2'(C?)735

2C: MATHS 2032

NC: COMPSCI 2024

$\frac{2n}{5n^2-3n-1} < \varepsilon$

$\frac{2n}{5n^2-3n-1} < \frac{2n}{n^2-3n-1} < \frac{2n}{n^2-3n-4} = \frac{2n}{(n-4)(n+1)}$

$\frac{1}{2}5n^2 \le 5n^2-3n-1 \le \frac{3}{2}5n^2$

$2n \le 2n^2$

$\frac{2}{15n^2} \le \frac{1}{5n^2-3n-1} \le \frac{2}{5n^2}$

$\frac{2n}{\dots} \le \frac{4}{5}$

a $\frac{1}{n^2} > 0, \searrow, \to 0$ abrol $\left(\frac{1}{n^r}\right)$

b. $(-1)^n \frac{1}{2^n} > 0, \searrow, \to 0$, abrol $\left(\frac{1}{a^n}\right)$

c) $(-1)^n \frac{1}{n+2} > 0, \searrow, \to 0$, ~~abrol~~ cond

d)

e) $(-1)^n \frac{1}{2^n(n+1)} > 0, \searrow, \to 0$, abrol. $\left(\frac{1}{a^n}\right)$

P: $x > y$ Converse:

Q: $x^3 > y^3$ $Q \to P$

$P \to Q$:   contrapositive

$\neg P \to \neg Q$

$x \le y \to x^3 \le y^3$

$15(n-15)(n+3)$

$\frac{2n}{5n^2-3n-1} - 3 = \frac{2n-15n^2+9n+3}{5n^2-3n-1} = -\frac{15n^2-11n-3}{5n^2-3n-1} < -\frac{15n^2-12n-3}{5n^2-3n-1} = \frac{-15(n-15)(n+3)}{5n^2-3n-1} < \frac{-15(n-1)(n+\frac{1}{5})}{5n^2-3n-2}$

$= \frac{-15(n-1)(n+\frac{1}{5})}{5(n-\frac{5}{5})(n+\frac{2}{5})}$

1. i) $3x > 2\cdot 3$ $3x-2y+\frac{1}{2z} > 2\cdot 3 - 2\cdot 4 + \frac{1}{8}$ $\sup((2,4)) = 4$

$2y < 2\cdot 4$ $\inf((2,4)) = 2$

$\frac{1}{2z} > \frac{1}{2\cdot 4}$

$3x < 4\cdot 3$ $3x-2y+\frac{1}{2z} < 4\cdot 3 - 2\cdot 2 + \frac{1}{2\cdot 2}$ non-

$-2y < -2\cdot 2$

$\frac{1}{2z} < \frac{1}{2\cdot 2}$

ii) Set $A = \left\{\frac{2n+5}{7n+7} \mid n \in \mathbb{N}\right\}$. For $n \in \mathbb{N}$, we have $\frac{2n+5}{7n+7} \geq \frac{2}{7}$ because, so $\frac{2}{7}$ is a lower bound for A

Set $\varepsilon > 0$ b a. For $n \in \mathbb{N}$, $\frac{2n+5}{7n+7} < \frac{2}{7} + \varepsilon \Leftrightarrow 2n+5 < (7n+7)\left(\frac{2}{7}+\varepsilon\right) = 2n+2+\varepsilon(7n+7) \Leftrightarrow 7n+7 > \frac{3}{\varepsilon} \Leftrightarrow n > \frac{3}{7\varepsilon} - 1$

$2(n+1) - 7\varepsilon(n+1)$

2. i) $\forall \varepsilon>0, \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, (n \geq n_0 \Rightarrow |a_n - L| < \varepsilon)$

$$-\varepsilon < a_n - L < \varepsilon$$

$$L-\varepsilon < a_n < \varepsilon + L$$

ii) $|f(x) - f(1)| = \left|\frac{x^2-1-x-2}{x+2}\right| = \left|\frac{x^2-x-3}{x+2}\right|$

$$\frac{x^2-1}{x+2} = \cancel{\frac{(x-1)(x+1)}{(x+2)}} = \cancel{x+1}$$

$$= \frac{(x-1)(x+1)}{(x+2)}$$

3. $\lim_{n\to\infty} \frac{4n^3+2n^2}{3n^4+(-1)^n} = \lim_{n\to\infty} \frac{4+\frac{2}{n}}{3n+\frac{(-1)^n}{n^3}} = \lim_{n\to\infty} \frac{4}{3n} = 0$

4. i) $\sum_{n=1}^{\infty} (-1)^n \frac{2n^2+1}{n^2}$

$(a_n)_{n=1}^{\infty} = \frac{2n^2+1}{n^2} \geq 0$ ~~and~~ $\frac{a_{n+1}}{a_n} = \frac{(2(n+1)^2+1)\, n^2}{(n+1)^2(2n^2+1)}$ ~~$\neq \frac{2n^4}{...}$~~

~~Because~~ $a_{n+1} - a_n = \frac{2(n+1)^2+1}{4n^2+2n+1} - \frac{2n^2+1}{n^2} = \frac{(2n^2+4n+3)n^2 - (2n^2+1)}{}$

$a_n - a_{n+1} = \frac{2n^2+1}{n^2} - \frac{2n^2+4n+3}{n}$

$\frac{a_n}{a_{n+1}} = \frac{(2n^2+1)(n+1)^2}{n^2(2(n+1)^2+1)} = \frac{(2n^2+1)(n+1)^2}{n^2(2n^2+4n+3)} = \frac{2n^4+4n^3+3n^2+2n+1}{2n^4+4n^3+3n^2} > 1$

$\frac{2n^2+1}{n^2} \to \frac{2}{1} = 2 \Rightarrow$ diverges

ii) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2} \leq \frac{1}{n^2} \to 0$

Comparison test $\Rightarrow$ absol. conv.

iii) $\frac{5^n}{3^n+3n!} > 0$ $\quad \frac{5 \cdot 5^n \cdot 3^n}{3 \cdot 3^n \cdot 5^n} > 1$ $\quad \frac{\cancel{5^n} \cdot 5 \cdot \cancel{n!}}{3(n+1)\cancel{n!} \cdot \cancel{5^n}} \to 0$

absol. converges

5. i) $\tan(\frac{\pi}{2}x) = y$

ii) Define $h(x) = f(x) - (1-x^2)$. $h(1) = f(1) - 0 \geq 0$
$h(0) = f(0) - 1 \leq 0$

Since f is continuous and $x \mapsto 1-x^2$ is continuous, $h(x)$ is continuous.

By the intermediate value theorem, there exists .... ✓

6. i)