U.PORTO
FEUP FACULDADE DE ENGENHARIA
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OPTIMIZATION

Faculdade de Engenharia da Universidade do Porto

FORMULATION OF LINEAR PROGRAMMING PROBLEMS

Mestrado em Engenharia Informática e Computação

CONTENTS

EXERCISE 1 - CORPORATION WEIGELT

The corporation Weigelt has three factories with excess of production capacity. Fortunately, the company has a new product ready to be produced, and all three factories can produce it. Thus, some of the excess of capacity can be used for the new product.

This product can be produced in three different sizes (large, medium, and small), with a unitary profit of 420€, 360€ and 300€, respectively. Factories 1, 2 and 3 have excess of capacity, which enables them to produce 750, 900 and 450 units of this product per day, respectively, independently of the produced size or the combination of different sizes.

The availability of storage space during the production process also imposes a limitation in the production of the new product. Factories 1, 2 and 3 have storage capacity of 13 000, 12 000 and 5 000 square meters, respectively, for a production day. Each unit of large, medium, and small product produced per day requires 20, 15 and 12 square meters, respectively.

The sales forecast indicates that 900, 1200 and 750 units of large, medium, and small sizes can be sold daily, respectively.

(i) The management wants to know how many units of each size to produce in each factory to maximize the profit. Formulate the linear programming model for this problem.

(ii) In each factory, some of the employees may be fired, unless most of the excess production could be used to produce the new product. To avoid firing employees, management decided that the three factories should use the same percentage of the excess capacity to produce the new product. How to incorporate this new constraint in the formulation?

SOLUTION

(i) Decision Variables: x_{ij} = number of products produced in factory i with size j

Objective function:

$$\text{Max } Z = 420\,x_{11} + 360\,x_{12} + 300\,x_{13} + 420\,x_{21} + 360\,x_{22} + 300\,x_{23} + 420\,x_{31} + 360\,x_{32} + 300\,x_{33}$$

Constraints:

$$x_{11} + x_{12} + x_{13} \leq 750$$

$$x_{21} + x_{22} + x_{23} \leq 900$$

$$x_{31} + x_{32} + x_{33} \leq 450$$

$$20\,x_{11} + 15\,x_{12} + 12\,x_{13} \leq 13000$$

$$20\,x_{21} + 15\,x_{22} + 12\,x_{23} \leq 12000$$

$$20\,x_{31} + 15\,x_{32} + 12\,x_{33} \leq 5000$$

$$x_{11} + x_{21} + x_{31} \leq 900$$

$x_{12} + x_{22} + x_{32} \leq 1200$

$x_{13} + x_{23} + x_{33} \leq 750$

$x_{ij} >= 0,\ i,j = 1,..3$

(ii)

$$\frac{1}{750}(x_{11} + x_{12} + x_{13}) - \frac{1}{900}(x_{21} + x_{22} + x_{23}) = 0$$

$$\frac{1}{750}(x_{11} + x_{12} + x_{13}) - \frac{1}{450}(x_{31} + x_{32} + x_{33}) = 0$$

We could also include the following constraint, but it is redundant considering the last 3 equations

$$\frac{1}{900}(x_{21} + x_{22} + x_{23}) - \frac{1}{450}(x_{31} + x_{32} + x_{33}) = 0$$

OPL code

File .mod

```
{string} Factories =...; // factories
{string} Prod_Sizes =...; // product sizes


float Profit[Prod_Sizes] = ...;
float SalesForecast [Prod_Sizes] = ...;
float SpaceRequired[Prod_Sizes] = ...;
float SpaceAvailable[Factories]= ...;
float ProdCapacity[Factories]= ...;

//decision variables
dvar float+ x[Factories][Prod_Sizes];

maximize sum(j in Prod_Sizes) sum(i in Factories) Profit[j] * x[i][j] ;
 subject to {
   // production capacity
  forall (i in Factories)
   sum (j in Prod_Sizes) x[i][j] <= ProdCapacity[i];
   //storage capacity
  forall (i in Factories)
   sum (j in Prod_Sizes) SpaceRequired[j]*x[i][j] <= SpaceAvailable[i];
   //sales
  forall (j in Prod_Sizes)
   sum (i in Factories) x[i][j] <= SalesForecast[j];

 }
```

EXERCISE 2 - COMFORTABLE HANDS

Comfortable hands is a company that has a line of gloves for the whole family - man, woman and child. The company is trying to decide which combination of these three types of gloves to produce. The workers of the company are unionized. Each full-time worker works 40 hours a week. In addition, the collective bargaining agreement stipulates that the number of full-time workers can never be less than 20. The company may also hire non-union part-time workers according to the following conditions:

- each part-time employee works 20 hours per week, and
- there must be at least 2 full-time employees for each part-time employee.

The three types of gloves are made of the same kind of genuine cowhide. The company have a long-term contract with a supplier of skin and receives 5,000 square decimeters of the material every week. The requirements of materials and labor and the gross profit by selling glove (not including labor costs) are provided in the table below.

Type of glove	Material requirements (m^2)	Labor requirements (minutes)	Gross profit (per pair of gloves)
Man	2,0	30	€8
Woman	1,5	45	€10
Child	1,0	40	€6

Each full-time worker earns € 13 per hour, while each part-time worker earns € 10 per hour. The management wants to know what combination of the three types of gloves should produce per week, as well as how many full-time and part-time workers should employ. They want to maximize their net income (sales gross profit - labor costs).

Formulate a linear programming model for this problem.

SOLUTION

Decision variables:

x_H = number of man gloves produced weekly,

x_M = number of woman gloves produced weekly,

x_C = number of child gloves produced weekly,

x_{TI} = number of full-time workers to hire,

x_{TP} = number of partial-time workers to hire.

Objective:

Max $Z = 8\,x_H + 10\,x_M + 6\,x_C - 13(40)x_{TI} - 10(20)\,x_{TP}$,

Constraints:

$2\,x_H + 1{,}5\,x_M + x_C \leq 5000$
$30\,x_H + 45\,x_M + 40\,x_C \leq 40(60)\,x_{TI} + 20(60)\,x_{TP}$
$x_{TI} \geq 20$
$x_{TI} \geq 2\,x_{TP}$
$x_H \geq 0,\ x_M \geq 0,\ x_C \geq 0,\ x_{TI} \geq 0,\ x_{TP} \geq 0$

EXERCISE 3 - COMPANY SLIM

The company Slim produces a complete nutritional range of weight loss drinks. One of their products is a strawberry milkshake which has the purpose of substituting a complete meal. The strawberry milkshake is made of several ingredients. The information of each ingredient is detailed below.

Ingredients	Fat calories (per spoon)	Total calories (per spoon)	Vitamins (mg/spoon)	Thickener (mg/spoon)	Cost (per spoon)
Strawberry flavour	1	50	20	3	10
Cream	75	100	0	8	8
Vitamin supplement	0	0	50	1	25
Artificial sweetener	0	120	0	2	15
Thickener	30	80	2	25	6

The nutritive requisites are the following:

1) The drink should have between 380 and 420 calories (inclusive).
2) At most, 20% of the calories should be fat.
3) Should have at least 50 mg of vitamins.
4) Should have 2 spoons of strawberry flavour for each spoon of artificial sweetener added.
5) To acquire the desired texture, it should have exactly 15 mg of thickener in each drink.

Management wants to decide how much of each ingredient to add in order to minimize the costs and, at the same time, meet the requirements.

Formulate the linear programming model for this problem.

EXERCISE 4 - PIG FARMER

A pig farmer wants to determine the amount of food to provide on a daily basis to each animal. He is considering using a combination of different types of food (A and B) available from local suppliers. He wants to feed his pigs at a low cost while ensuring that each pig receives the appropriate amount of calories and vitamins.

The data regarding the cost, number of calories and vitamins of each type of food is listed in the table below.

Content	**Type of food A**	**Type of food B**
Calories (per Kg)	800	1.000
Vitamins (per Kg)	140 units	70 units
Cost (per Kg)	€0,40	€0,80

Each pig needs 8,000 calories per day and at least 700 units of vitamin. In addition, the amount of food type A cannot exceed one third of the diet (by weight), because it contains an ingredient that is toxic if consumed in large quantities.

a) Formulate a linear programming model for this problem.

b) Solve the problem graphically.

EXERCISE 5 - COFFEE SHOP

In a coffee shop, which is open only on weekdays, workers work four days per week followed by a day off. This scheme is repeated every 5 days and therefore a worker has the same day off every week.

The need for workers is given in the table below and must be met or exceeded in every day.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Demand	3	5	9	2	7

a) Formulate a linear programming model for this problem that minimizes the total number of workers needed to meet the daily demand.

b) Suppose it is necessary to ensure that at least 40% of workers have a free Friday. Enter this restriction on the problem.

EXERCISE 6 – FLORIST SHOP

A florist plants and grows red and white roses. Each red rose costs € 8 per year and each white rose € 2 per year. The bare-roots of red and white roses require an area of 5dm2 and 4dm2 per root, respectively. The work required per year for a red rose plant is 1 man hour, and for a white rose plant is 5 man hours. Red roses make a unitary profit of € 2, and white roses a unitary profit of € 3. The available land has a maximum area of 6100dm2, the total cash available is € 8000. Annual work availability is at most of 5000 man hours.

a) Formulate the linear problem that allows to determine how many roses of each type should be planted to maximize the profit?

b) Solve the problem graphically

i. Identify the optimal solution

ii. Identify are the active constraints?

iii. Among what values can the profit of the red rose vary while maintaining the same optimal solution?

iv. The florist has the possibility to increase the available capital through a loan with 0% interest. Should the florist apply for the loan?

v. The florist is considering increasing the available land. Knowing that 1 m2 costs 40 €, should the owner do it?

c) Solve the problem using the Simplex method.

EXERCISE 7 - NEWAGE FACTORY

The NewAge factory manufactures a type of electrical component for which the demand is variable. The estimated demand for this component for the next four months is 2600, 2900, 3400 and 3000, respectively. In the months when demand is lower, there may be excess production and creation of stock to cover demand for the posterior months, when demand is greater.

The factory can produce 2400 components per month under normal shifts. Resorting to extraordinary shifts the production can be increased by 600 components per month. Each component manufactured in the extraordinary shift has an additional cost of € 7 relating to its manufacture in a normal shift. In addition, it is estimated a storage charge of € 3 / month for each component stock.

It is intended to determine a production plan for the next four months to minimize the total costs of production and storage.

EXERCISE 8 – COMPUTER CENTRE 1

The director of a computer centre wants to create the schedules of the centre workers who provide support to the users. The computer centre is open from 8:00 to 24:00. The personnel needs are presented in the following table:

Day period	Minimum number of workers
From 8:00 to 12:00	4
From 12:00 to 16:00	8
From 16:00 to 20:00	10
From 20:00 to 24:00	6

The workers can be hired in full-time or part-time.

Full-time employees work 8 consecutive hours in one of the following shifts: Morning (from 8:00 to 16:00), Afternoon (from 12:00 to 20:00) or Night (from 16:00 to 24:00) and get 14€/hour.

Part-time workers can work in any of the 4 hours shifts presented in the table above and receive 12€/hour.

The director of the centre intends that, at any period of the day, the centre has two full-time employees working per each part-time employee working in the same period.

How many full-time and part-time works must be hired in order to minimise costs?

EXERCISE 9 – COMPUTER CENTRE 2

The director of a computer centre wants to plan the schedules of the staff who help the centre users. The director identified the following needs regarding the number of workers:

Month	Minimum number of workers
Jan/Feb/Mar	18
Apr/May/Jun	12
Jul/Aug/Sep	30
Oct/Nov/Dec	16

The employees can be hired with a permanent contract or a short-term contract.

Short-term workers are hired for a fixed period of 6 months. A permanent worker earns 650€/month and a short-term worker earns 500€/month.

To maintain a level of continuity in the services, the director established that at least 75% of its employees should be permanent.

a) Formulate a linear programming model to determine how many permanent and short-term staff are to be recruited every three months in order to meet the requirements of the centre minimising the costs for the year in question.

b) Due to bureaucratic issues related to the hiring of permanent staff, the director would like to have access to a plan that seeks to minimize the maximum number of employees in this regime contracted in a single month. Change the model to according to this plan.

EXERCISE 10 - URBAN SOLID WASTE MANAGEMENT COMPANY

The Urban Solid Waste Management Company RESITRATA has contracts with the nearest municipalities to receive the Urban Solid Waste (USW) generated by the population and for their disposal in a sanitary landfill. To facilitate the USW reception process, the company has five transfer centres (A, B, C, D and E), where the municipalities can deliver the amount of USW collected.

After weighted and compacted, the waste needs to be transported to the landfill (O). The following amount of USW is accumulated daily in each centre:

A empresa de Gestão de Resíduos Sólidos Urbanos RESITRATA tem contratos com os municípios mais próximos para receção dos Resíduos Sólidos Urbanos (RSU) gerados pela população e para a sua deposição num aterro sanitário. Para facilitar o processo de receção de RSU, a empresa dispõe de cinco centros de transferência (A, B, C, D e E), nos quais os municípios podem entregar a quantidade de RSU recolhida. Depois de pesados e compactados, os resíduos necessitam de ser transportados para o aterro sanitário (O). A seguinte quantidade de RSU é acumulada diariamente em cada centro:

Transfer Centre	Amount of USW accumulated daily (Ton)
A	80
B	30
C	25
D	65
E	50

Every day a set of vehicles travels through the different transfer centres to transport USW to the landfill. Four possible routes were pre-established by the company for transportation:

Route W: O – A (8) – C (5) – E (5) – O

Route Y: O – A (8) – D (7) – B (3) – O

Route Z: O – E (6) – D (8) – C (4) – O

Route K: O – B (5) – C (8) – D (5) – O

In parentheses is the amount (in tons) transported from each centre to the landfill along the route. Each route is performed by one vehicle and there are only 12 vehicles available for transportation.

Formulate the problem that allows to determine how many vehicles are used per day to complete each route, ensuring that all accumulated USW is transported to the landfill. The objective is to minimize the total distance, and the distance travelled in each route is indicated as follows:

Route	Distance (km)
W	130
Y	180
Z	70
K	250

EXERCISE 11 - ETHEREALSMELL

The company EtherealSmell produces the perfumes Bloom, a feminine scent and Dream, a masculine scent. The raw material to produce each type of perfume can be purchased for 3 € / kg. Processing 1 kg of raw material requires one hour of laboratory. Each kg of processed raw material can produce 3 bottles of Bloom Standard and 4 bottles of Dream Standard simultaneously. Each bottle of Bloom Standard is sold at 7€ and each bottle of Dream Standard is sold at 6 €.

EtherealSmell also has the option to improve the standard versions of each perfume in order to produce Bloom Luxury, sold for € 18 a bottle and Dream Luxury, sold at € 14 a bottle.

The production of 1 bottle of Bloom Luxury requires the processing of a bottle of Bloom Standard for 3 extra hours of laboratory and costs € 4 in processing costs. The production of 1 bottle of Dream Luxury requires the processing of a bottle of Dream Standard for 2 extra hours of laboratory and costs € 4 in processing costs.

Each year EtherealSmell has 6,000 laboratory hours available and can purchase up to 4000Kg of raw material. However, once acquired, all raw material must be processed.

EtherealSmell's research department has also developed a new unisex perfume called Droom Luxury. Each bottle of this new perfume is obtained by processing half bottle of Standard Bloom and half bottle of Dream Standard. This new perfume will be sold at 20 € the bottle and requires 1 hour of laboratory to be developed and an extra processing cost of 5 €. It was also decided that the production of this perfume should correspond to 10% of the total production.

Formulate a Linear Programming problem that can be used to determine how EtherealSmell can maximize its profits. Assume that the cost of laboratory hours is a fixed cost.

EXERCISE 12 – SUN&BEACH BAR

The Sun & Beach Bar in the Caribbean is improving the area of paid umbrellas. This bar is trying to introduce a novelty in the market and will distinguish between three types of areas of paid umbrellas. The table below summarizes the characteristics of each area.

Area	Designation	Price per umbrella (€/day)	Space occupied by umbrella (unid./m^2)	Costs for each area (€/ano.m^2)	Maximum area (m^2)	Estimated average use rate

A	Lounger without pad	10	0,3	30	1000	60 %
B	Lounger with pad	15	0,2	50	1000	50 %
C	Personalised service	30	0,1	150	700	40 %

The bar management decides that the annual costs must not exceed 150 000 € and also that the total area must be inferior to 2500 m2. Finally, the direction wants the C-type area to be at least one third of the total area used.

a) Formulate the area dimensioning of the Sun & Beach bar as a linear programming problem. Describe any assumptions you may need.

b) Sol & Praia would like to incorporate, in the same optimization problem, the number of employees that will need to ensure the proper functioning of these areas. Employee data are described in the table below. Make the changes that you think are necessary to incorporate these decisions. Describe any assumptions you may need.

Designation	Cost per employee (€/month)	Minimum number of employees (unit./m^2)
Area A	600	0,004
Area B	700	0,005
Area C	800	0,01

Over years of observation, Sol & Praia realized that, for each employee exceeding the minimum required in each area, the " Estimated average use rate " increases by about 5%. Is it possible to introduce this empirical observation into the formulation and maintain the linear optimization problem? Justify.

EXERCISE 13 - EXAM OF PROF. MARCELO

Professor Marcelo recently gave his students a special exam. The exam was composed of 4 items (two per question). There were 3 students on the exam and their classifications (from 0 to 100) per sub-question are shown in table 1.

Table 1 – Classifications of students per item

	1 a)	1 b)	2 a)	2 b)
Student 1	30	80	10	90
Student 2	50	30	0	100
Student 3	20	70	30	70

The weight of each item is now under consideration of Professor Marcelo. Given the special nature of this exam, the Professor is thinking of formulating a linear programming model to test various objectives. However, all these scenarios will have to respect the following constraints:

i. The weight of each item may not be less than 10 percent.
ii. The first question should correspond to at least 40 percent of the total weight.
iii. None of the questions may exceed 60 percent of the total weight.

a) Identify the decision variables of this problem and write its constraints.
b) How would you write the objective function if Professor Marcelo wanted to maximize the average of the grades?
c) What would be the complete formulation (objective function and constraints) if Professor Marcelo wanted to maximize the lowest grade.

EXERCISE 14 - PJ&T COSMETICS

PJ&T Cosmetics is launching a new solar product for the next summer. After considering possible advertising media and the market to be covered, PJ&T decided that the first month's advertising be restricted to five media. At the end of the month, PJ&T will re-evaluate its strategy based on the month's results.

PJ&T collected data on the number of potential customers reached, the cost per advertisement, the maximum number of times each medium is available, and the exposure quality rating for each of the five media.

The quality rating is measured in terms of an exposure quality unit, a measure of the relative value of one advertisement in each of the media. The information collected is presented in Table 1. PJ&T has an advertising budget of €30,000 for the first month's campaign.

In addition, they imposed the following restrictions on how to allocate these funds: at least 10 television commercials must be used, at least 50,000 potential customers must be reached, and at least 10% of them must be reached through radio advertisements. Finally, no more than €18,000 may be spent on television advertisements.

Formulate the problem as a Linear Programming model.

Table 1 - PJ&T Collected data

Advertising Media	**Number of Potential Customers Reached**	**Cost (€$) per Advertisement**	**Maximum Times Available per Month***	**Exposure Quality Units**
Daytime TV (1 min), station TVPlus	1000	1500	15	65
Evening TV (30 sec), station TVPlus	2000	3000	10	90

Daily newspaper (full page), *The Morning Press*	1500	400	25	40
Sunday newspaper magazine(0,5 page color) *The Sunday Journal*	2500	1000	4	60
Radio, 8:00 a.m. or 5:00 p.m. news(30 sec), station SpotFM	300	100	30	20

SOLUTION

Variáveis de decisão

DTV, ETV, DN, SN, R : número de anúncios a usar em cada meio de comunicação

DTV (TV horário diurno), ETV (TV horário noturno), DN (Jornal diário), SN (Jornal de Domingo), R (rádio) .

Max 65 DTV + 3000 ETV + 400 DN + 1000 SN + 100 R

s.a

1500 DTV + 3000 ETV + 400 DN + 1000 SN + 100 R <= 30000

1500 DTV + 3000 ETV <= 18000

DTV + ETV >= 10

1000 DTV + 2000 ETV + 1500 DN + 2500 SN + 300 R >= 50000

0.1 (1000 DTV + 2000 ETV + 1500 DN + 2500 SN + 300 R) >= 300 R

0 <= DTV <= 15

0 <= ETV <= 10

0 <= DN <= 25

0 <= SN <= 4

0 <= R <= 30

EXERCISE 15 - PORTUGUESE OLYMPIC COMMITTEE

To avoid repeating the recent adaptation problems some teams faced this Soccer World Cup, the Portuguese Olympic Committee intends to make a pre stage with the three main Olympic teams in Brazil to prepare their participation for the Olympic Games that will take place at Rio de Janeiro in 2016.

The flight will take 9 hours with a plane that can carry 100 people. The three teams are Swimming, Gymnastics and Cycling. These teams have the following number of members and trainers (respectively): Swimming 42 and 12; Gymnastics 22 and 14; Cycling 34 and 16. There must be at least one swimming trainer accompanying every three swimmers on the plane. Similarly, there must be at least one gymnastics trainer for every two gymnasts on the plane. Cyclists tend to be older and can travel by themselves without trainers. Swimming and cycling associations are equally paying for the trip and they require that at least the 70% of the seats are allocated to swimmers, cyclists and their trainers. Moreover, the total number of swimmers and their trainers must equal to the total number of cyclist and their trainers.

a) Provide an LP formulation to minimize the number of people that cannot be put on this flight.
b) Suppose that leaving out a gymnast costs three times as much as leaving out a swimmer or a cyclist. And also suppose that the cost of leaving out trainers is negligible. Modify your answer to a) to minimize the cost of people left behind (not put on the plane) in these new conditions.

SOLUTION

a)

Variáveis de decisão:

Xn - número de nadadores colocados no avião.

Xg - número de ginastas colocados no avião.

Xc - número de canoístas colocados no avião.

Tn - número de treinadores de nadadores colocados no avião.

Tg - número de treinadores de ginastas colocados no avião.

Tc - número de treinadores de canoistas colocados no avião.

Função Objetivo:

Min 140 – (Xn + Xg+ Xc + Ts + Tg + Tc)

Restrições:

$Xn - 3Tn \leq 0$

$Xg - 2Tg \leq 0$

$Xn + Tn + Xc + Tc \geq 70$

$Xn + Tn - Xc - Tc = 0$

$Xn \leq 42$

$Xg \leq 22$

$Xc \leq 34$

$Tn \leq 12$

$Tg \leq 14$

$Tc \leq 16$

$Xn, Xg, Xc, Tn, Tg, Tc >= 0$

b) Modificar a função objetivo para Min $(42 - Xn) + 3(22 - Xg) + (34 - Xc)$

EXERCISE 16 – MARKETING MANAGER

As product marketing managers, one of our tasks is to prepare recommendations to the Executive Board on how to allocate advertising expenses. Last year's advertising budget of € 40,000 was spent in identical increments over the four quarters. Initial expectations are to repeat this plan next year. However, the Executive Board would like to know if there are any other allocations that are advantageous by maintaining or lowering the overall budget.

The product is sold for € 40 and costs € 25 to produce. Sales in the past have had seasonality, and our consultants have estimated the following seasonal adjustment factors for units sold:

T1 - 90%	T3 - 80%
T2 - 110%	T4 - 120%

(A seasonal adjustment factor measures the percentage of the quarterly average demand in a given quarter).

Clearly, advertising can increase sales. Our consultants estimated the relationship between advertising and sales a few years ago. Converting this relation to the current conditions results in the following formula:

$$Quarter\ sales\ =\ 3000\ + 0{,}1 \times Advertising \times Seasonal\ Adjustment\ Factor$$

In addition to production costs, the cost of the sales force (estimated at € 34,000 per year, allocated as follows: T1 and T2, € 8,000 each, T3 and T4, € 9,000 each), the advertising costs and other costs (estimated at around 15 percent of revenues). Revenues result from the multiplication of sales by the selling price.

a) Formulate a linear programming problem that allows you to prepare recommendations for the Executive Board on how to allocate advertising expenses.

b) Although it is clear that advertising increases sales, there are limits to this impact. So the consultants re-read the relationship between advertising and sales and arrive at the following expression:

$$Quarter\ sales\ =\ 35 \times Seasonal\ Adjustment\ Factor \times \sqrt{3000 + Advertising}$$

Considering this new expression comment the possibility of being able to reformulate the previous model and use the algorithm Simplex to solve the problem.

EXERCISE 17 - WEIRDPIECES

WeirdPieces is a factory that produces wood-pressed parts. On one of its production lines it uses quadrangular plates, like the one shown in figure 1.

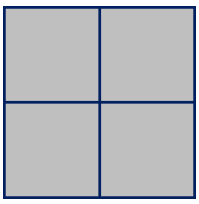
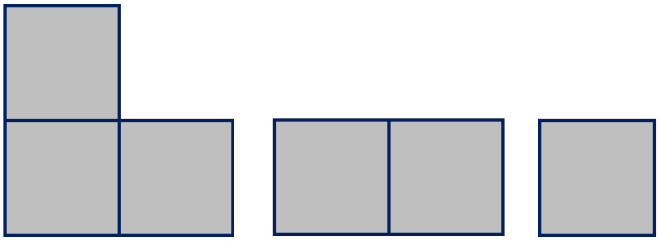


Figure 1 – Quadrangular plate

WeirdPieces received an order for manufacturing parts with three different shapes, cut from the quadrangular plates. The formats of the pieces to cut are shown in Figure 2. To satisfy the order it will be necessary to manufacture 50 pieces with the format 1, 80 pieces with the format 2 and 200 pieces with the format 3.



Format 1 *Format 2* *Format 3*

Figure 2 – Formats of the pieces to cut

Formulate the problem as Linear Programming, in order to guarantee the satisfaction of the order minimizing the use of the necessary raw material (quadrangular plates). Tip: Try to identify what cutting patterns a plate might have to make the necessary pieces.

EXERCISE 18 - POMPEU GAS STATION

Mr. Pompeu owns a gas station near Vila do Conde and he wants to improve his business through a complete redesign of the gas station operation.

Before taking any decision, Mr. Pompeu decided to gather some significant information:

- The gas station must be open to the public 24 hours a day
- The work day will be divided into six-hour shifts
- Employees must work two consecutive shifts of six hours
- Employees will be paid € 10 per hour
- Each employee can serve on average 20 customers per hour
- The influx of customers at the gas station varies throughout the day, but Mr. Pompeu established values for the following intervals, which coincide with the shifts intervals.

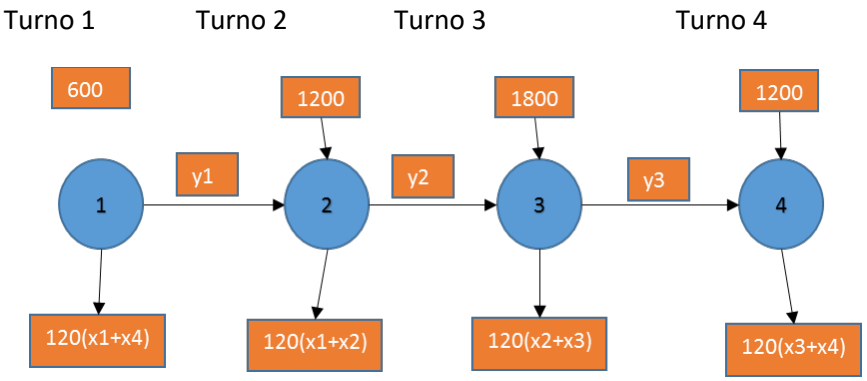
Shift	Interval	Nr. of clients arriving to the gas station
1	00h00-06h00	600
2	06h00-12h00	1200
3	12h00-18h00	1800
4	18h00-24h00	1200

- Whenever it is not possible to attend a customer during a given shift, he must be attended in the next shift(s).
- In order to model the customer's impatience, it is considered that any customer who is in the gas station at the end of a shift will cost € 5.
- It is also assumed that by midnight of each day (at the end of the shift 4) all customers must be served, so the first shift always begins with zero customers in the gas station.

Based on this information:

a) Identify and describe the decision variables of the problem
b) Represent mathematically the constraints of the problem
c) Identify and describe the objective function of the problem

SOLUTION



Variáveis de decisão:

Xi: funcionários que iniciam o turno i, i=1,2,3,4

Yi: clientes não satisfeitos no fim do turno i, i=1,2,3

Média de clientes atendidos por hora por cada funcionário: 20

Restrições:

120 (x1+x4) + y1 = 600

120(x1+x2) + y2 = 1200 + y1

120(x2+x3)+y3 = 1800+y2

120(x3+x4) = 1200+y3

Função Objetivo:

min 60 x1+60x2+60x4+60x4+5y1+5y2+5y3

x1	x2	x3	x4	y1	y2	y3	
60	60	60	60	5	5	5	
5	5	10	0	0	0	0	1200

120			120	1			600	=	600
120	120			-1	1		1200	=	1200
	120	120			-1	1	1800	=	1800
		120	120			-1	1200	=	1200

EXERCISE 19 - HIGHWAY CONTRACTOR

A highway contract requires a contractor to alter the terrain of a section of road work. The work involves cut and fill of soil so that the original profile will be much flatter. The original profile and the finished profile are shown in Figure 1.

The roadway is divided into five sections. For example, 10 x 10^3 m^3 of fill is required in section 1; 18 x 10^3 m^3 of cut in section 2; and so on.

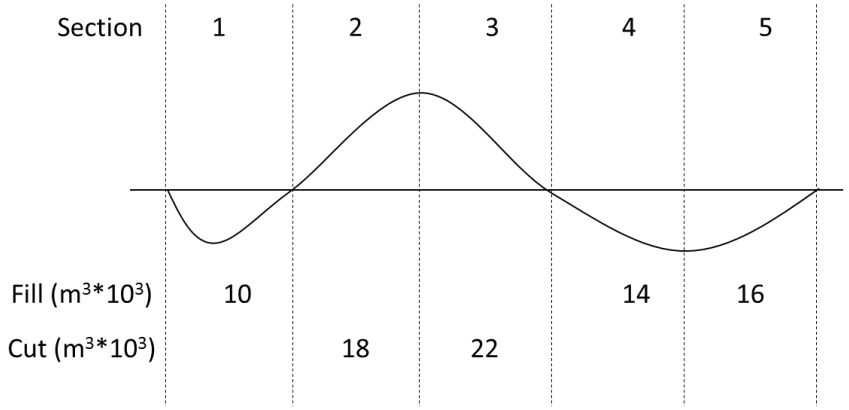


Figure 1

The cost for cutting including loading is € 8 per m^3 and that for filling including compaction is € 12 per m^3.

The unit cost of transporting soil by trucks from one section to another is €2 per m^3 per section travelled.

Our task is to determine how much earth should be moved from where to where so as to optimize the soil moving operations.

Based on this information:

a) Identify and describe the decision variables of the problem
b) Represent mathematically the constraints of the problem
c) Identify and describe the objective function of the problem

SOLUTION

Variáveis de decisão: Xij: terra a transportar da secção i para a secção j (em milhares de metros cúbicos)

		Fill Sections		
	Sections	1	4	5
Cut Sections	2	x21	x24	x25
	3	x31	x34	x35

Cálculo dos custos unitários (em € /m^3)

C21 = 8 (custo de cortar na secção 2)+ 12 (custo de encher na secção 1) + 2 x1 (custo de transportar da secção 2 para a secção 1) = 22 € /m^3

C24 = 8 + 12 + 2 x 2 = 24 € /m^3

C25 = 8 + 12 + 2 x 3 = 26 € /m^3

C31 = 8 + 12 + 2 x 2 = 24 € /m^3

C34 = 8 + 12 + 1 x 2 = 22 € /m^3

C35 = 8 + 12 + 1 x 2 = 22 € /m^3

Função objectivo: minimizar custos

F(xij) = $\sum Cij * Xij$ = 22 x21 + 24 x24 + 26 x25 + 24 x31 + 22 x34 + 22 x35

Restrições:

Para cada secção que vai encher de terra:

Para a secção 1: x21 + x31 >= 10

Para a secção 4: x24 + x34 >= 14

Para a secção 5: x25 + x35 >= 16

Para cada secção que vai remover terra:

Para a secção 2: x21 + x24 + x25 <= 18

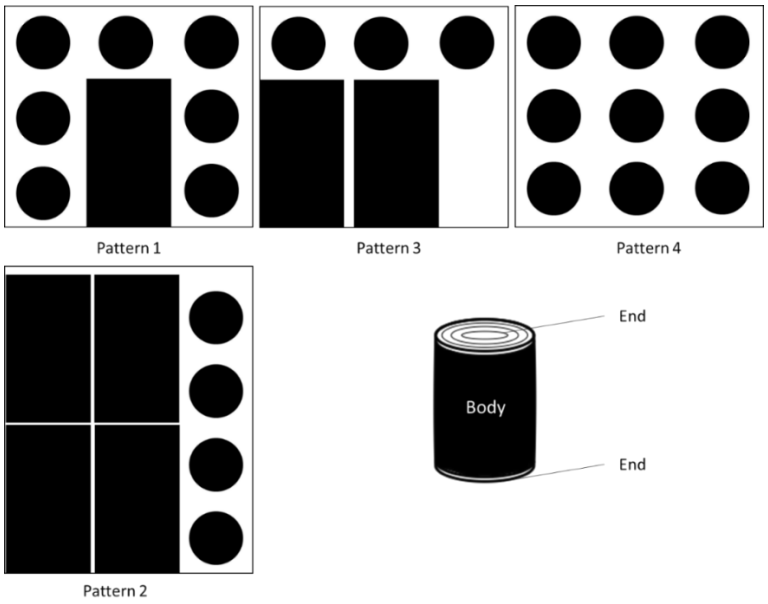
Para a secção 3: x31 + x34 + x35 <= 22

Xij >= 0, i,j

EXERCISE 20 – STAMPED CANS

Consider the production of stamped cans from metal leaves. To create a can, you need a main body and two ends. At the moment, there are 4 types of cut for the possible stamping to be used (involving 2 different types / sizes of metal leaves), as shown in the figure below.

Consider that 40 hours of work per week are available, as well as 15 Type-1 metal leaves per week and 7 Type-2 metal leaves. The company wants to plan the production in a way that minimizes waste.



	Type of cut			
Characteristics	**1**	**2**	**3**	**4**
Type of metal leaf	1	2	1	1
Number of bodies	1	4	2	0
Number of ends	7	4	3	9
Waste (m^2)	5	3	4	6
Effort (hours)	3	4	2	1