Mapping

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Also, a very special Thank You to: ***Prof. Nuno Lau, IEETA, U. Aveiro***

Background

- **Localization** – Where am I?
- **Mapping** – My (dynamic?) surroundings
- **Navigation** – How do I get where I want to go?
- TREND: **SLAM –**
  - Simultaneous Localization and Mapping

Mapping

- **Mapping is the process of building an internal estimate of the metric map of the environment**
- **What should be represented?**
  - Each cell is occupied or unoccupied
  - Each cell has a continuous value (high-occupied, low unoccupied)
  - Probability that each cell is occupied P(H) and probability that each cell is unoccupied P(~H)

    0<=P(H)<=1

    1-P(H) = P(~H)

Mapping

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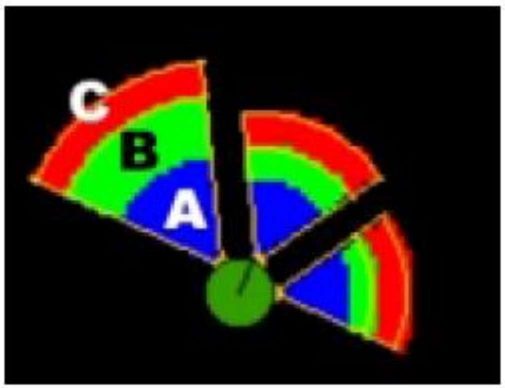
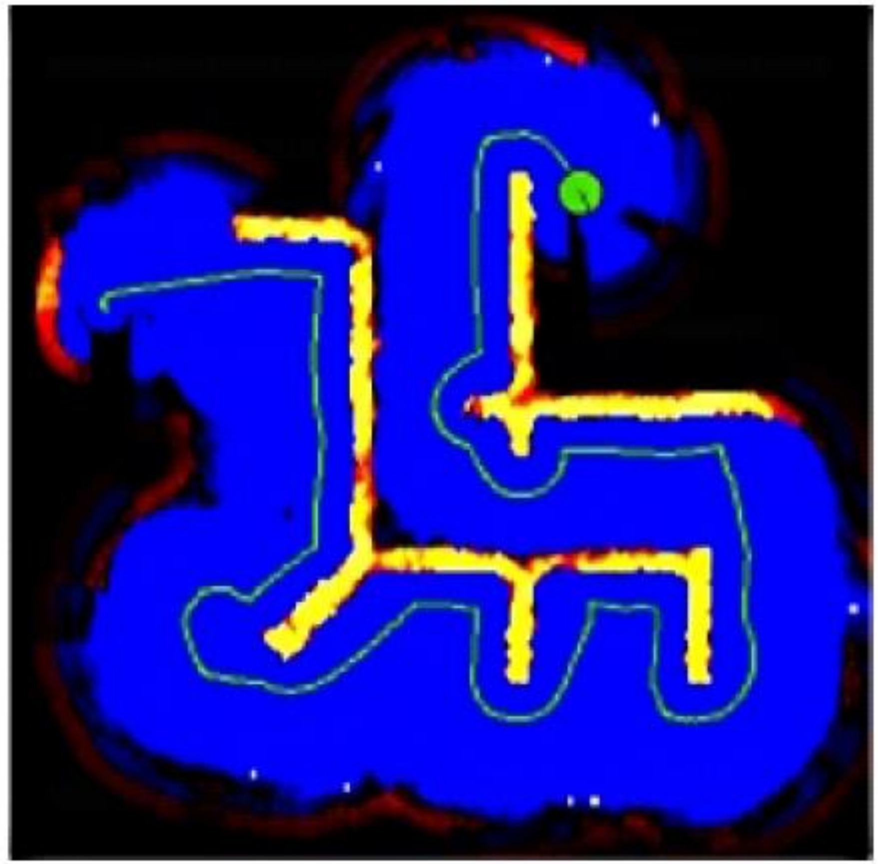
  - Probability that each cell is occupied P(H) and probability that each cell is unoccupied P(~H)

    $0 <= P(H) <= 1$

    $1-P(H) = P(\sim H)$

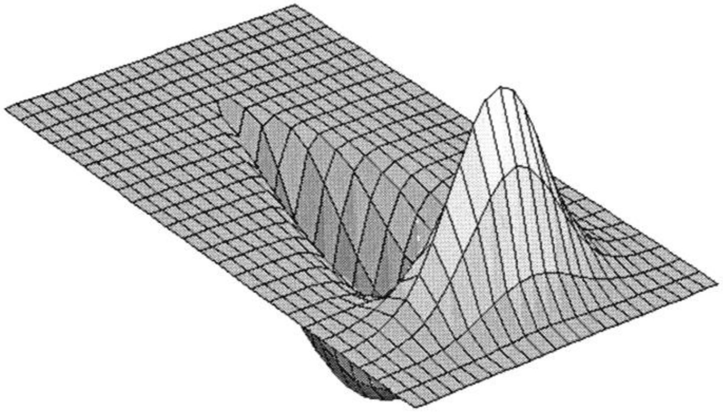
Mapping





Mapping

- **Conditional probabilities**
  - We want to determine P(H|s)
    - Probability cell is occupied given a certain measure s
  - Let’s start by determining P(s|H)
    - Probability of getting measure s if there is H is occupied
      - In general P(H|s) is not equal to P(s|H)
    - This is the sensor model



Mapping

- **Conditional probabilities**
  - We want to determine P(H|s)
  - From Bayes' Rule:

$$P(H \mid s) = \frac{P(s \mid H).P(H)}{P(s \mid H)P(H) + P(s \mid \sim H)P(\sim H)}$$

  - P(s|H) and P(s|~H) are known from the sensor model
  - P(H) and P(~H) are unconditional probabilities or prior probabilities
    - If no information is available P(H)=P(~H)=0.5 can be assumed

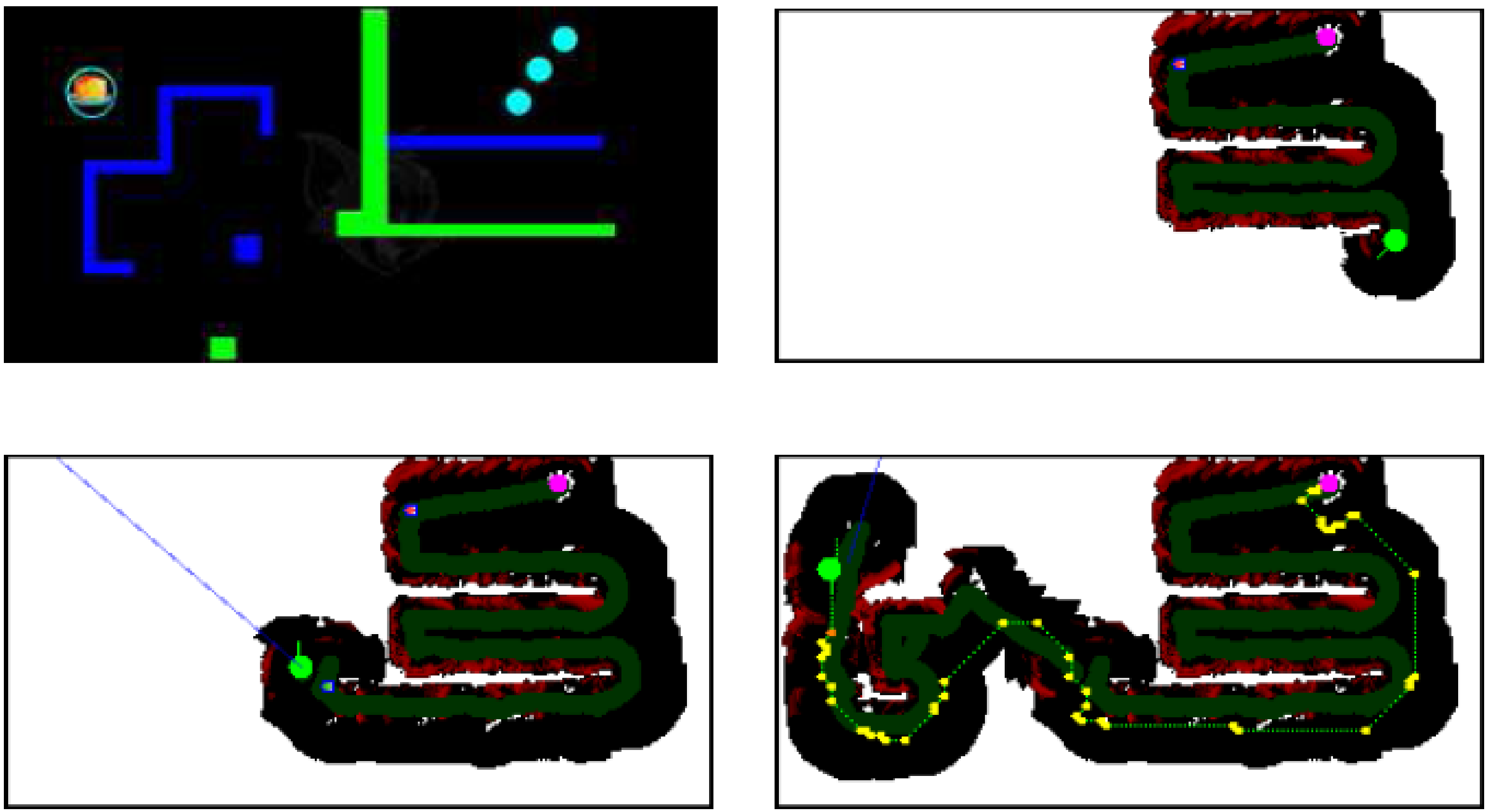
Mapping

- **Updating with the Bayes' rule**
  - How to fuse the computed probabilities with new sensor readings?

$$P(H \mid s_n) = \frac{P(s_n \mid H).P(H \mid s_{n-1})}{P(s_n \mid H)P(H \mid s_{n-1}) + P(s_n \mid \sim H)P(\sim H \mid s_{n-1})}$$

  - This is the recursive version of the update formula
  - Each time a new observation is made it can be employed to update the occupancy grid

Mapping



Mapping

- **Dempster-Schafer Theory**
  - Belief functions instead of probabilities
    - Measure belief mass
    - Each sensor contributes with a belief mass of 1.0
    - Can distribute the mass to a set of propositions
  - Set of propositions is the Frame of Discernement (FOD)
    - In the case of occupancy grid FOD={Occupied, Empty}
    - It may include non exclusive propositions
    - A sensor reading may be considered as ambiguous

Mapping

- **Dempster-Schafer Belief function properties**
  - Bel(X) measures the likelihood that previous evidence supports X
  - Bel({}) = 0
  - Bel (set of all subsets of FOD) = 1
  - In the case of the occupancy grid:

    Bel = m({Occupied}), m({Empty}), m(dontknow)

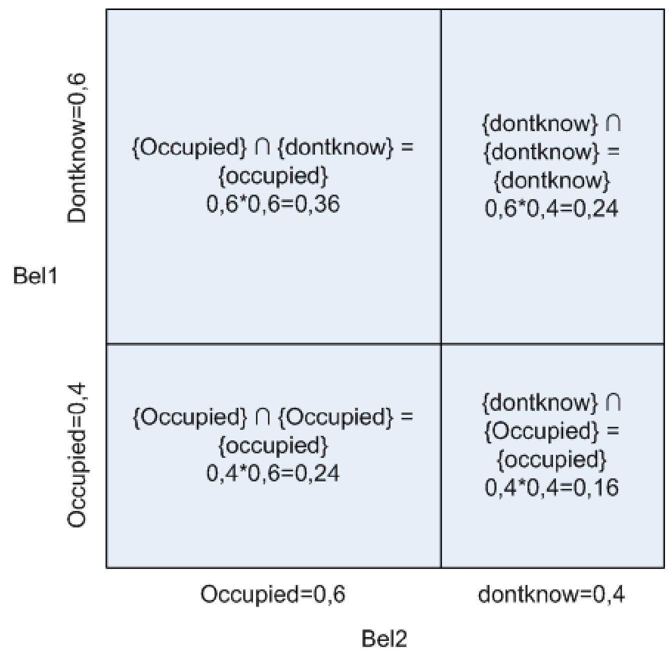
    dontknow={Occupied,Empty}

Mapping

- **Belief function for sonar**
  - Region that supports evidence of having an obstacle
    - m(occupied)=evidence
    - m(empty)=0
    - m(dontknow)=1-evidence
  - Region that supports evidence of being empty
    - m(occupied)=0
    - m(empty)=evidence
    - m(dontknow)=1-evidence
  - The main difference from probabilities is that uncertainties in the reading count as belief mass for dontknow

Mapping

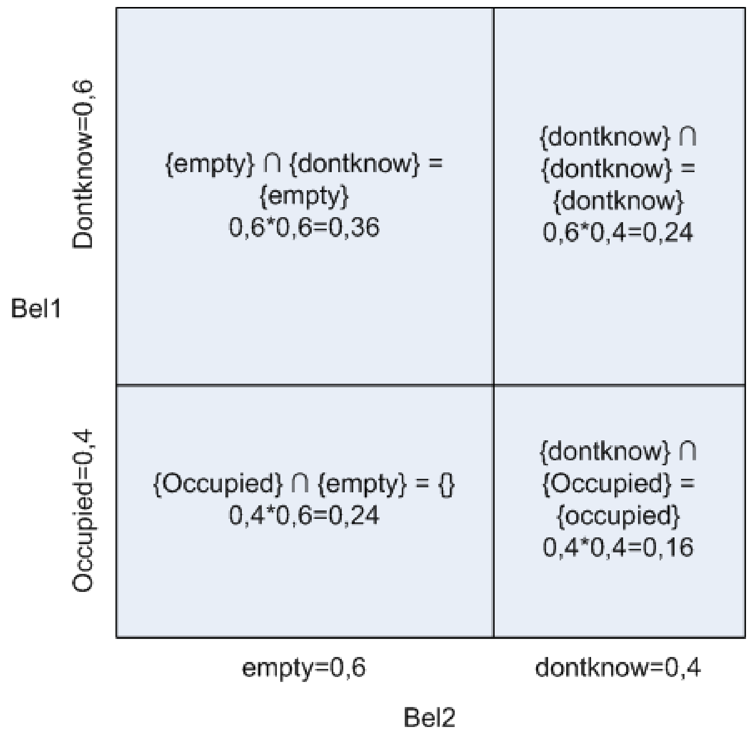
- **Rule of combination**
  - Bel_1 = m(occupied)=0.4, m(empty)=0, m(dontknow)=0.6
  - Bel_2 = m(occupied)=0.6, m(empty)=0, m(dontknow)=0.4



  - Bel_3 = m(occupied)=0,76, m(empty)=0,0, m(dontknow)=0,16

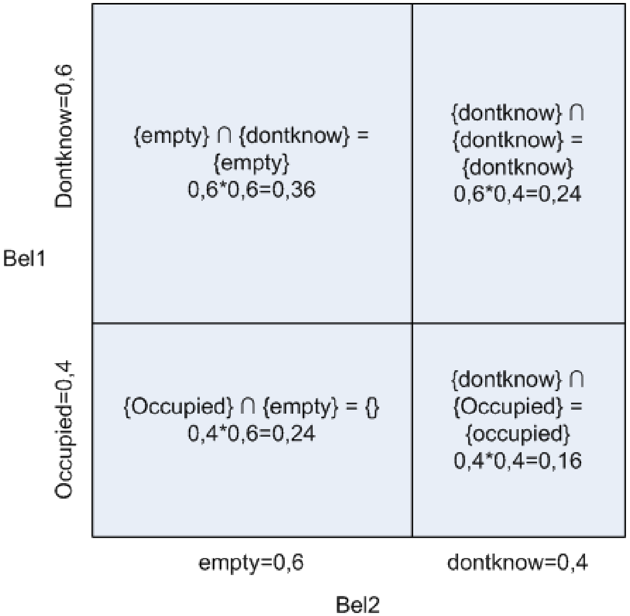
Mapping

- **Rule of combination**
  - Bel_1 = m(occupied)=0.4, m(empty)=0, m(dontknow)=0.6
  - Bel_2 = m(occupied)=0.0, m(empty)=0,6, m(dontknow)=0.4



Mapping

- **Rule of combination**



  - As m({}) must be 0, a normalization is needed
  - Bel_3 = m(occupied)=0,16/(1-0,24)=0,21
    m(empty)=0,36/(1-0,24)=0,47
    m(dontknow)=0,24/(1-0,24)=0,32

Mapping

- **Rule of combination**

$$m(C_k) = \frac{\sum_{A_i \cap B_j = C_k;\, C_k \neq \{\}} m(A_i).m(B_j)}{1 - \sum_{A_i \cap B_j = \{\}} m(A_i).m(B_j)}$$

  - It must be repeated for every subset of the orthogonal sum

Additional Sources

Mapping

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