IART - Artificial Intelligence

Exercise 1: Formulation of Search Problems

Luís Paulo Reis, Henrique Lopes Cardoso

LIACC – Artificial Intelligence and Computer Science Lab.
DEI/FEUP – Informatics Engineering Department, Faculty of Engineering of the University of Porto, Portugal
APPIA – Portuguese Association for Artificial Intelligence

Exercise: Bucket Filling Problem

Two buckets, with capacities c1 (ex: 4 liters) and c2 (ex: 3 liters), respectively, are initially empty. The buckets have no intermediate markings.

The only operations you can perform are:

- empty a bucket
- fill (completely) a bucket
- pour one bucket into the other until the second is full
- pour one bucket into the other until the first is empty

The objective is to determine which operations to carry out so that the first bucket contains n liters (example: 2 liters)?

a) Formulate the Problem as a search problem

b) Solve the problem through a tree search

Search Problem Formulation

- **State Representation**
  - Typically a combination of some variables, arrays and matrixes
  - In this case obviously only two variables needed
- **Initial (Current) State**
- **Objective Test (defines the desired states)**
  - Typically define the function

    bool objective_test(State)
  - In this case we have a clear objective state

Search Problem Formulation

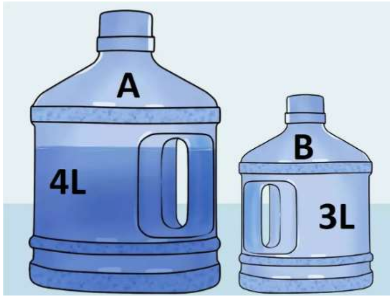
- **Operators (Name, Preconditions, Effects, cost)**
  - For each operator define a name for it and define the functions:

    bool preconditions(State, Operator)

    and
    State effects(State, Operator)
    and the cost of each operator
- **Solution Cost**
  - Typically the sum of the cost of the operators used to get from the initial state to an objective state

Exercise: Bucket Filling Problem

Two buckets, with capacities c1 (ex: 4 liters) and c2 (ex: 3 liters), respectively, are initially empty. The buckets have no intermediate markings.



The only operations you can perform are:

- empty a bucket
- fill (completely) a bucket
- pour one bucket into the other until the second is full
- pour one bucket into the other until the first is empty

The objective is to determine which operations to carry out so that the first bucket contains n liters (example: 2 liters)?

a) Formulate the Problem as a search problem

b) Solve the problem through a tree search

Bucket Filling Problem Formulation

- **State Representation:**

  [W1/W2]   W1:0..4; W2:0..3

- **Initial State:**

  [0/0]

- **Objective State:**

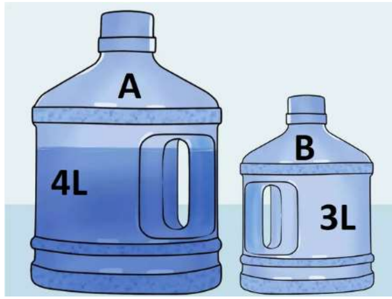
  [2/_] or in general case [NL/_]

- **Operators (3 possibilities):**

  Emp1, Emp2, Fill1, Fill2, Pour12a, Pour 12b, Pour21a, Pour21b

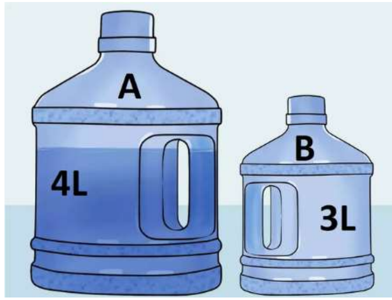
  Emp1, Emp2, Fill1, Fill2, Pour12, Pour21

  Emp(x), Fill(x), Pour(x,y)



Bucket Filling Problem Formulation

- **Operators (1st possibility):**



Emp1 – empty bucket 1

Emp2 – empty bucket 2

Fill1 – fill bucket 1

Fill2 – fill bucket 2

Pour12a – pour bucket 1 to 2 until 2 is full

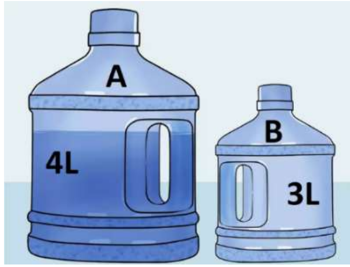
Pour 12b – pour bucket 1 to 2 until 1 is empty

Pour21a – pour bucket 2 to 1 until 1 is full

Pour21b – pour bucket 2 to 1 until 2 is empty

Typically lots of modelling possibilities for state and operators

Bucket Filling Problem Formulation

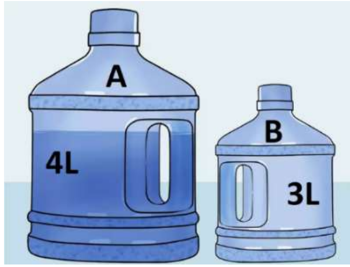


Operators (using possibility 1):

Emp1, Emp2, Fill1, Fill2, Pour12a, Pour12b, Pour21a, Pour21b

Name PreCond Effects Cost

Bucket Filling Problem Formulation

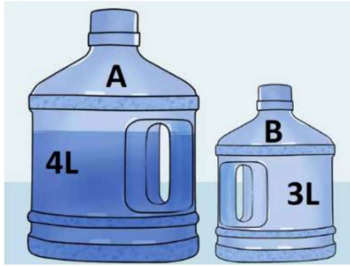


Operators (using possibility 1):

Emp1, Emp2, Fill1, Fill2, Pour12a, Pour12b, Pour21a, Pour21b

Name	PreCond	Effects	Cost
Emp1	W1>0	W1=0	1
Fill1	…		

Bucket Filling Problem Formulation

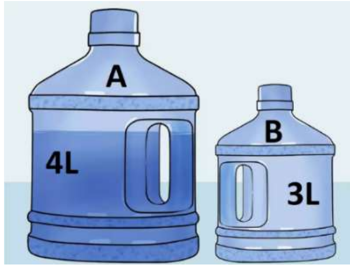


Operators (using possibility 1):

Emp1, Emp2, Fill1, Fill2, Pour12a, Pour12b, Pour21a, Pour21b

Name	PreCond	Effects	Cost
Emp1	W1>0	W1=0	1
Emp2	W2>0	W2=0	1
Fill1	W1<4	W1=4	1
Fill2	W2<3	W2=3	1
Po12a	…		

Bucket Filling Problem Formulation

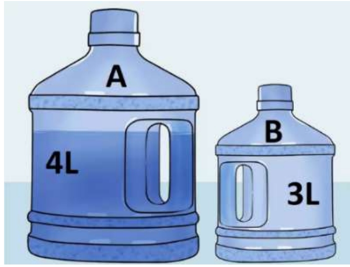


Operators (using possibility 1):

Emp1, Emp2, Fill1, Fill2, Pour12a, Pour12b, Pour21a, Pour21b

Name	PreCond	Effects	Cost
Emp1	W1>0	W1=0	1
Emp2	W2>0	W2=0	1
Fill1	W1<4	W1=4	1
Fill2	W2<3	W2=3	1
Po12a	W1>0/\W2<3/\W1>=3-W2	W1=W1-(3-W2);W2=3	1
Po12b	…		
Po21a	…		
Po21b	…		

Bucket Filling Problem Formulation

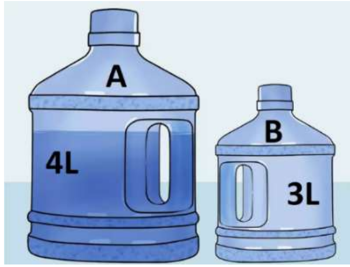


Operators (using moddeling 1):

Emp1, Emp2, Fill1, Fill2, Pour12a, Pour12b, Pour21a, Pour21b

Name	PreCond	Effects	Cost
Emp1	W1>0	W1=0	1
Emp2	W2>0	W2=0	1
Fill1	W1<4	W1=4	1
Fill2	W2<3	W2=3	1
Po12a	W1>0/\W2<3/\W1>=3-W2	W1=W1-(3-W2);W2=3	1
Po12b	W1>0/\W2<3/\W1<3-W2	W2=W1+W2;W1=0	1
Po21a	W2>0/\W1<4/\W2>=4-W1	W2=W2-(4-W1);W1=4	1
Po21b	W2>0/\W1<4/\W2<4-W1	W1=W1+W2;W2=0	1

Bucket Filling Problem Formulation



General Case (capacities C1, C2)

Operators (using moddeling 1):

Emp1, Emp2, Fill1, Fill2, Pour12a, Pour12b, Pour21a, Pour21b

Name	PreCond	Effects	Cost
Emp1	W1>0	W1=0	1
Emp2	W2>0	W2=0	1
Fill1	W1<C1	W1=C1	1
Fill2	W2<C2	W2=C2	1
Po12a	W1>0/\W2<C2/\W1>=C2-W2	W1=W1-(C2-W2);W2=C2	1
Po12b	W1>0/\W2<C2/\W1<C2-W2	W2=W1+W2;W1=0	1
Po21a	W2>0/\W1<C1/\W2>=C1-W1	W2=W2-(C1-W1);W1=C1	1
Po21b	W2>0/\W1<C1/\W2<C1-W1	W1=W1+W2;W2=0	1

Bucket Filling Problem Formulation

Other possible Costs:

C2 -Water Poured from tap ; C3 - Water wasted

Operators (using possibility 1):

Emp1, Emp2, Fill1, Fill2, Pour12a, Pour12b, Pour21a, Pour21b

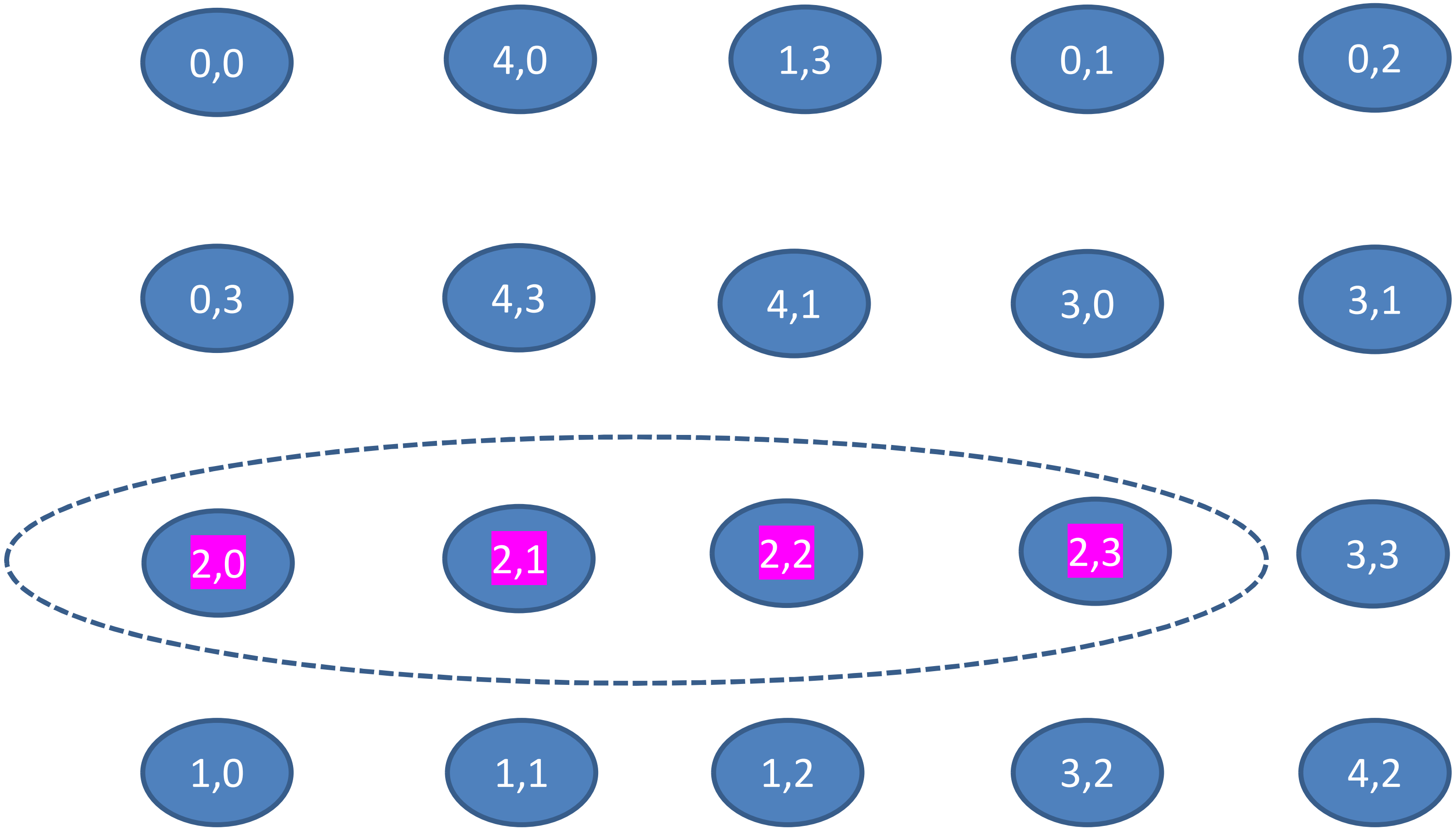
Name	PreCond	Effects	**C2**	**C3**
Emp1	W1>0	W1=0	**0**	**W1**
Emp2	W2>0	W2=0	**0**	**W2**
Fill1	W1<C1	W1=C1	**C1-W1**	**0**
Fill2	W2<C2	W2=C2	**C2-W2**	**0**
Po12a	W1>0/\W2<C2/\W1>=C2-W2	W1=W1-(C2-W2);W2=C2	**0**	**0**
Po12b	W1>0/\W2<C2/\W1<C2-W2	W2=W1+W2;W1=0	**0**	**0**
Po21a	W2>0/\W1<C1/\W2>=C1-W1	W2=W2-(C1-W1);W1=C1	**0**	**0**
Po21b	W2>0/\W1<C1/\W2<C1-W1	W1=W1+W2;W2=0	**0**	**0**

State Space

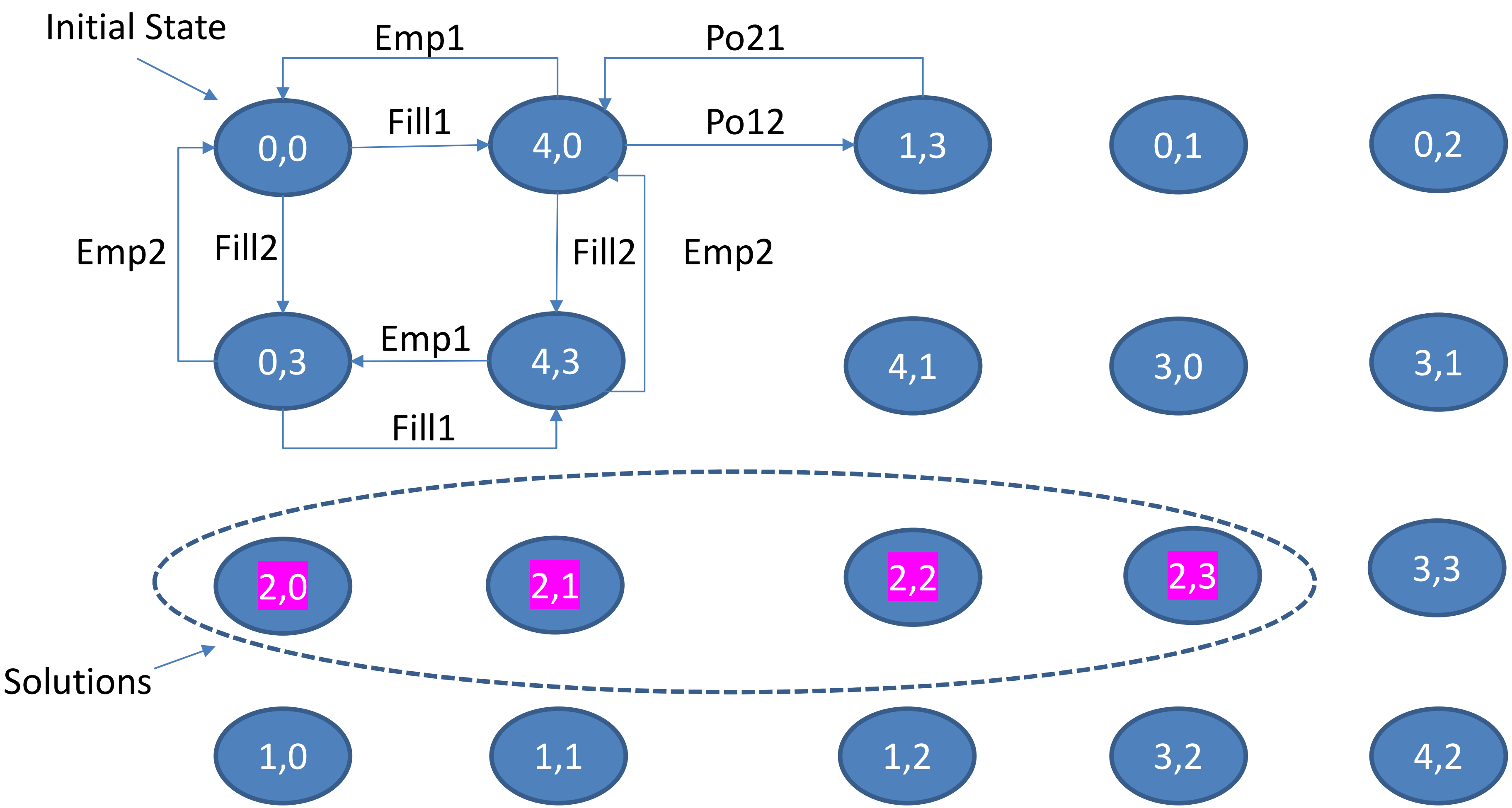
- What is the State Space Size for this problem with two buckets of capacity 4, 3?

State Space

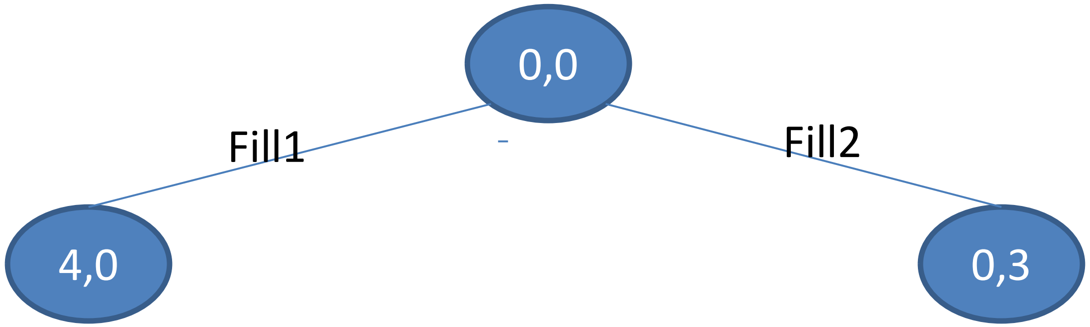
- What is the State Space Size for this problem with two buckets of capacity 4, 3? Size = 5*4 = 20 states



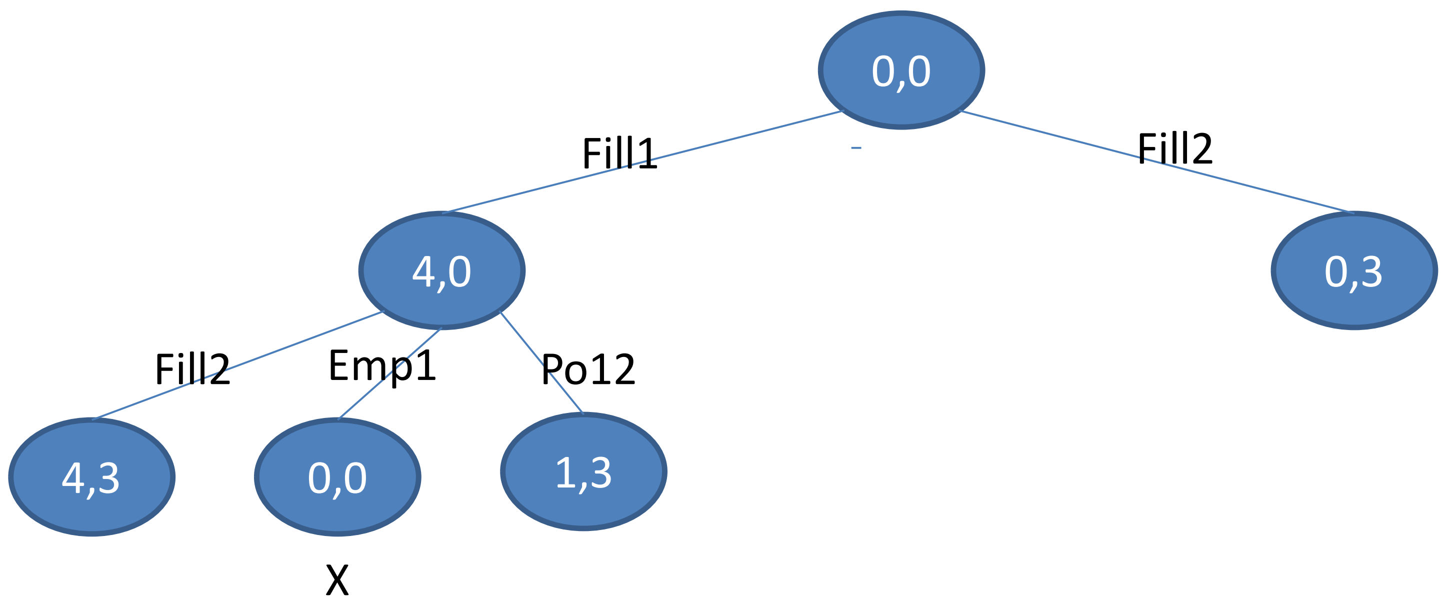
State Space


Initial State
Emp1
Po21
Fill1
Po12
0,0
4,0
1,3
0,1
0,2
Emp2
Fill2
Fill2
Emp2
Emp1
0,3
4,3
4,1
3,0
3,1
Fill1
2,0
2,1
2,2
2,3
3,3
Solutions
1,0
1,1
1,2
3,2
4,2

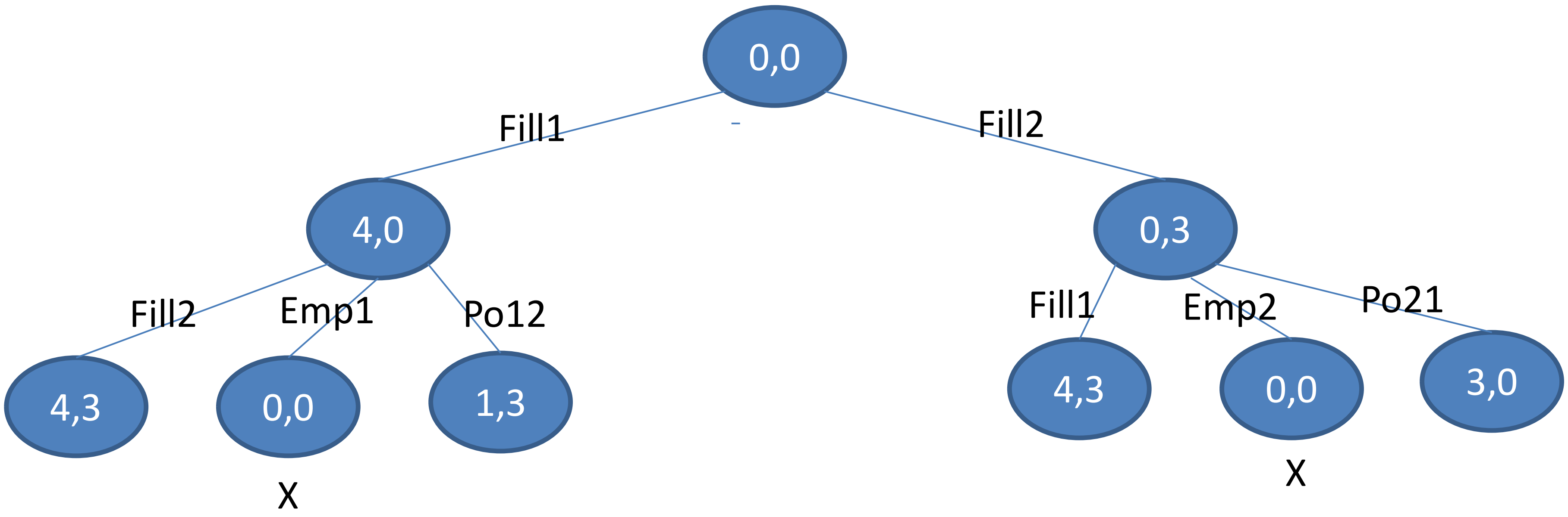
Breadth First Search



Breadth First Search



Breadth First Search



Exercise: Missionaries and Cannibals

3 missionaries and 3 cannibals are on one side of the river with a boat that only takes 2 people.

The objective is to find a way to take the 6 to the other side of the river without ever leaving more cannibals than missionaries on one of the banks during the process!

a) Formulate this problem as a search problem, defining the representation of the state, initial state, the operators (and respective preconditions and effects), the objective test and the cost of the solution.

b) Solve the problem through a tree search

Missionaries/Cannibals Prob. Formulation

- **State Representation:**

  [NM/NC/NB]   NM:0..3; NC:0..3 ; NB:0..1

  // Number of missionaries, cannibals and boats on the initial margin. On the other margin: 3-X

- **Initial State:**

  [3/3/1]

- **Objective State:**

  [0/0/_] or obviously [0/0/0]

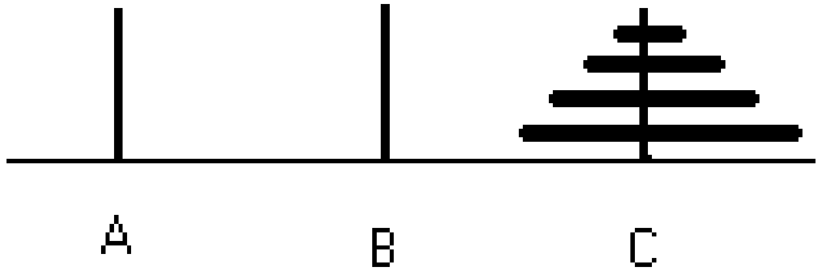
- **Operators (3 possibilities):**

  MM1, CC1, MC1, M1, C1, MM2, CC2, MC2, M2, C2

  MM(Dir), CC(Dir), MC(Dir), M(Dir), C(Dir)

  Trip(NM,NC,Dir)

Exercise: Hanoi Towers



a) Formulate the problem of the Towers of Hanoi as a search problem.

Notes:

- In this version of the problem, you have 3 towers (A, B and C) and 4 disks (D1 to D4).
- Initially the disks are in tower C and the objective is to transfer them to tower A.
- In each move, the player can move a disk from one tower to another tower, if he does not place that disk on a smaller disk.

b) Suppose that the number of disks and the number of towers can be different (n disks and m towers) and formulate this generic version of the problem as a search problem.

c) Solve the problem through a tree search

U.PORTO

FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO

IART - Artificial Intelligence

Exercise 1: Search Problems

Luís Paulo Reis, Henrique Lopes Cardoso

LIACC – Artificial Intelligence and Computer Science Lab.
DEI/FEUP – Informatics Engineering Department, Faculty of Engineering of the University of Porto, Portugal
APPIA – Portuguese Association for Artificial Intelligence