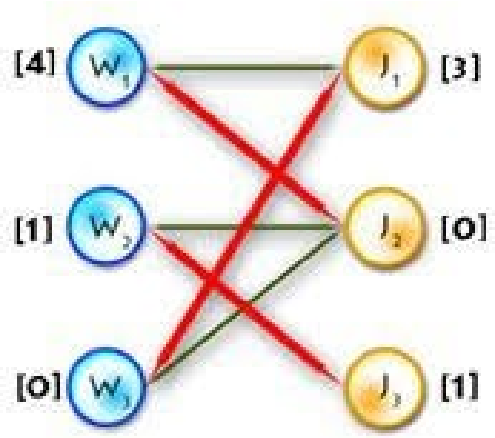

OPERATIONS RESEARCH

Faculdade de Engenharia da Universidade do Porto

ASSIGNMENT PROBLEMS

Mestrado em Engenharia Informática e Computação

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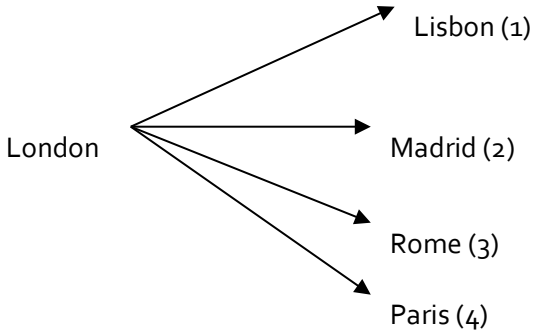
EXERCISE 1 – AIRLINE COMPANY

A British airline company intends to organize trips to 4 distinct cities during Easter that correspond to the points where the largest number of tourists depart to visit London. Schematically, we will have:



For this purpose, the company has four different airplanes in which the number of seats and the number of crew members vary, as well as the consumption of each airplane.
Hence, we will have different costs for each type of airplane and for each trip. The costs are given in the following table:

		Planes			
		1	**2**	**3**	**4**
Cities	**1**	48	50	44	46
	2	56	60	68	60
	3	40	48	50	52
	4	95	85	90	84

Find the optimal assignment (minimum cost) of the planes to the cities.

SOLUTION:

	1	**2**	**3**	**4**
1	48	50	44	46
2	56	60	68	60
3	40	48	50	52
4	95	85	90	84

4	6	0	2
0	4	12	4
0	8	10	12
11	1	6	0

Subtract the smallest element of each row

4	5	0	2
0	3	12	4
0	7	10	12
11	0	6	0

Smallest uncut element: 3

7	5	0	2
0	0	9	1
0	4	7	9
14	0	6	0

Minimum number of lines: 4 (= dimension of the matrix)

The solution is optimal with cost = 40+60+44+84=228

EXERCISE 2 – MARKET RESEARCH

A research company received market research requests from 4 clients. The company has to allocate researchers to the 4 new market studies for which they have 4 researchers. However, the time to complete each study depends on the experience of the researcher assigned to the project. The times needed by each individual to complete each market study are shown in the following table:

		Clients			
		A	**B**	**C**	**D**
Researchers	**1**	3	6	5	7
	2	3	7	2	4
	3	4	3	4	5
	4	6	4	5	6

a) Knowing that the company intends to minimize the total number of days to complete the 4 market studies, which should be the assignment plan?

b) Suppose now that the studies are done simultaneously. Define the assignment plan allowing the studies to be completed as soon as possible.

SOLUTION

a)

	A	**B**	**C**	**D**
1	3	6	5	7
2	3	7	2	4
3	4	3	4	5
4	6	4	5	6

0	3	2	4
1	5	0	2
1	0	1	2
2	0	1	2

0	3	2	2
1	5	0	0
1	0	1	0
2	0	1	0

Or

0	3	2	2
1	5	0	0
1	0	1	0
2	0	1	0

N.º of days =3+3+2+6=3+4+2+5=14

b) Bottleneck Assignment: We choose any assignment, for example, the optimal solution obtained in the previous question.

	A	B	C	D
1	3	6	5	7
2	3	7	2	4
3	4	3	4	5
4	6	4	5	6

TMax = 6

0	1	0	1
0	1	0	0
0	0	0	0
1	0	0	1

Tmax=5

0	1	1	1
0	1	0	0
0	0	0	1
1	0	1	1

Tmax = 4

0	1	1	1
0	1	0	1
1	0	1	1
1	1	1	1

Optimal solution: nr. of lines inferior than the dimension of the matrix

EXERCISE 3 – ASSIGNING WORKERS TO MACHINES

A company with 5 workers and 5 machines, intends to distribute the workers by the machines in order to maximize the total productivity. Knowing that the daily productivity of each worker on each machine is represented in the following table, what should be the assignment plan?

		Workers				
		A	**B**	**C**	**D**	**E**
Machines	**1**	6	4	3	10	5
	2	4	2	2	5	6
	3	11	4	6	8	7
	4	5	7	3	4	4
	5	0	2	1	0	4

SOLUTION

Transform the maximization problem to a minimization problem. The highest element in the matrix is 11. Replace all elements in the matrix a_{ij} by $(11-a_{ij})$.

5	7	8	1	6
7	9	9	6	5
0	7	5	3	4
6	4	8	7	7
11	9	10	11	7

Solving the minimization problem

4	6	7	0	5
2	7	7	1	0
0	7	5	3	4
2	0	4	3	3
4	2	3	4	0

Number of lines less than the dimension of the matrix.
The solution is not optimal.
Subtract the smallest element of each column.

4	6	4	**0**	5
2	7	4	1	**0**
0	7	2	3	4
2	**0**	1	3	3
4	2	**0**	4	0

Optimal solution

EXERCISE 4 – JOB APPLICANTS

Seven candidates applied for seven jobs. In the following matrix are the competences of each candidate to the place (0- approved; 1 - not approved). Is it possible to fill all the places with suitable candidates for them? If so, determine how to do it.

0	1	1	0	1	1	0
1	0	0	0	1	1	1
1	0	1	1	0	1	1
1	1	1	0	0	0	1
0	0	1	1	1	0	1
0	1	0	1	0	1	1
1	1	0	1	1	0	0

SOLUTION

0	1	1	0	1	1	0
1	**0**	0	0	1	1	1
1	0	1	1	**0**	1	1
1	1	1	**0**	0	0	1
0	0	1	1	1	**0**	1
0	1	**0**	1	0	1	1
1	1	0	1	1	0	**0**

EXERCISE 5 – LOCATIONS FOR MACHINES

A factory has four locations (1,2,3,4) to receive three new machines (A, B, C). The location 4 is too small to contain the machine A.

The costs of handling the materials being processed on the machines for each location (in hundreds of euros per hour) are as follows::

	1	**2**	**3**	**4**
A	5	1	3	-
B	3	1	4	3
C	3	3	4	2

It is intended to determine which location must occupy each of the new machines, in order to minimize the total cost of material handling.

SOLUTION

	1	**2**	**3**	**4**
A	5	1	3	∞
B	3	1	4	3
C	3	3	4	2
*****	0	0	0	0

4	0	2	∞
2	0	3	2
1	1	2	0
0	0	0	0

tmin = 2

2	0	**0**	∞
0	**0**	1	0
1	3	2	**0**
0	2	0	0

Or

2	**0**	0	∞
0	0	1	0
1	3	2	**0**
0	2	**0**	0

Total cost= 1+3+2=6

EXERCISE 6 – EXTRAWING AIRLINE COMPANY

The ExtraWing airline company bought 3 new small planes. After a market study, 4 possible destinations for the new flights were identified: Monte Carlo, Canary Islands, Biarritz, and the Greek Isles. For each destination, the profit (in Millions of euros) of each airplane would was estimated:

Destination	A1	A2	A3
Monte Carlo	8	11	10
Canary Islands	10	9	9
Biarritz	9	4	8
Greek Isles	6	7	5

During a meeting, the manager of the ExtraWing (who owns an apartment in Biarritz) decided that Biarritz would necessarily be the destination of one of the three planes.

On the other hand, the Marketing Director considered that, as a matter of strategy, one should target as many destinations as possible, thus not sending more than one plane to each destination.
The Maintenance director drew attention to the fact that A1 and A3 aircrafts were not able to land on the Greek Isles. Decide which plane to assign to each destination.

SOLUTION

Destination	A1	A2	A3	*
Monte Carlo	8	11	10	0
Canary Islands	10	9	9	0
Biarritz	9	4	8	$-\infty$
Greek Isles	$-\infty$	7	$-\infty$	0

Transform the maximization problem into a minimization one. The highest element in the matrix is 11:

3	0	1	11
1	2	2	11
2	7	3	∞
∞	4	∞	11

3	0	1	11
0	1	1	10
0	5	1	∞
∞	0	∞	7

3	**0**	0	4
0	1	0	3
0	5	**0**	∞
∞	0	∞	**0**

or

3	**0**	0	4
0	1	**0**	3
0	5	0	∞
∞	0	∞	**0**

Profit=10+11+8=29

EXERCISE 7 – APPLIANCE FACTORY

An appliance factory has 6 employees (A, B, C, D, E and F) and 5 machines (M1, M2, M3, M4 and M5). The insufficient qualification of employees E, A and C does not allow them to work with machines M2, M3 and M4, respectively.

The Production Manager intends to define a man-machine assignment plan that minimizes the total cost of production. The assignment costs (in euros) are as follows:

Employee \ Machine	M1	M2	M3	M4	M5
A	8	7	-	10	6
B	7	12	9	6	8
C	5	7	7	-	7
D	10	10	8	5	4
E	13	-	8	8	9
F	6	9	10	9	6

Find the optimal assignment plan and comment the solution.

SOLUTION

	M1	M2	M3	M4	M5	*
A	8	7	∞	10	6	0
B	7	12	9	6	8	0
C	5	7	7	∞	7	0
D	10	10	8	5	4	0
E	13	∞	8	8	9	0
F	6	9	10	9	6	0

3	0	∞	5	2	0
2	5	2	1	4	0
~~0~~	~~0~~	~~0~~	~~∞~~	~~3~~	~~0~~
~~5~~	~~3~~	~~1~~	~~0~~	~~0~~	~~0~~
8	∞	1	3	5	0
1	2	3	4	2	0

Subtract the minimum element of each column. Nr. of lines less than the dimension of the matrix; smallest uncut element = 1

2	**0**	∞	4	1	0
1	5	1	**0**	3	0
0	1	0	∞	3	1
5	4	1	0	**0**	1
7	∞	**0**	2	4	0
0	2	2	3	1	**0**

Optimal assignment cost: 7+6+5+4+8+0=30

EXERCISE 8: LUSO BANK

The CEO of Luso Bank intends to launch 5 new financial products, P1, P2, P3, P4 and P5. For that purpose, he asked the Commercial Department to develop an advertising campaign for these products.
The Marketing Director asked the 7 main media companies operating in the Portuguese market and requested budgets for audiovisual ads for the different products.
He received the following proposals, in thousand euros per second of advertising:

	E1	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**
P1	35	29	36	28	27	34	36
P2	28	30	32	29	38	33	40
P3	30	24	25	21	34	33	24
P4	40	28	20	21	38	27	30
P5	32	34	32	30	27	29	30

Considering that the Marketing Director of Luso Bank wants to advertise each financial product in one single media company, what is the best choice in order to minimize the total cost of the advertising campaign?

SOLUTION

	E1	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**
P1	35	29	36	28	27	34	36
P2	28	30	32	29	38	33	40
P3	30	24	25	21	34	33	24
P4	40	28	20	21	38	27	30
P5	32	34	32	30	27	29	30
*	0	0	0	0	0	0	0
**	0	0	0	0	0	0	0

8	2	9	1	0	7	9
0	2	4	1	10	5	12
9	3	4	0	13	12	3
20	8	0	1	18	7	10
5	7	5	3	0	2	3
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Smallest uncut element: 1

8	1	9	0	0	6	8
0	1	4	0	10	4	11
10	3	5	0	14	12	3
20	7	0	0	18	6	9
5	6	5	2	0	1	2
1	0	1	0	1	0	0
1	0	1	0	1	0	0

Smallest uncut element: 1

7	**0**	8	0	0	5	7
0	1	4	1	11	4	11
9	2	4	**0**	14	11	2
20	7	**0**	1	19	6	9
4	5	4	2	**0**	0	1
1	0	1	1	2	**0**	0
1	0	1	1	2	0	**0**

Optimal assignment cost = 29+28+20+21+27=125

EXERCISE 9 - 24 HOURS OF KARTING

In the race "24 hours of Karting for Universities and Polytechnics", one of the sponsors of the competition intends to pay the registration of the 5 most competitive teams in the tests timed with the 5 karts available. On the day of the selection test, 6 groups were present and timed tests were performed with the elements of each group, using each of the 5 karts. The best times of each team with each kart were as follows (in minutes):.

	Kart1	**Kart 2**	**Kart 3**	**Kart 4**	**Kart 5**
Grupo1	12	13	10	13	12
Grupo2	10	11	-	9	13
Grupo3	16	12	14	12	12
Grupo4	16	15	10	15	11
Grupo5	15	11	11	18	17
Grupo6	13	14	13	12	16

a) Find the optimal assignment plan in terms of global competitive performance.

b) Find the optimal assignment plan that minimizes the worst time.

SOLUTION

a)

12	13	10	13	12	0
10	11	∞	9	13	0
16	12	14	12	12	0
16	15	10	15	11	0
15	11	11	18	17	0
13	14	13	12	16	0

2	2	0	4	1	0
0	0	∞	0	2	0
6	1	4	3	1	0
6	4	0	6	0	0
5	0	1	9	6	0
3	3	3	3	5	0

Smallest uncut element: 1

1	2	0	3	0	0
0	1	∞	0	2	1
5	1	4	2	0	0
6	5	1	6	0	1
4	0	1	8	5	0
2	3	3	2	4	0

Smallest uncut element: 1

0	2	0	2	0	0
0	2	∞	0	3	2
4	1	4	1	0	0
5	5	1	5	0	1
3	0	1	7	5	0
1	3	3	1	4	0

Smallest uncut element: 1

0	3	0	2	1	1
0	3	∞	**0**	4	3
3	1	3	0	**0**	0
4	5	**0**	4	0	1
2	**0**	0	6	5	0
0	3	2	0	4	**0**

Optimal time =12+11+10+9+12=54

b) Bottleneck Assignment Problem
Choose any assignment, for example:

12	13	10	13	12	0
10	11	∞	**9**	13	0
16	12	14	12	**12**	0
16	15	**10**	15	11	0
15	**11**	11	18	17	0
13	14	13	12	16	**0**

1	1	0	1	1	0
~~0~~	~~0~~	~~1~~	~~0~~	~~1~~	~~0~~
1	1	1	1	1	0
~~1~~	~~1~~	~~0~~	~~1~~	~~0~~	~~0~~
~~1~~	~~0~~	~~0~~	~~1~~	~~1~~	~~0~~
1	1	1	1	1	0

TMAX = 12
The solution is optimal because the number of lines is inferior than the dimension of the matrix.

EXERCÍCIO 10 - POSITIONS IN THE UNIVERSITY

A certain university accepts the enrollment of 4 more students to fill 2 positions in the Chemistry course, 1 in the Mathematics course and 1 in the Management course. These courses have the following minimum marks: 13, 14 and 15, respectively.

The marks presented by the 5 candidate students, as well as their preferences (on a scale of 0 to 5, where 5 represents the maximum preference) are as follows:

	A1	A2	A3	A4	A5
Student Average marks	14	15	16	13	14
Chemistry	3	5	4	3	0
Mathematics	5	2	0	5	1
Management	4	1	5	2	5

Find the distribution of the candidates to the courses in order to maximize overall satisfaction.

	A1	**A2**	**A3**	**A4**	**A5**
Chemistry 1	3	5	4	3	0
Chemistry 2	3	5	4	3	0
Mathematics	5	2	0	-∞	1
Management	-∞	1	5	-∞	-∞
Not admitted	0	0	0	0	0

SOLUÇÃO

2	0	1	2	5
2	0	1	2	5
0	3	5	∞	4
∞	4	0	∞	∞
5	5	5	5	5

2	0	1	2	5
2	0	1	2	5
0	3	5	∞	4
∞	4	0	∞	∞
0	0	0	0	0

Smallest uncut element: 1

1	0	0	1	4
1	0	0	1	4
0	4	5	∞	4
∞	5	0	∞	∞
0	1	0	0	0

Smallest uncut element: 1

0	0	0	**0**	3
0	**0**	0	0	3
0	5	6	∞	4
∞	5	**0**	∞	∞
0	2	1	0	**0**

Student 1 – Mathematics
Student 2 – Chemistry 1
Student 3 - Mnagement
Student 4 – Chemistry 2
Student 5 – Not admitted

EXERCISE 11 – MEDLEY RACE

The 4×100 meters medley relay is a medley race in which each of four swimmers on a team swims a 100-metre leg of the relay, each swimming a different stroke, in the following sequence: Backstroke, Breaststroke, Butterfly and Freestyle.

Mr. José Piscinas, the coach of the Mar Alto Club, has 6 good swimmers whose usual times in the 100 meters are as follows (the * symbol indicates that the respective swimmer does not practice the style).

	Backstroke	**Breaststroke**	**Butterfly**	**Freestyle**
Swimmer 1	65	73	63	57
Swimmer 2	67	70	65	57
Swimmer 3	68	*	70	59
Swimmer 4	71	69	*	57
Swimmer 5	69	71	66	59

a) How should the trainer assign swimmers to the styles in order to minimize the total time?

b) Suppose that, in order to promote the regularity of the teams, the organization of the competition decided to give an award to the team that, in the worst of its tests made the best time. How to assign the swimmers in this case?

SOLUTION

a)

65	73	63	57	0
67	70	65	57	0
68	∞	70	59	0
71	69	∞	57	0
69	71	66	59	0

0	4	0	0	0
2	1	2	0	0
3	∞	7	2	0
6	0	∞	0	0
4	2	3	2	0

Smallest uncut element: 2

0	6	0	2	2
0	1	0	0	0
1	∞	5	2	0
6	0	∞	0	0
2	2	1	2	0

Smallest uncut element: 1

0	6	0	2	3
0	1	0	**0**	1
0	∞	4	1	**0**
6	**0**	∞	0	1
1	1	**0**	1	0

b) Bottleneck Assignment Problem

65	73	63	57	0
67	70	65	**57**	0
68	∞	70	59	**0**
71	**69**	∞	57	0
69	71	66	59	0

0	1	0	0	0
0	1	0	0	0
0	∞	1	0	0
1	1	∞	0	0
1	1	0	0	0

TMAX = 69

This solution is also optimal for the bottleneck assignment problem.

EXERCISE 12 - FELIZ ENTERRO DO MARIALVA

The marital agency "*Feliz Enterro do Marialva* (FEM)" disclosed to four customers classified information about five young boys that appeared in their files. The ladies (the customers) had no difficulty in classifying

their preferences on a scale of 1 to 5 (5 corresponds to the maximum preference). The agency detected physiological incompatibilities that prevented the consummation of two marriages, indicated in the table with a *.

	Marialva 1	**Marialva 2**	**Marialva 3**	**Marialva 4**	**Marialva 5**
Lady 1	5	1	3	2	4
Lady 2	*	4	3	5	2
Lady 3	5	2	3	4	1
Lady 4	4	5	*	3	2

a) Help FEM to select the 4 couples that maximize the overall satisfaction of ladies' preferences.

b) Which other criteria could be reasonably applied? Would the result be different?

SOLUTION

a)

From maximization to minimization – (displeasure)

0	4	2	3	1
∞	1	2	0	3
0	3	2	1	4
1	0	∞	2	3

Balancing the problem

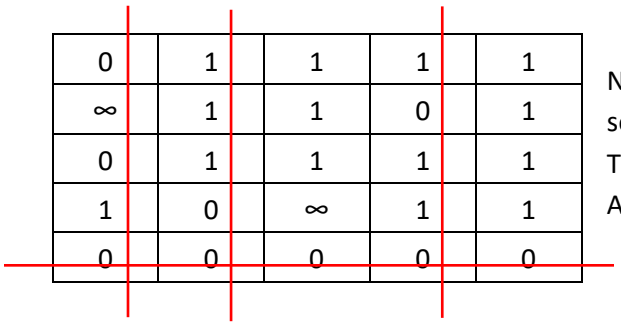
0	4	2	3	1
∞	1	2	0	3
0	3	2	1	4
1	0	∞	2	3
0	0	0	0	0

0	4	2	3	1
∞	1	2	0	3
0	3	2	1	4
1	0	∞	2	3
0	0	0	0	0

0	3	1	3	**0**
∞	0	1	**0**	2
0	2	1	1	3
2	**0**	∞	3	3
1	0	**0**	1	0

min=1

b) A different criterion would be the minimization of the maximum displeasure, as the Bottleneck Assignment problem. Using for example, the optimal solution of the previous problem, with maximum displeasure = 1:



Nº lines inferior to the dimension of the matrix, so it is not possible to find a zero cost assignment. The solution is optimal for the bottleneck Assignment problem.

EXERCISE 13 –CONSRAPID COMPANY

The manager of the construction company, Consrapid SA, has to deliver a work for which he is responsible in a very short time. Since Consrapid SA does not have the manpower or equipment needed to complete the work, the manager requested 4 different companies to submit proposals for the missing works. The following table shows the average forecast time, in days, for the completion of the work, provided by the possible companies to be subcontracted. The symbol * indicates that company C has no painters qualified for the intended service.

	Painting	Varnishing	Cleaning
Company A	3	4	6
Company B	5	8	5
Company C	*	6	6
Company D	7	5	4

The companies' proposals present the following additional information: Companies A, B and C can only perform one task each, and company D can perform two tasks.

a) The manager of Consrapid SA intends to minimize the total time associated with carrying out the different works. Determine the optimum assignment plan and the minimum number of days required to complete the work.

b) The manager of Consrapid SA received a new proposal from a company that proposes to carry out the works simultaneously, with a duration of 4 days. Assuming that companies A, B, C, and D can also work simultaneously, say whether or not the manager should accept the new proposal. Justify your answer by using an appropriate procedure.

c) The companies also provided the company manager Consrapid SA with information on the average number of employees required to perform each task. The following table presents this information. Knowing that the manager of Consrapid SA intends to maximize the productivity of the employees who work in his work, say if the plan of affectation determined in a) remains optimal. (Note: if you did not solve point a) determine the optimal assignment plan for this problem.

	Painting	Varnishing	Cleaning
Company A	1	2	5
Company B	5	2	4
Company C	*	3	3
Company D	5	4	1

SOLUTION

a)

	Painting	Varnishing	Cleaning	Task1	Task2
Company A	3	4	6	0	0
Company B	5	8	5	0	0
Company C	*	6	6	0	0
Company D	7	5	4	0	0
Company D	7	5	4	0	0

0	0	2	0	0
2	4	1	0	0
*	2	2	0	0
4	1	0	0	0
4	1	0	0	0

0	0	3	1	1
1	3	1	**0**	0
*	1	2	0	**0**
3	0	**0**	0	0
3	**0**	0	0	0

Optimal solution

Nº lines (4) < dimension (5)

Minimum uncut element=1

Optimal solution:

Company A: Paints; Company B: excluded; Company C: excluded; Company D: Varnishes and cleans

Minimum total time: 3+4+5=12 days

b) Bottleneck Assignment Problem

Using the optimal solution of the previous problem, the maximum time is 5 days.

0	0	1	0	0
1	1	1	0	0
*	1	1	0	0
1	1	0	0	0
1	1	0	0	0

Nº lines (4) < matrix dimension (5). The solution is optimal

c) A possible performance measure is productivity, calculated as (average number of days / average number of employees)

We want to maximize productivity.

	Painting	Varnishing	Cleaning	Task1	Task2
Company A	3/1=3	4/2=2	6/5=1.2	0	0
Company B	5/5=1	8/2=4	5/4=1.25	0	0
Company C	-∞	6/3=2	6/3=2	0	0
Company D	7/5=1.4	5/4=1.25	4/1=4	0	0
Company D	7/5=1.4	5/4=1.25	4/1=4	0	0

Here, Cmax = 4.

1	2	2.8	4	4
3	0	2.75	4	4
∞	2	2	4	4
2.6	2.75	0	4	4
2.6	2.75	0	4	4

0	1	1.8	3	3
3	0	2.75	4	4
∞	0	0	2	2
2.6	2.75	0	4	4
2.6	2.75	0	4	4

0	1	1.8	1	1
3	0	2.75	2	2
∞	0	0	0	0
2.6	2.75	0	2	2
2.6	2.75	0	2	2

Min uncut element = 1

0	1	1.8	0	0
3	0	2.75	1	1
∞	1	1	0	0
2.6	2.75	0	1	1
2.6	2.75	0	1	1

Min uncut element = 1

0	2	2.8	0	0
2	**0**	2.75	0	0
∞	2	2	**0**	0
1.6	2.75	0	0	**0**
1.6	2.75	**0**	0	0

Optimal solution: Company A: painting; Company B: Varnishing: Company C: excluded; Company D: cleaning.

Total time: 11 days/employee.

This solution is different from b)

EXERCISE 14 – HIRING EMPLOYEES

A company is recruiting staff to perform certain functions. The owner of the company intends to assign the functions according to the preferences of the candidates. In the recruitment interview the candidates were asked to rate the preferences for the different types of functions on a scale of 1 to 5 (5 represents the maximum preference). The table below summarizes the information collected.

	Courier	Telephone operator	Receptionist	Doorman	Supervisor
Candidate A	5	2	3	-	4
Candidate B	1	5	4	3	2
Candidate C	4	5	3	2	1
Candidate D	3	1	5	4	2
Candidate E	1	3	2	5	4

a) Determine the optimal assignment plan in order to maximize the overall satisfaction of employee preferences.

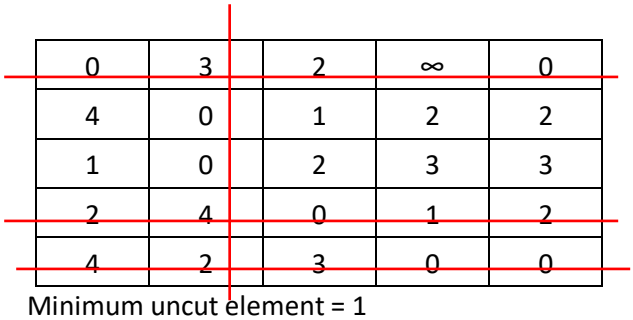
b) At the time of the selection of candidates, the company owner also had the following information in mind:
- Officer C does not have the necessary qualifications to perform Supervisor duties;
- The function of Receiver has already been assigned to an internal employee of the company;
- Employee D must necessarily be assigned a task because he has the best curriculum.

Formulate the minimization problem based on this information.

SOLUTION

Transform the maximization problem into a minimization problem (cij mas = 5)

0	3	2	∞	1
4	0	1	2	3
1	0	2	3	4
2	4	0	1	3
4	2	3	0	1



Minimum uncut element = 1

0	4	2	∞	**0**
3	**0**	0	1	1
0	0	1	2	2
2	5	**0**	1	2
4	3	3	**0**	0

Optimal solution: A: Supervisor; B: Telephone operator; C: Courier; D: Receptionist; E: Doorman

Maximum total satisfaction: 4+5+5+5+4=23

b)

Maximization problem

	Courier	Telephone operator	Receptionist	Doorman	Supervisor
Candidate A	5	2	-∞	4	0
Candidate B	1	5	3	2	0
Candidate C	4	5	2	-∞	0
Candidate D	3	1	4	2	-∞
Candidate E	1	3	5	4	0

Corresponding minimization problem (cij max = 5)

	Courier	Telephone operator	Receptionist	Doorman	Supervisor
Candidate A	5	2	-∞	4	0
Candidate B	1	5	3	2	0
Candidate C	4	5	2	-∞	0
Candidate D	3	1	4	2	-∞
Candidate E	1	3	5	4	0

EXERCISE 15 –DRIVE-AND-FLY GOLF CLUB

The Drive-and-Fly golf club is hosting a peer tournament for next weekend.
Contrary to normal, management decided that the pair would be made by the club, rather than the players themselves being organized. In this sense and since the direction counts with a great affluence, the coordinator of tournament planning was asked to automate the process of constitution of the pairs of players. In the act of inscription each player has to disclose his handicap. The handicap is a numerical measure that shows the ability of each player. The higher the handicap, the worse this player is in relation to players with a lower handicap. To validate the planning the management decided to use the following 5 players (note that there is a player that will be left out):

Player	1	2	3	4	5
Handicap	19	1	5	24	8

a) Formulate the problem of finding the participating pairs as an assignment problem, where the goal is to minimize the greater of the absolute differences between the handicaps within a pair. (Example: Player 1 - Player 2 has a difference of 18 in handicap).
b) Solve the problem you have just formulated and identify the pairs formed as well as the value of the solution found. To solve this problem consider the pairs (Player 1 / Player 2, Player 3 / Player 4 and Player 5 outside the tournament) as the initial solution.
c) How would you change the table you used to formulate the problem in paragraph a) so that the player with the lowest handicap was always chosen to participate in the tournament.

SOLUTION

a) and b) You just need to calculate the difference between potential pairs. It's a bottleneck assignment problem. The initial solution is marked in red.

	1	2	3	4	5	*
1	-	**18**	14	5	11	0
2	**18**	-	4	23	7	0
3	14	4	-	**19**	3	0
4	5	23	**19**	-	16	0
5	11	7	3	16	-	**0**
*	0	0	0	0	**0**	-

Tmax = 19

	1	2	3	4	5	*
1	-	0	0	**0**	0	0
2	0	-	**0**	1	0	0
3	0	0	-	1	**0**	0
4	**0**	0	1	-	0	0
5	0	0	0	0	-	**0**
*	0	**0**	0	0	0	-

Nr of line is equal to the dimension of the matrix (not represented). There exists an assignemt with cost zero. In this case, Tmax = 5

	1	2	3	4	5	*
1	-	1	1	1	1	0
2	1	-	0	1	1	0
3	1	0	-	1	0	0
4	1	1	1	-	1	0
5	1	1	0	1	-	0
*	0	0	0	0	0	-

Optimal solution:

Pair 1-4 was selected.

Then we could choose pair 3-5 leaving out player 2 or we could choose pair 2-3 leaving out player 5.

b) We just need to put infinite in cells (2,*) e (*,2)

EXERCISE 16 – GUITAR CONCERTS

The G3 is an annual concert series featuring the world's top 3 guitarists. Each concert consists of three parts, one for each guitarist. For the selection of the musicians, the candidates have to show their virtuosity in the rehearsal of the 3 parts. The following table illustrates the quality of the performance of each of the pre-selected musicians in each part, evaluated on a scale from 0 (worst) to 100 (best).

	Part 1	**Part 2**	**Part 3**
J. Petrucci	85	45	85
S. Vai	92	90	66
Z. Wylde	80	60	70
A. Laiho	90	70	85
R. Cooley	83	78	92

a) Knowing that the G3 organization wants to give the best possible performance, indicate which musician will play in each part of the show of this edition of the G3 tour, justifying properly with the calculations to be made.

b) Due to overwork on his solo project, musician R. Cooley will not be able to play part 3. To compensate for this problem, both S. Vai and J. Petrucci are available to play two parts in the Tour, if necessary. Formulate this problem as an assignment problem.

SOLUTION

a) Transform the maximization problem into a minimization problem (max Cij = 92)

7	47	7	0	0
0	2	26	0	0
12	32	22	0	0
2	22	7	0	0
9	14	0	0	0

7	47	7	0	0
0	2	26	0	0
12	32	22	0	0
2	22	7	0	0
9	14	0	0	0

The solution is not optimal because the nr. of lines (4) is inferior than the dimension of the matriz (5).

Subtract the minimum element of each column.

7	45	7	0	0
0	0	26	0	0
12	30	22	0	0
2	20	7	0	0
9	12	0	0	0

The solution is not optimal because the nr. of lines (4) is inferior than the dimension of the matriz (5)

Minimum uncut element = 2

5	43	5	**0**	0
0	**0**	26	2	2
10	28	20	0	**0**
0	18	5	0	0
9	12	**0**	2	2

Optimal solution

EXERCISE 17 – TRUE OR FALSE

Mark the following statements as TRUE or FALSE (note that a penalization will be given to wrong answers):

1. Applying the transportation algorithm to an assignment problem that not guarantee that we will obtain the optimal solution.
2. The Hungarian method is not suitable to deal with the degeneracy of the assignment problem.
3. If one wants to minimize the average assignment cost, it is possible to use the same formulation and, therefore, use the same solution method that is used to minimize the total assignment cost.
4. In an assignment problem it is necessary to have a constraint forcing that all decision variable are in the {0,1} set.
5. In most cases, when it is necessary to add a set of lines or columns to make the matrix square, we should add a very large unitary cost to these fictitious lines/columns.
6. To solve a problem where we aim at maximizing the profit of the assignment with the Hungarian method, it is necessary to find the symmetric of the profits and add the largest unitary profit. These operations will convert the problem into a minimization one.
7. Add a constant to all costs of assigning a given resource does not change the optimal solution.
8. It is possible that the same assignment matrix give the same solution for the classic assignment problem and for the bottleneck assignment problem.
9. In some cases it is possible to obtain a better solution for an assignment problem by using the simplex method instead of using the Hungarian method.
10. Solving an assignment problem with the simplex method implies the utilization of the simplex of two phases.

SOLUTION

1	F
2	F
3	V
4	F
5	F

6	V
7	V
8	V
9	F
10	V

EXERCISE 18 – DIGITAL DESIGN PROJECTS

Students of the Digital Design course must perform a work project in teams formed by two students. The themes of the works are published in advance and each student sets their preferences for each theme using a scale of 1 to 5 (where 1 is the minimum and the 5 is the maximum value). The students who, for any reason, cannot be assigned to a team, will join students from other classes that are in the same conditions. However, this process will be treated in a later stage.

In one of the classes, consisting of 7 students, there are three themes. The preferences of the students for each of the themes are presented in Table 1.

a) What should be the theme assigned to each student, considering the maximization of the overall student satisfaction? Justify your answer carefully.
b) What other reasonable criterion could be applied? The solution would be the same? Justify your answer carefully.

	Theme 1	Theme 2	Theme 3
Student 1	4	2	5
Student 2	3	5	1
Student 3	2	1	4
Student 4	2	2	4
Student 5	5	3	4
Student 6	1	4	1
Student 7	3	4	5

SOLUTION

a) This is a maximization problem. We need to transform it into a minimization problem and balance the matrix (Cij max = 5).

1	1	3	3	0	0	5
2	2	0	0	4	4	5
3	3	4	4	1	1	5
3	3	3	3	1	1	5
0	0	2	2	1	1	5
4	4	1	1	4	4	5
2	2	1	1	0	0	5

Nr. of lines (4) inferior than the dimension of the matrix (7), so the solution is not optimal.
Subtract the minimum element of each row.

1	1	3	3	0	0	5
2	2	0	0	4	4	5
2	2	3	3	0	0	4
2	2	2	2	0	0	4
0	0	2	2	1	1	5
3	3	0	0	3	3	4
2	2	1	1	0	0	5

Nr. of lines (6) inferior than the dimension of the matrix (7), so the solution is not optimal.
Subtract the minimum element of each column.

1	1	3	3	0	0	1
2	2	0	0	4	4	1
2	2	3	3	0	0	0
2	2	2	2	0	0	0
0	0	2	2	1	1	1
3	3	0	0	3	3	0
2	2	1	1	0	0	1

Nr. of lines (6) inferior than the dimension of the matrix (7), so the solution is not optimal.
Subtract the minimum uncut element (1)

0	**0**	2	2	0	0	1
2	2	**0**	0	5	5	2
1	1	2	2	**0**	0	0
1	1	1	1	0	0	**0**
0	0	2	2	2	2	2
3	3	0	**0**	4	4	1
1	1	0	0	0	**0**	1

Optimal solution: nº of lines equal to the dimension of the matrix. It is possible to find a zero cost assignment.

EXERCISE 19 – QATAR 2022

The major stadium in Qatar 2022 football world cup, named "Lusail Iconic" will be designed by several teams. The organization of the event is currently requesting several proposals to companies in order to elaborate the following projects: architecture, acoustic, structures, electrical facilities.

The table below shows the duration, in days, to develop the projects presented by the companies. The symbol (-) means that a company is not able to carry out the project. Note that company A may develop two distinct projects, whereas the others can only develop one project.

	Architecture	Structures	Acoustic	Electrical facilities
Company Q	100	150	-	85
Company A	90	135	95	100
Company T	110	160	90	90
Company R	105	110	80	-

a) Formulate the problem as an assignment problem.
b) Indicate the optimal assignment plan and the minimum duration to develop the projects.
c) Suppose that the projects can be elaborate simultaneously due to the most recent information technologies implemented in the construction industry (Building Information Modelling). Indicate the optimal assignment plan that enables to minimize the overall time of projects' elaboration.

SOLUTION

a)

	Architecture	Structures	Acoustic	Electrical facilities	Project
Company Q	100	150	+∞	85	0
Company A	90	135	95	100	0
Company A	90	135	95	100	0
Company T	110	160	90	90	0
Company R	105	110	80	+∞	0

b)

	Architecture	Structures	Acoustic	Electrical facilities	Project
Company Q	10	40	+∞	0	0
Company A	0	25	15	15	0
Company A	0	25	15	15	0
Company T	20	50	10	5	0
Company R	15	0	0	+∞	0

Non optimal solution because the number of lines (4) is less than the dimension of the matrix (5)

Lowest uncut element: 10

	Architecture	Structures	Acoustic	Electrical facilities	Project
Company Q	10	30	+∞	**0**	0
Company A	0	15	5	15	**0**
Company A	**0**	15	5	15	0
Company T	20	40	**0**	5	0
Company R	25	**0**	0	+∞	10

Optimal Assignment solution: Compan

Minimum total time: 85+90+90+135=400

c) Bottleneck Assignment Problem. Consider any assignement, for example the optimal solution of the previous question:

	Architecture	Structures	Acoustic	Electrical facilities	Project
Company Q	100	150	+∞	**85**	0
Company A	90	135	95	100	**0**
Company A	**90**	135	95	100	0
Company T	110	160	**90**	90	0
Company R	105	**110**	80	+∞	0

Longest time: 110 days

	Architecture	Structures	Acoustic	Electrical facilities	Project
Company Q	0	1	1	0	0
Company A	0	1	0	0	0
Company A	0	1	0	0	0
Company T	1	1	0	0	0
Company R	0	1	0	1	0

Minimum nr. of lines (4) < dimension of the matrix (5), so the solution is optimal.

EXERCISE 20 – SWIMMING WORLD CUP

The Special Swimming World Cup includes several races that include four different styles of swimming: Butterfly (Mp), Backstroke (Co), Breaststroke (Br) and Crawl (Cr). At each swim, only 4 swimmers from each country can participate. However, for different events different swimmers may be chosen if the Olympic Committee of each country so wishes. The Portuguese Olympic Committee intends to build a team for the next World Cup and has drawn up the following list. The following tables show the speed of each swimmer in each style (greater number = faster).

	Mp	Co	Br	Cr
A	3	6	7	8

	Mp	Co	Br	Cr
B	5	6	7	5

	Mp	Co	Br	Cr
C	7	4	5	6

	Mp	Co	Br	Cr
D	5	6	6	7

Due to a recent shoulder injury, swimmer A will not be able to compete in the breaststroke style.

a) The first competition includes all swimming styles. Swimmers from each country swim simultaneously, each in a particular style. The final classification of the country is attributed by the speed with which the set of the swimmers of each country completes the test. Determine what would be the best swimmers choice for this event.

b) Meanwhile the Olympic Committee decided to put one more player (E) in the list:

	Mp	Co	Br	Cr
E	4	6	9	8

The second competition consists of a 4-step swimming marathon, and each stage consists of a certain distance with a proper swimming style. When a swimmer finishes his or her step, the teammate responsible for the next step continues the race immediately. Swimmer D, known for his unusual physical ability, can compete in two different stages. Determine the best choice of swimmers for this event.

SOLUTION

a) Bottleneck Assignement Problem

	Mp	Co	Br	Cr
A	3	6	- ∞	8
B	5	6	7	5
C	7	4	5	6
D	5	6	6	7

	Mp	Co	Br	Cr
A	5	2	∞	**0**
B	3	2	**1**	3
C	**1**	4	3	2
D	3	**2**	2	1

	Mp	Co	Br	Cr
A	1	1	1	0
B	1	1	0	1
C	0	1	1	1
D	1	1	1	0

Optimal assignment: Minimum speed = 6

Swimmer A – Crawl; Swimmer B – Breaststroke; Swimmer C – Butterfly; Swimmer D – Backstroke

b)

Maximization problem

	Mp	Co	Br	Cr	*	*
A	3	6	- ∞	8	0	0
B	5	6	7	5	0	0
C	7	4	5	6	0	0
D	5	6	6	7	0	0
D	5	6	6	7	0	0
E	4	6	9	8	0	0

Cij max = 9

	Mp	Co	Br	Cr	*	*
A	6	5	∞	1	9	9
B	4	3	2	4	9	9
C	2	5	4	3	9	9
D	4	3	3	2	9	9
D	4	3	3	2	9	9
E	5	3	0	1	9	9

	Mp	Co	Br	Cr	*	*
A	5	4	∞	0	8	8
B	2	1	0	2	7	7
C	0	3	2	1	7	7
D	2	1	1	0	7	7
D	2	1	1	0	7	7
E	5	3	0	1	9	9

Non optimal solution; nr of lines (3) < dimension of the matrix (6)

Subtract the minimum element in each column

	Mp	Co	Br	Cr	*	*
A	5	3	∞	**0**	1	1
B	2	0	0	2	**0**	0
C	**0**	2	2	1	0	**0**
D	2	0	1	0	0	0
D	2	**0**	1	0	0	0
E	5	2	**0**	1	2	2

Optimal solution (not unique): Swimmer A – Crawl; Swimmer C – Butterfly; Swimmer D – Backstroke; Swimmer E – Breaststroke; Swimmers B and C: excluded

Total speed= 30

EXERCISE 21 – MULTIPLE CHOICE

1) Which of the following methods **cannot** be used to solve Assignment problems?

   a) Hungarian Method
   b) Transportation Algorithm
   c) Simplex Method
   d) All can be used

2) A generic Bottleneck Assignment Problem has the objective of:

   a) Minimizing the total time of the selected tasks
   b) Minimizing the number of tasks to select
   c) Minimizing the time of the longest selected task
   d) None of the answers

3) After two iterations of the Hungarian method, an assignment (not optimal) was chosen in the resulting matrix. The sum of the values corresponding to the selected affectation is 20. It is known that the value 2 was subtracted from each column and the value 1 was subtracted from each row. Knowing that the problem matrix has dimension 4, what is the value of the selected assignment, considering the original values of the problem?

   a) 20
   b) 23
   c) 32
   d) 48

4) 4) Consider the formulation presented below for an assignment problem, for which it is intended to define in which machines (A, B, C, D) must be cut 4 metal plates (C1-C4). After an upgrade in machine

B, it was able to cut two metal plates at the same time, unlike the other machines that only cut one. Given this new information, select the statements that are correct.

a) The initial matrix does not need to change
b) It is enough to add an extra column to the matrix
c) When choosing the final assignment, we must choose two products from column B
d) None of the previous answers

	A	B	C	D
C1				
C2				
C3				
C4				

SOLUTION

1-d; 2-c; 3-b; 4-d