

FB1:

The limit in question does not exist with just any value of d since the denominator evaluates to 0 when 3 is input as x . Since one cannot divide by zero, the function has to be simplified, which is usually done by factoring both the numerator and denominator. The denominator can be factored as $x(x - 3)$. Since the $(x - 3)$ part is the one which evaluates to 0 when x is 3, it would be very convenient if the numerator was also a multiple of $(x - 3)$. This can be achieved by dividing the numerator by $(x - 3)$ and making the remainder equal 0. Therefore,

$$\frac{x^2 + 2dx - d + 6}{x - 3} = x + 2d + 3,$$

with the remainder of $5d + 15$. When equating the remainder to 0, d is found to be -3 . Hence, the limit becomes

$$\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x(x - 3)} = \lim_{x \rightarrow 3} \frac{(x - 3)^2}{x(x - 3)} = \lim_{x \rightarrow 3} \frac{x - 3}{x} = \frac{0}{3} = 0$$

Another way to find the value of d is by remembering the condition for the use of L'Hôpital's rule, which is that both the numerator and the denominator have to evaluate to 0 or infinity. As such, the numerator can be equated with 0, further supporting that d is -3 .

$$3^2 + 2 \cdot 3d - d + 6 = 0$$

$$5d = -15$$

$$d = -3$$

Therefore, the limit exists when d is -3 , and the limit's value is 0.

FB2:

Let the statement that n^3 is a multiple of 2 be called P, and the statement that the integer n is also a multiple of 2 be called Q. Then the task is to prove

$$P \Rightarrow Q$$

Since n is an integer, its cube will also be an integer. Furthermore, all integers are either even or odd, even integers being multiples of 2 and odd ones not. Let us consider the negation of Q. If n were not a multiple of 2, in other words, an odd number, it could be expressed as

$$n = 2k - 1$$

Therefore, its cube would be

$$n^3 = (2k - 1)^3 = 8k^3 - 12k^2 + 6k - 1$$

Since the first three terms are multiples of 2 and the last term is not, n^3 comes out as an odd number, in other words, not a multiple of 2, making statement P false. Since P cannot be false as it is the given statement, Q cannot be assumed to be false. Thus, statement Q, or the fact that the integer n is a multiple of 2, is proven true by contradiction.

FB3:

Let us draw a Venn diagram of the given situation. The easiest thing to start with is with the fact that A and C have no elements in common; therefore, they can be represented as two distinct circles.



Figure 1. Distinct subsets A and C.

Furthermore, A is a subset of B, which is a statement for which the representation is irrelevant because it is not included in the conclusion.

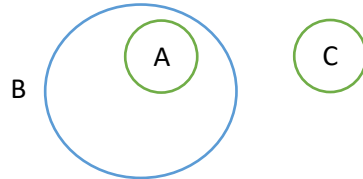


Figure 2. A is a subset of B and has no elements in common with C.

Finally, C is a subset of D, which is a statement that can be drawn in multiple ways, for example, as distinct from A and B, distinct from only A, i.e., having elements in common with only C and B, and having common elements with A, shown below.

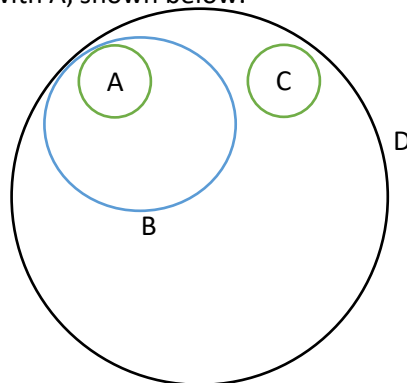


Figure 3. C is a subset of D in a way that A has elements in common with D.

An example would be:

$$A = \{1\}$$

$$B = \{1, 2\}$$

$$C = \{3\}$$

$$D = \{1, 2, 3, 4\}$$

Therefore, the assertion that A and D have no elements in common is false.