

# Network Routing

Redes de Computadores

2021/22

Pedro Brandão

## References

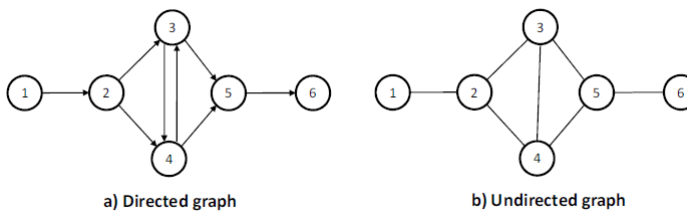
- These slides are from “Computer Networking: A Top Down Approach 5th edition. Jim Kurose, Keith Ross Addison-Wesley, April 2009”
- And “Computer Networks, 5th edition. Andrew S. Tanenbaum, David J. Wetherall, Prentice Hall, 2011”
- With adaptations/additions by Manuel Ricardo and Pedro Brandão

## Driving questions...

- What is a graph?
- What is a spanning tree?
- What is a shortest path tree?
- How are paths defined in a network?
- How does the Dijkstra algorithm work?
- How does a link state routing protocol work?
- How does a node learn about neighbours?
- How does the Bellman-Ford algorithm work?
- How does a distance vector work?
- What are the limitations of the layer 2 network of switches?
- How does the IEEE spanning tree protocol work?
- What is the maximum capacity of a flow network?

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## Graph – Directed and Undirected



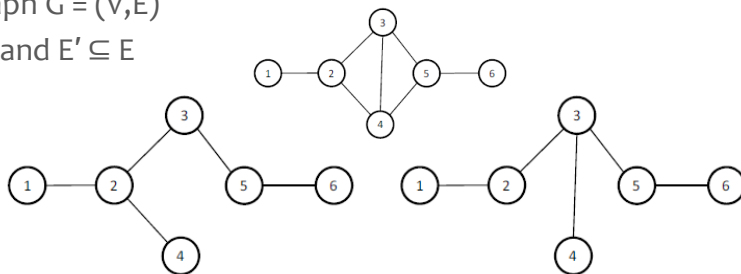
$$\begin{aligned}
 G &= (V, E) \\
 V &= \{v_1, v_2, v_3, v_4, v_5, v_6\}, & |V| &= 6 \\
 E &= \{(v_1, v_2), (v_2, v_3), (v_2, v_4), (v_3, v_4), \\
 &\quad (v_4, v_3), (v_3, v_5), (v_4, v_5), (v_5, v_6)\}, & |E| &= 8
 \end{aligned}$$

$$\begin{aligned}
 G &= (V, E) \\
 V &= \{v_1, v_2, v_3, v_4, v_5, v_6\}, & |V| &= 6 \\
 E &= \{(v_1, v_2), (v_2, v_3), (v_2, v_4), (v_3, v_4), \\
 &\quad (v_3, v_5), (v_4, v_5), (v_5, v_6)\}, & |E| &= 7
 \end{aligned}$$

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## Tree

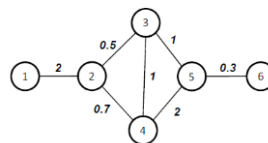
- Trees  $T = (V, E)$ 
  - graph with **no cycles**
  - number of edges  $|E| = |V| - 1$
  - any two vertices of the tree are connected by exactly **one** path
- A tree  $T$  is said to span a graph (spanning tree) if:
  - Given graph  $G = (V, E)$
  - $T = (V, E')$  and  $E' \subseteq E$



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## Shortest Path Trees

- Graphs and Trees can be weighted
  - $G = (V, E, w)$
  - $T = (V, E', w)$
  - $w: E \rightarrow \mathbb{R}$
- Total cost of a tree  $T \rightarrow C_{total}(T) = \sum_{i=1}^{|E|} w(e_i)$
- Minimum Spanning Tree  $T^* \rightarrow C_{total}(T^*) = \min(C_{total}(T))$ 
  - algorithms used to compute MST: Prim, Kruskal
- Shortest Path Tree (SPT) Rooted at Vertex  $s$ 
  - tree composed by the union of the shortest paths between  $s$  and each of other vertices of  $G$
  - Algorithms used to compute SPT: Dijkstra, Bellman-Ford
- Computer networks use Shortest Path Trees



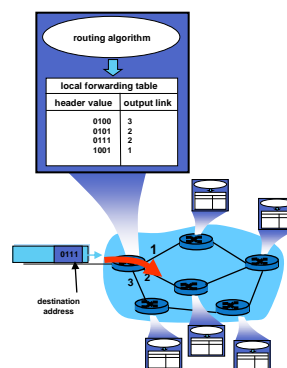
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# Routing in Layer 3 Networks

(Routing)

## Forwarding, Routing

- Forwarding → data plane
  - directing packet from input to output link
  - using a forwarding table
- Routing → control plane
  - computing paths the packets will follow
  - routers exchange messages
  - each router creates its forwarding table



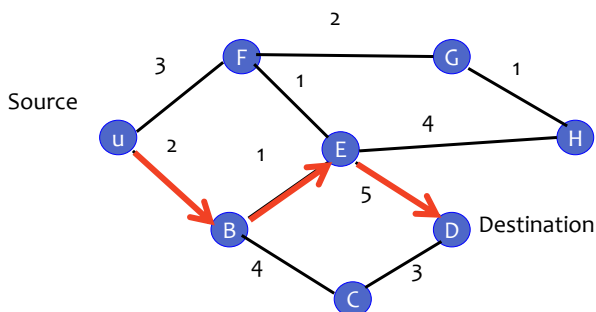
## Importance of Routing

- End-to-end performance
  - path affects quality of service
  - delay, throughput, packet loss
- Use of network resources
  - balance traffic over routers and links
  - avoiding congestion by directing traffic to less-loaded links
- Transient disruptions
  - failures, maintenance
  - limiting packet loss and delay during changes

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## Shortest-Path Routing

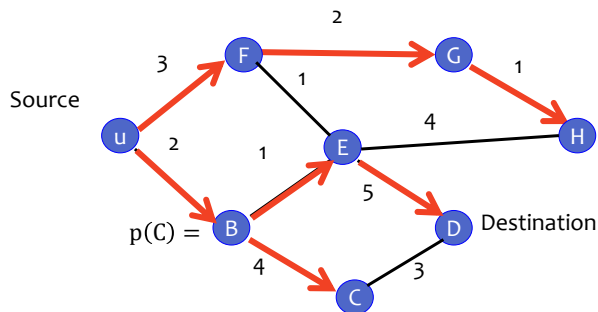
- Path-selection model
  - Destination-based
  - Load-insensitive (e.g., static link weights)
  - Minimum hop count or minimum sum of link weights



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## Shortest-Path Routing

- Given a network topology with link costs
  - $c(x, y)$  - link cost from node  $x$  to node  $y$
  - Infinity ( $\infty$ ) if  $x$  and  $y$  are not direct neighbours
- Compute the least-cost paths from source  $u$  to all nodes
  - $p(v)$  - node predecessor of node  $v$  in the path to  $u$



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## Dijkstra's Shortest-Path Algorithm

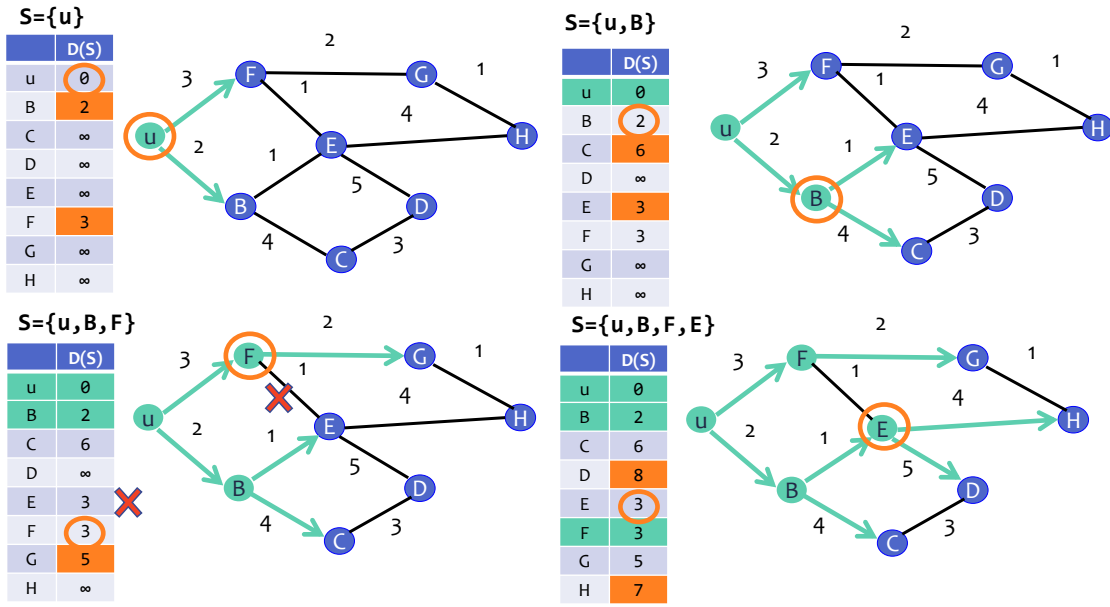
- Iterative algorithm
  - After  $k$  iterations  $\rightarrow$  known least-cost paths to  $k$  nodes
- $S \rightarrow$  set of nodes for which least-cost path is known
  - Initially,  $S = \{u\}$ , where  $u$  is the source node
  - Add one node to  $S$  in each iteration
- $D(v) \rightarrow$  current cost of path from source to node  $v$ 
  - Initially
    - $D(v) = c(u, v)$  for all nodes  $v$  adjacent to  $u$
    - $D(v) = \infty$  for all other nodes  $v$
  - Continually update  $D(v)$  when shorter paths are learned

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# Dijkstra's Algorithm

- 1 Initialization:
- 2  $S = \{u\}$
- 3 for all nodes  $v$
- 4 if  $v$  adjacent to  $u$  {
- 5  $D(v) = c(u,v)$
- 6 }
- 7 else  $D(v) = \infty$
- 8 Loop
- 9 find node  $w$  not in  $S$  with the smallest  $D(w)$
- 10 add  $w$  to  $S$
- 11 update  $D(v)$  for all  $v$  adjacent to  $w$  and not in  $S$ :
- 12  $D(v) = \min\{D(v), D(w) + c(w,v)\}$
- 13 until all nodes in  $S$

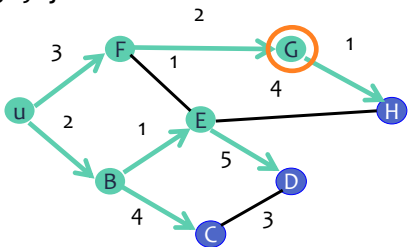
## Dijkstra's Algorithm – Example – 1



# Dijkstra's Algorithm – Example – 2

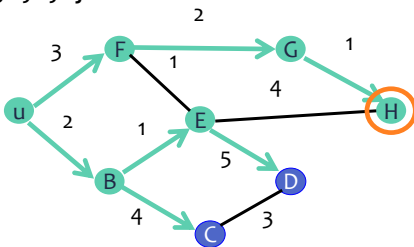
S={u,B,F,E,G}

	D(S)
u	0
B	2
C	6
D	8
E	3
F	3
G	5
H	6



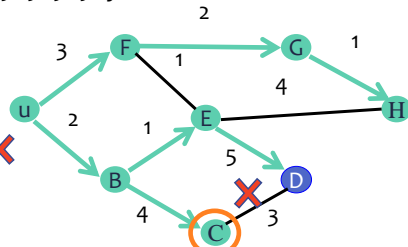
S={u,B,F,E,G,H}

	D(S)
u	0
B	2
C	6
D	8
E	3
F	3
G	5
H	6



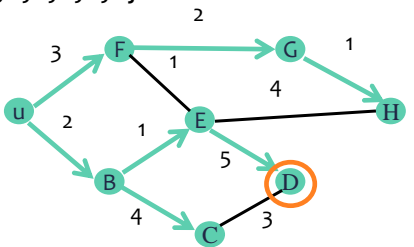
S={u,B,F,E,G,H,C}

	D(S)
u	0
B	2
C	6
D	8
E	3
F	3
G	5
H	6



S={u,B,F,E,G,H,C,D}

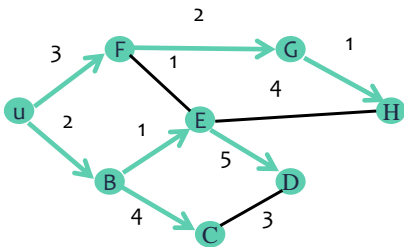
	D(S)
u	0
B	2
C	6
D	8
E	3
F	3
G	5
H	6



# Dijkstra's Algorithm – Example – 3

	D(S)	Next Hop
u	0	---
B	2	B
C	6	B
D	8	B
E	3	B
F	3	F
G	5	F
H	6	G

Forwarding table at u



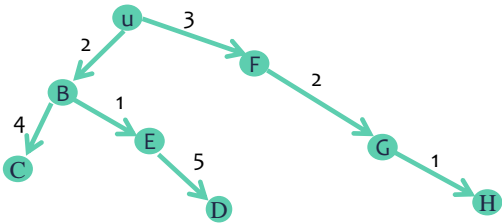
Shortest-path tree from u



# Dijkstra’s Algorithm – Example – 4

	D(S)	Next Hop
u	0	---
B	2	B
C	6	B
D	8	B
E	3	B
F	3	F
G	5	F
H	6	G

Forwarding table at u



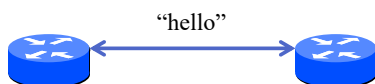
Shortest-path tree from u

# Link-State Routing

- Each router keeps track of its incident links
  - link up, link down
  - cost on the link
- Each router broadcasts link state
  - every router gets a complete view of the graph
- Each router runs Dijkstra’s algorithm, to
  - compute the shortest paths
  - construct the forwarding table
- Example protocols
  - Open Shortest Path First (OSPF), [STD 54](#), [RFC 2328](#), [OSPF Version 2](#)
  - Intermediate System – Intermediate System (IS-IS)
    - [ISO/IEC 10589:2002](#), Information technology — Telecommunications and information exchange between systems — Intermediate System to Intermediate System intra-domain routeing information exchange protocol for use in conjunction with the protocol for providing the connectionless-mode network service (ISO 8473)
    - [RFC 1195](#), Use of OSI IS-IS for routing in TCP/IP and dual environments

## Detection of Topology Changes

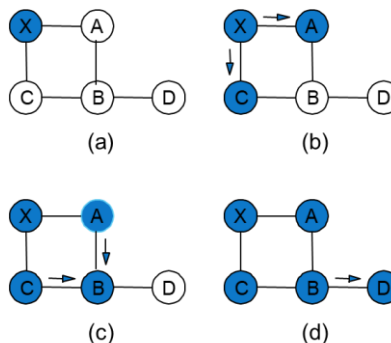
- Beacons generated by routers on links
  - Periodic “hello” messages in both directions
  - few missed “hellos” → link failure



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## Broadcasting the Link State

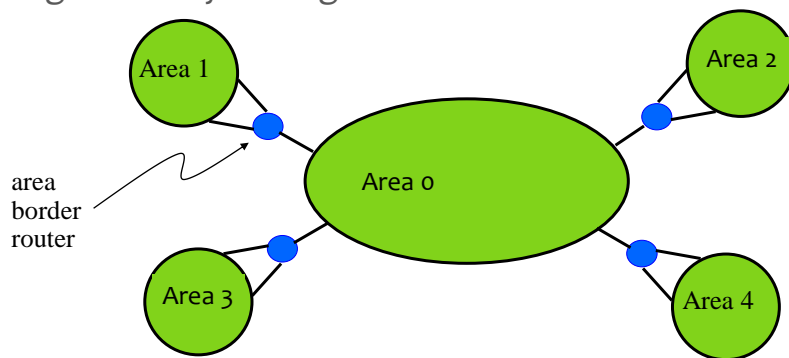
- How to Flood the link state?
  - every node sends link-state information through adjacent links
  - next nodes forward that info to all links
    - except the one where the information arrived
- When to initiate flooding?
  - Topology change
    - link or node failure/recovery
    - link cost change
  - Periodically
    - refresh link-state information
    - typically 30 minutes



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## Scaling Link-State Routing

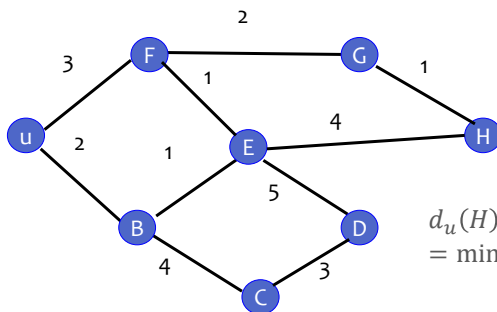
- Overhead of link-state routing
  - flooding link-state packets throughout the network
  - running Dijkstra's shortest-path algorithm
- Introducing hierarchy through “areas”



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## Bellman-Ford Algorithm

- Define distances at each node  $x$ 
  - $d_x(y)$  = cost of least-cost path from  $x$  to  $y$
- Update distances based on neighbours
  - $d_x(y) = \min(c(x, n) + d_n(y))$  over all neighbours  $n$  of  $x$



$$d_u(H) = \min(c(u, F) + d_F(H), c(u, B) + d_B(H))$$

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## Distance Vector Algorithm

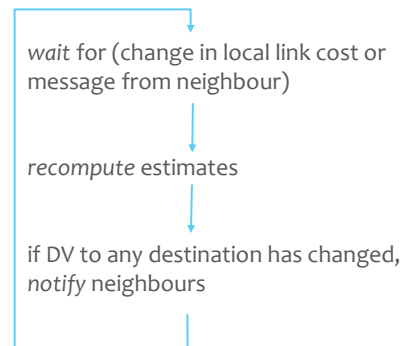
- $c(x,n)$  = cost for direct link from  $x$  to  $n$ 
  - node  $x$  maintains costs of direct links  $c(x,n)$
  - $n$  are  $x$ 's neighbours
- $D_x(y)$  = estimate of least cost from  $x$  to  $y$ 
  - node  $x$  maintains **distance vector**  $D_x = [D_x(y): y \in V]$
  - $V$  is all nodes
- Node  $x$  maintains also its neighbours' distance vectors
  - for each neighbour  $n$ ,  $x$  maintains  $D_n = [D_n(y): y \in V]$
- Each node  $x$  periodically sends  $D_x$  to its neighbours
  - and neighbours update their own distance vectors
  - $D_z(y) \leftarrow \min_x \{c(z,x) + D_x(y)\}$  for each node  $y \in V$ 
    - $z$  is a neighbour of  $x$
- Over time, the distance vector  $D_x$  converges

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## Distance Vector Algorithm

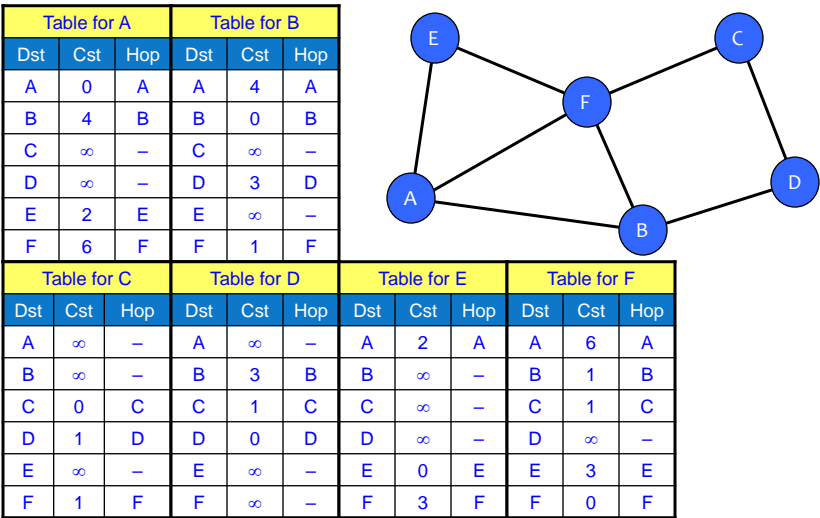
- Iterative, asynchronous
  - each local iteration caused by:
    - local link cost change
    - distance vector update message from neighbour
- Distributed
  - node notifies neighbours
    - only when its DV changes
- Neighbours then
  - notify their neighbours,
  - if necessary

- Each node:

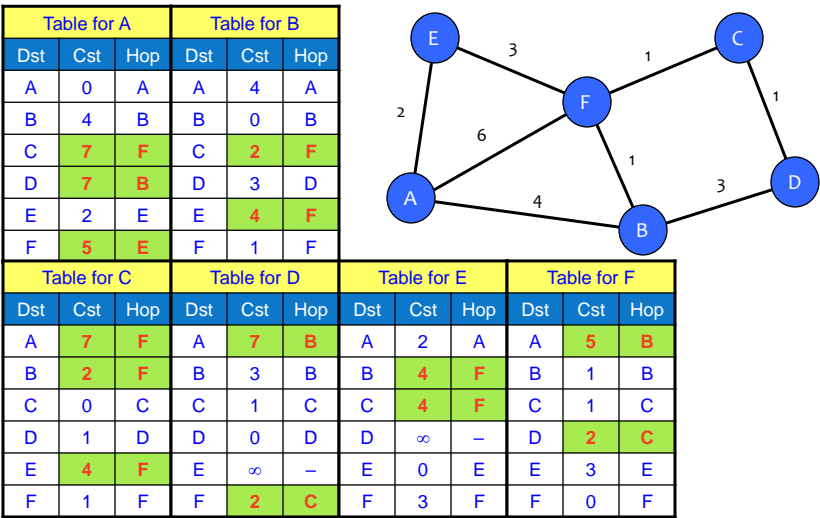


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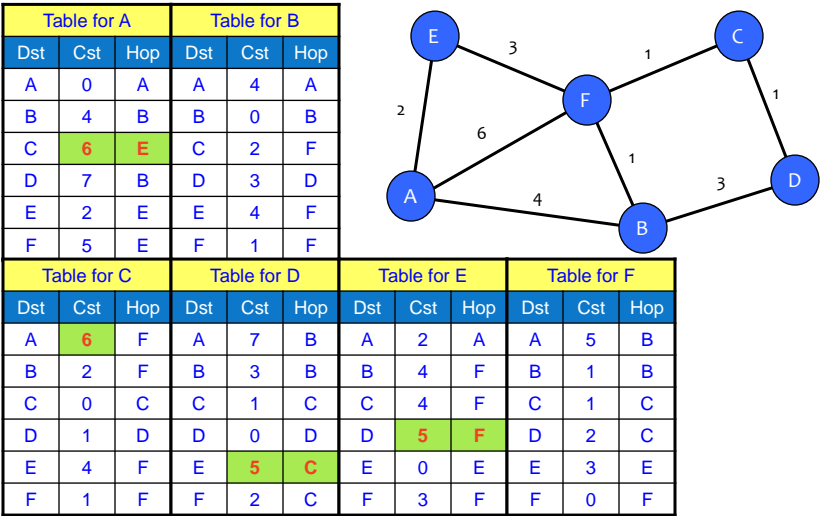
# Distance Vector Example - Step 0



# Distance Vector Example - Step 1



## Distance Vector Example - Step 2



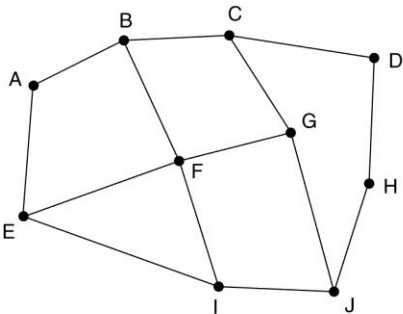
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## Routing Information Protocol (RIP)

- [STD 56, RFC 2453, RIP Version 2](#)
- Distance vector protocol
  - nodes send distance vectors every 30 seconds
  - or, when an update causes a change in routing
- RIP is limited to small networks

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# BGP – The Exterior Gateway Routing Protocol



(a)  
A set of BGP routers.

Information F receives  
from its neighbors about D

- From B: "I use BCD"
- From G: "I use GCD"
- From I: "I use IFGCD"
- From E: "I use EFGCD"

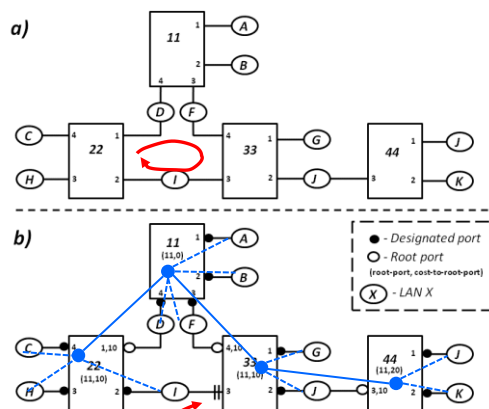
(b)  
Information sent to F

# Unique Spanning Tree in Ethernet Networks

Routing

## L2 Networking - Single Tree Required

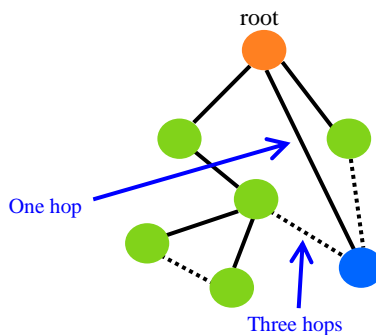
- Ethernet frame
  - No hop-count
  - Could loop forever
  - broadcast frame, mis-configuration
- Layer 2 network
  - Required to have tree topology
  - Single path between every pair of stations
- Spanning Tree Protocol (STP)
  - Running in bridges
  - Helps building the spanning tree
  - Blocks ports



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## Constructing a Spanning Tree

- Distributed algorithm
  - switches need to elect a “root”
    - the switch with the smallest identifier
  - each switch identifies if its interface is on the shortest path from the root
  - messages (Y, d, X)
    - from node X
    - claiming Y is the root
    - and the distance is d



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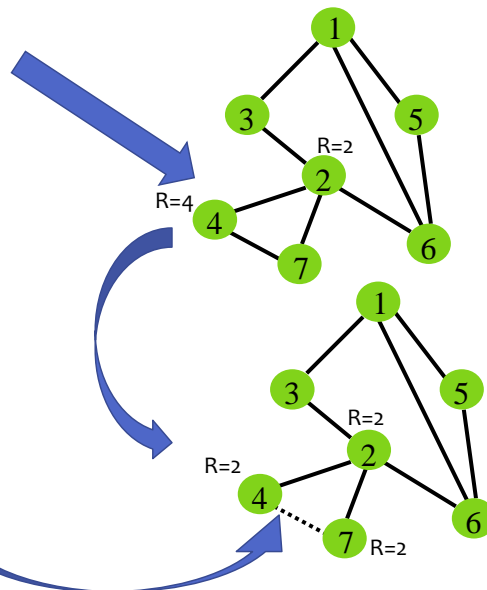
## Steps in Spanning Tree Algorithm

- Initially, each switch thinks it is the root
  - switch sends a message out every interface
  - identifying itself as the root with distance 0
  - example: switch X announces (X, 0, X)
- Other switches update their view of the root
  - upon receiving a message, check the root id
  - if the new id is smaller, start viewing that switch as the root
- Switches compute their distance from the root
  - add 1 to the distance received from a neighbour
  - identify interfaces not on a shortest path to the root and exclude them from the spanning tree

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## Example - Switch #4's Viewpoint

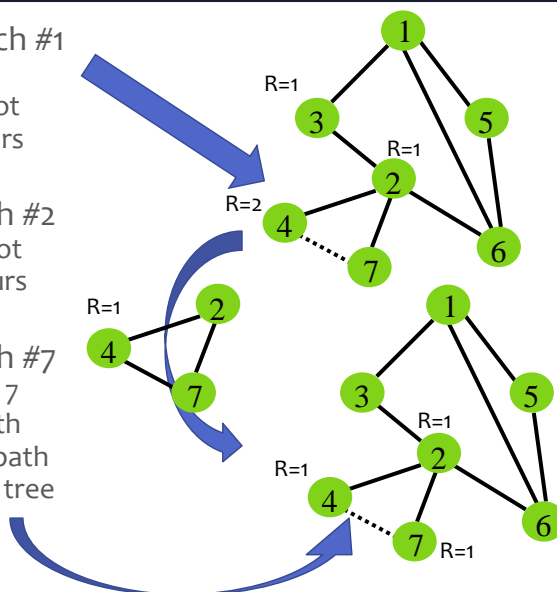
- Switch #4 thinks it is the root
  - sends (4, 0, 4) message to 2 and 7
- Then, switch #4 hears from #2
  - receives (2, 0, 2) message from 2
  - ... and thinks that #2 is the root
  - and realizes it is just one hop away
- Then, switch #4 hears from #7
  - receives (2, 1, 7) from 7
  - and realizes this is a longer path
  - so, prefers its own one-hop path
  - and removes 4-7 link from the tree



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## Example - Switch #4's Viewpoint

- Switch #2 hears about switch #1
  - switch 2 hears (1, 1, 3) from 3
  - switch 2 starts treating 1 as root
  - and sends (1, 2, 2) to neighbours
- Switch #4 hears from switch #2
  - switch 4 starts treating 1 as root
  - and sends (1, 3, 4) to neighbours
- Switch #4 hears from switch #7
  - switch 4 receives (1, 3, 7) from 7
  - and realizes this is a longer path
  - so, prefers its own three-hop path
  - and removes 4-7 link from the tree



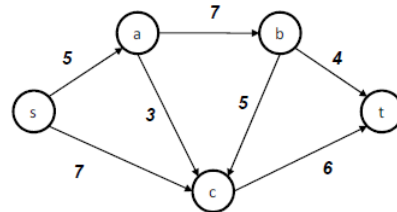
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# Maximum Flow of a Network

Routing

## Flow Network Model

- Flow network
  - source  $s$
  - sink  $t$
  - nodes  $a$ ,  $b$  and  $c$
- Edges are labelled with **capacities**
  - (e.g. bit/s)
- Communication networks are not flow networks
  - they are queue networks
  - flow networks enable to determine limit capacity values



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## Maximum Capacity of a Flow Network

- Max-flow min-cut theorem
  - maximum amount of flow transferable through a network
  - equals minimum value among all simple cuts of the network
- Cut → split of the nodes  $V$  into two disjoint sets  $S$  and  $T$ 
  - $S \cup T = V$
  - there are  $2^{|V|-2}$  possible cuts
- Capacity of cut  $(S, T)$ : 
$$c(S, T) = \sum_{(u,v) | u \in S, v \in T, (u,v) \in E} c(u, v)$$

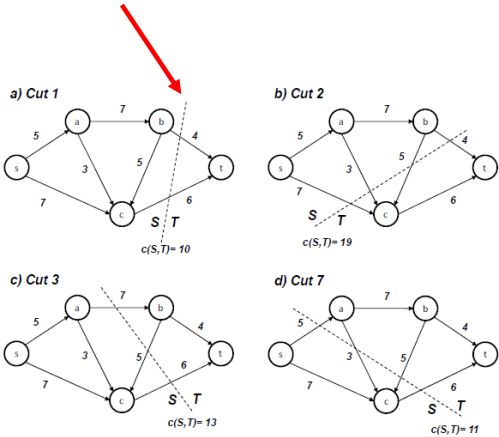
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# Max-flow Min-cut - Example

$2^{|5|-2} = 8$  possible cuts

Cut	Vertices					$c(S,T)$	Feasibility
	s	a	b	c	t		
1	S	S	S	S	T	10	✓
2	S	S	S	T	T	19	✓
3	S	S	T	S	T	13	✓
4	S	S	T	T	T	17	✓
5	S	T	S	S	T	-	×
6	S	T	S	T	T	-	×
7	S	T	T	S	T	11	✓
8	S	T	T	T	T	12	✓

Maximum flow = 10



## Homework

- 1.Review slides
- 2.Read from Tanenbaum
  - Section 5.2 – Routing algorithms
  - Section 4.8.3 Spanning Tree Bridges
- 3.Answer questions at Moodle

# End of Routing