

OPTIMIZATION

Faculdade de Engenharia da Universidade do Porto

DECISION THEORY PROBLEMS

Mestrado em Engenharia Informática e Computação

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EXERCISE 1 WARREN BUFFY INVESTMENTS

Warren Buffy is a great investor who has built his fortune in stock market investments. It was offered to him 3 major investments and he would like to choose one. The first is a conservative investment that would perform very well in a growing economy and suffer only a small loss in a recessionary economy. The second is a speculative investment that would perform extremely well in a growing economy but would perform very poorly in a recessionary economy. The third is a counter-cyclical investment in which he would lose some money in a growing economy but would perform very well in a recessionary economy.

Warren thinks that there are 3 possible scenarios during the period of these investments: growth economy, stagnant and in recession. He is pessimistic about the direction of the economy and assigns 3 probabilities of 0.1, 0.5, and 0.4 to each of these scenarios, respectively. He also estimated that the profits for each of these scenarios would be the ones given in the table below (in millions of dollars):

Alternatives	States of nature		
	Growing economy	Stagnated economy	Economy in recession
Conservative Investment	30	5	-10
Speculative Investment	40	10	-30
Counter-cycle Investment	-10	0	15
Probabilities	0,1	0,5	0,4

- What investment should Warren make on each of the following criteria??
 - Maximin
 - Maxmax
 - Laplace
 - Maximum Expected Utility. Present a decision tree.
- Determine the value of perfect information, and explain its meaning.

SOLUTION:

1.a) Maximin

Alternatives	Growing economy	Stagnated economy	Economy in recession	Min
Conservative	30	5	-10	-10
Speculative	40	10	-30	-30
Counter-cycle Investment	-10	0	15	-10

b) Maximax

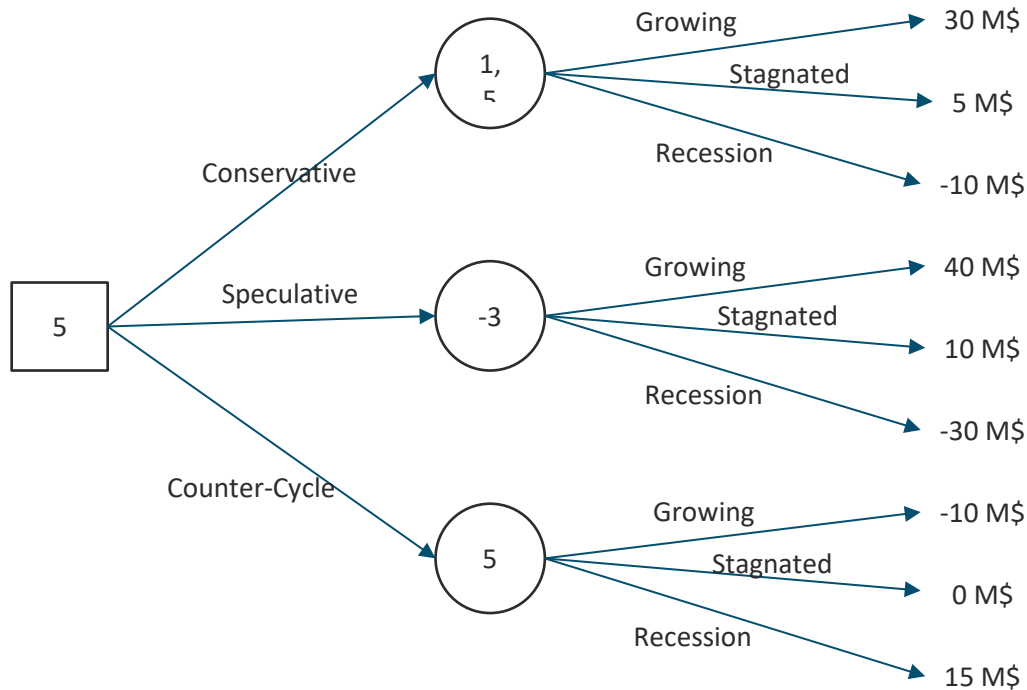
Alternatives	Growing	Stagnated economy	Economy in recession	Max
Conservative	30	5	-10	30
Speculative	40	10	-30	40
Counter-cycle Investment	-10	0	15	15

c) Laplace (or average)

Alternatives	Growing economy	Stagnated economy	Economy in recession	Média
Conservative	30	5	-10	8,33
Speculative	40	10	-30	6,66
Counter-cycle Investment	-10	0	15	1,66

d) Maximum Expected Utility or Bayes' Criterion

Alternatives	Growing economy	Stagnated economy	Economy in recession	V. E.
Conservative	30	5	-10	1,5
Speculative	40	10	-30	-3
Counter-cycle Investment	-10	0	15	5
Probabilities	0,1	0,5	0,4	



2. The expected value with perfect information is:

$$VEIP = (40 \times 0.1 + 10 \times 0.5 + 15 \times 0.4) - ME = 15 - 5 = 10$$

EXERCISE 2 SILICOM DYNAMICS

Silicom Dynamics has developed a new chip that will make it possible to manufacture and market a new personal computer, if they wish it. Alternatively they can sell the rights to the chip for \$ 15.million. If the company decides to produce the computers, the profitability will depend on the company's ability to market the computer during the first year. The company has access to specialty stores that can guarantee sales of 10,000 computers. On the other hand, if the computer is successful, the company can sell 100,000 machines. For analysis purposes, these are the two levels of possible sales of computers, but their probabilities are unknown. The cost of setting up the production line is \$ 6 million. The difference between the selling price and the cost of each computer is \$ 600.

- Formulate this problem of decision analysis identifying the decision alternatives, the states in nature and the table of results.
- What is the decision to make if the maximax criterion is used?
- What is the decision to make if the maximin criterion is used?
- Assuming that the odds of the two levels of sales are 0.5 each, what decision should be made?

If you could hire a specialist in market analysis that would give you perfect information, what would be the maximum amount to pay to that specialist?

SOLUTION:

a)

	States of nature	
Alternatives	Sell 10.000	Sell 100.000
Produce the computer chip	$0 = 10000 \cdot 600 - 6000000$	$54 = 100000 \cdot 600 - 6000000$
Sell the rights	15	15

Values in \$ M

b) Maximax:

	States of nature		
Alternatives	Sell 10.000	Sell 100.000	Max
Produce the computer chip	0	54	54
Sell the rights	15	15	15

Using the maximax criterion, we should produce the computer chip

c) Maximin:

	States of nature		
Alternatives	Sell 10.000	Sell 100.000	Min
Produce the computer chip	0	0	0
Sell the rights	15	15	15

Using the maximax criterion, we should sell the rights.

d) MVE

	States of nature		
Alternatives	Sell 10.000	Sell 10.000	V. E.
Produce the computer chip	0	0	27
Sell the rights	15	15	15
Probabilities	0,5	0,5	

We should produce the computer chip (with an expected value of \$ 27M).

e) The maximum we should pay to a specialist is the expected value with perfect information.

$$VEIP = (54 \times 0.5 + 15 \times 0.5) - MV E = 34,5 - 27 = \$ 7,5$$

EXERCISE 3 BUILDING A FACTORY

Consider that you want to make an investment in a new product line. In order to analyze its feasibility, the market evolution as well as the size of the factory to be built need to be taken into account. The different alternatives are presented below, as well as the income associated with them

Alternatives	Favorable market	Unfavorable market
Build large factory	200.000 m.u.	-180.000 m.u.
Build small factory	100.000 m.u.	-20.000 m.u.
Do not invest	0 m.u.	0 m.u.

Determine the recommended decisions considering the following criteria:

- a) Maximax.
- b) Maximin
- c) Average

SOLUTION:

a. b. c.

Alternatives	Maximax	Maximin	Average
Build large factory	200.000 u.m.	-180.000 u.m.	10.000 u.m.
Build small factory	100.000 u.m.	-20.000 u.m.	40.000 u.m.
Do not invest	0 u.m.	0 u.m.	0 u.m.

EXERCISE 4 STUDENTS TRIP

A group of finalists intends to raise funds for the end-of-course trip. Having agreed that it was not possible to carry out several activities simultaneously, they concluded that viable activities would be: selling flowers at airports and tourist sites, selling newspapers in a suitable place, taking pictures at tourist sites or selling small toys and games at the park. While trying to determine the expected profit from each of the previous activities, the group realized that the profit would depend heavily on the weather conditions. Thus, they decided to build four scenarios corresponding to: mostly cloudy, sun, frequent fog or rain. By

consulting several dealers as to estimate the costs and profits, they were able to collect the information from the table.

Estimated Profit	Weather conditions			
Activity	Cloudy	Sun	Fog	Rain
Flowers	20	50	0	10
Newspapers	10	10	10	10
Pictures	0	40	0	0
Games	10	30	0	0

Which activity should be chosen by the group?

- a) Use the pessimistic criteria
- b) Use the optimistic criteria
- c) Use the Savage criteria

SOLUTION

a. b.

Alternatives	Pessimistic	Optimistic
Flowers	0	50
Newspapers	10	10
Pictures	0	40
Games	0	30

C-Regret Matrix:

	Weather conditions				
Activity	Cloudy	Sun	Fog	Rain	
Flowers	0	0	10	0	10
Newspapers	10	40	0	0	40
Pictures	20	10	10	10	20
Games	10	20	10	10	20

EXERCISE 5 NEW COMPUTER CHIP

One company developed a new computer chip. They can manufacture it or sell its development rights for 800,000 m.u. If they choose to manufacture the computers, the following sales may happen with equally probability:

- The sale of at least 1,000 computers is guaranteed;
- If it goes well, they can sell about 10,000 computers.

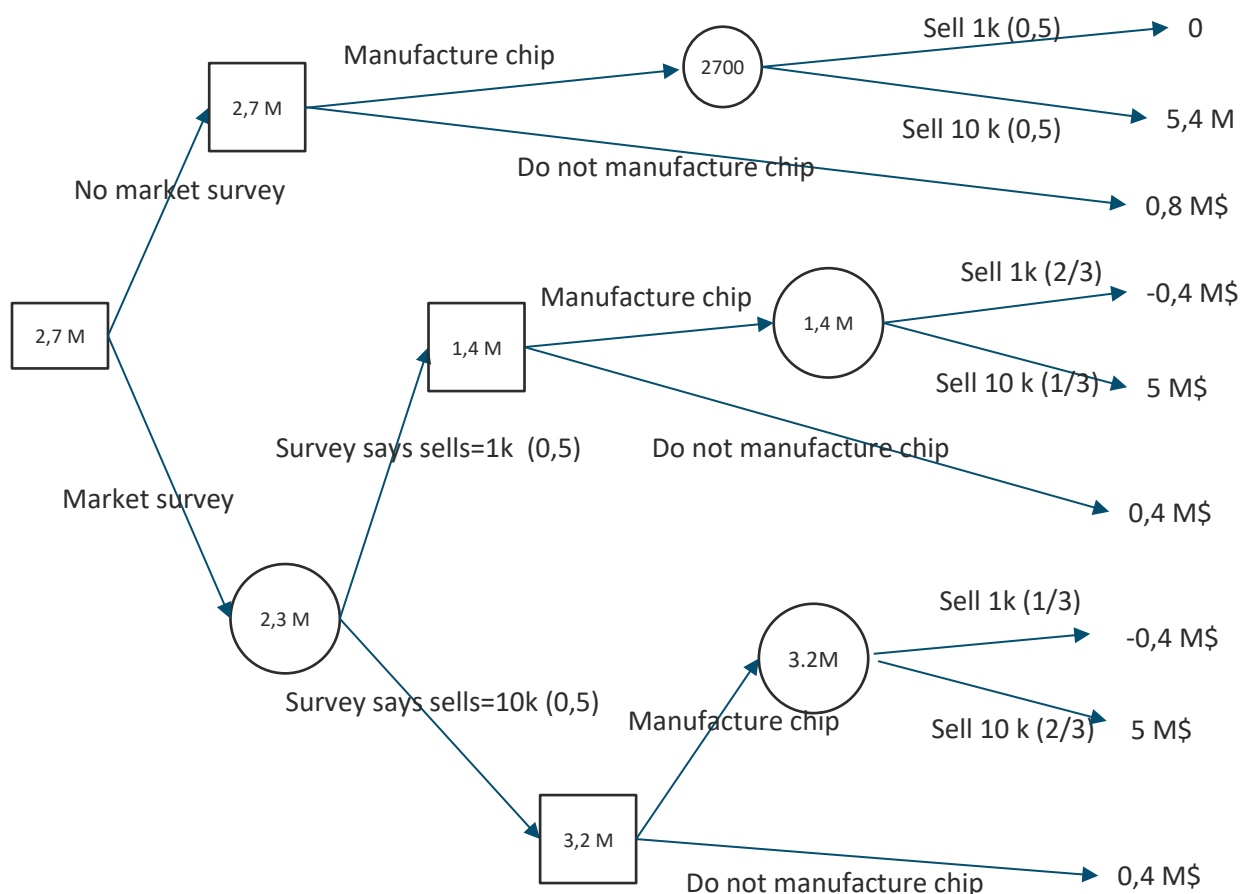
The assembly line cost to manufacture the computers is 600,000 m.u.

The operational profit margin is 600 m.u.

To determine the most realistic level of demand, a market survey can be carried out with a cost of 400,000 m.u. Past experiences indicate that market research is correct two-thirds of the time.

Use a decision tree to analyze the problem.

SOLUTION



The best decision is to not perform the market survey and manufacture the chips.

EXERCISE 6 NEW LESTO PRODUCT

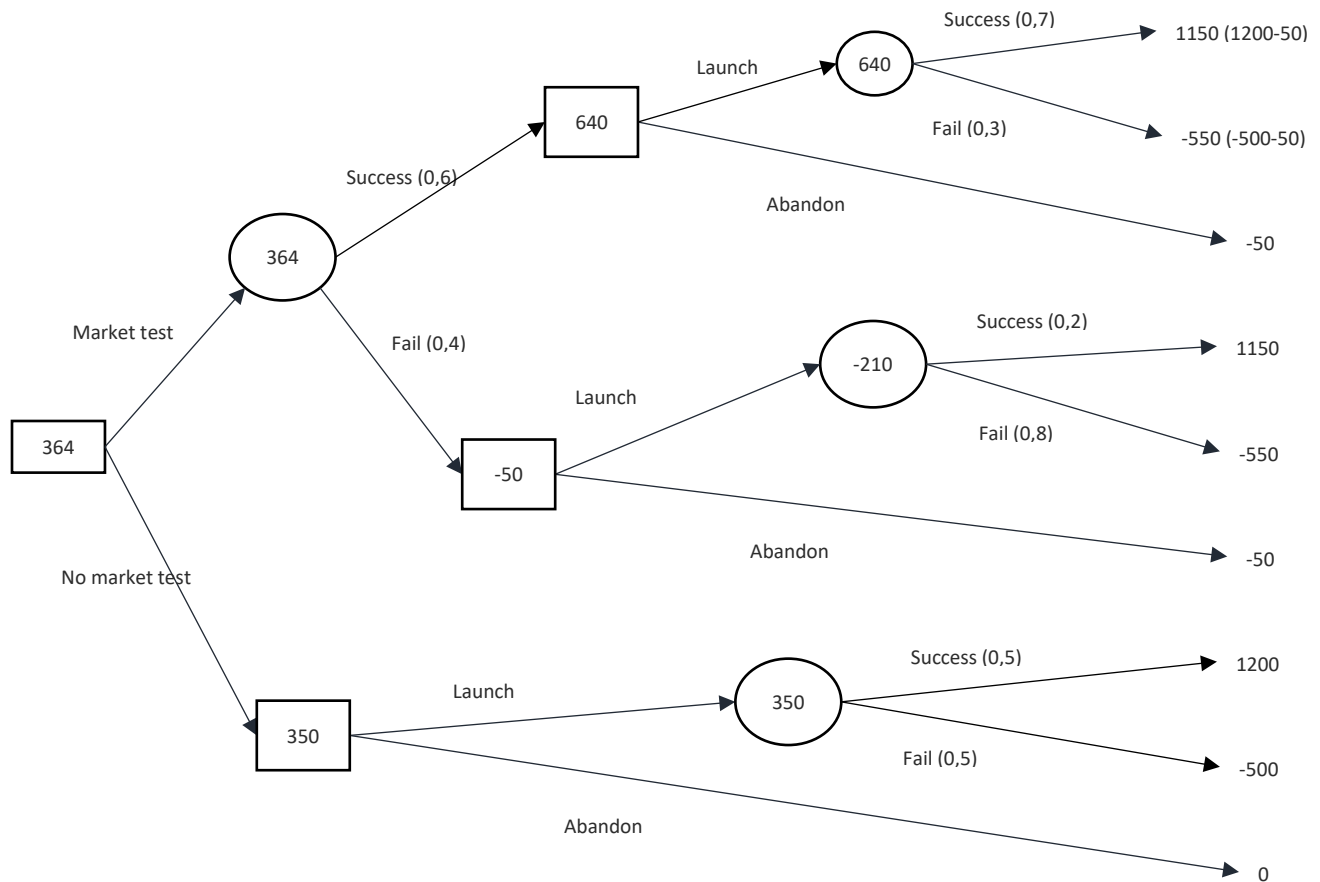
The commercialization of LESTO, a new product (that already has name) of the company Expedita begins to take form. However, as is often the case in launching new brands, there is a considerable risk associated with it - will the product hold up well? In a prudent position, Expedita finds it convenient to introduce LESTO only at the regional level, for market testing, before a national launch. Therefore, the first decision to be taken concerns the conduct (or otherwise) of the market test. The company estimates the cost of 50 000 euros for the market study. If this is the option, they should wait for the results of the test. Then they will decide, supported in the test, if they will launch the commercialization of the LESTO in all the country. On the other hand, if the initial option is not to carry out the tests, then the final decision - whether or not to launch the product at national level, may already be taken. Expedita evaluates the success of the product in the national market at 1 200 000 euros, and a failure to derive a cost to the company of 500 000 euros.

The probabilities to associate with the various events will reflect some scientific knowledge and company's experience with similar products. Thus, Expedita evaluates the likelihood of success (or failure) of LESTO at a national level by 0.5, without any information from market tests. However, if a test is performed and aim for success then the company believes that the probability of success at the national level of the product will be 0.7, whereas otherwise (the test points to failure), the probability of success in the domestic market will be only 0.2. Finally, it is assumed that the probability of the test pointing to a success is 0.6.

- a) What is the appropriate strategy to adopt (criterion of maximum expected value)?
- b) The function of the market test is to obtain more accurate information referring to the national market, in the form of probabilities. Based on the available data, what is the maximum Expedita should pay for this market test?
- c) An increase in the value attributed to the success of LESTO (1200000 euros) will have some consequence for the answer in a.?

SOLUTION

a.



The strategy to adopt is the make the market test which would get an expected value of 364 k euros.

b.

$$((1200 - x) \times 0.7 + (-500 - x) \times 0.3) \times 0.6 - 0.4 \times x \geq 350$$

The maximum value to pay is 64 k euros.

c.

$$[(x - 50) \times 0.7 - 550 \times 0.3] \times 0.6 - 0.4 \times 50 \leq x \times 0.5 - 500 \times 0.5$$

$$x \geq 1375$$

Beyond 1375 k euros, the best strategy is to sell now without testing.

EXERCISE 7 REAL ESTATE BUILDINGS

The public administration of a locality of a certain island intend to promote the construction of a housing estate. There are three zones with conditions for the deployment of the housing estate, identified by A, B and C. In the meantime, a planning is in motion to launch infrastructures and equipment on the island, with the support of the regional government and the European Community, according to which one of

the three zones could benefit from general improvements, which would greatly reduce the costs of building the housing estate. Waiting for the infrastructure plan decision involves a cost of 50 thousand euros.

The following table shows the estimated costs (in thousands of euros) for the construction of the housing estate in question for scenarios A, B or C of the area benefiting from the above improvements. To better support its decision, the regional planning organization, which assigns the probabilities indicated in the table, was consulted on the occurrence of the infrastructure plan in each of the zones with conditions for the implementation of the housing estate.

Cost (thousands of euros)	Infrastructure plan include zone		
Zone for construction	A	B	C
A	400	900	950
B	900	500	800
C	700	700	650
Probabilities	20%	60%	20%

Consider this problem as a problem with two decision variables. Formulate and solve in a context of risk.

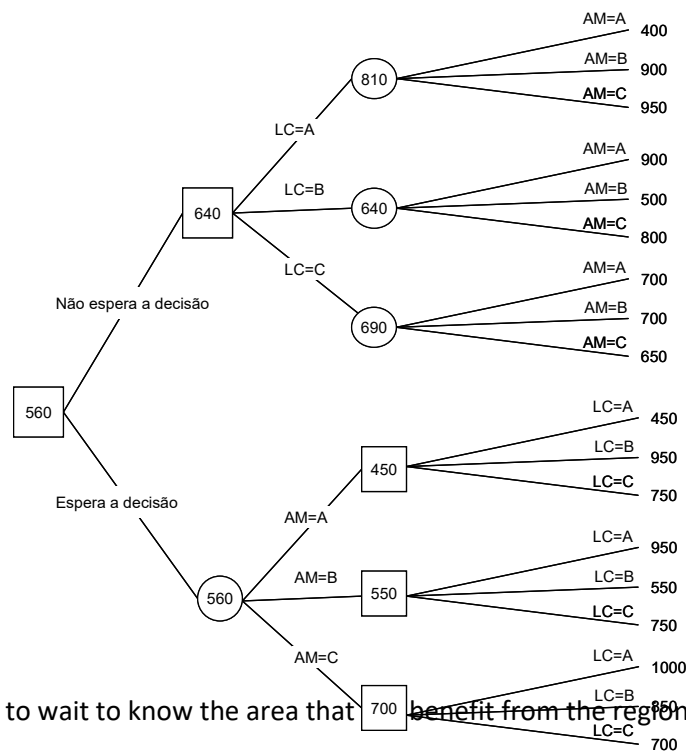
SOLUTION

Decision variables: LC – local where the building is going to be constructed

E – decision about waiting or not for the decision on the infrastructures plan.

States of the nature: AM – Area that will benefit from the investment

Costs (mil €)



The best decision is to wait to know the area that will benefit from the regional development.

GOFERBROKE COMPANY owns a land that may contain oil. A consulting geologist told the board he believes there is a possibility of finding oil of one in four.

Due to this possibility, another oil company offered 90,000 u.m. to buy the land. However, Goferbroke is considering the possibility of remaining with the land in order to drill it itself into oil. If oil is found, the expected revenue will be \$ 800,000, so that the company's expected profit (after deduction of the cost of drilling) will be \$ 700,000, and the company will bear a loss of 100.00 one (the cost of drilling) if the land is dry (no oil).

However, before deciding whether to drill or sell the land, another option is to conduct a detailed seismic survey of the land to obtain a better estimate of the likelihood of finding oil that has a cost of 30,000 u.m.

- a) Apply the maximin criteria to the first Goferbroke Co. problem.
- b) Apply the maximax criteria to the first Goferbroke Co problem.
- c) Apply the maximum expected value criteria to the first problem of Goferbroke Co.
- d) Determine the value perfect information.

SOLUTION:

a.b.c.

Alternatives	Profit	
	Oil	No oil
Drill the terrain	700.000	-100.000
Sell the terrain	90.000	90.000
Probability	0,25	0,75

Alternatives	Maximin	Maximax	MEV
Drill the terrain	-100.000	700.000	100.000
Sell the terrain	90.000	90.000	90.000

d.

Expected value of perfect information = Profit with perfect information- expected profit with risk

Profit with perfect information = $0,25 \times 700.000 + 0,75 \times 90.000 = 242.500$ u.m.

Expected profit with risk = 100.000 u.m.

Expected value of perfect information = 142.500 u.m.

