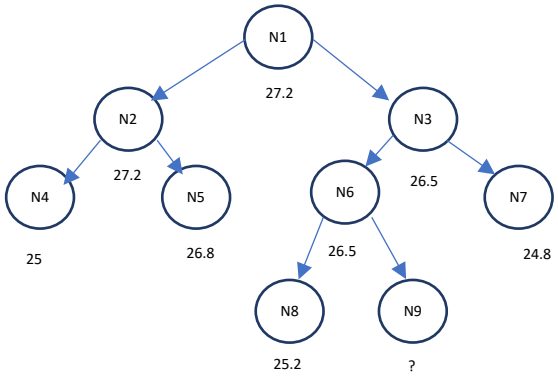
Integer Programming

Additional Exercises

Problem 1

We are using Branch-and-Bound to solve an Integer Program with an objective function in maximization form. All coefficients of the objective function are integer valued.
We currently have the following Branch-and-Bound tree, where nodes are labeled N1,..N_9 and the numbers below each node indicate the value of its LP relaxation. The incumbent solution was obtained in solving the LP at N4. The optimal LP solution was feasible for the IP and had objective value 25.



(a) Let v9 be the optimum value of the LP associated with node N9. Choose the best answer. (It is the answer that is correct and provides the most information.)

i. v9 ≤ 27.2
ii. v9 ≤ 26.5
iii. v9 = 26.5
iv. v9 ≥ 25.2

(b) With the information that we currently have, what are the best upper and lower bounds that we can give on the value v* of the optimal solution for the integer program?

(c) For each of the following nodes of the tree, say whether it is active (A) or fathomed (F) or whether there is not enough information (NEI) to know. We recall that fathoming is the same as pruning.

i. N4
ii. N5
iii. N7
iv. N8
v. N9

Problem 2

A computational application that implements a branch and bound algorithm provided the following list of solutions (in this order, before being interrupted) for a pure integer programming problem. The criterion of choice of the branch variable is the increasing sequence of the index of the variables.

a) Indicate if the integer problem is a maximization or minimization. Justify.
b) Construct the binary tree of sub-problems associated with this list of solutions (indicate the nodes number), including the restrictions of each branch. What was the node selection strategy of this branch-and-bound tree?
c) Which nodes can be pruned ("fathomed or pruned")? Justify.
d) Is the optimal solution known in the current situation? If not, what is the next branch to be analysed?

Solution (node)	Z	X1	X2	X3	X4	X5
1	250,667	28	2,667	0	0	0
2	248,667	27,333	3	0	0	0
3	Infeasible	28	3	0	0	0
4	247,875	27	3	0,25	0	0
5	245,5	26	3	1	0	0
6	247,667	27	3,167	0	0	0
7	242,667	25,333	4	0	0	0
8	247,5	27	3	0	0	0,25
9	242	25,75	3	0	0	1
10	247	27	3	0	0,143	0
11	237	25	3	0	1	0
12	246	27	3	0	0	0
13	248	28,5	2	0	0	0
14	245,333	29	1,333	0	0	0
15	247,5	28	2	0	0,5	0

Problem 3

Consider the following capital budgeting problem: We have a set of six possible investments with the following characteristics:

Investment	1	2	3	4	5	6
NPV Added	$33	$45	$25	$17	$39	$23
Cash Required	$10	$14	$8	$6	$12	$8

(a) Write an integer program to determine the optimal set of investments that maximizes the net present value.

(b) (Use the Excel file). Start with the incumbent solution (best feasible solution so far) x1 =1, x2 =0, x3 =0, x4 = 1, x5 = 1, and x6 = 0. Its objective value is 89. Solve for the first five nodes of the Branch and Bound tree as given on the spreadsheet. (The spreadsheet is already set up to solve the linear program.) You should adjust upper or lower bounds in each case to solve the LP. Write the solutions in the spaces indicated on the spreadsheet. Also enter the objective values manually after the solutions to the nodes of the tree. (Don't copy cell K19 because it contains a formula). Indicate in column M whether the nodes of the tree are fathomed or not.

Problem 4

Consider the following integer programming problem, presented in Figure 1:

$$
\begin{aligned}
max\ Z = & -x_1 + 4x_2 \\
& -10x_1 + 20x_2 \leq 22 \\
& 5x_1 + 10x_2 \leq 49 \\
& x_1 \leq 5 \\
& x_1, x_2 \geq 0, integer
\end{aligned}
$$

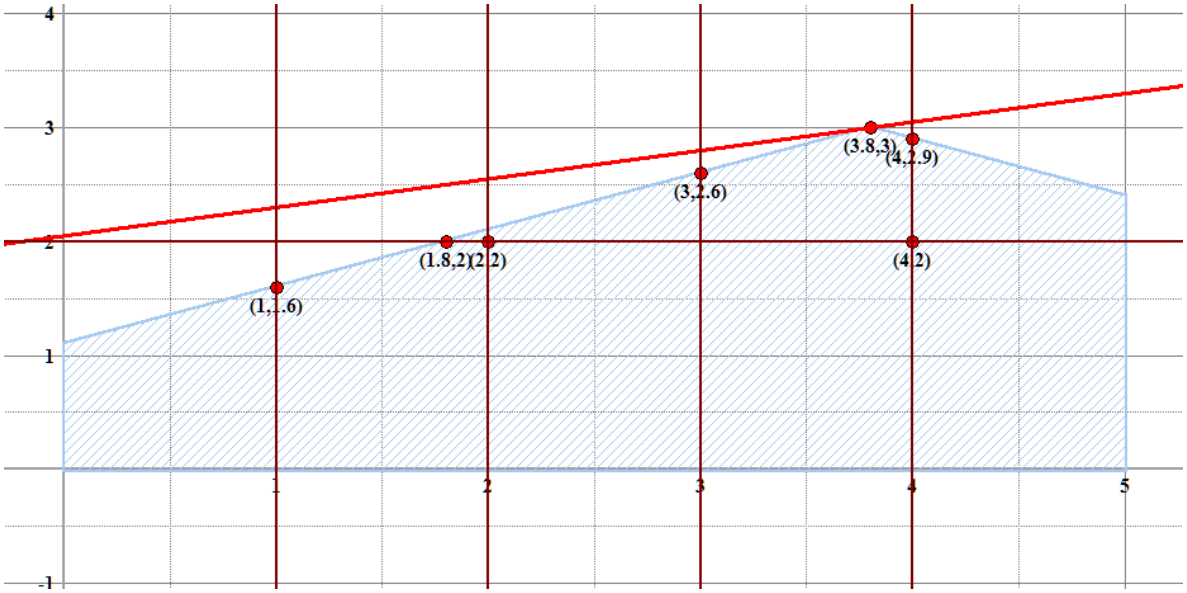


Figure 1

Using the Branch&Bound method the following solutions were obtained (Table 1). Analysing the graphical resolution presented, indicate the branch& bound tree for this problem. Was the optimal solution obtained? Justify your answer.

X	Y
1	1.6
1.8	2
2	2
3	2.6
3.8	3
4	2
4	2.9