University of Glasgow

Tuesday, 24 April 2018
9.30 am – 11.00 am
(1 hour 30 minutes)

DEGREES of MSci, MEng, BEng, BSc, MA and MA (Social Sciences)

ALGORITHMICS I (H)

Answer all 4 questions

This examination paper is worth a total of 60 marks.

The use of calculators is not permitted in this examination.

INSTRUCTIONS TO INVIGILATORS: Please collect all exam question papers and exam answer scripts and retain for school to collect. Candidates must not remove exam question papers.

1. **(a)** Define what is meant by a border of a string. Give an example of a string of length at least 3 whose longest border is of length 1.

[3]

(b) Describe the contents of the border table that is used in the Knuth-Morris-Pratt (KMP) string searching algorithm for a string or pattern $s = s_0 s_1 \ldots s_{n-1}$. Explain briefly how this table is used to determine the appropriate action when a mismatch is detected between the ith character of the text and the jth character of the string or pattern being searched for.

[9]

(c) State two advantages that the KMP algorithm has over a naive 'brute-force' approach.

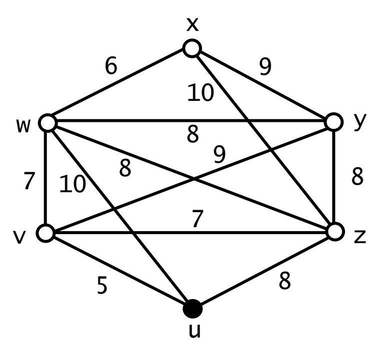
[3]

2. For a weighted graph $G = (V, E, wt)$ explain the following concepts:

(a) a *spanning tree* of G; [2]

(b) the *weight* of a spanning tree of G; [1]

(c) Apply Dijkstra's refinement of the Prim-Jarnik algorithm to find a minimum weight spanning tree of the weighted graph G shown below, using vertex u as the starting tree-vertex.



wt({u,v}) =5
wt({u,w})=10
wt({u,z})=8
wt({v,w})=7
wt({v,y})=9
wt({v,z})=7
wt({w,x})=6
wt({w,y})=8
wt({w,z})=8
wt({x,y})=9
wt({x,z})=10
wt({y,z})=8

Include in your answer the steps performed by the algorithm through the changes to the attribute "best tree vertex" for each vertex of the graph. [9]

(d) Is the minimum weight spanning tree unique? Justify your answer. [3]

3. **(a)** What is meant by each of the following:

(i) the class NP; [1]

(ii) a polynomial-time reduction; [2]

(iii) the statement that a given decision problem Π is NP-complete. [3]

(b) Explain carefully the implications, from the algorithmic point of view, of proving that a decision problem is NP-complete. [2]

(c) Consider the following two decision problems:

Name: Hamiltonian Path (HP)
Instance: undirected (unweighted) graph G;
Question: is there a path in G that visits every vertex exactly once?

Name: Degree-constrained spanning tree (DCST)
Instance: undirected (unweighted) graph G and target integer K;
Question: does G have a spanning tree in which all vertices have degree $\leq K$?
(i.e. the number of edges in the tree incident on any vertex is less than or equal to K)

Suppose that you have a proof that **HP** is NP-complete, present a formal proof showing that **DCST** is NP-complete. [7]

Hint: Consider the correspondence between a spanning tree and a Hamiltonian path?

4. **(a)** Explain the language represented by the regular expression $a|(a(a|b)^*a)$ and then give a deterministic finite state automaton over the alphabet $\Sigma = \{a, b\}$ that accepts this language. [6]

(b) Design a pushdown automaton that recognises the strings $\{a^n b^{2n} \,|\, n \geq 0\}$ over the alphabet $\Sigma = \{a, b\}$. Assume the stack symbols are 1 together with the special symbol \$. [9]

END OF QUESTION PAPER