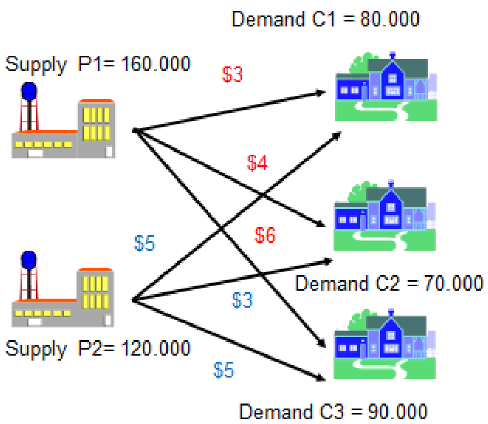


OPTIMIZATION

Faculdade de Engenharia da Universidade do Porto

TRANSPORTATION PROBLEMS

Mestrado em Engenharia Informática e Computação

CONTENTS

EXERCISE 1: ONENOTE CO.

The Onenote Co. produces only one type of product in three factories for three customers. The three factories will produce 140, 110, and 100 units, respectively, during the next season. The company assumed the compromise of selling 100 units to customer 1, 50 to customer 2 and 200 to customer 3. The unitary costs of transportation between the factories and the customers are given in the table below.

		Clients		
		1	**2**	**3**
Factories	**1**	4	3	3,2
	2	2,2	2,4	3,4
	3	2,2	2,5	3,6

They want to know how many units should be sent from each factory to each customer in order to minimize the transportation costs.

SOLUTION:

Applying the minimum cost method:

		Clients			
		1	2	3	Offer
Factories	1	0	0	140	140
	2	0	50	60	110
	3	100	0	0	100
	Demand	100	50	200	**350**

Cost=100 x 2,2+50 x 2,4+140 x 3,2+60 x 3,4=992

	1	2	3	u_i
1	4 2,1 0	3 0,8 0	3,2 140	0^*
2	2,2 0,1 0	2,4 50	3,4 60	0,2
3	2,2 100	2,5 0^*	3,6 0,1 0	0,3
v_i	1,9	2,2	3,2	

Optimal solution.

EXERCISE 2: WHEAT, BARLEY AND OATS

Suppose that England, France and Spain are the only producers of wheat, barley and oats in the whole world. The world demand of wheat corresponds to 125 Millions of acres of soil. With the same objective it is necessary 60 Millions of acres for barley and 75 Millions of acres for oats. The total soil available for this purpose in England, France and Spain, respectively, is 70 Million, 110 Million and 80 Millions of acres.

The number of working hours necessary to produce 1 acre of wheat are 18h in England, 13 in France and 16 in Spain. For the barley are 15h in England, 12 in France and Spain. For the oats are 12h in England, 10h in France and 16h in Spain. The cost of one hour of work for wheat production is 3, 2.4, 3.3 in England, France and Spain, respectively. In the case of barley is 2.7, 3.0, 2.8 in England, France and Spain. In the case of oats is 2.3, 2.5 and 2.1 in England, France and Spain

Formulate the problem as a Transportation Problem, building the cost and requirements matrix and solving the problem with the objective of minimizing the total production costs and meeting the global needs for wheat, barley and oats.

SOLUTION:

Applying the NW corner method:

	Wheat	Barley	Oats	**Offer**
England	70	0	0	70
France	55	55	0	110
Spain	0	5	75	80
Needs	125	60	75	**260**

Cost= 70x54+55x31,2+55x36+5x33,6+75x33,6=10164

	Wheat	Barley	Oats	u_i
England	54 $70^{-\Theta}$	40,5 -18,3 0	27,6 -31,2 $0^{+\Theta}$	0^*
France	31,2 $55^{+\Theta}$	36 $55^{-\Theta}$	25 -11 0	-22,8
Spain	52,8 24 0	33,6 $5^{+\Theta}$	33,6 $75^{-\Theta}$	-25,2
v_i	54	58,8	58,8	

$\Theta = 55$

	Wheat	Barley	Oats	u_i
England	54 $15^{-\Theta}$	40,5 12,9 0	27,6 $55^{+\Theta}$	0^*
France	31,2 110	36 31,2 0	25 -4,8 0	-22,8
Spain	52,8 -7,2 $0^{+\Theta}$	33,6 60	33,6 $20^{-\Theta}$	6
v_i	54	27,6	27,6	

$\Theta = 15$

	Wheat		Barley		Oats		u_i
England	54 7,2	0	40,5 12,9	0	27,6 7	70	0^*
France	31,2	110	36 24	0	25 13	0	-15,6
Spain	52,8	15	33,6	60	33,6	5	6
v_i	46,8		27,6		27,6		

Optimal solution.

Cost=110x31,2+15x52,8+60x33,6+70x7+5x33,6= 8340

EXERCISE 3

Consider a problem whose cost matrix, supply and demand at the origins and destinations are the following:

$$C_{ij} = \begin{array}{|c|c|c|c|c|} \hline 7 & 9 & 5 & 3 & 6 \\ \hline 6 & 4 & 6 & 7 & 5 \\ \hline 8 & 6 & 4 & 5 & 5 \\ \hline \end{array}$$

ai	7	9	11		
bj	4	6	4	8	5

By applying the transportation algorithm

1) Find basic feasible solution (by one of the methods studied)

2) Find the optimal solution

SOLUTION:

1) Applying the NW corner Method (for example):

	1	**2**	**3**	**4**	**5**	
1	4	3	0	0	0	7
2	0	3	4	2	0	9
3	0	0	0	6	5	11
	4	6	4	8	5	**27**

Cost=4x7+3x9+3x4+4x6+2x7+6x5+5x5=160

2)

	1		2		3		4		5		u_i
1	7		9		5		3		6		0^{*}
		4		$3^{-\Theta}$	-6	0	-9	$0^{+\Theta}$	-6	0	
2	6		4		6		7		5		-5
	4	0		$3^{+\Theta}$		4		$2^{-\Theta}$	-2	0	
3	8		6		4		5		5		-7
	8	0	4	0	0	0		6		5	
v_i	7		9		11		12		12		

Θ = 2

	1		2		3		4		5		u_i
1	7		9		5		3		6		0^{*}
		4		$1^{-\Theta}$	-6	0		$2^{+\Theta}$	3	0	
2	6		4		6		7		5		-5
	4	0		$5^{+\Theta}$		$4^{-\Theta}$		0	7	0	
3	8		6		4		5		5		2
	-1	0	-5	0	-9	$0^{+\Theta}$		$6^{-\Theta}$		5	
v_i	7		9		11		3		3		

Θ = 1

	1		2		3		4		5		u_i
1	7		9		5		3		6		0^{*}
		$4^{-\Theta}$	9	0	3	0		$3^{+\Theta}$	3	0	
2	6		4		6		7		5		4
	-5	$0^{+\Theta}$		6		$3^{-\Theta}$	0	0	-2	0	
3	8		6		4		5		5		2
	-1	0	4	0		$1^{+\Theta}$		$5^{-\Theta}$		5	
v_i	7		0		2		3		3		

Θ = 3

	1		2		3		4		5		u_i
1	7		9		5		3		6		0^{*}
		$1^{-\Theta}$	4	0	3	0		$6^{+\Theta}$	3	0	
2	6		4		6		7		5		-1
		3		6	5	0	5	0	3	0	
3	8		6		4		5		5		2
	-1	$0^{+\Theta}$	-1	0		4		$2^{-\Theta}$		5	
v_i	7		5		2		3		3		

Θ = 1

	1	2	3	4	5	u_i
1	7 1 0	9 5 0	5 3 0	3 7	6 3 0	0^*
2	6 3	4 6	6 4 0	7 4 0	5 2 0	0
3	8 1	6 0 0	4 4	5 1	5 5	2
v_i	6	4	2	3	3	

Custo=3x6+1x8+6x4+4x4+7x3+1x5+5x5=117

EXERCISE 4: MOVE-IT COMPANY

Move-It Company has three factories producing forklifts that are then shipped to four distribution centres. The production costs are the same in both factories and the transport costs for each forklift are shown for each factory-distribution centre combination:

		Distribution Centres			
		1	**2**	**3**	**4**
Factories	**1**	464	513	654	867
	2	352	416	690	791
	3	995	682	388	685

The demand for forklifts in each distribution centre is:

Distribution Centres			
1	**2**	**3**	**4**
80	65	70	85

The production capacity in each factory is:

Factories		
1	**2**	**3**
75	125	100

Define the number of forklifts that should be sent from each factory to each distribution centre.

SOLUTION:

Applying the minimum cost method:

		Distribution Centres				
		1	2	3	4	Offer
Factories	1	0	20	0	55	75
	2	8 0	45	0	0	125
	3	0	0	70	30	100
	Demand	80	65	70	85	**300**

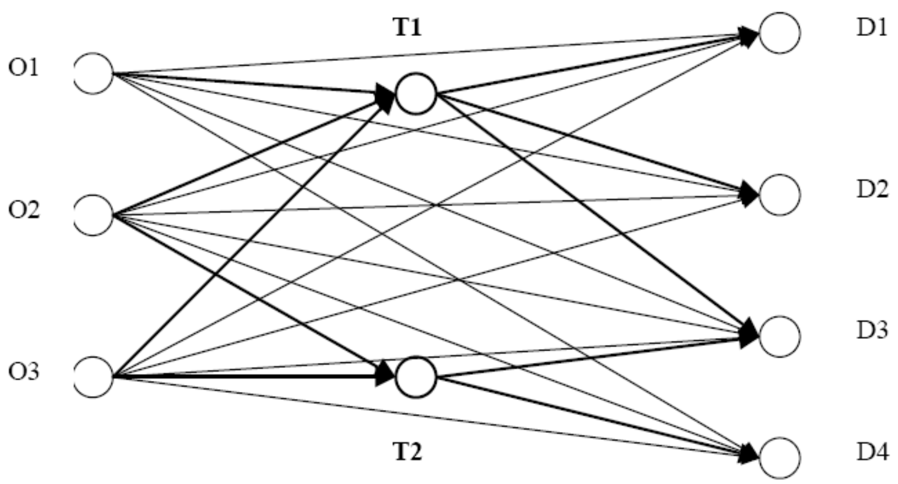
Cost=80x352+20x513+45x416+70x388+55x867+30x685=152535

	1	2	3	4	u_i
1	464 15 0	513 20	654 84 0	867 55	0^*
2	352 80	416 45	690 217 0	791 21 0	-97
3	995 728 0	682 351 0	388 70	685 30	-182
v_j	449	513	570	867	

Optimal solution.

EXERCISE 5: TRANSHIPMENT 1

Consider the following problem with 3 origins, 4 destinations and 2 transfer points, and the possible routes as depicted in the figure below:



The cost matrices and the supply/demand quantities are the following:

	D1	D2	D3	D4
O1	2	3	4	5
O2	5	4	3	1
O3	1	3	3	2

	T1	T2
O1	1	-
O2	1	3
O3	3	1

	D1	D2	D3	D4
T1	1	1	2	-
T2	-	-	1	2

Note: “-“ means that the route is not allowed

ai	6	8	10	
bj	4	6	8	6

Define the cost matrix and provide a solution for the problem by applying the North West Corner Rule.

SOLUTION:

Cost matrix:

	D1	D2	D3	D4	T1	T2
O1	2	3	4	5	1	∞
O2	5	4	3	1	1	3
O3	1	3	3	2	3	2
T1	1	1	2	∞	0	∞
T2	∞	∞	1	2	∞	0

NW corner rule:

	D1	**D2**	**D3**	**D4**	**T1**	**T2**	
O1	4	2	0	0	0	0	6
O2	0	4	4	0	0	0	8
O3	0	0	4	6	0	0	10
T1	0	0	0	0	24	———	24
T2	0	0	0	0	———	18	18
	4	6	8	6	24	18	**66**

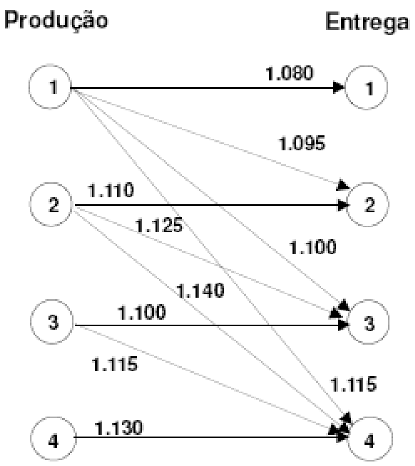
EXERCISE 6: AIRCRAFT ENGINES

A company that builds aircraft engines intends to plan the production of an engine for the next 4 months. In order to meet the contractual delivery dates it needs to supply the engines in the quantities indicated in the second column of the table. The maximum number of engines the company produces per month, as well as the cost of each engine (in millions of dollars) are given in the third and fourth columns of the table. Given the variations in production costs, it may be worth producing some engines one or more months before the scheduled delivery dates. If this hypothesis is chosen, the engines will be stored until the month of delivery, with an additional cost of 0.015 million dollars / month.

Month	Units to deliver	Maximum production	Unitary production cost	Unitary storage cost
1	10	25	1.08	-
2	15	35	1.11	0.015
3	25	30	1.10	0.015
4	20	10	1.13	0.015

The production manager wants to know how many engines to manufacture each month (and for which delivery months) in order to minimize the overall costs of production and storage. Formulate the problem as a transportation problem.

SOLUTION:

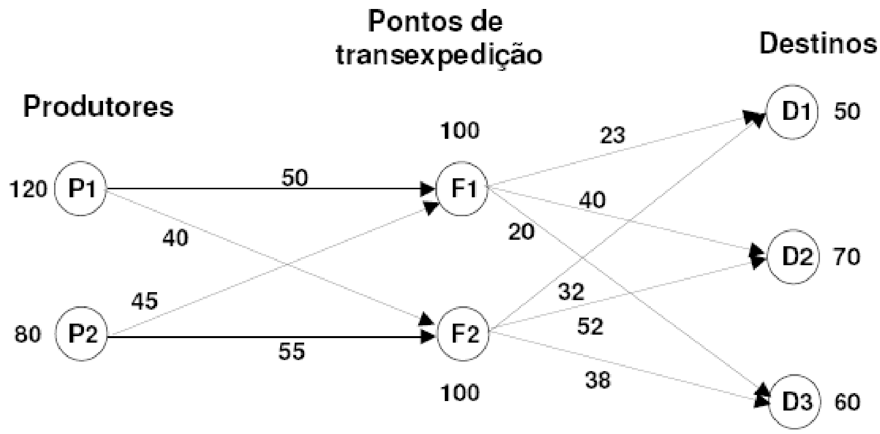


Delivery Month

Production Month	1	2	3	4	x	
1	1.08	1.095	1.110	1.125	0	25
2	∞	1.110	1.125	1.140	0	35
3	∞	∞	1.110	1.115	0	30
4	∞	∞	∞	1.130	0	10
	10	15	25	20	30	100

EXERCISE 7: TRANSHIPMENT 2

Formulate the transhipment problem of as a transportation problem. The following diagram indicates the capacity constraints in the nodes, as well as the costs associated with each flow.



SOLUTION:

Costs matrix:

	D1	D2	D3	F1	F2	X	
P1	∞	∞	∞	50	40	0	120
P2	∞	∞	∞	45	55	0	80
F1	23	40	20	0	∞	∞	100
F2	34	52	38	∞	0	∞	100
	50	70	60	100	100	20	

EXERCISE 8: CAR TIRES MANUFACTURING

The director of a large company is responsible for the production section of three separate factories (F1, F2 and F3), which produce car tires. The manufacturing factories produce the same quality of tires, but they have different daily production capacities. Factories F1, F2 and F3 can produce 1500, 2500 and 2000 tires, respectively. There are 4 supplier markets for this product, although, for commercial and tax reasons, not all factories can serve all markets. F1 can supply the 4 markets, M1, M2, M3 and M4, with unitary transportation costs of 4, 3, 2 and 5 euros, respectively. F2 can only supply M1, M2 and M3 markets with unitary transportation costs of 2, 1 and 4 euros, respectively. F3 factory can only supply M2, M3 and M4 markets with unitary transportation costs of 4, 2 and 7 euros, respectively. M1, M2 and M3 markets require 1500, 1200 and 1800 tires, respectively. M4 needs at least 500 tires.

Formulate the transportation problem and determine the optimal solution. Comment the solution obtained.

SOLUTION:

Costs matrix:

	M1	**M2**	**M3**	**M4**	**M4***	
F1	4	3	2	5	5	1500
F2	2	1	4	∞	∞	2500
F3	∞	4	2	7	7	2000
	1500	1200	1800	500	1000	**6000**

Minimum cost method:

	M1	**M2**	**M3**	**M4**	**M4***	
F1	200	0	0	500	800	1500
F2	1300	1200	0	0	0	2500
F3	0	0	1800	0	200	2000
	1500	1200	1800	500	1000	

	M1	M2	M3	M4	M4*	u_i
F1	4 200-Θ	3 0 0	2 2 0	5 500	5 800+Θ	0^*
F2	2 1300+Θ	1 1200-Θ	4 0	∞ ∞ 0	∞ ∞ 0	-2
F3	∞ ∞ 0	4 -1 0+Θ	2 1800	7 0 0	7 200-Θ	2
v_i	4	3	0	5	5	

Θ = 200

	M1	M2	M3	M4	M4*	u_i
F1	4 0 0	3 0^*	2 0 0	5 500	5 1000	0^*
F2	2 1500	1 1000	4 4 0	∞ ∞ 0	∞ ∞ 0	-2
F3	∞ ∞ 0	4 200	2 1800	7 1 0	7 1 0	1
v_i	4	3	1	5	5	

Cost=5x500+5x1000+2x1500+1x1000+4x200+2x1800=15900

EXERCISE 9: MEDICINE MEMO+

Suppose three chemical companies, A, B and C, sell the medicine Memo+ to hospitals H1, H2, H3 and H4, whose daily needs are 100, 125, 190 and 70 units, respectively. The production capacity of companies A, B and C is 220, 250 and 180 units, respectively. The daily unit costs of transporting this medicine from company A to H1, H2, H3 and H4 are 2, 3, 1 and 6 euros, respectively. From company B to H1, H3 and H4 are 7, 3 and 4 euros, respectively, and from company C to H2, H3 and H4 are 8, 2 and 5 euros, respectively. For reasons of commercial nature, companies B and C do not provide hospitals H2 and H1, respectively. The storage costs for companies A, B and C for each not transported unit of Memo+ are 1, 1.5 and 2 euros, respectively.

Formulate the problem as a transportation problem and determine the optimum solution. Comment the obtained solution.

SOLUTION:

Costs matrix:

	H1	**H2**	**H3**	**H4**	*****	
A	2	3	1	6	1	220
B	7	∞	3	4	1.5	250
C	∞	8	2	5	2	180
	100	125	190	70	165	**650**

Applying the NW corner rule:

	H1	**H2**	**H3**	**H4**	*****	
A	100	120	0	0	0	220
B	0	5	190	55	0	250
C	0	0	0	15	165	180
	100	125	190	70	165	**650**

	H1	H2	H3	H4	*	u_i
A	2 $100^{-\Theta}$	3 ∞ $120^{+\Theta}$	1 ∞ 0	6 ∞ 0	1 0	0^*
B	7 -∞ $0^{+\Theta}$	∞ $5^{-\Theta}$	3 190	4 0	1,5 1,5 0	∞
C	∞ -2 0	8 - 0	2 2 0	5 165	2 165	∞
v_i	2	3	-∞	-∞	-∞	

Θ = 5

	H1	H2	H3	H4	*	u_i
A	2 95	3 125	1 3 0	6 7 0	1 5 0	0^*
B	7 5	∞ ∞ 0	3 $190^{-\Theta}$	4 $55^{+\Theta}$	1,5 0,5 0	5
C	∞ ∞ 0	8 -1 0	2 -2 $0^{+\Theta}$	5 $15^{-\Theta}$	2 165	6
v_i	2	3	-2	-1	-4	

Θ = 15

	H1	H2	H3	H4	*	u_i
A	2 95	3 125	1 3 0	6 7 0	1 3 0	0^*
B	7 5	∞ ∞ 0	3 $175^{-\Theta}$	4 70	1,5 -1,5 $0^{+\Theta}$	5
C	∞ ∞ 0	8 1 0	2 $15^{+\Theta}$	5 0	2 $165^{-\Theta}$	4
v_i	2	3	-2	-1	-2	

Θ = 165

	H1	H2	H3	H4	*	u_i
A	2 95	3 125	1 3 0	6 7 0	1 4,5 0	0^*
B	7 5	∞ ∞ 0	3 10	4 70	1,5 165	5
C	∞ ∞ 0	8 1 0	2 180	5 2 0	2 1,5 0	4
v_i	2	3	-2	-1	-3,5	

Cost=95x2+125x3+5x7+10x3+70x4+165x1,5+180x2=1517,5

EXERCISE 10

In the following transportation problem the total demand exceeds the total supply. Assume that the unsatisfied cost per unit of demand is 5, 3 and 2 euros respectively for destinations 1, 2 and 3.

	1	2	3	**Offer**
1	5	1	7	10
2	6	4	6	80
3	3	2	5	15
Demand	75	20	50	

SOLUTION:

Applying the Minimum cost method:

	1	**2**	**3**	**Offer**
1	0	10	0	10
2	70	0	10	80
3	5	10	0	15
4	0	0	40	40
Demand	75	20	50	**145**

	1	2	3	u_i
1	5 3 0	1 10	7 5 0	0^*
2	6 $70^{-\Theta}$	4 -1 $0^{+\Theta}$	6 10	4
3	3 $5^{+\Theta}$	2 $10^{-\Theta}$	5 2 0	1
*	5 3 0	3 2 0	2 40	0
v_i	2	1	2	

$\Theta = 10$

	1	2	3	u_i
1	5 2 0	1 10	7 4 0	0^*
2	6 60	4 10	6 10	3
3	3 15	2 1 0	5 2 0	0
*	5 3 0	3 3 0	2 40	-1
v_i	3	1	3	

Cost=1x10+6x60+4x10+6x10+3x15+2x40=595

EXERCISE 11: DISTRIBUTION VEHICLES

Suppose that you have 45 vehicle available for transportation purposes in 4 different depots (A, B, C and D). There are 5 vehicles available in depot A, 12 in B, 13 in C and 15 in D. The vehicles are required at locations L1, L2, L3 and L4 in the following quantities: 10, 15, 11 and 14 respectively.

The minimum distances (km) between the mentioned locations are shown in the table below.

At least 12 vehicles must supplied to the L4 location. Display the matrix to calculate an initial basic solution.

	L1	**L2**	**L3**	**L4**
A	12	16	22	33
B	15	25	21	40
C	11	14	19	28
D	15	20	23	32

SOLUTION:

	L1	L2	L3	L4(minimum	L4(above)	Offer
A	12	16	22	33	33	5
B	15	25	21	40	40	12
C	11	14	19	28	28	13
D	15	20	23	31	31	15
*	0	0	0	∞	0	5
Demand	10	15	11	12	2	**50**

EXERCISE 12: TRUE/FALSE

Verify if the following sentences are true or false:

1. A transportation problem always has a solution.
2. A transportation model with 3 origins and 4 destinations, with total offer higher than the total demand has basic solutions with 6 basic variables.
3. A transportation problem can always be represented by a balanced model.
4. An unbalanced transportation model may require both a dummy origin and dummy destination.
5. If a transportation model is unbalanced, with a total demand higher than the total offer, the technical offer constraints are of for the "=" or "≥" type and the demand constraints are of the "≤" type.

6. If a transportation model is unbalanced, with a total demand lower than the total offer, the technical offer constraints are of the "≤" type and the demand constraints are of the "=" or "≥" type.
7. In an unbalanced transportation problem, with more supply than demand, in the matrix to calculate the initial basic solution we insert a dummy destination whose constraint is the sum of the slack variables of the supply constraints.
8. In the basic solution of a transportation problem, the value of the variables associated with a dummy source represent quantities not received at the destinations associated with those variables.
9. The transportation model allows the operation with a single good or service.
10. The northwest corner method is one of the methods of optimizing the value of f (x).
11. In the transportation model there are no degenerate solutions.
12. A balanced transportation model may not have a feasible solution.

SOLUTION:

1. T
2. F
3. V
4. F
5. V
6. V
7. V
8. V
9. V
10. F
11. F
12. F

EXERCISE 13: SHIRTS FOR THE ARMY

A government army department requests 300 shirts to warehouse A, 400 to warehouse B and 300 to warehouse C.

Manufacturer X is able to supply up to 800 shirts, manufacturer Y is able to supply up to 500 and manufacturer Z up to 300. The shirts unitary costs depend on the prices charged by each manufacturer and the transportation costs to the warehouses. For reasons that were not specified, manufacturer Y must be ordered at least 400 shirts.

The unitary costs, expressed in an appropriate monetary unit, of transporting a shirt from each manufacturer to each different warehouse are shown in Table 1.

From the 200 shirts, manufacturer X charges an extra fee of 1 monetary unit for each additional shirt.

	A	B	C
X	9	11	13
Y	12	15	15
Z	13	14	16

Formulate this problem as a Transportation problem and present the matrix to calculate an initial basic solution

SOLUTION

	A	B	C	*
X	9	11	13	0
Y	12	15	15	0
Z	13	14	16	

	A	B	C	*	
X1	9	11	13	0	200
X2	10	12	14	0	600
Y1	12	15	15	∞	400
Y2	12	15	15	0	100
Z	13	14	16	0	300
	300	400	300	600	

EXERCISE 14: HOLYWATER COMPANY

The HolyWater Company holds the concession of water distribution in a certain geographical area.

As the region is very arid, the Company has to buy and transport water from other regions. The origins of water are the rivers Temptation, Sacrament, and Salvation.

The main clients of the Company are the municipalities of “High Anger”, “Laziness Place”, “Evil of envy” and “Greedy Heights”.

It is possible to supply any of the villages with water from any of the rivers, with only one exception: that of the river Salvation to “Greedy Heights”.

The HolyWater Company accepts to supply to each locality a minimum amount of water, necessary to satisfy the basic needs of the respective populations, except “Evil of envy”, which has an autonomous fountain.

The values of water transportation costs from each river to each village are expressed in the following table. The table also includes the available capacities of each river, the quantities of water requested by the villages and the minimum values to satisfy their basic needs.

The Company intends to distribute all available water from the three rivers in order to satisfy at least the basic needs of each locality.

Assuming, of course, that the Company wants to achieve this goal at the minimum transportation costs, formulate the problem.

	High Anger	Laziness Place	Evil of envy	Greedy Heights	Capacity
Temptation River	16	13	22	17	50
Sacrament River	14	13	19	15	60
Salvation River	19	20	23	-	50
Minimum	30	70	0	10	
Requested	50	70	30	60	

SOLUTION

Initial Solution:

	High Anger (min)	High Anger (requested)	Laziness Place	Evil of envy	Greedy Heights (min)	Greedy Heights (requested)	Offer
Temptation River	16	16	13 50	22	17	17	50
Sacrament River	14 30	14 10*	13 20	19	15	15	60
Salvation River	19	19 10	20	23 30	∞ 10	∞ 0	50
Not provided	∞	0	∞	0	∞	0 50	50
	30	20	70	30	10	50	

Optimal solution:

	High Anger (min)	High Anger (requested)	Laziness Place	Evil of envy	Greedy Heights (min)	Greedy Heights (requested)	Offer
Temptation River	16	16	13 50	22	17	17	50
Sacrament River	14	14	13 20	19	15 10	15 30	60
Salvation River	19 30	19 20	20 0	23	∞	∞	50
Not provided	∞	0	∞	0 30	∞	0 20	50
	30	20	70	30	10	50	

EXERCISE 15: HEALTH CENTRE UNIT

A Health Centre Unit is open to the public from Monday to Friday, from 8 am to 4 pm and on Saturdays from 8 am to 12 noon.

To ensure the presence of a doctor when the center is open to the public, the health center the unit provides the services of 5 doctors (A, B, C, D, E).

The table below shows the availability and remuneration agreed with each of the five doctors..

A	Up to 5 h/week 30 € /h	From 6 to 12 h/week 40 €/h	A is not available on Mondays
B	Up to 9 h/ week 32 €/h		B is only available for 2 hours on Tuesdays
C	Up to 4 h/ week 25 €/h	From 5 to 10 h/ week 37 €/h	
D	Up to 12 h/ week 35 €/h		
E	Up to 4 h/ week 25 €/h	From 5 to 10 h/ week 37 €/h	E is not available on Saturdays

SOLUTION

Solution A

	2ª	3ª	4ª	5ª	6ª	Sáb	*	**	
A	M	30	30	30	30	30	0	M	5
A’	M	40	40	40	40	40	0	M	7
B	32	M	32	32	32	32	0	0	9
B’	M	32	M	M	M	M	M	0	2
C	25	25	25	25	25	25	0	M	4
C’	37	37	37	37	37	37	0	M	6
D	35	35	35	35	35	35	0	M	12
E	25	25	25	25	25	M	0	M	4
E’	37	37	37	37	37	M	0	M	6
	8	8	8	8	8	4	9	2	

Solution B (simpler than A)

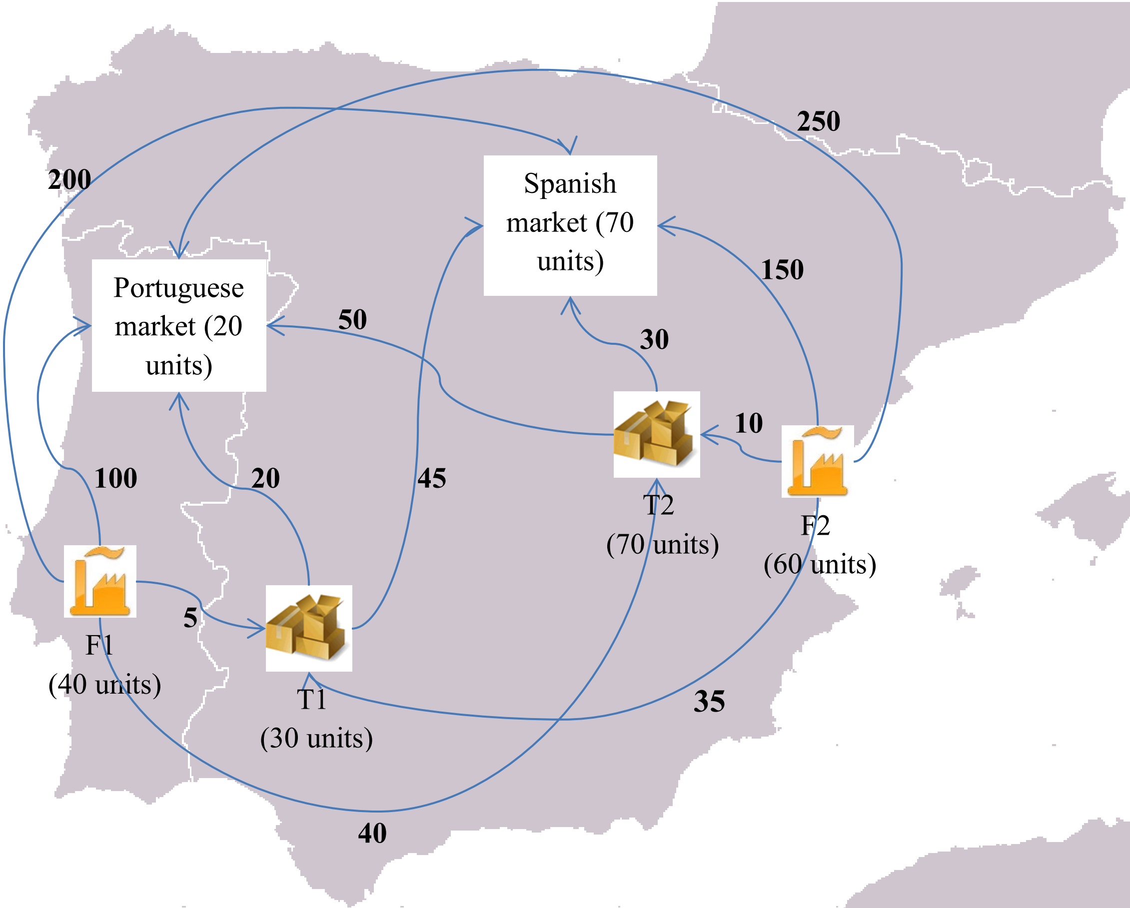
	2ª	3ª	4ª	5ª	6ª	Sáb	*	
A	M	30	30	30	30	30	0	5
A'	M	40	40	40	40	40	0	7
B	32	M	32	32	32	32	0	7
B'	32	32	32	32	32	32	0	2
C	25	25	25	25	25	25	0	4
C'	37	37	37	37	37	37	0	6
D	35	35	35	35	35	35	0	12
E	25	25	25	25	25	M	0	4
E'	37	37	37	37	37	M	0	6
	8	8	8	8	8	4	9	

EXERCISE 16: YOGURT FACTORY

The firm PéptidosActivos produces yogurts in two different factories that fill the market of the Iberian Peninsula.

The distribution policy of the company favours the production flow through the transhipment centres but, at a higher cost, also allows the direct satisfaction of the demand of final customers through the factories.

The consultancy team assigned to this project estimated the costs of each possible link (number in bold), the capacities of the plants (F1 and F2), the capacities of the transfer centres (T1 and T2) and the demand in each one of the markets as shown in the figure below.



a) Present the initial matrix for the formulation of a transportation problem to determine the yoghurt distribution in order to minimize the total cost of transportation, meeting the capacity constraints of the factories and transhipment centres as well as the Portuguese and Spanish market demands.
b) Obtain an initial basic solution to the transportation problem using the Northwest corner rule.
c) Solve the problem by using the transportation algorithm until you obtain a solution with a finite total cost.
d) How would be the initial matrix if the consultants wanted to incorporate the following restriction: "The F1 and F2 plants must divide in equal amounts the amount of overcapacity they have cumulatively."

SOLUTION:

a) e b)

	M1	M2	T1	T2	X	
F1	100 **20**	200 **20**	5 **0**	40 **0**	0 **0**	**40**
F2	250 **0**	150 **50**	35 **10**	10 **0**	0 **0**	**60**

T1	20 **0**	45 **0**	0 **20**	∞ **10**	∞ **0**	**30**
T2	50 **0**	30 **0**	∞ **0**	0 **60**	∞ **10**	**70**
	20	**70**	**30**	**70**	**10**	

c) We have two alternative paths

Table 1 - Path 1a) Step 1.

	M1	M2	T1	T2	X	
F1	100 **20**	200 -θ **20**	5 **0** -80	40 +θ **0** -∞	0 **0** -∞	**40** 0
F2	250 **0** 200	150 +θ **50**	35 -θ **10**	10 **0** -∞	0 **0** -∞	**60** -50
T1	20 **0** 5	45 **0** -70	0 +θ **20**	∞ -θ **10**	∞ **0** N/D	**30** -85
T2	50 **0** ∞	30 **0** ∞	∞ **0** ∞	0 **60**	∞ **10**	**70** -∞
	100 **20**	200 **70**	85 **30**	∞ **70**	∞ **10**	

u1 + v1 = 100 -> u1 = 0 e v1 = 100

u1 + v2 = 200 -> v2 = 200

u2 + v2 = 150 -> u2 = -50

u2 + v3 = 35 -> v3 = 85

u3 + v3 = 0 -> u3 = -85

u3 + v4 = ∞ -> v4 = ∞

u4 + v4 = 0 -> u4 = -∞

u4 + v5 = ∞ -> v5 = ∞

Table 2 - Path 1a) Step 2.

	M1	M2	T1	T2	X	
F1	100 **20**	200 **10**	5 **0***	40 -θ **10**	0 +θ **0** -∞	**40** 0
F2	250 **0** 200	150 **60**	35 **0** 80	10 **0** 20	0 **0** -∞	**60** -50
T1	20 **0** -75	45 **0** -150	0 **30**	∞ **0** ∞	∞ **0** N/D	**30** -5
T2	50 **0**	30 **0**	∞ **0**	0 +θ **60**	∞ -θ **10**	**70**

	M1	M2	T1	T2	X	
	-10	-130	∞			-40
	100 20	200 70	5 30	40 70	∞ 10	

This solution is degenerate. So once again here are several possible paths. If the student chooses (as it seems to be intuitive) a zero of the possible ones that will become basic with a cost inferior to ∞ then he will reach immediately the end of the exercise.

$u1 + v1 = 100 \rightarrow u1 = 0$ e $v1 = 100$
$u1 + v2 = 200 \rightarrow v2 = 200$
$u1 + v4 = 40 \rightarrow v4 = 40$
$u2 + v2 = 150 \rightarrow u2 = -50$
$u3 + v3 = 0 \rightarrow u3 = -5$
$u4 + v4 = 0 \rightarrow u4 = - 40$
$u4 + v5 = \infty \rightarrow v5 = \infty$
$u1 + v3 = 5 \rightarrow v3 = 5$

Table 3 - Path 1a) Step 3 OK

	M1	M2	T1	T2	X	
F1	100 **20**	200 **10**	5 **0**	40 **0**	0 **10**	**40**
F2	250 **0**	150 **60**	35 **0**	10 **0**	0 **0**	**60**
T1	20 **0**	45 **0**	0 **30**	∞ **0**	∞ **0**	**30**
T2	50 **0**	30 **0**	∞ **0**	0 **70**	∞ **0**	**70**
	20	**70**	**30**	**70**	**10**	

Table 4 - Path 1b) Step 1

	M1	M2	T1	T2	X		
F1	100 **20**	200 **20**	5 **0** -80	40 **0** -∞	0 **0** -∞	**40**	0
F2	250 **0** 200	150 **50**	35 -θ **10**	10 +θ **0** -∞	0 **0** -∞	**60**	-50
T1	20 **0** 5	45 **0** -70	0 +θ **20**	∞ -θ **10**	∞ **0** N/D	**30**	-85
T2	50 **0** ∞	30 **0** ∞	∞ **0** ∞	0 **60**	∞ **10**	**70**	-∞
	100 **20**	200 **70**	85 **30**	∞ **70**	∞ **10**		

Table 5 Path 1b) Step 2

M1	M2	T1	T2	X

F1	100 **20**	200 **20**	5 **0***	40 **0**	0 **0** -∞	**40**	0
F2	250 **0**	150 **50**	35 **0**	10 **10**	0 **0** -∞	**60**	-50
T1	20 **0**	45 **0**	0 **30**	∞ **0**	∞ **0**	**30**	-5
T2	50 **0**	30 **0**	∞ **0**	0 **60**	∞ **10**	**70**	-60
	100 **20**	200 **70**	5 **30**	60 **70**	∞ **10**		

This solution is degenerate. So once again here are several possible paths. If the student chooses (as it seems to be intuitive) a zero of the possible ones that will become basic with a cost inferior to ∞ then he will reach immediately the end of the exercise. Step 3 is very similar to the same step in path 1a).

u1 + v1 = 100 -> u1 = 0 e v1 = 100
u1 + v2 = 200 -> v2 = 200
u2 + v2 = 150 -> u2 = -50
u2 + v4 = 10 -> v4 = 60
u3 + v3 = 0 -> u3 = -5
u4 + v4 = 0 -> u4 = -60
u4 + v5 = ∞ -> v5 = ∞
u1 + v3 = 5 -> v3 = 5

Table 6 - Path 2a) Step 1

	M1	**M2**	**T1**	**T2**	**X**		
F1	100 **20**	200 -θ **20**	5 **0** -80	40 **0** -∞	0 +θ **0** -∞	**40**	0
F2	250 **0** 200	150 +θ **50**	35 -θ **10**	10 **0** -∞	0 **0** -∞	**60**	-50
T1	20 **0** 5	45 **0** -70	0 +θ **20**	∞ -θ **10**	∞ **0** N/D	**30**	-85
T2	50 **0** ∞	30 **0** ∞	∞ **0** ∞	0 +θ **60**	∞ -θ **10**	**70**	-∞
	100 **20**	200 **70**	85 **30**	∞ **70**	∞ **10**		

Table 7 - Path 2a) Step 2

	M1	**M2**	**T1**	**T2**	**X**

F1	100 **20**	200 **10**	5 **0**	40 **0**	0 **10**	**40**
F2	250 **0**	150 **40**	35 **0**	10 **0**	0 **0**	**60**
T1	20 **0**	45 **0**	0 **30**	∞ **0**	∞ **0**	**30**
T2	50 **0**	30 **0**	∞ **0**	0 **70**	∞ **0**	**70**
	20	**70**	**30**	**70**	**10**	

Table 8 - Path 2b) Step 1

	M1	**M2**	**T1**	**T2**	**X**	
F1	100 **20**	200 **20**	5 **0** -80	40 **0** -∞	0 **0** -∞	**40** 0
F2	250 **0** 200	150 **50**	35 -θ **10**	10 **0** -∞	0 +θ **0** -∞	**60** -50
T1	20 **0** 5	45 **0** -70	0 +θ **20**	∞ -θ **10**	∞ **0** N/D	**30** -85
T2	50 **0** ∞	30 **0** ∞	∞ **0** ∞	0 +θ **60**	∞ -θ **10**	**70** -∞
	100 **20**	200 **70**	85 **30**	∞ **70**	∞ **10**	

Table 9 - Path 2b) Step 2

	M1	**M2**	**T1**	**T2**	**X**	
F1	100 **20**	200 **20**	5 **0**	40 **0**	0 **0**	**40**
F2	250 **0**	150 **50**	35 **0**	10 **0**	0 **10**	**60**
T1	20 **0**	45 **0**	0 **30**	∞ **0**	∞ **0**	**30**
T2	50 **0**	30 **0**	∞ **0**	0 **70**	∞ **0**	**70**
	20	**70**	**30**	**70**	**10**	

c)

M1	**M2**	**T1**	**T2**	**X1**	**X2**

F1	100	200	5	40	0	∞	**40**
F2	250	150	35	10	∞	0	**60**
T1	20	45	0	∞	∞	∞	**30**
T2	50	30	∞	0	∞	∞	**70**
	20	**70**	**30**	**70**	**5**	**5**	

EXERCISE 17: XPERTS CONSULTANT

The consulting company Xperts has a new project with an expected duration of four weeks (1 month) needing a total of 80 hours per week in human resources (consultants). The company director has 4 consultants to carry out this work and they have different availabilities and costs (table 1) but similar skills.

a) Formulate the problem described as a transportation problem in order to minimize the total costs

b) What methods could you use to solve this problem? Indicate which one you would choose and the reason (s) that motivate your choice.

Consultants	Costs until 80h/month	Costs from 80 to 100h/ month	Non-availability
A	42 €/hour	50 €/ hour	2nd week
B	40 €/ hour	40 €/ hour	3rd week
C	45 €/ hour	45 €/ hour	-
D	40 €/ hour	-	1st and 4th week

SOLUTION

a)

Consultants	1	2	3	4	*	
A	42	M	42	42	0	80
A’	50	M	50	50	0	20
B	40	40	M	40	0	100
C	45	45	45	45	0	100
D	M	40	40	M	0	80
	80	80	80	80	60	

b) The Simplex method or the Transport Algorithm could be used. However, the transport algorithm would potentially be faster.

EXERCISE 18: PAPERPLUS

PaperPlus is a paper factory, producing paper from both recycled pulp and virgin pulp, resulting in two different product types: recycled paper (R) and normal (N) paper. For the paper production the factory has four machines, M1, M2, M3 and M4, each one with a production capacity of 100 tons per month. The production costs of each type of paper in each of the machines are shown in the table below. A malfunction in the normal paper silo temporarily prevents the M3 machine from producing this type of paper.

Paper\Machine	M1	M2	M3	M4
N	5	4	7	3
R	6	5	6	3

PaperPlus is currently developing the production plan for next month, aiming to come up with a solution that minimizes its operating costs. To do this, the production manager asked the sales department for an estimate of the demand of the two types of paper for the next month, who indicated a forecast of 100 tonnes of plain paper and 80 tonnes of recycled paper sold.

a) Formulate the production planning problem described as a Transportation Problem.

b) Determine the optimal production plan, using the minimum cost rule to generate the initial solution.

c) The production manager, after reviewing the plan, decided to implement two new planning rules. The first rule requires that the M1 machine produces at least 60 tons of Paper. The second rule limits the monthly production of paper in the group of machines M2 and M4 to 120 tons Reformulate the problem in order to incorporate these new constraints.

SOLUTION

	N	R	Surplus	
M1	5	6	0	100
M2	4	5	0	100
M3	∞	6	0	100
M4	3	3	0	100
	100	80	220	

a) Initial solution (Cost: 700 u.m.)

	N		R		Surplus		
M1	5		6		0		100
						100	
M2	4		5		0		100
		0		80		20	
M3	∞		6		0		100
						100	
M4	3		3		0		100
		100					
	100		80		220		

1ª iteration:

	0		**1**		**-4**		
4	5		6		0		100
	1		1		0	100	
4	4	+Θ	5	- Θ	0		100
	0	0	0	80	0	20	
4	∞		6		0		100
	∞		1		0	100	
3	3	- Θ	3	+ Θ	0		100
	0	100	-1		1		
	100		80		220		

Θ = min (100, 80) = 80

2ª iteration:

	0		**0**		**-4**		
4	5		6		0		100
	1		2		0	100	
4	4		5		0		100
	0	80	1		0	20	
4	∞		6		0		100
	∞		2		0	100	
3	3		3		0		100
	0	20	0	80	1		
	100		80		220		

Optimal solution (Cost: 620 u.m.)

b)

	N	R	M2/M4	Surplus	
M1-min	5	6	∞	∞	60
M1	5	6	∞	0	40
M2	4	5	0	0	100
M3	∞	6	∞	0	100
M4	3	3	0	0	100
	100	80	80	140	

EXERCISE 19: "THE SEAGULL" RECREATIONAL ASSOCIATION

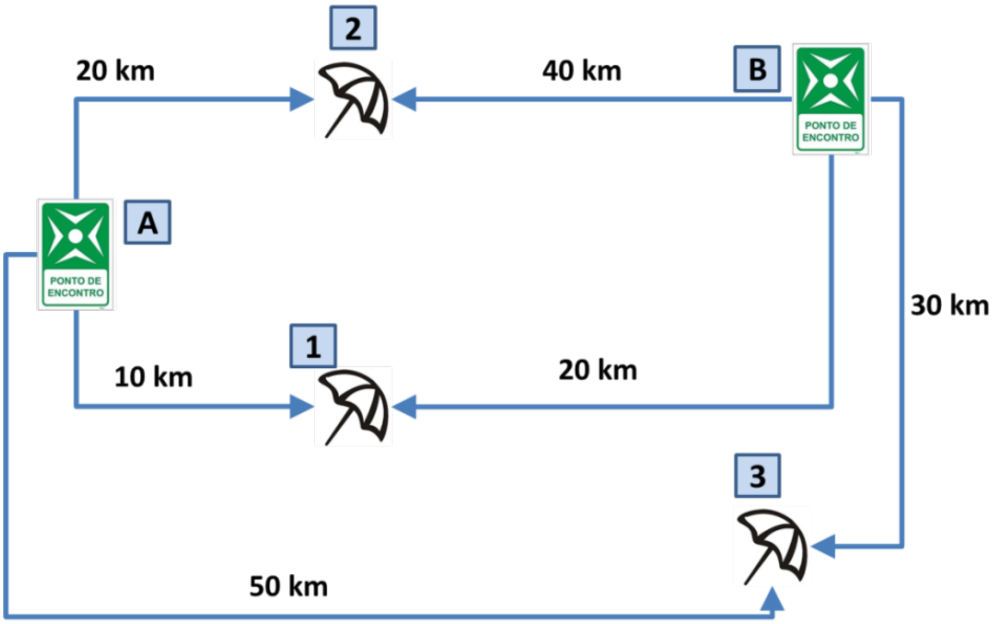
The "The Seagull" Recreational Association had the initiative of organizing summer activities for the children of its members. Some of these activities will be carried out on beaches, having already been selected 3 beaches for this purpose. Children will be collected from two meeting points (designated by Points A and B) and will be distributed across the three beaches (Beach 1, 2 and 3). The Association "The Seagull" intends to study the best way to transport the children to the beaches, taking into account the following conditions:

1) At meeting points A and B there will be, respectively, 100 and 120 children waiting to be transported.

2) Beaches 1, 2 and 3 are prepared to receive 80, 120 and 100 children, respectively.

3) The transport costs were estimated according to the number of trips (round trip) required, taking into account the distances between the places (see next figure) and a cost of 0.5 € / km.

4) Each car has a capacity to take 20 children per trip.



a) Formulate this problem as a transportation problem. Find a feasible basic solution by the northwest corner method.
b) Find the optimal solution of the problem using the transportation algorithm. What is the optimal solution value? Is the solution found unique? Justify.

SOLUTION

a)

	1	2	3	oferta
A	10 4	20 1	50 0	5
B	20 0	40 5	30 1	6
*	0 0	0 0	0 4	4
procura	4	6	5	

b) Alternative 1

	1	2	3	u_i
A	10 $4^{-\Theta}$	20 $1^{+\Theta}$	50 40 0	0^*
B	20 -10 $0^{+\Theta}$	40 $5^{-\Theta}$	30 1	20
*	0 0 0	0 -10 0	0 4	-10
v_j	10	20	10	

$\Theta=4$

	1	2	3	u_i
A	10 10 0	20 5	50 40 0	0^*
B	20 4	40 $1^{-\Theta}$	30 $1^{+\Theta}$	20
*	0 10 0	0 -10 $0^{+\Theta}$	0 0 $4^{-\Theta}$	-10
v_i	0	20	10	

Θ=1

	1	2	3	u_i
A	10 0 0	20 5	50 30 0	0^*
B	20 4	40 10 0	30 2	10
*	0 10 0	0 1	0 3	-20
v_i	10	20	20	

Total Cost = 240

b) Alternative 2

	1	2	3	u_i
A	10 4	20 1	50 40 0	0^*
B	20 -10 0	40 $5^{-\Theta}$	30 $1^{+\Theta}$	20
*	0 0 0	0 -10 $0^{+\Theta}$	0 $4^{-\Theta}$	-10
v_i	10	20	10	

Θ=4

	1	2	3	ui
A	10 4-Θ	20 1+Θ	50 40 0	0*
B	20 -10 0+Θ	40 1-Θ	30 5	20
*	0 10 0	0 4	0 10 0	-20
vi	10	20	10	

Θ=1

	1	2	3	u_i
A	10 3	20 2	50 30 0	0*
B	20 1	40 10 0	30 5	10
*	0 10 0	0 4	0 0 0	-20
v_i	10	20	20	

Total cost = 240

EXERCISE 20 –BOATARDE RETAIL COMPANY EXAM 2013-2014

The retail company BoaTarde will launch a promotional campaign for bananas in its 5 stores. Bananas can be purchased from 3 different suppliers that deliver directly to the stores. The forecasted demand for bananas with promotional price for the 5 stores, the availability of bananas in the 3 suppliers and the unitary transportation costs are described in the table below.

BoaTarde also wants to ensure that the lost demand in stores 1, 2 and 3 does not exceed 10% of the estimated demand.

a) Formulate the problem of distributing the promotional bananas of BoaTarde as a transportation problem. Find a basic feasible solution with the minimum cost method.

b) The solution found in a) is optimal? Justify. If not, find a new solution to the problem using the transportation algorithm.

		Stores					
		1	**2**	**3**	**4**	**5**	**Offer**
Suppliers	**A**	10	20	10	15	50	**50**
	B	20	40	20	20	30	**50**
	C	10	10	5	10	15	**100**
	Demand	**50**	**60**	**40**	**50**		

SOLUTION

a)

		Lojas								
		1	1'	2	2'	3	3'	4	5	Oferta
Fornecedores	A	10 45	10 0	20 0	20 0	10 0	10 0	15 5	50 0	50
	B	20 0	20 0	40 0	40 0	20 0	20 0	20 0	30 50	50
	C	10 0	10 0	10 54	10 0	5 36	5 0	10 10	15 0	100
	*	M 0	0 5	M 0	0 6	M 0	0 4	0 35	0 0	50
	Procura	45	5	54	6	36	4	50	50	

b)

		Stores								
		1	1’	2	2’	3	3’	4	5	u_i
Suppliers	A	10 45	10 30 0	20 5 0	20 40 0	10 0 0	10 30 0	15 5	50 80 0	0
	B	20 -50 0	20 -20 0	40 -35 0	40 $0^{*-\Theta}$	20 -50 0	20 -20 0	20 **-55** $0^{+\Theta}$	30 50	60
	C	10 25 0	10 35 0	10 54	10 35 0	5 36	5 30 0	10 10	15 50 0	-5
	*	M M 0	0 5	M M 0	0 $6^{+\Theta}$	M M 0	0 4	0 $35^{-\Theta}$	0 10 0	20
	v_i	10	-20	15	-20	10	-20	15	-30	

EXERCISE 21 – LIFEGUARDS EXAM 2014-2015

In the northern part of the country, the ISN (Ocean Rescue Institute) has three rescue training centres. The lifeguards formed by these centres will be available to watch over three of the region's most important beaches.

The unit costs (in euros) associated with the training of lifeguards depend on the beach to which each one is associated, due to the transport of the trainees and their equipment. These costs are presented in the following table:

	Beach 1	Beach 2	Beach 3
Centre 1	5	3	3
Centre 2	5	4	4
Centre 3	5	3	4

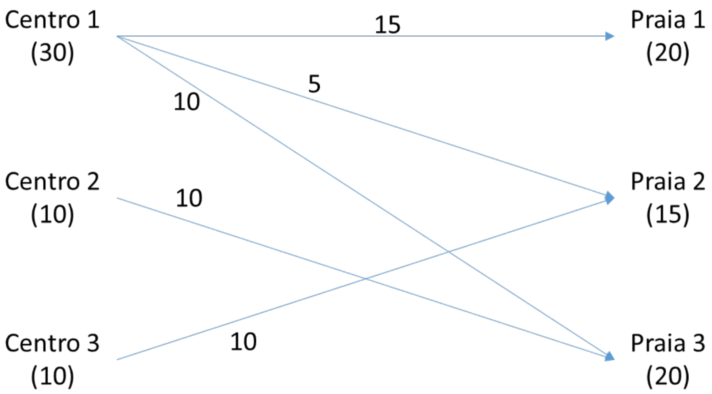
The number of lifeguards provided for each of the training centres, as well as the requirements for each of the beaches, are presented in the following tables:

Centre 1	Centre 2	Centre 3
30	10	10

Beach 1	Beach 2	Beach 3
20	15	20

a) Formulate the problem as a transportation problem;

b) The next figure presents a possible distribution of the lifeguards across the three beaches. Verify that this distribution matches the optimal solution. If it does not, determine the optimal solution using the transportation algorithm.



c) Is the optimal solution unique? Justify your answer.

d) Consider now that the usual sea conditions on beaches 2 and 3 they require that lifeguards receive additional training which can only be given at two specialized advanced training centres. Half of the graduates of the centre 1 have already received this training, but the rest will have to receive this additional training. For safety reasons, beach 2 must receive all required lifeguards.

The unitary transportation costs from each training centres to the specialized advanced training centres and from there to each beache (only beaches 2 and 3) are € 4.

Formulate again the problem, considering these new constraints.

SOLUTION

a)

	Beach 1	Beach 2	Beach 3	
Centre1	5	3	3	30
Centre2	5	4	4	10
Centre3	5	3	4	10
*	0	0	0	5
	20	15	20	

b)

	Beach 1	Beach 2	Beach 3	
Centre1	5 - 15	3 5	3 + 10	u1=0
Centre2	5 + -1 0	4 0 0	4 - 10	u2=1
Centre3	5 0 0	3 10	4 1 0	u3=0
*	0 5	0 2 0	0 2 0	u4=-5
	v1=5	v2=3	v3=3	

teta=10

	Beach 1	Beach 2	Beach 3	
Centreo1	5 5	3 5	3 20	u1=0
Centre2	5 10	4 1 0	4 1 0	u2=0
Centre3	5 0 0	3 10	4 1 0	u3=0
*	0 5	0 2 0	0 2 0	u4=-5
	v1=5	v2=3	v3=3	

Solução

c) Não é única, dado que delta31=0, ou seja, o coeficiente da f. o. correspondente à variável não básica x31 é nulo. Assim, esta variável poderá entrar na base sem que o valor da f.o se altere.

d)

	Beach 1 (Normal)	Beach 2 Min (Especial)	Beach 2 (Special)	Beach 3 (Special)	AdvForm1	AdvForm2	
Centre1 (Normal)	5	M	M	M	4	4	15
Centre1 (spec)	M	3	3	3	M	M	15
Centre2	5	M	M	M	4	4	10
Centre3	5	M	M	M	4	4	10
AdvForm1	M	4	4	4	0	M	35
AdvForm2	M	4	4	4	M	0	35
*	0	M	0	0	0	0	10
	20	10	10	20	35	35	

Optimal solution (by curiosity)

	Beach 1 (Normal)	Beach 2 Min (Especial)	Beach 2 (Special)	Beach 3 (Special)	AdvForm1	AdvForm2	
Centre1 (Normal)	15	0	0	0	0	0	15
Centre1 (Espec)	0	0	0	15	0	0	15
Centre2	5	0	0	0	5	0	10

Centre3	0	0	0	0	10	0	10
AdvForm1	0	10	0	5	20	0	35
AdvForm2	0	0	0	0	0	35	35
*	0	0	10	0	0	0	10
	20	10	10	20	35	35	

EXERCISE 22 – RIO 2016 EXAM 2015-2016

In the Olympic Games RIO 2016 will be put on sale promotional sweaters with the logo of the event. The organization plans to distribute a total of 100 000 sweaters that will be available in 4 different stores in the Olympic zone: Barra, Copacabana, Deodoro and Maracanã. The number of shirts to distribute to each of the stores is 30 000, 15 000, 20 000, 35 000, respectively.

The sweaters will be specially produced for the event in 4 different factories (F1, F2, F3, F4). The factories that will produce the promotional sweaters have the following capacities: 25 000, 15 000, 30 000, 25 000 sweaters, respectively. The unitary transportation costs of the sweaters (in euros) are shown in the table below and vary according to the distance from the factory to the store.

	F1	F2	F3	F4
Barra	5	4	4	3
Copacabana	4	6	5	4
Deodoro	2	4	5	6
Maracanã	5	3	4	3

a) Formulate the problem as a transportation problem in the standard form.
b) Determine an initial basic solution by applying the Northwest corner rule.
c) Perform an iteration of the transportation algorithm.
d) Indicate the reduction of the total cost (in euros) that is obtained at the end of the first iteration of the transportation algorithm.
e) Change the formulation proposed in (a) in order to include the following constraints:
Before going to the shops, the sweaters have to go through one of two prints (E1, E2) to print the logo of the Olympic Games. Admit that the two prints have the ability to engrave the logo on all sweaters arriving from the factories (F1, F2, F3, F4). The cost of stamping the logo on a sweater is 2 euros on the print (E1) and 1.5 euros on the print (E2). The unit costs of transporting the sweaters from each factory to each print and from each print to each store are presented in the tables below.
f) In addition, the organization has to ensure that at least 30,000 sweaters are actually delivered at the Maracanã store, as this will be the stage for the most media Olympic ceremonies.

	Barra	Copacabana	Deodoro	Maracanã
E1	5	4	3	3
E2	4	-	5	2

	F1	F2	F3	F4
E1	5	4	-	3
E2	-	3	5	3

- it is not possible to perform this transport

SOLUTION

a)

	Barra	Copacabana	Deodoro	Maracanã	
F1	5	4	2	5	**25**
F2	4	6	4	3	**15**
F3	4	5	5	4	**30**
F4	3	4	6	3	**25**
Dummy	0	0	0	0	**5**
	30	**15**	**20**	**35**	

b)

	Barra	Copacabana	Deodoro	Maracanã
F1	5 **25**	4 **0**	2 **0**	5 **0**
F2	4 **5**	6 **10**	4 **0**	3 **0**
F3	4 **0**	5 **5**	5 **20**	4 **5**
F4	3 **0**	4 **0**	6 **0**	3 **25**
Dummy	0 **0**	0 **0**	0 **0**	0 **5**

Total Cost= 25*5+5*4+10*6+5*5+20*5 = €425 000

c)

	Barra	Copacabana	Deodoro	Maracanã	Ui
F1	5 -θ **25**	4 **0** -3	2 +θ **0** -5	5 **0** -1	U1=0*
F2	4 +θ **5**	6 -θ **10**	4 **0** -2	3 **0** -2	U2=-1
F3	4 **0** 1	5 +θ **5**	5 -θ **20**	4 **5**	U3=-2
F4	3 **0** 1	4 **0** 0	6 **0** 2	3 **25**	U4=-3
Dummy	0 **0** 1	0 **0** -1	0 **0** -1	0 **5**	U5=-6
Vj	V1=5	V2=7	V3=7	V4=6	

Θ = min {25, 10, 20} = 10

	Barra	**Copacabana**	**Deodoro**	**Maracanã**	**Ui**
F1	5 **15**	4 **0** -2	2 **10**	5 **0** -4	U1=0*
F2	4 **15**	6 **0** 5	4 **0** 3	3 **0** 3	U2=-1
F3	4 **0** 2	5 **15**	5 **10**	4 **5**	U3=3
F4	3 **0** 0	4 **0** 4	6 **0** 2	3 **25**	U4=2
Dummy	0 **0** -6	0 **0** -1	0 **0** -1	0 **5**	U5=-1
Vj	V1=5	V2=2	V3=2	V4=1	

Total Cost= 15*5+10*2+15*4+15*5+10*5+5*4+25*3 = €375 000

d) Cost reduction after the 1st iteration: €425.000 - €375.000 = €50.000

e)

	Barra	**Copaca**	**Deodoro**	**Maracanã**	**Maracanã**	**E1**	**E2**	
F1	∞	∞	∞	∞	∞	5	∞	**25**
F2	∞	∞	∞	∞	∞	4	3	**15**
F3	∞	∞	∞	∞	∞	∞	5	**30**
F4	∞	∞	∞	∞	∞	0	∞	**25**
E1	7	6	5	5	5	0	∞	**65**
E2	5.5	∞	6.5	3.5	3.5	∞	0	**55**
Dummy	0	0	0	∞	0	0	0	**5**
	30	**15**	**20**	**30**	**5**	**65**	**55**	

A clothing wants to open 3 new stores in the country. These stores can be supplied by 3 warehouses from the company. Due to internal reasons, the warehouse A cannot supply store 2 and warehouse B cannot supply store 1. The estimated demand of each store, the availability of each warehouse and the unitary transportation costs are described in Table 1. Consider a unitary cost of 35 for unsatisfied demand.

		Stores			
		1	2	3	Offer
Warehouses	A	15	-	25	**50**
	B	-	20	30	**65**
	C	20	15	10	**40**
	Demand	**55**	**45**	**70**	

a) Formulate the problem of stores supply as a transportation problem.
b) Find a basic feasible solution using the minimum cost method. Indicate the cost of the solution found.
c) Is the solution found previously optimal? Justify. If is not optimal indicate what would be the next step.
d) Consider that the company warehouses are used only as transshipment points and that they can receive apparel from 2 factories. Table 2 indicate the availability of each factory and the unitary transportation costs between the factories and warehouses. Formulate the transshipment problem as a transportation problem. (Consider that the data previously provided is the same)

		Warehouses			
		A	B	C	Offer
Factories	F1	45	20	30	**85**
	F2	20	15	35	**70**

SOLUTION

a)

	Store 1	Store 2	Store 3	
A	15	∞	25	**50**
B	∞	20	30	**65**
C	20	15	10	**40**
Dummy	35	35	35	**15**
	55	**45**	**70**	

b)

	Store 1	Store 2	Store 3	
A	15 **50**	∞ **0**	25 **0**	**50**
B	∞ **0**	20 **45**	30 **20**	**65**
C	20 **0**	15 **0**	10 **40**	**40**
Dummy	35 **5**	35 **0**	35 **10**	**15**
	55	**45**	**70**	

Total cost=50*15+45*20+20*30+40*10+5*35+10*35=3175

c)

	Store 1	Store 2	Store 3	Ui
A	15 **50**	∞ **0** ∞	25 **0** 10	**U1=0***
B	∞ **0** ∞	20 **45**	30 **20**	**U2=15**
C	20 **0** 10	15 **0** 15	10 **40**	**U3= -5**
Dummy	35 **5**	35 **0** 10	35 **10**	**U4=20**
Vi	**V1=15**	**V2=5**	**V3=15**	

This is the optimal solution. All Δij ≥ 0.

d)

	Store 1	Store 2	Store 3	A	B	C	
A	15	∞	25	0	∞	∞	**50**
B	∞	20	30	∞	0	∞	**65**
C	20	15	10	∞	∞	0	**40**
F1	∞	∞	∞	45	20	30	**85**
F2	∞	∞	∞	20	15	35	**70**
Dummy	35	35	35	∞	∞	∞	**15**
	55	**45**	**70**	**50**	**65**	**40**	

The XPTO Company, known for its luxury cars, has 3 warehouses and 5 stands in the northern part of the country. Every week a vehicle transfer plan is defined between the warehouses and the stands in order to minimize the total costs of the operation.

In order to define next week's plan, it is known that warehouses A, B and C have 30, 50, 20 vehicles for transfer respectively, and that stands 1, 2, 3, 4 and 5 are intended to receive 10, 15 , 30, 10, 25 vehicles, respectively. If warehouses A, B and C do not transfer all the vehicles they will have a cost of stock per vehicle of 3u.m, 5u.m. and 4u.m, respectively.

The table below shows the unit transport costs between warehouses and stands.

		Stands				
		1	**2**	**3**	**4**	**5**
Warehouses	**A**	5	2	7	9	2
	B	4	6	3	3	8
	C	2	8	5	4	5

a) Formulate this problem as a transportation problem.

b) Find a feasible basic solution by the northwest corner method. Please indicate the cost of this solution.

c) Is the solution found in the previous paragraph optimal? Justify. If it is not optimal, indicate which variables to enter and leave the database at the next iteration.

d) Now consider that warehouses A, B and C are intermediate points between the factories of the vehicles (F1 and F2) and the stands. It is known that F1 and F2 produced 50 and 60 vehicles, respectively, and that the cost of stock per vehicle in the factories is 10u.m.
The unitary transportation costs between the factories and the warehouses are indicated in the table below. Given the proximity of the factory F1 to the stand 2 this can directly send the vehicles to this stand, without going through a warehouse, with a cost of 2u.m. (Consider the demand for the stands, the stock costs in warehouses and the transportation costs between warehouses and stands the same as the previous ones). Formulate the transhipment problem as a transportation problem.

		Intermediate		
		A	**B**	**C**
Factories	**F1**	5	10	7
	F2	12	6	9

e) The XPTO Company is developing three new car models (XY1, XY2 and XY3). A prototype of each model was produced, with the exception of the XY3 model, for which two prototypes were produced. The company intends to exhibit these prototypes on its stands, but no more than one prototype per stand. After conducting market studies, the following table indicates the level of exposure that each model should have at each stand. However, due to internal constraints, Stand 4 must have one of the models and model XY2 cannot be exhibited in stands 3 and 5.

		Stands				
		1	**2**	**3**	**4**	**5**
Models	**XY1**	10	5	7	12	9
	XY2	6	11	5	8	4
	XY3	5	9	12	8	3

e1. Formulate the problem as an assignment problem.

e2. Find the optimal assignment in order to maximise the exposition of the prototypes. Indicate the maximum level of exposure that can be achieved.

SOLUTION

a)

		Stands						
		1	**2**	**3**	**4**	**5**	**Stock**	**Offer**
Warehouses	**A**	5	2	7	9	2	3	30
	B	4	6	3	3	8	5	50
	C	2	8	5	4	5	4	20
	Demand	10	15	30	10	25	10	100

b)

		Stands						
		1	**2**	**3**	**4**	**5**	**Stock**	**Offer**
Warehouses	**A**	10	15	5	0	0	0	30
	B	0	0	25	10	15	0	50
	C	0	0	0	0	10	10	20
	Demand	10	15	30	10	25	10	

Cost=10*5+15*2+5*7+25*3+10*3+15*8+10*5+10*4=430 c)

		Stands							
		1	2	3	4	5	Stock	Offer	u
Warehouses	A	5 10	2 15	7 5-θ	9 0 2	2 0+θ -10	3 0 -8	30	0*
	B	4 0 3	6 0 8	3 25+θ	3 10	8 15-θ	5 0 -2	50	-4
	C	2 0 4	8 0 13	5 0 5	4 0 4	5 10	4 10	20	-7
	Demand	10	15	30	10	25	10		
	v	5	2	7	7	12	11		

The solution is not optimal because there are some variables with Δij <0.

The entering variable is $X_{A.5}$, with $\theta = \min\{5,15\} = 5$, and the leaving variable is $X_{A.3.}$

d)

		Stands					Warehouses				
		1	2	3	4	5	A	B	C	Stock	Offer
Factories	F1	-	2	-	-	-	5	10	7	10	50
	F2	-	-	-	-	-	12	6	9	10	60
Warehouses	A	5	2	7	9	2	0	-	-	3	90
	B	4	6	3	3	8	-	0	-	5	90
	C	2	8	5	4	5	-	-	0	4	90
	Demand	10	15	30	10	25	90	90	90	20	380

e.1)

		Stands				
		1	2	3	4	5
Models	XY1	10	5	7	12	9
	XY2	6	11	-	11	-
	XY3	5	9	10	8	5
	XY3	5	9	10	8	5
Unfilled demand		0	0	0	-	0

e.2)

		Stands				
		1	2	3	4	5
Modelos	XY1	2	7	5	**0**	3
	XY1	5	**0**	-	0	-
	XY2	**0**	3	2	4	0
	XY2	0	3	2	4	**0**
Não recebe		0	0	**0**	-	0

Exposure level=12+11+5+0+5=33