

Laboratory Sheet 3

This lab sheet contains material based on Lectures 7-9. This exercise is not assessed but should be completed to gain sufficient experience in the implementation of elementary abstract data types, and the testing thereof.

Exercise

You are to implement the Dequeue abstract data type (ADT) using two different data structures. Then, you will use this ADT to define generic Stack and Queue ADTs. Finally, you will use a stack to implement a non-recursive quicksort.

Part 1

- a) Implement in Java the merge sort algorithm for linked lists introduced in Lecture 7 (slide 25). Use the following class structure to implement linked lists.

```
public class LinkedList<Item>{
    private Node<Item> head;

    private static class Node<Item>{
        private Item key;
        private Node<Item> next;
    }

    public LinkedList(){
        head = null;
    }

    ...
}
```

The implementation is straightforward following the pseudocode in slides 27, 29 and 36.

Part 2

- a) Implement in Java the Dequeue abstract data type introduced in Lecture 9 (slide 35). Use resizable arrays in a circular fashion (slide 16) in the class below. What is the time complexity of each operation (i.e. PUSH-BACK, PUSH-FRONT, POP-BACK, POP-FRONT)?

```
public class ResizingDequeue<Item>{
    private Item[] q;
    private int n; // number of elements in the dequeue
    private int tail;
    private int head;

    public ResizingDequeue (){
        q = (Item[]) new Object[2];
        n = 0;
        head = 0;
        tail = 0;
    }

    ...
}
```

See Lecture 9 for implementation details. All operations have amortised $O(1)$ time complexity.

- b) Implement the Dequeue abstract data type using a circular doubly linked list. Modify the class structure given in 1a for singly linked lists to include `prev` pointers and a sentinel. What is the time complexity of each operation?

```
public class LinkedDeque<Item>{
    private Node<Item> nil; // sentinel
    private int n; // number of elements in the dequeue

    private static class Node<Item>{
        private Item key;
        private Node<Item> next;
        private Node<Item> prev;
    }

    public LinkedDeque(){
        nil = new Node();
        nil.prev = nil;
        nil.next = nil;
        n = 0;
    }

    ...
}
```

See Lecture 7 for the implementation details of a circular doubly linked list with sentinel. All operations have $O(1)$ time complexity.

Part 3

- a) Implement the Stack ADT using the Dequeue you implemented in 2b.

Implement `Pop` with `PopFront` and `Push` with `PushFront`.

- b) Implement the Queue ADT using the Dequeue you implemented in 2b.

Implement `Dequeue` with `PopFront` and `Enqueue` with `PushBack`.

- c) Implement the Queue ADT using two stacks. What is the time complexity of each operation?

There are several possible ways to do this. Below, the sketch of one solution in which stack `S1` always stores the oldest element on the top.

ENQUEUE(Q,x)

1. While `S1` is not empty, push everything from `S1` to `S2`
2. Push `x` to `S1`
3. Push everything back to `S1`

DEQUEUE(Q)

1. If `S1` is empty, then underflow error
2. Pop an item from `S1` and return it

In this implementation, `ENQUEUE` is $O(n)$ while `DEQUEUE` is $O(1)$.

Part 4

Using an auxiliary stack, implement an iterative version of quicksort. To reduce the stack size, first push the indexes of the smaller subarray.

Below is a simple solution without the optimisation on the stack size.

```
void quicksortIter (int a[], int p, int r){
    Stack<int> stack = new Stack<int>();

    stack.push(p);
    stack.push(r);

    while (!stack.isEmpty()){
        r = stack.pop();
        p = stack.pop();
        int q = partition(a, p, r);

        if (q-1 > p){
            stack.push(p);
            stack.push(q-1);
        }

        if (q+1 < r){
            stack.push(q+1);
            stack.push(r);
        }
    }
}
```