

Duration: 2h30

Version A

No consultation is allowed, other than the supplied document.

No electronic means are allowed (computer, cellphone, ...).

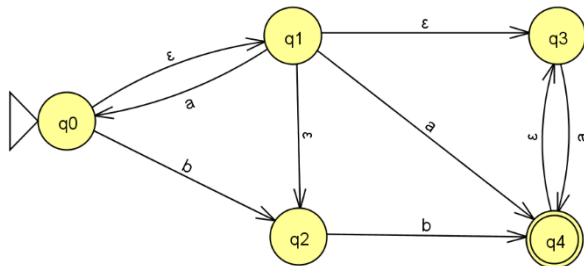
Fraud attempts lead to the annulment of the exam for all participants.

Answer each group in separate sheets!

Write your full name and exam version in all sheets!

Group I: [4.5 Points] Finite Automata and Regular Expressions

Consider the following ε -NFA.



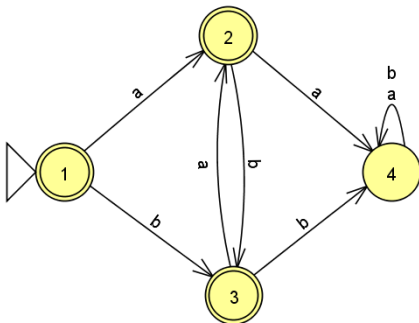
a) Determine the ε -closure of each of the ε -NFA states.

b) Obtain the equivalent DFA for the ε -NFA. Show the table of transitions and the state diagram of the DFA.

c) Minimize the obtained DFA. Show the table of distinguishable states and the state diagram for the

minimized DFA.

Consider the following DFA.

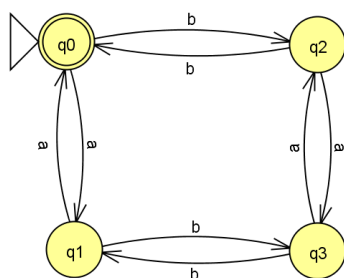


d) Obtain a regular expression for the language defined by the DFA using the state elimination method. Show all the intermediate steps.

e) Show the regular expressions for the terms $R_{24}^{(0)}$, $R_{11}^{(0)}$, $R_{12}^{(0)}$, $R_{44}^{(0)}$, $R_{21}^{(0)}$, $R_{24}^{(1)}$, $R_{12}^{(1)}$ and $R_{24}^{(1)}$ obtained by the method of path constructions for the conversion of DFAs to regular expressions.

Suppose that we need a DFA for the language $L = \{w \in \{a,b,c\}^* \mid \text{no. of } a\text{'s is even, no. of } b\text{'s is even and the no. of } c\text{'s is even}\}$.

The DFA for the language $L1 = \{w \in \{a,b\}^* \mid \text{no. of } a\text{'s is even and the no. of } b\text{'s is even}\}$ is given (see the DFA below).



f) Explain how you can determine a DFA for L based on the DFA of $L1$ and considering the closure operations of the regular languages.

We intend to specify as a regular expression the language representing registers of persons. Each register uses a line and there might exist 0 or more registers in the same file.

g) Show a regular expression that allows the validation of the format of data assuming that each line consists of: name of the person, followed by the birthday date, separated by ':', as we can see in the examples below. Use the symbols 'M' to represent an uppercase letter,

'm' for a lowercase letter (both including letters with accents), 'D' for a digit, 'E' for a space and 'R' for a new line. Indicate the alphabet used in the expression and in the case you use extra symbols explain their meaning. The birthday dates must be represented by: DD/MM/AAAA or DD-MM-AAAA, and the leftmost zeros can be omitted and the fields DD and MM may have values from 1 to 31 and from 1 to 12, respectively. Examples:

Dionísio Adalberto da Silva Côte-Real : 01/04/1999

Leonilde Maria do Ouro dos Anjos e Ramos da Árvore : 4/08/2000

Group II: [2 Pts] Properties of Regular Languages

Prove that if each of the following languages is regular or not. In case a language is regular, show a DFA, NFA or ε -NFA to recognize the language. Note: if necessary, assume that the language $\{a^n b^n \mid n \geq 1\}$ has been proved as a non-regular language.

- a) $L = \{w \mid w \in \{o, n, y, e, s\}^* \text{ and } w \text{ has the same number of "no" and "yes" substrings}\}$.
- b) $L = \{w \mid w \in \{a, b, c\}^* \text{ and contains an even number of a's and an odd number of b's}\}$.

Group III: [4.5 Pts] Context-Free Grammars (CFG) and Push-Down Automata (PDA)

$S \rightarrow \varepsilon \mid (S)S$

Consider the CFG G in the left.

- a) Show the syntax tree and a leftmost derivation for the string: $(())()$.
- b) The CFG G is ambiguous? Justify. If it is ambiguous, modify the grammar in order to eliminate the ambiguity.
- c) Suppose one wants to represent the L language given by G with a CFG for which all the syntax trees are binary trees. Indicate the new CFG for L .
- d) Indicate a PDA accepting by final state of G .
- e) The previous PDA is deterministic or non-deterministic? Justify your answer.
- f) Indicate a sequence of instantaneous descriptions that result in the acceptance of the string: $()$.
- g) Considering that given a language L , the set of prefixes of L , denoted by $\text{Pref}(L)$, contains all the prefixes of the strings in L (i.e., a string x belongs to $\text{Pref}(L)$ when it exists another string y and xy belongs to L), indicate a grammar for $\text{Pref}(L(G))$.

Group IV: [4 Pts] Turing Machine

We intend to implement a Turing Machine TM to recognize the strings of the language in the alphabet $\{a, b, c\}$ in the format $\{a^n a^p c^{(n-p)} \mid n, p > 0, n \geq p\}$.

Ex.: the strings $abbaa$ and $abbbaac$ belong to the language while the strings $abbbaaac$ and $abbaca$ do not belong. Note: it is not needed to maintain the input string in the end of the computations.

- a) Describe a strategy to implement a TM to perform the recognition of strings of the language.
- b) Draw a possible TM.
- c) Indicate the computing trace when the input to the TM is: $abbac$.

Group V: [5 Pts] Statements about Languages (V/F: 20%, justification: 80%; wrong answer = reduction of 50%)

Indicate, justifying succinctly (with 1 or 2 sentences or a counter example), whether each of the following statements is True or False.

- a) If we show based on the pumping lemma for regular languages that a language defined by a subset of strings of a given language L is non-regular, it is immediately proved that L is non-regular.
- b) The equality $L_1 \cap L_2 = \Sigma^* \setminus ((\Sigma^* \setminus L_1) \cup (\Sigma^* \setminus L_2))$ is wrong (note: " \setminus " stands for the subtraction operation).
- c) The language $L = \{0^i 10^j 10^k \mid k < i+j\}$ is a context-free language (CFL);
- d) If A is an NFA then $L(A)$, i.e., the language represented by A , is a context-free language (CFL).
- e) The intersection of two context-free languages does not result always in a context-free language.
- f) We can determine a regular expression that represents the language of a PDA by converting the PDA to a DFA and then use the state elimination conversion method.
- g) If a context-free grammar (CFG) is ambiguous then the PDA that implements the language of that grammar must be always non-deterministic.
- h) If the tape of a Turing Machine (TM) is finite then the languages that that TM recognizes can be recognized by deterministic PDAs.