

# OPTIMIZATION

## LESSON 2.2

### **Formulation Exercises**

# FORMULATING LINEAR PROBLEMS

# A product-mix problem

- A furniture manufacturer produces **tables** and **chairs**. The process involves machining, sanding, and assembling the pieces to make the tables and chairs.
- It takes **5h** to machine the pieces for a table, **4h** to sand the pieces, and **3h** to assemble a table.
- A chair requires **2h** to machine the pieces, **3h** to sand the pieces, and **4h** to assemble a chair.
- There are **270h** available for machining the pieces, **250h** for sanding the pieces, and **200h** for assembling.
- If the profit for a table is **€100** and for a chair **€60**, how many tables and chairs should the manufacturer produce in order to **maximize** the overall **profit**?
- What if there is an additional requirement to produce four chairs for each table?

# LP MODEL

Resource/Item	Per Unit Product		Resource Availability (h)
	Table	Chair	
Machining	5	2	270
Sanding	4	3	250
Assembly	3	4	200
Profit per unit (€)	100	60	

Maximize  $Z = 100T + 60C$

Subject to

$$5T + 2C \leq 270 \quad \text{Machining}$$

$$4T + 3C \leq 250 \quad \text{Sanding}$$

$$3T + 4C \leq 200 \quad \text{Assembly}$$

$$T \geq 0 \text{ and } C \geq 0 \quad \text{Nonnegativity}$$

# Excel Model

# Excel Model

Decision variables	Table	Chair	Objective function	
Number of units to produce	0	0	0	
Profit per unit	100	60		
Constraints			LHS	RHS
Machining	5	2	0	270
Sanding	4	3	0	250
Assembly	3	4	0	200

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method  
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

# Excel Solution

Decision variables	Table	Chair	Objective function	
Number of units to produce	48.5714	13.5714	5671.428571	
Profit per unit	100	60		
Constraints			LHS	RHS
Machining	5	2	270	270
Sanding	4	3	235	250
Assembly	3	4	200	200

Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$L\$11	Number of units to produce		
11	Objective function	0	5671.428571

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$I\$11	Number of units to produce			
1	Table	0	48.57142857	Contin
\$J\$11	Number of units to produce			
11	Chair	0	13.57142857	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$K\$15					
15	Machining LHS	270	\$K\$15<=\$L\$15	Binding	0
\$K\$16					
16	Sanding LHS	235	\$K\$16<=\$L\$16	Not Binding	15
\$K\$17					
17	Assembly LHS	200	\$K\$17<=\$L\$17	Binding	0

**IBM ILOG CPLEX  
Optimization  
Studio**



# How to install IBM CPLEX

- Go to <https://www.ibm.com/products/ilog-cplex-optimization-studio>
- Choose “No-cost academic edition”
- Sign up with the FEUP email address
- Fill the form and check the email to complete the registration

Enter your academic institution issued email to begin

Only the students and faculty of participating academic institutions are eligible to access this website. Please enter your academic institution issued email below to register.

Your academic institution issued email

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Size 4MB

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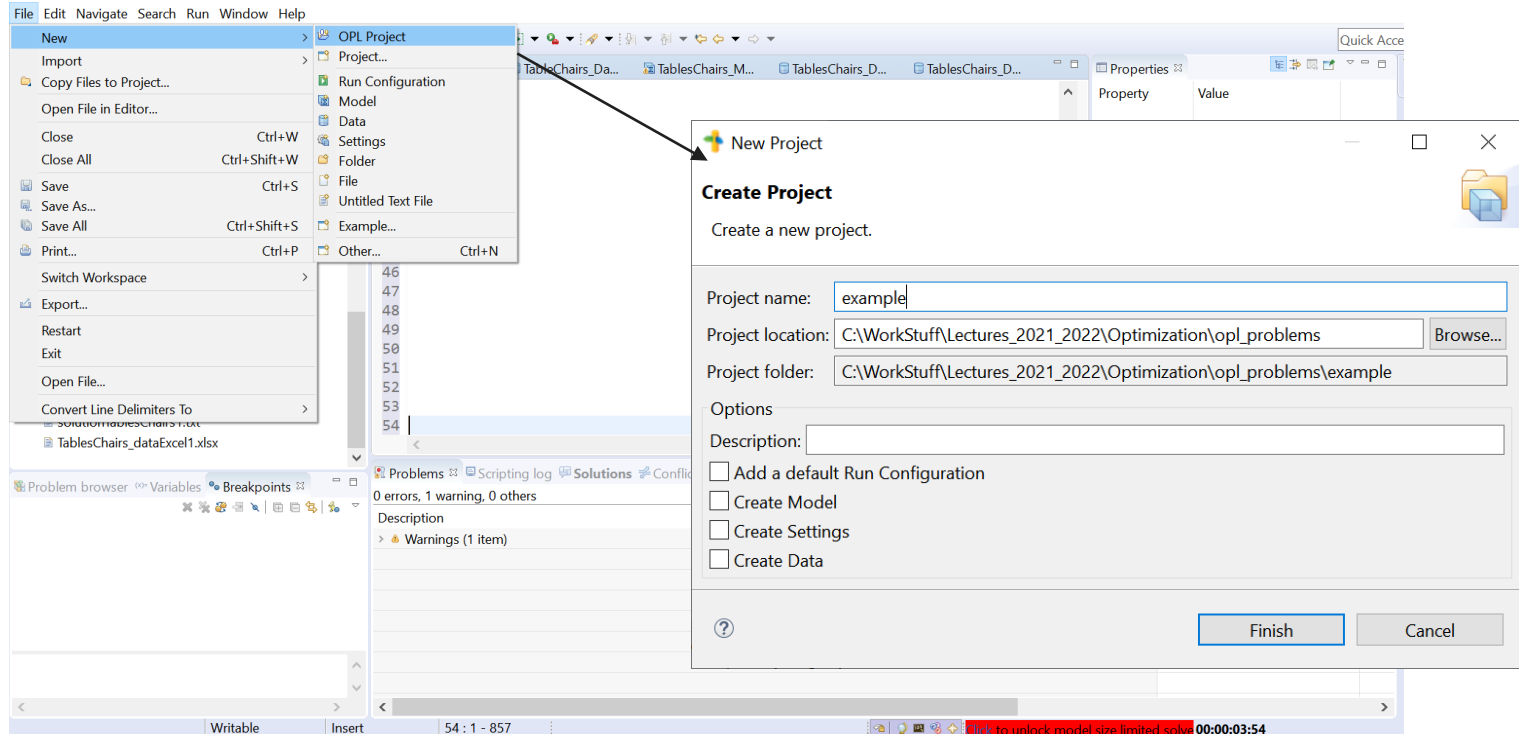
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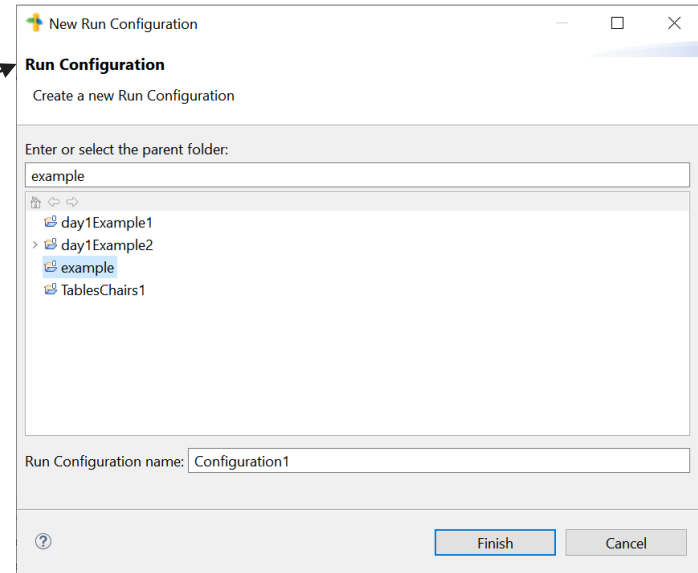
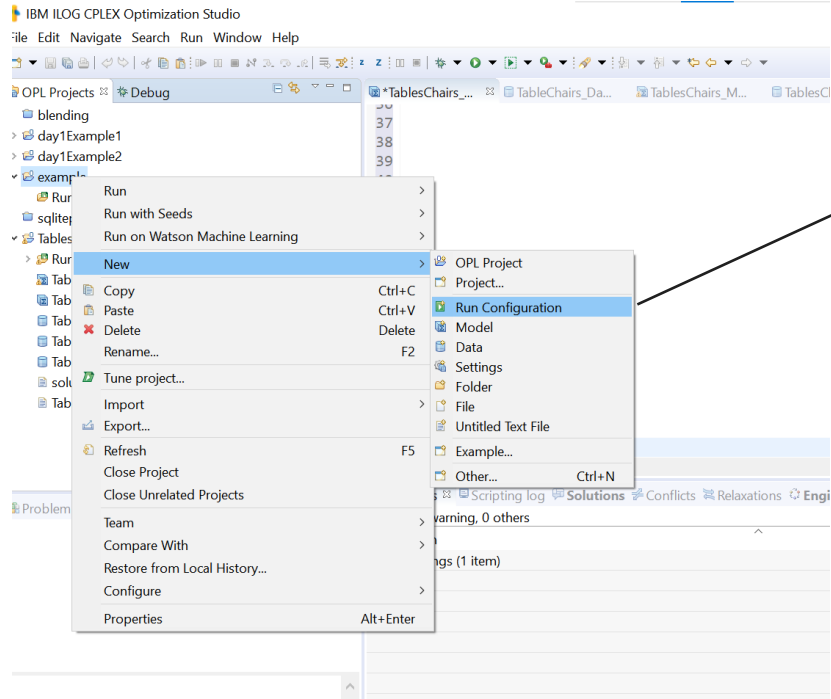
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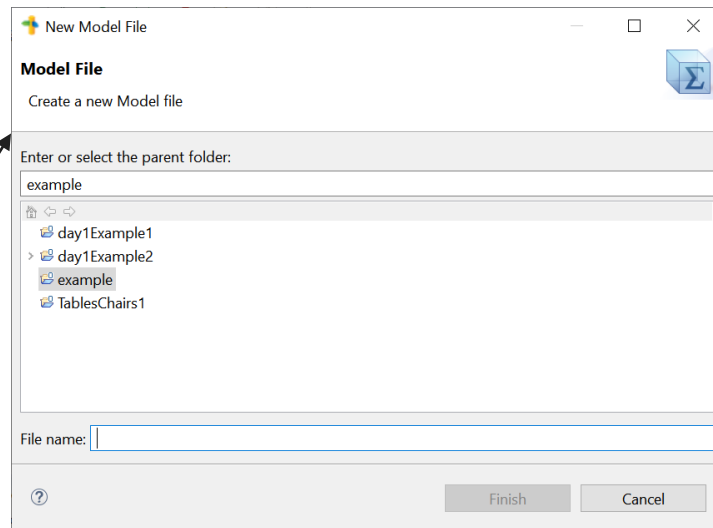
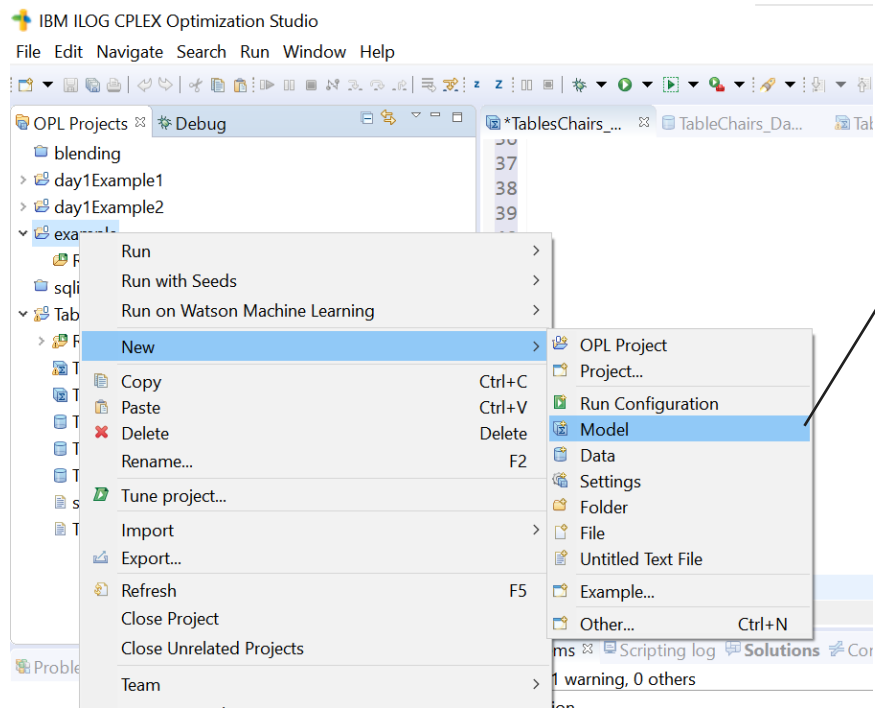
# 1. Creating a new project



## 2. Adding a run configuration



# 3. Creating a model



# 3.1 Model 1

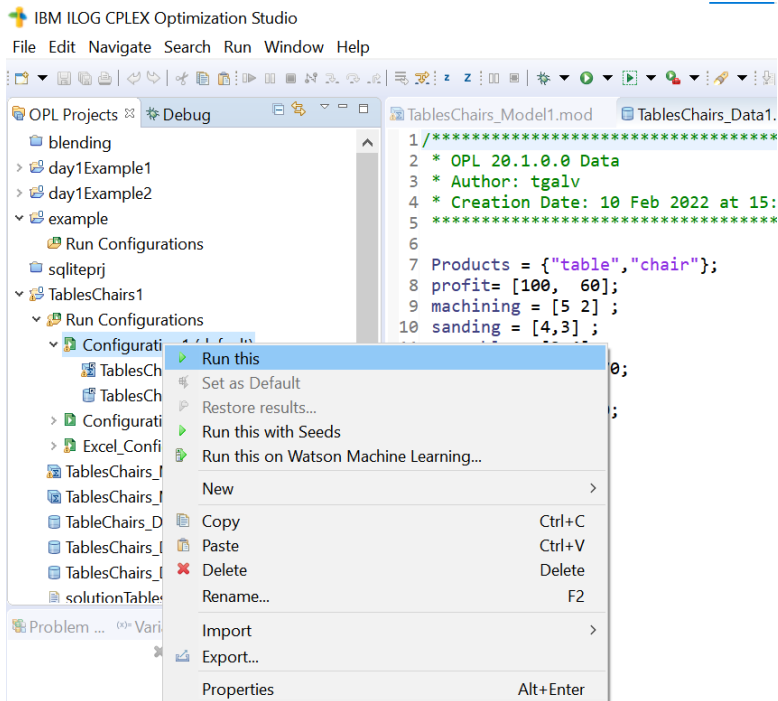
## File .mod – with the model

```
TablesChairs_Model1.mod
1 /*****
2  * OPL 20.1.0.0 Model
3  * Author: tgalv
4  * Creation Date: 10 Feb 2022 at 14:06:35
5  *****/
6 //declare data
7
8 {string} Products = ...; // products
9
10 float profit[Products] = ...;
11 float machining[Products] = ...;
12 float sanding[Products] = ...;
13 float assembly[Products] = ...;
14 float timeMachining = ...;
15 float timeSanding = ...;
16 float timeAssembly = ...;
17
18 //decision variables
19 dvar float+ x[Products];
20
21 maximize sum(i in Products) profit[i] * x[i] ;
22
23 subject to {
24     sum (i in Products) machining[i]* x[i] <= timeMachining;
25     sum (i in Products) sanding[i]* x[i] <= timeSanding;
26     sum (i in Products) assembly[i]* x[i] <= timeAssembly;
27 }
28
29 execute OUTPUT_RESULTS {
30     var file = new IloOplOutputFile ("solutionTablesChairs1.txt", true);
31     file.writeln ("Objective Function = ", cplex.getObjValue ());
32     for (i in Products)
33         file.writeln ("Variable x[" , i , "] = ", x[i]);
34 }
35
```

## File .dat – with the data

```
TablesChairs_Model1.mod  TablesChairs_Data1.dat
1 /*****
2  * OPL 20.1.0.0 Data
3  * Author: tgalv
4  * Creation Date: 10 Feb 2022 at 15:38:49
5  *****/
6
7 Products = {"table", "chair"};
8 profit= [100, 60];
9 machining = [5 2] ;
10 sanding = [4,3] ;
11 assembly = [3,4] ;
12 timeMachining = 270;
13 timeSanding = 250;
14 timeAssembly = 200;
```

# 3.1 Running Model 1



```
odel1.mod: CPLEX  
ata1.dat  
  
on1  
1.mod: CPLEX  
2.mod: CPLEX  
  
at  
dat  
cel1.dat  
s1.txt  
  
Breakpoi...  
  
11 float machining[Products] = ...;  
12 float sanding[Products] = ...;  
13 float assembly[Products] = ...;  
14 float timeMachining = ...;  
15 float timeSanding = ...;  
16 float timeAssembly = ...;  
17  
  
Solutions  
// Quality There are no bound infeasibilities.  
// There are no reduced-cost infeasibilities.  
// Maximum Ax-b residual = 2.84217e-14  
// Maximum c-B'pi residual = 0  
// Maximum |x| = 48.5714  
// Maximum |slack| = 15  
// Maximum |pi| = 15.7143  
// Maximum |red-cost| = 0  
// Condition number of unscaled basis = 1.3e+01  
//  
x = [48.571  
13.571];
```

## 3.2 Model 2

### File **.mod** – with the model

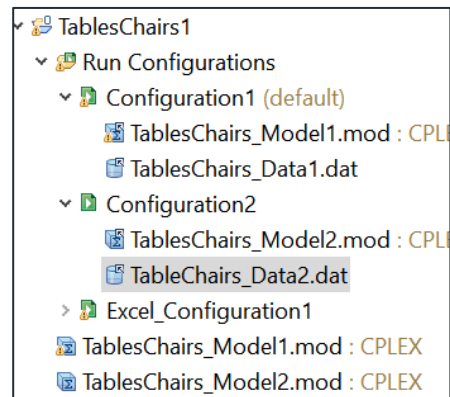
```
TablesChairs_Model2.mod  TableChairs_Data2.dat
1 /*****
2  * OPL 20.1.0.0 Model
3  * Author: tgalv
4  * Creation Date: 10 Feb 2022 at 17:52:22
5  *****/
6 //declare data
7
8 {string} Products =...; // products
9 {string} Sections =...; // sections
10
11 float Profit[Products] = ...;
12 float TimePerSection[Sections][Products] = ...;
13
14 float AvailableTime[Sections] = ...;
15
16 //decision variables
17 dvar float+ x[Products];
18
19 maximize sum(i in Products) Profit[i] * x[i] ;
20
21 subject to {
22   forall (i in Sections)
23     sum (j in Products) TimePerSection[i,j]* x[j] <= AvailableTime[i];
24 }
25
26 execute OUTPUT_RESULTS {
27   var file = new IloOplOutputFile ("solutionTablesChairs1.txt", true);
28   file.writeln ("Objective Function = ", cplex.getObjValue ());
29   for (i in Products)
30     file.writeln ("Variable x[" , i , "] = ", x[i]);
31 }
32
33 --
```

The sections are  
not defined in the  
model

### File **.dat** – with the data

```
TablesChairs_Model2.mod  TableChairs_Data2.dat
1 /*****
2  * OPL 20.1.0.0 Data
3  * Author: tgalv
4  * Creation Date: 10 Feb 2022 at 18:01:58
5  *****/
6 Products = {"table","chair"};
7 Sections = {"machining","sanding","assembly"};
8
9 Profit= [100, 60];
10 TimePerSection = [[5,2],[4,3],[3,4]];
11 AvailableTime =[270,250,200];
12
13
```

To run this new  
model we will need  
to create a new  
Run Configuration





## 3.3 Model 3 – reading data from a excel file

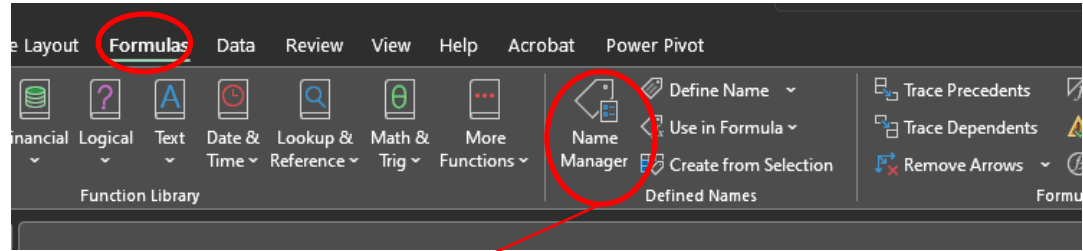
File **.mod** is the same of Model 1  
(for simplicity)

```
TablesChairs_Model1.mod  TablesChairs_DataExcel1.dat
1 /*****
2  * OPL 20.1.0.0 Model
3  * Author: tgalv
4  * Creation Date: 10 Feb 2022 at 14:06:35
5  *****/
6 //declare data
7
8 {string} Products =...; // products
9
10 float profit[Products] = ...;
11 float machining[Products] = ...;
12 float sanding[Products] = ...;
13 float assembly[Products] = ...;
14 float timeMachining = ...;
15 float timeSanding = ...;
16 float timeAssembly = ...;
17
18 //decision variables
19 dvar float+ x[Products];
20
21 maximize sum(i in Products) profit[i] * x[i] ;
22
23 subject to {
24     sum (i in Products) machining[i]* x[i] <= timeMachining;
25     sum (i in Products) sanding[i]* x[i] <= timeSanding;
26     sum (i in Products) assembly[i]* x[i] <= timeAssembly;
27 }
```

File **.dat** – links to the excel file

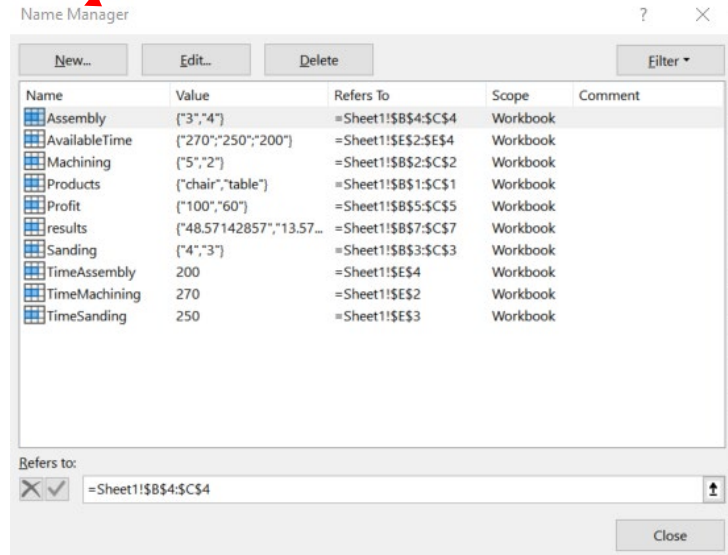
```
TablesChairs_Model1.mod  TablesChairs_DataExcel1.dat
1 /*****
2  * OPL 20.1.0.0 Data
3  * Author: tgalv
4  * Creation Date: 10 Feb 2022 at 16:50:16
5  *****/
6 //n=4;
7 //m=3;
8
9 SheetConnection my_sheet("TablesChairs_dataExcel1.xlsx");
10
11 Products from SheetRead(my_sheet, "Products");
12 machining from SheetRead(my_sheet, "Machining");
13 sanding from SheetRead(my_sheet, "Sanding");
14 assembly from SheetRead(my_sheet, "Assembly");
15 profit from SheetRead(my_sheet, "Profit");
16
17 timeMachining from SheetRead(my_sheet, "TimeMachining");
18 timeSanding from SheetRead(my_sheet, "TimeSanding");
19 timeAssembly from SheetRead(my_sheet, "TimeAssembly");
20
21 x to SheetWrite(my_sheet, "results");
```

### 3.3 Model 3 – reading data from a excel file



In the excel file we need to define names for the cells that characterize the problem

	A	B	C	D	E	F	G
1	Products	chair	table		AvailableTime		
2	Machining	5	2		270		
3	Sanding	4	3		250		
4	Assembly	3	4		200		
5	Profit	100	60				
6							
7	results	48.57143	13.57143				
8							
9							
10							
11							



## 3.4 Model 4 – Exercise

Add the new constraint (4 chairs for each table) and solve the integer problem

(i) in excel

(ii) in CPLEX

# HOMEWORK ASSIGNMENT

# PROBLEM 1 - CORPORATION WEIGELT

- The corporation Weigelt has three factories with excess of production capacity. Fortunately, the company has a new product ready to be produced, and all three factories can produce it. Thus, some of the excess of capacity can be used for the new product.
- This product can be produced in three different sizes (large, medium and small), with a unitary profit of 420€, 360€ and 300€, respectively. Factories 1, 2 and 3 have excess of capacity, which enables them to produce 750, 900 and 450 units of this product per day, respectively, independently of the produced size or the combination of different sizes.
- The availability of storage space during the production process also imposes a limitation in the production of the new product. Factories 1, 2 and 3 have storage capacity of 13 000, 12 000 and 5 000 square meters, respectively, for a production day. Each unit of large, medium and small product produced per day requires 20, 15 and 12 square meters, respectively.
- The sales forecast indicates that 900, 1200 and 750 units of large, medium and small sizes can be sold daily, respectively.
- The management wants to know how many units of each size to produce in each factory in order to maximize the profit.

# PROBLEM 1 - FORMULATION

**Decision Variables:**  $x_{ij}$  = number of products produced in factory  $i$  with size  $j$

**Objective function:**

$$\text{Max } Z = 420 x_{11} + 360 x_{12} + 300 x_{13} + 420 x_{21} + 360 x_{22} + 300 x_{23} + 420 x_{31} + 360 x_{32} + 300 x_{33}$$

**Constraints:**

$$x_{11} + x_{12} + x_{13} \leq 750$$

$$x_{21} + x_{22} + x_{23} \leq 900$$

$$x_{31} + x_{32} + x_{33} \leq 450$$

→ Production capacity

$$20 x_{11} + 15 x_{12} + 12 x_{13} \leq 13000$$

$$20 x_{21} + 15 x_{22} + 12 x_{23} \leq 12000$$

$$20 x_{31} + 15 x_{32} + 12 x_{33} \leq 5000$$

→ Storage capacity

$$x_{11} + x_{21} + x_{31} \leq 900$$

$$x_{12} + x_{22} + x_{32} \leq 1200$$

$$x_{13} + x_{23} + x_{33} \leq 750$$

→ Sales forecast

$$x_{ij} \geq 0, i, j = 1, \dots, 3$$

# Problem 1 – Excel

xij		number of units produced in factory i with size j								
		Product sizes								
		Size L	Size M	Size S	Production /Factory		Production Capacity	Storage occupied		Storage Space
Factories	Factory 1	0	0	0	0	<=	750	0	<=	13000
	Factory 2	0	0	0	0	<=	900	0	<=	12000
	Factory 3	0	0	0	0	<=	450	0	<=	500
	Space required	20	15	12						
	Sales	0	0	0						
		<=	<=	<=						
	Sales forecast	900	1200	750			Total profit			
	Unitary Profit	€ 420.00	€ 360.00	€ 300.00			€ -			

# PROBLEM 1 - CPLEX

## File weigelt.mod

```
6 {string} Factories =...; // factories
7 {string} Prod_Sizes =...; // product sizes
8
9
10 float Profit[Prod_Sizes] = ...;
11 float SalesForecast [Prod_Sizes] = ...;
12 float SpaceRequired[Prod_Sizes] = ...;
13 float SpaceAvailable[Factories]= ...;
14 float ProdCapacity[Factories]= ...;
15
16 //decision variables
17 dvar float+ x[Factories][Prod_Sizes];
18
19 maximize sum(j in Prod_Sizes) sum(i in Factories) Profit[j] * x[i][j] ;
20 subject to {
21     // production capacity
22     forall (i in Factories)
23         sum (j in Prod_Sizes) x[i][j] <= ProdCapacity[i];
24     //storage capacity
25     forall (i in Factories)
26         sum (j in Prod_Sizes) SpaceRequired[j]*x[i][j] <= SpaceAvailable[i];
27     //sales
28     forall (j in Prod_Sizes)
29         sum (i in Factories) x[i][j] <= SalesForecast[j];
30 }
31 }
```

## File weigelt.dat

```
6 SheetConnection my_sheet("Weigelt1.xlsx");
7
8 Factories from SheetRead(my_sheet, "Factories");
9 Prod_Sizes from SheetRead(my_sheet, "Prod_Sizes");
10 ProdCapacity from SheetRead(my_sheet, "ProdCapacity");
11
12 Profit from SheetRead(my_sheet, "Profit");
13 SalesForecast from SheetRead(my_sheet, "SalesForecast");
14 SpaceAvailable from SheetRead(my_sheet, "SpaceAvailable");
15 SpaceRequired from SheetRead(my_sheet, "SpaceRequired");
16
17
18 x to SheetWrite(my_sheet, "results");
```

Use the previous excel file, but do not forget to **define names** for the parameters of the problem



# PROBLEM 1 - CORPORATION WEIGELT

A new constraint:

In each factory, some of the employees may be fired, unless most of the excess production could be used to produce the new product. To avoid firing employees, management decided that the three factories should use the same percentage of the excess capacity to produce the new product.

# PROBLEM 1 - FORMULATION

New constraints:

$$\frac{1}{750}(x_{11} + x_{12} + x_{13}) - \frac{1}{900}(x_{21} + x_{22} + x_{23}) = 0$$
$$\frac{1}{750}(x_{11} + x_{12} + x_{13}) - \frac{1}{450}(x_{31} + x_{32} + x_{33}) = 0$$

We could also include the following constraint, but it is redundant considering the last 3 equations

$$\frac{1}{900}(x_{21} + x_{22} + x_{23}) - \frac{1}{450}(x_{31} + x_{32} + x_{33}) = 0$$

**Homework:** adapt the excel file (and OPL project) to these new constraints. Comment the results