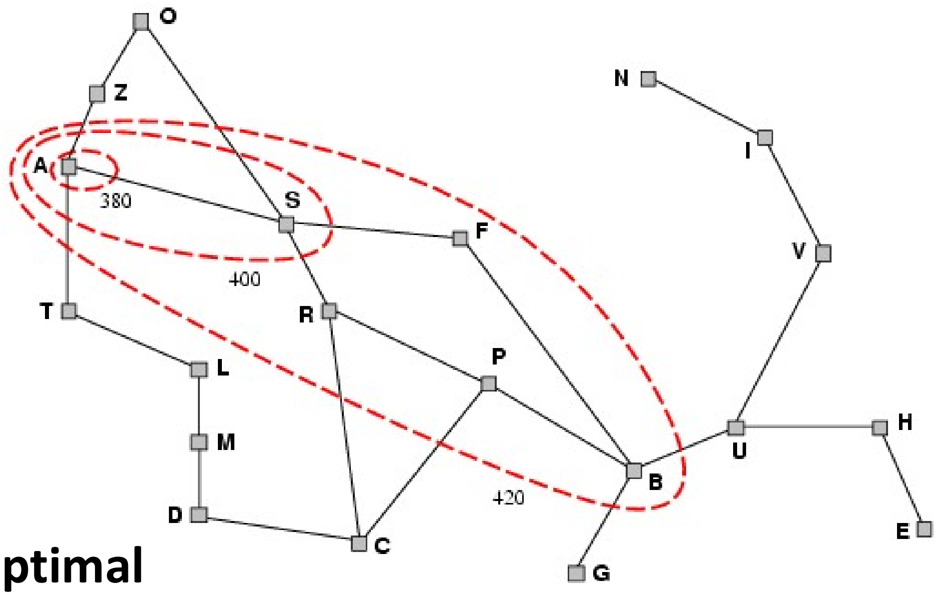


A* Algorithm Optimality

- **Assuming that a sub-optimal G2 objective has been generated and is on the list. n being an unexpanded node that leads to the optimal goal G1**

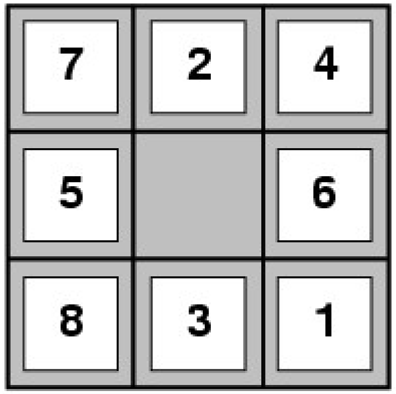




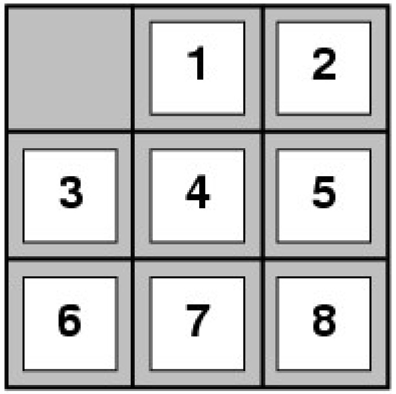
- **f(G2) = g(G2) bacause h(G2)=0**
- **f(G2) > g(G1) because G2 is sub-optimal**
- **f(G2) >= f(n) because h is an admissible heuristic**
- **Thus, the A* Algorithmo never chooses G2 for expansion!**

Heuristic Functions - 8 Puzzle

- Typical 20-step solution with average branching factor: 3
- Number of states: 3^{20} = 3.5 * 109
- Nº States (without repeated states) = 9! = 362880
- Heuristics:
  - h1 = Number of pieces outside the correct placement
  - h2 = Sum of pieces distances to their correct positions



Start State



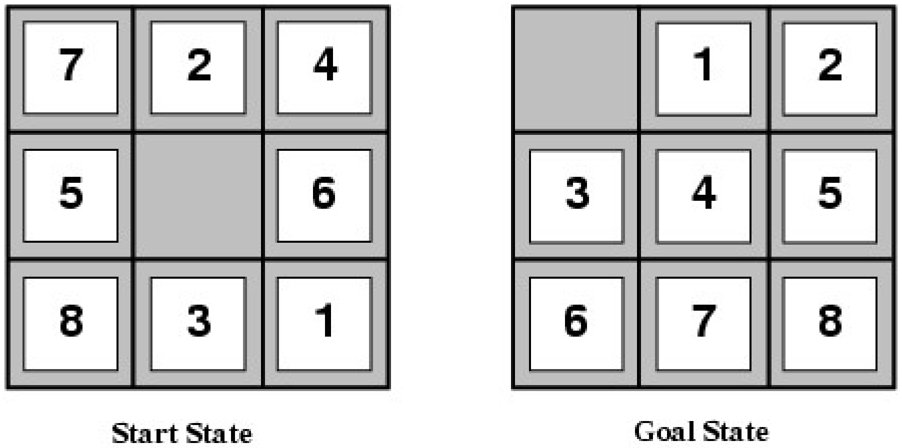
Goal State

Heuristic Functions - 8 Puzzle (2)

Problem Relaxation as a way to invent heuristics:

Piece can move from A to B if A is adjacent to B and B is empty

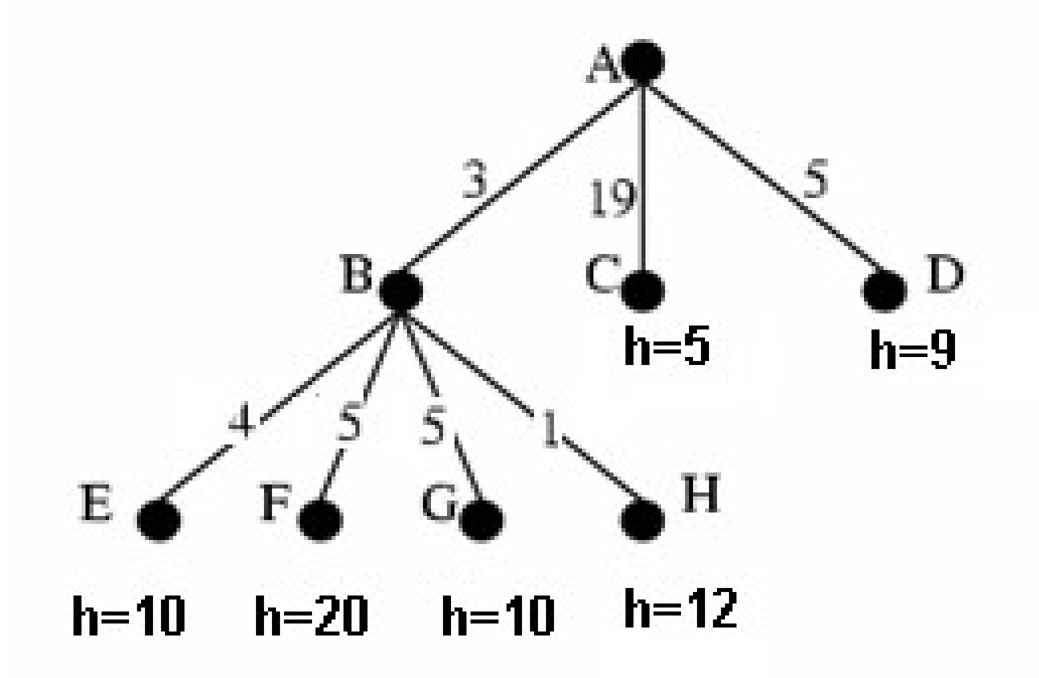
- **a) Piece can move from A to B if A is adjacent to B**
- **b) Piece can be moved from A to B if B is empty**
- **c) Piece can be moved from A to B**



Exercise

Assuming the following search tree in which each arc shows the cost of the corresponding operator, indicate justifying, which node is expanded next using each of the following methods:

a) Breadth-First Search; b) Depth-first Search; c) Uniform Cost search; d) Greedy Search; e) A* Algorithm



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