FB1:

The limit in question does not exist with just any value of *d* since the denominator evaluates to 0 when 3 is input as x. Since one cannot divide by zero, the function has to be simplified, which is usually done by factoring both the numerator and denominator. The denominator can be factored as $\mathrm{x}(\mathrm{x}-3)$. Since the $(x-3)$ part is the one which evaluates to 0 when x is 3, it would be very convenient if the numerator was also a multiple of $(x-3)$. This can be achieved by dividing the numerator by $(x-3)$ and making the remainder equal 0. Therefore,

$$\frac{x^2+2dx-d+6}{x-3}=x+2d+3,$$

with the remainder of $5d\ +15$. When equating the remainder to 0, *d* is found to be $-3$. Hence, the limit becomes

$$\lim_{x\to 3}\frac{x^2-6x+9}{x(x-3)}=\lim_{x\to 3}\frac{(x-3)^2}{x(x-3)}=\lim_{x\to 3}\frac{x-3}{x}=\frac{0}{3}=0$$

Another way to find the value of *d* is by remembering the condition for the use of L'Hôpital's rule, which is that both the numerator and the denominator have to evaluate to 0 or infinity. As such, the numerator can be equated with 0, further supporting that *d* is $-3$.

$$3^2+2\cdot 3d-d+6=0$$

$$5d=-15$$

$$d=-3$$

Therefore, the limit exists when *d* is $-3$, and the limit's value is 0.

FB2:

Let the statement that $n^3$ is a multiple of 2 be called P, and the statement that the integer $n$ is also a multiple of 2 be called Q. Then the task is to prove

$$P\Rightarrow Q$$

Since $n$ is an integer, its cube will also be an integer. Furthermore, all integers are either even or odd, even integers being multiples of 2 and odd ones not. Let us consider the negation of Q. If $n$ were not a multiple of 2, in other words, an odd number, it could be expressed as

$$n=2k-1$$

Therefore, its cube would be

$$n^3=(2k-1)^3=8k^3-12k^2+6k-1$$

Since the first three terms are multiples of 2 and the last term is not, $n^3$ comes out as an odd number, in other words, not a multiple of 2, making statement P false. Since P cannot be false as it is the given statement, Q cannot be assumed to be false. Thus, statement Q, or the fact that the integer $n$ is a multiple of 2, is proven true by contradiction.

FB3:

Let us draw a Venn diagram of the given situation. The easiest thing to start with is with the fact that A and C have no elements in common; therefore, they can be represented as two distinct circles.


*Figure 1. Distinct subsets A and C.*

Furthermore, A is a subset of B, which is a statement for which the representation is irrelevant because it is not included in the conclusion.

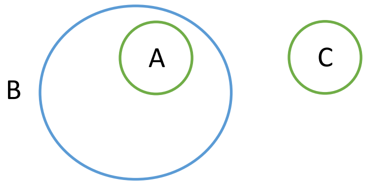


*Figure 2. A is a subset of B and has no elements in common with C.*

Finally, C is a subset of D, which is a statement that can be drawn in multiple ways, for example, as distinct from A and B, distinct from only A, i.e., having elements in common with only C and B, and having common elements with A, shown below.

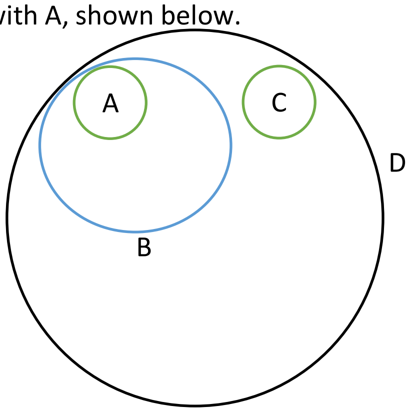


*Figure 3. C is a subset of D in a way that A has elements in common with D.*

An example would be:

A = {1}
B = {1, 2}
C = {3}
D = {1, 2, 3, 4}

Therefore, the assertion that A and D have no elements in common is false.