FB1: Find a parametric equation of the line passing through the points A(1, 2, 4) and B(11,-8, 26) and find the point where this line intersects the line

$$L1: x = 1 + s, y = 2 - s, z = 3s,$$

by solving a system of linear equations.

To find the line that passes through both points, one needs a point on the line and a vector parallel to the line. The vector can be found by calculating $\overrightarrow{AB}$, a vector from point A to point B,

$$\begin{aligned} &\overrightarrow{AB} = \boldsymbol{d} = \langle 11 - 1, -8 - 2, 26 - 4 \rangle \\ &= \langle 10, -10, 22 \rangle. \end{aligned}$$

Furthermore, any of the two given points can be chosen to make a position vector required for this line, for example, let vector **a** be

$$\boldsymbol{a} = \langle 1, 2, 4 \rangle.$$

Thus, the second line L2's vector equation is

$$L2: \boldsymbol{a} + \boldsymbol{d}t$$

$$\Rightarrow L2: \boldsymbol{r} = \langle 1, 2, 4 \rangle + \langle 10, -10, 22 \rangle t,$$

where *t* is a real number.

To find the intersection of both lines, a system of linear equations can be formed by using the parametric equations of L1 and equating them with the equations of L2:

$$\begin{aligned} &1 + s = 1 + 10t \\ &2 - s = 2 - 10t \\ &3s = 4 + 22t. \end{aligned}$$

By performing elementary row operations 3R2 + R3, one finds that

$$\begin{aligned} &6 = 10 - 8t \\ &\Rightarrow t = \frac{1}{2}. \end{aligned}$$

By substituting *t* as $\frac{1}{2}$ in R2:

$$s = 5.$$

Then substituting *t* and *s* in R1 to check for consistency,

$$\begin{aligned} &1 + 5 = 1 + 10 \cdot \frac{1}{2} \\ &\Rightarrow 6 = 6. \end{aligned}$$

Thus, the intersection of L1 and L2 is

$$\begin{aligned} &(1 + 5, 2 - 5, 3 \cdot 5) \\ &\Rightarrow (6, -3, 15). \end{aligned}$$

FB2: Consider an arbitrary 2 x 2 matrix with real entries, A, and let B be the matrix

$$B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

a) What restrictions must be placed on the entries of A in order for tr(A) = tr(AB)?

Let A be defined as

$$A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}.$$

Then the matrix product AB is

$$AB = \begin{bmatrix} a_1 + a_2 & a_1 + a_2 \\ a_3 + a_4 & a_3 + a_4 \end{bmatrix}.$$

Since $\mathrm{tr(A)} = a_1 + a_4$ and $\mathrm{tr(AB)} = a_1 + a_2 + a_3 + a_4$, the equation is

$$a_1 + a_4 = a_1 + a_2 + a_3 + a_4,$$

which means that

$$a_2 + a_3 = 0.$$

b) Show that if det (A) = det (AB), then A is not invertible.

Since $\det(\mathrm{A}) = a_1 a_4 - a_2 a_3$ and

$$\det(\mathrm{AB}) = (a_1 + a_2)(a_3 + a_4) - (a_1 + a_2)(a_3 + a_4) = 0,$$

$$\det(\mathrm{A}) = 0,$$

which means that A is not invertible.

FB3:

a) Write down and simplify the expansion of $\left(a + \frac{b}{a}\right)^6$, for $a, b \in R \setminus 0$. (*Hint*: Use the Binomial Theorem.)

By using the Binomial Theorem, the expansion is

$$\binom{6}{0} a^6 \left(\frac{b}{a}\right)^0 + \binom{6}{1} a^5 \left(\frac{b}{a}\right)^1 + \binom{6}{2} a^4 \left(\frac{b}{a}\right)^2 + \binom{6}{3} a^3 \left(\frac{b}{a}\right)^3 + \binom{6}{4} a^2 \left(\frac{b}{a}\right)^4 + \binom{6}{5} a^1 \left(\frac{b}{a}\right)^5 + \binom{6}{6} a^0 \left(\frac{b}{a}\right)^6$$

$$= a^6 + 6a^4 b + 15a^2 b^2 + 20b^3 + 15\frac{b^4}{a^2} + 6\frac{b^5}{a^4} + \frac{b^6}{a^6}$$

b) What is the coefficient of the b^3 term?

The coefficient of the b^3 term is 20.

c) Let b = 1. What is the simplified form of the expression now?

If b = 1, the expression is

$$a^6 + 6a^4 + 15a^2 + 20 + \frac{15}{a^2} + \frac{6}{a^4} + \frac{1}{a^6}.$$