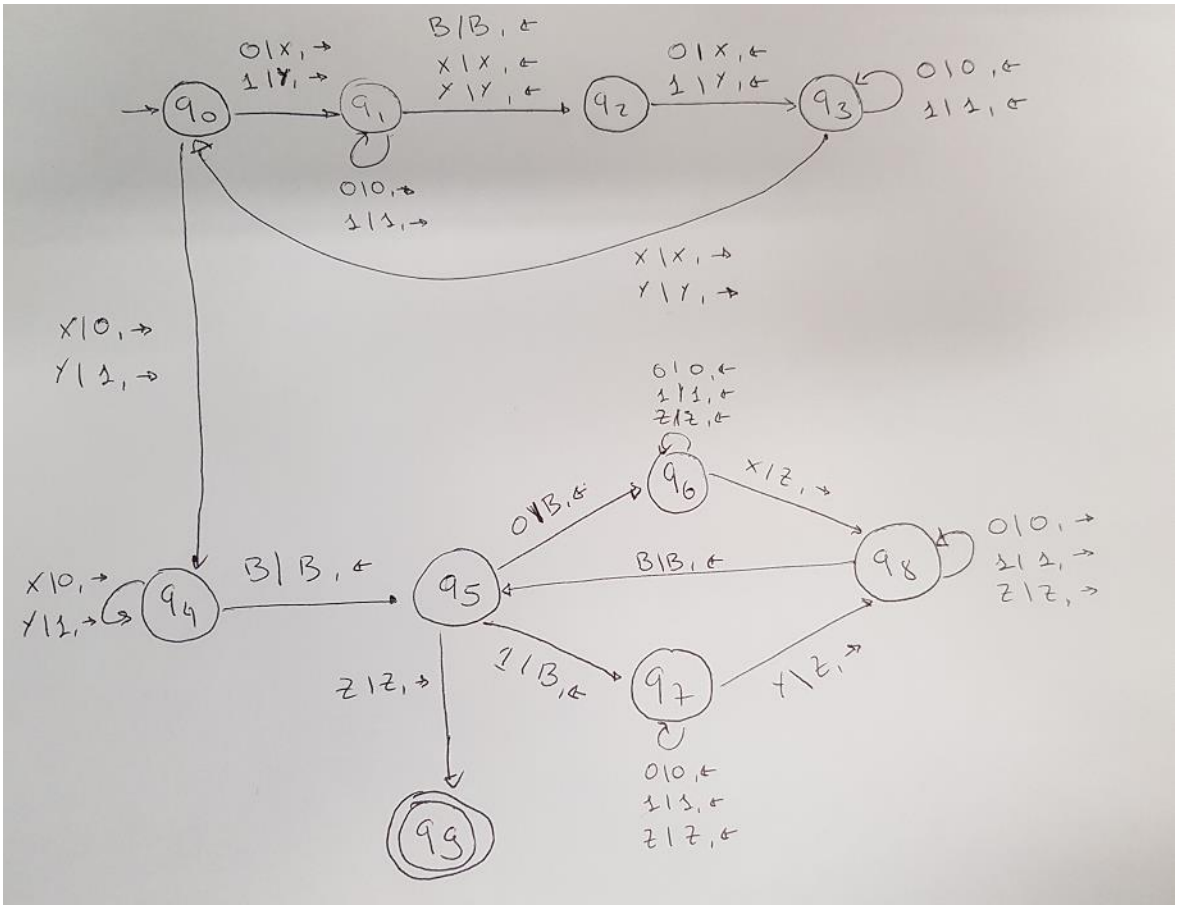
Correction Challenge Activity 3 – Turing Machine

1. As you know, the language L1 = {ww | w is in {0, 1}*} is not a context-free language (CFL). Considering L1, answer to the following questions:

2. Show a TM able to recognize L1; (4 points)

Correction: First, we find the middle of the string, replacing all 0s and 1s with Xs and Ys. If the string has an odd number of characters, the string is not accepted. After finding the middle of the string, the second half of the string is converted back to 0s and 1s. After this, we are at the end of the string. The TM reads the last character, erases it, and travels back until it finds an X or a Y. If the last character was a 0, the TM will look for an X; if it was a 1, it will look for a Y. The character X or Y is replaced by the character Z, then we go again to the end of the string and repeat, until there are no more 0s or 1s.



3. Show the trace of computation of your TM when the input is 0101; (0.5 points)

Correction: (q0)0101⊢ X(q1)101⊢ X1(q1)01⊢ X10(q1)1⊢ X101(q1)B⊢ X10(q2)1⊢ X1(q3)0Y⊢ X(q3)10Y⊢ (q3)X10Y⊢ X(q0)10Y⊢ XY(q1)0Y⊢ XY0(q1)Y⊢ XY(q2)0Y⊢ X(q3)YXY⊢ XY(q0)XY⊢ XY0(q4)Y⊢ XY01(q4)B⊢ XY0(q5)1⊢ XY(q7)0⊢ X(q7)Y0⊢ XZ(q8)0⊢ XZ0(q8)B⊢ XZ(q5)0⊢ X(q6)Z⊢ (q6)XZ⊢ Z(q8)Z⊢ ZZ(q8)B⊢ Z(q5)Z⊢ ZZ(q9) – String accepted

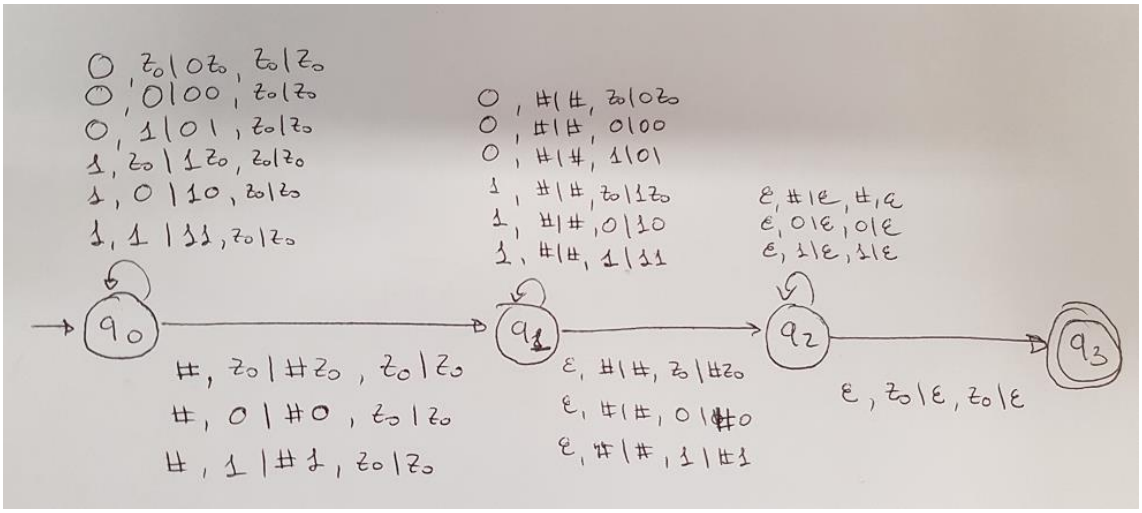
4. Show the trace of computation of your TM when the input is 10; (0.5 points)

Correction: (q0)10⊢ Y(q1)0⊢ Y0(q1)B⊢ Y(q2)0⊢ (q3)YX⊢ Y(q0)X⊢ Y0(q4)B⊢ Y(q5)0⊢ (q6)Y – String is not accepted

5. It is known that a PDA with two stacks is, in terms of computing power, equivalent to a Turing Machine. Consider the language L2 = {w#w | w is in {0, 1}*} and answer to the following questions

6. Show a possible PDA with two stacks that can implement L2. Describe the transition function that you propose for your extended PDA; (4 points)

Correction: The PDA will store the characters that appear before the # in the first stack. Then, it stores the remaining characters in the second stack. After there are no more characters, the PDA compares the contents of the stacks, which must be the same for the string to be accepted.



The transition function depends on the value that is on the top of the two stacks and is of the form $\delta(a, Z1, Z2) = (q, S1, S2)$ where a is the character currently being read in the string, Z1 and Z2 the top of the first and the second stack, respectively, q is the state to where the PDA will transition and S1 and S2 are the characters that will replace the top of the first and second stack, respectively.

7. Show the processing steps with the instantaneous descriptions of your PDA when the input is 01#01; (1 point)

Correction: (q0,01#01,Z0,Z0) → (q0,1#01,0Z0,Z0) → (q0,#01,10Z0,Z0) → (q1,01,#10Z0,Z0) → (q1,1,#10Z0,0Z0) → (q1,ε,#10Z0,10Z0) → (q2,ε,#10Z0,#10Z0) → (q2,ε,10Z0,10Z0) → (q2,ε,0Z0,0Z0) → (q2,ε,Z0,Z0) → (q3,ε,ε,ε) – String is accepted