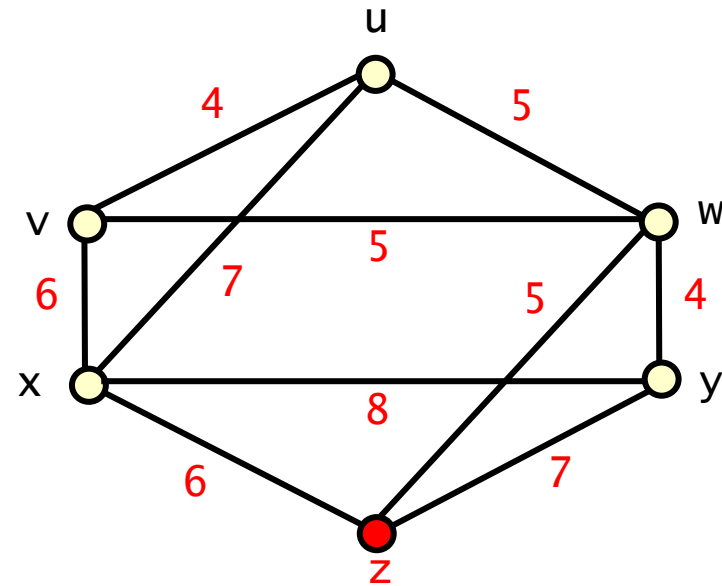


# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$



● tree vertices:  $z$

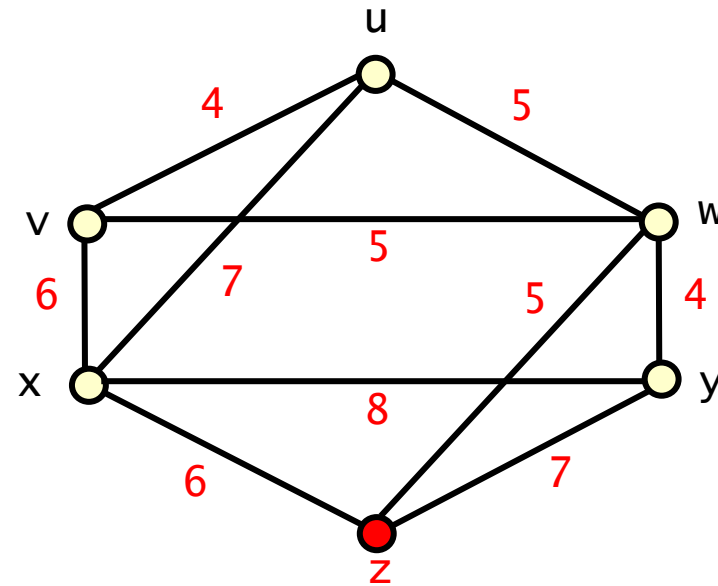
○ non-tree vertices:  $u, v, w, x, y$

# Dijkstra's refinement – Example

## Weighted graph **G**

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	z	$\infty$
v	z	$\infty$
w	z	5
x	z	6
y	z	7
z	–	–



● **tree vertices:** z

○ **non-tree vertices:** u, v, w, x, y

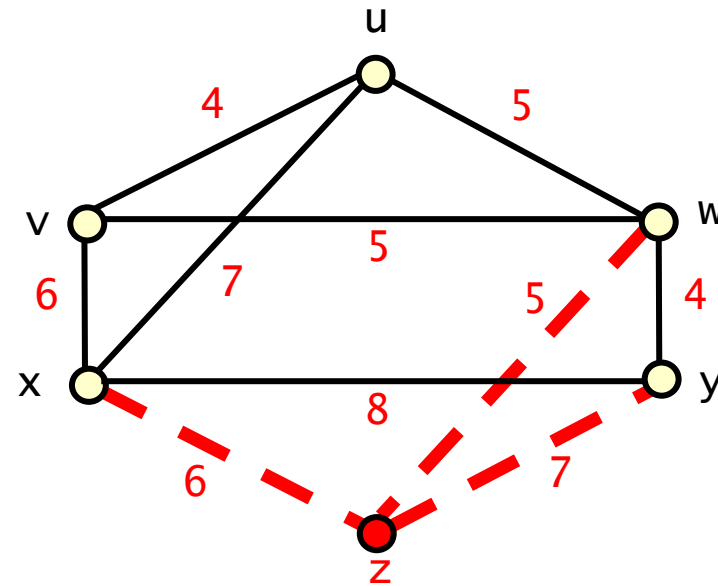
initialise **bestTV** to the only **tv** z

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	z	$\infty$
v	z	$\infty$
w	z	5
x	z	6
y	z	7
z	–	–



● tree vertices: z

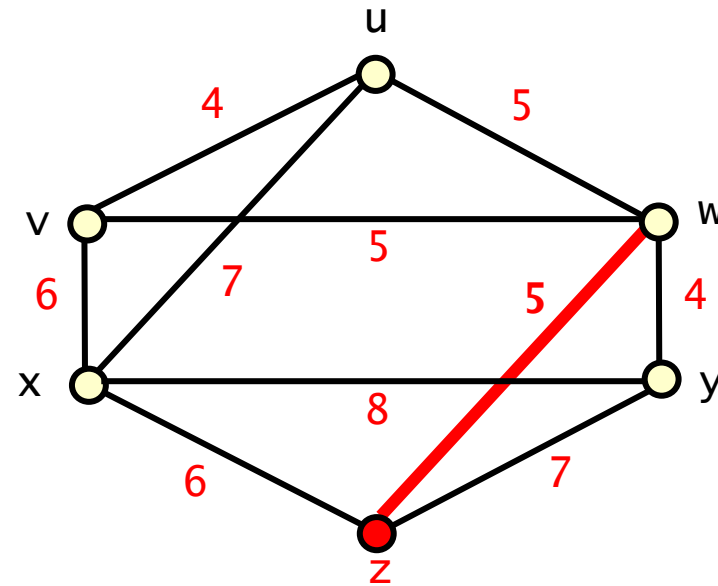
○ non-tree vertices: u, v, w, x, y

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	z	$\infty$
v	z	$\infty$
<b>w</b>	<b>z</b>	<b>5</b>
x	z	6
y	z	7
z	–	–



● tree vertices: z

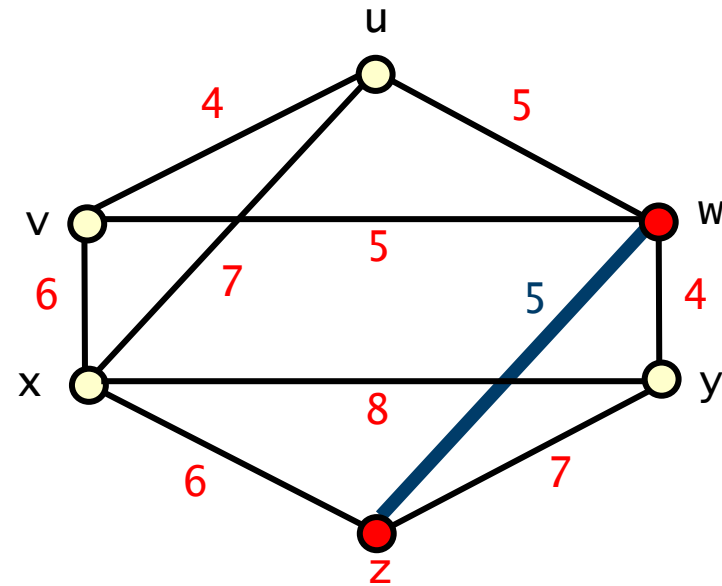
○ non-tree vertices: u, v, w, x, y

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $wt(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$wt(\{q.\text{bestTV}, q\})$
u	z	$\infty$
v	z	$\infty$
w	z	5
x	z	6
y	z	7
z	–	–



● tree vertices: w, z

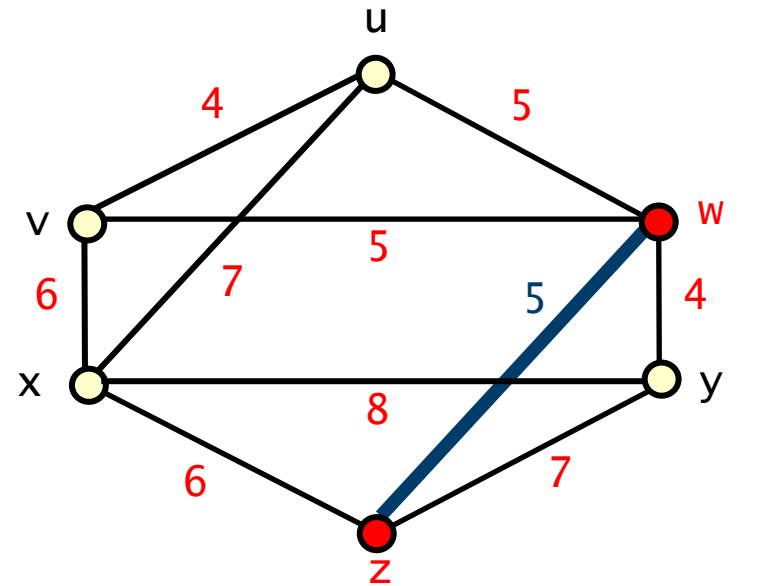
○ non-tree vertices: u, v, x, y

# Dijkstra's refinement – Example

## Weighted graph **G**

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	$z \rightarrow w$	$\infty \rightarrow 5$
v	$z \rightarrow w$	$\infty \rightarrow 5$
w	–	–
x	z	6
y	$z \rightarrow w$	$7 \rightarrow 4$
z	–	–



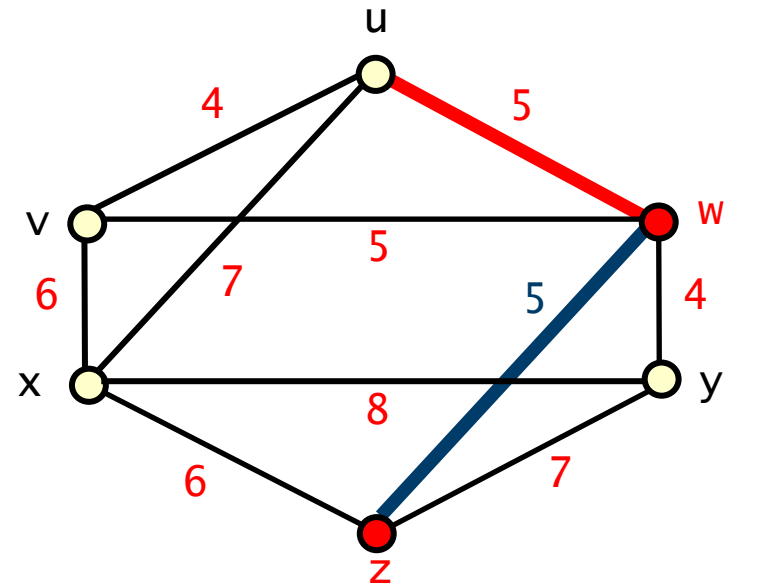
● tree vertices: w, z  
○ non-tree vertices: u, v, x, y

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
<b>u</b>	<b><math>z \rightarrow w</math></b>	<b><math>\infty \rightarrow 5</math></b>
v	$z \rightarrow w$	$\infty \rightarrow 5$
w	–	–
x	z	6
y	$z \rightarrow w$	$7 \rightarrow 4$
z	–	–



● tree vertices: w, z

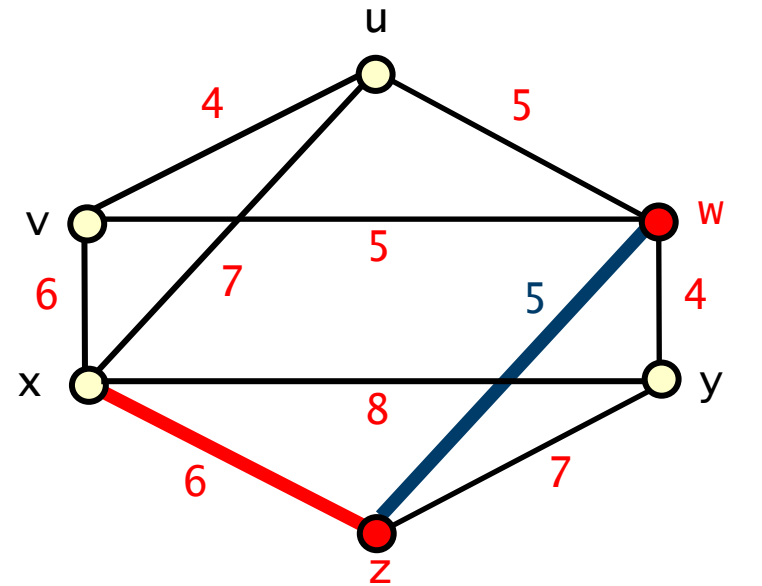
○ non-tree vertices: u, v, x, y

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
$u$	$z \rightarrow w$	$\infty \rightarrow 5$
$v$	$z \rightarrow w$	$\infty \rightarrow 5$
$w$	–	–
<b><math>x</math></b>	<b><math>z</math></b>	<b><math>6</math></b>
$y$	$z \rightarrow w$	$7 \rightarrow 4$
$z$	–	–



● tree vertices:  $w, z$

○ non-tree vertices:  $u, v, x, y$

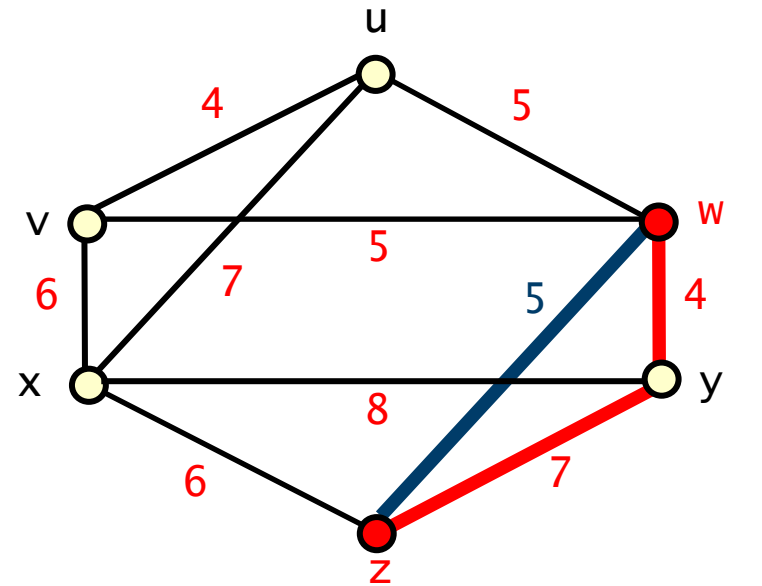


# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	$z \rightarrow w$	$\infty \rightarrow 5$
v	$z \rightarrow w$	$\infty \rightarrow 5$
w	–	–
x	z	6
<b>y</b>	<b><math>z \rightarrow w</math></b>	<b><math>7 \rightarrow 4</math></b>
z	–	–



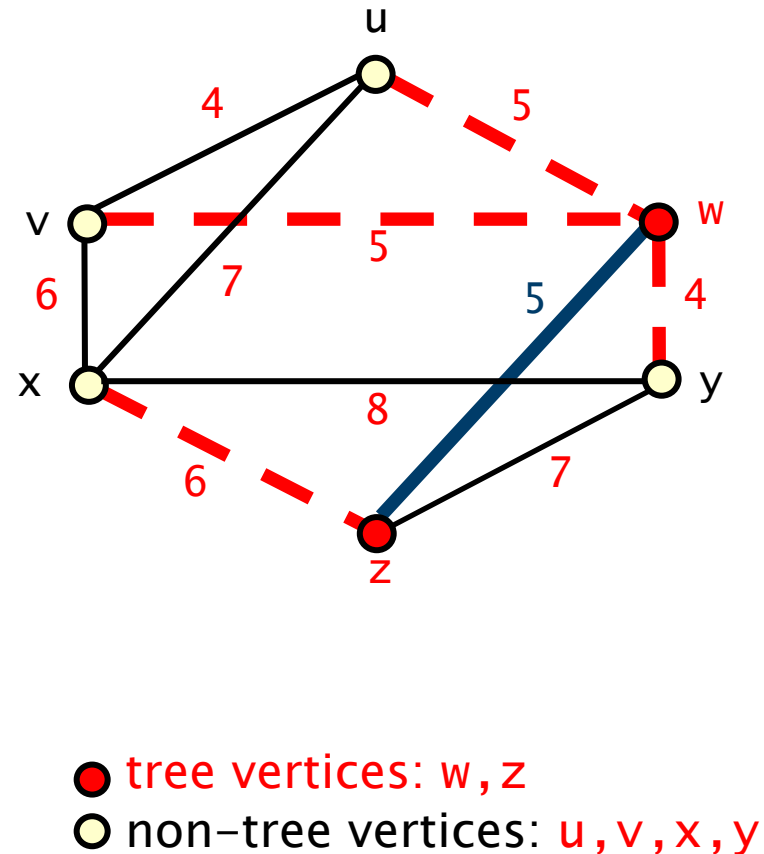
● **tree vertices:** w, z  
○ **non-tree vertices:** u, v, x, y

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	w	5
v	w	5
w	–	–
x	z	6
y	w	4
z	–	–

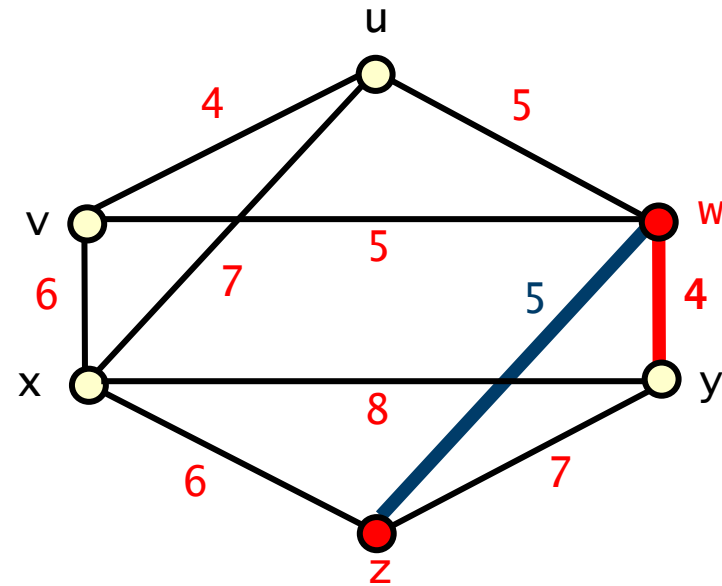


# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	w	5
v	w	5
w	–	–
x	z	6
y	w	4
z	–	–



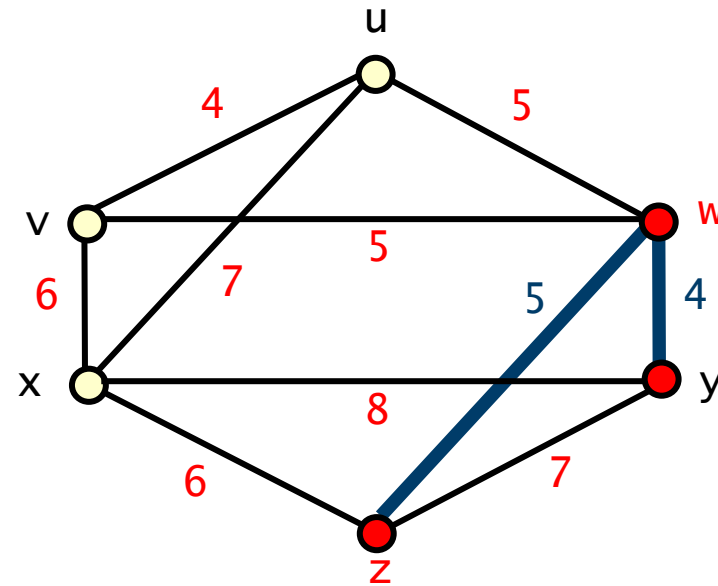
● tree vertices: w, z  
○ non-tree vertices: u, v, x, y

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	w	5
v	w	5
w	–	–
x	z	6
y	w	4
z	–	–



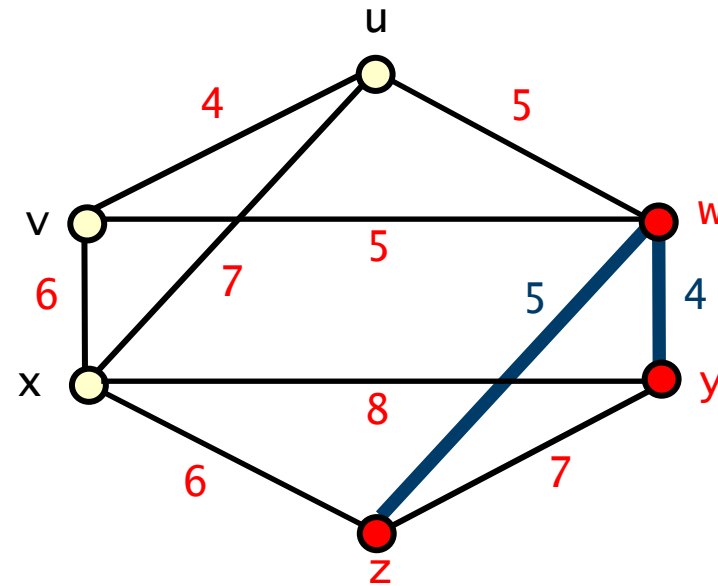
● tree vertices: w, y, z  
○ non-tree vertices: u, v, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	w	5
v	w	5
w	–	–
x	z	6
y	–	–
z	–	–



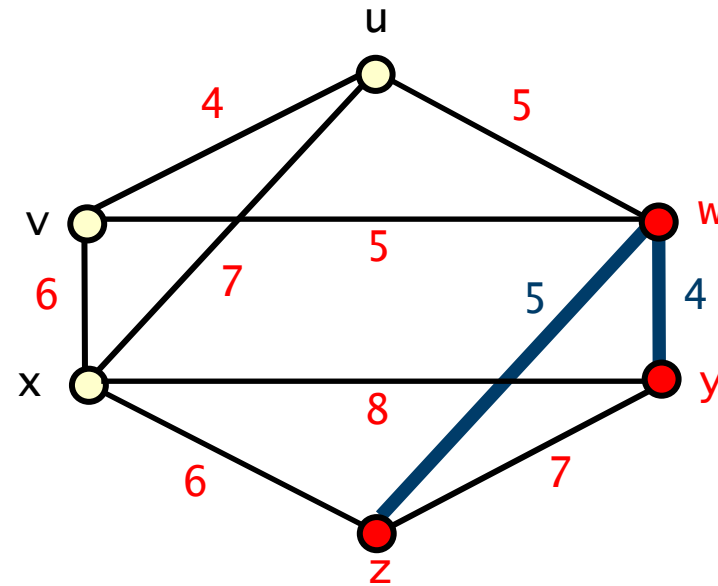
● tree vertices: w, y, z  
○ non-tree vertices: u, v, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
<b>u</b>	<b>w</b>	<b>5</b>
<b>v</b>	<b>w</b>	<b>5</b>
w	–	–
x	z	6
y	–	–
z	–	–



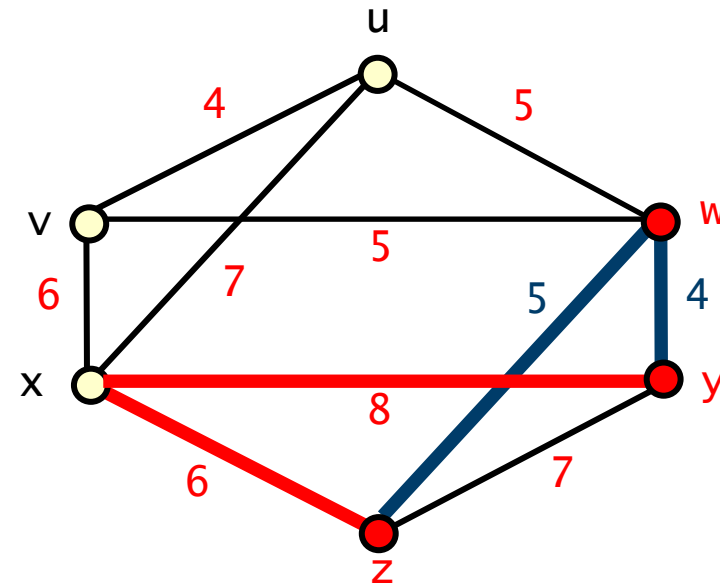
● **tree vertices:** w, y, z  
○ **non-tree vertices:** u, v, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	w	5
v	w	5
w	–	–
<b>x</b>	<b>z</b>	<b>6</b>
y	–	–
z	–	–



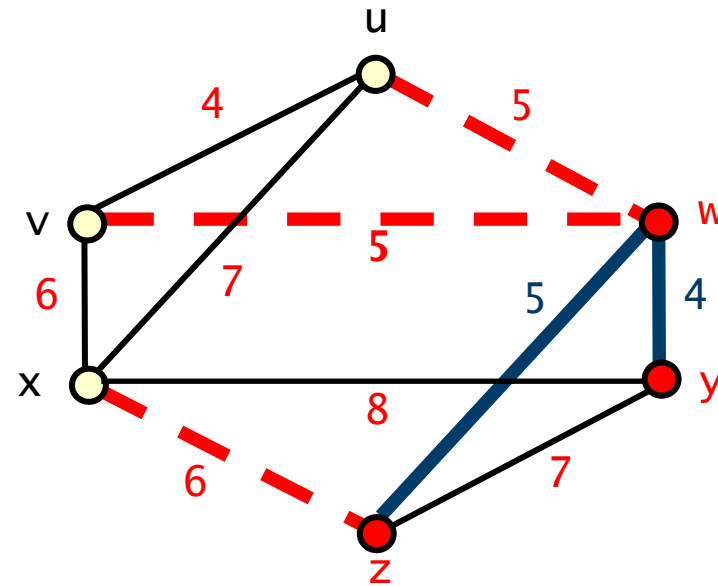
● **tree vertices:** w, y, z  
○ **non-tree vertices:** u, v, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	w	5
v	w	5
w	–	–
x	z	6
y	–	–
z	–	–



● tree vertices: w, y, z  
○ non-tree vertices: u, v, x

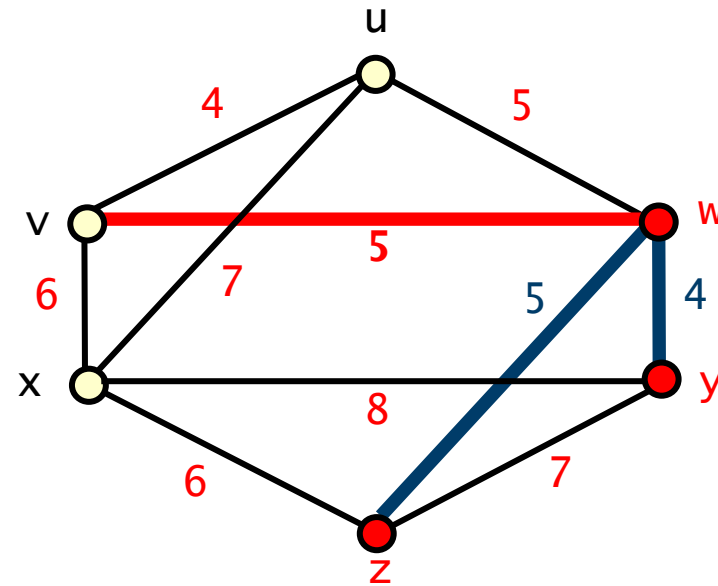


# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	w	5
<b>v</b>	<b>w</b>	<b>5</b>
w	–	–
x	z	6
y	–	–
z	–	–



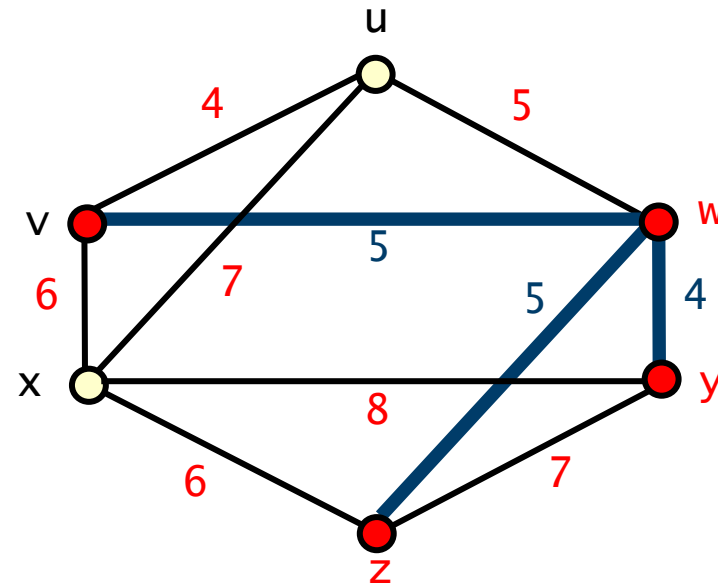
● tree vertices: w, y, z  
○ non-tree vertices: u, v, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $wt(\{q, q.bestTV\})$  is minimal and make  $q$  a **tv**
- update  $s.bestTV$  for all **ntv**  $s$

$q$	$q.bestTV$	$wt(\{q.bestTV, q\})$
u	w	5
v	w	5
w	–	–
x	z	6
y	–	–
z	–	–



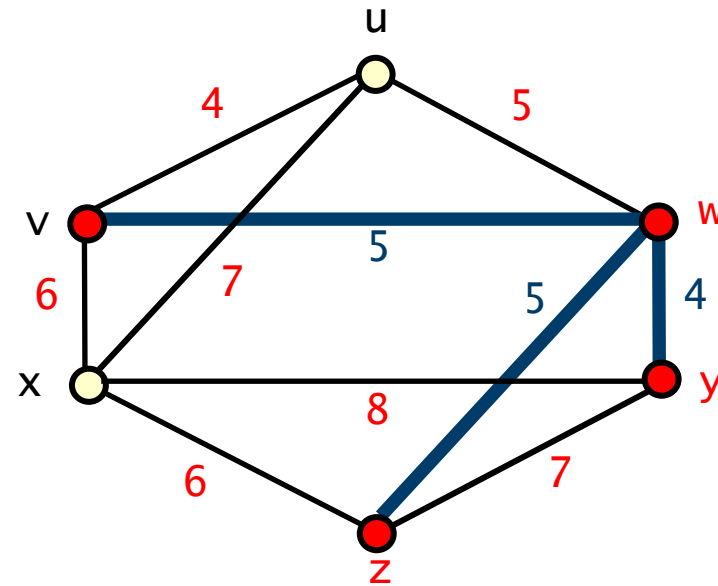
● tree vertices: v, w, y, z  
○ non-tree vertices: u, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
$u$	$w \rightarrow v$	$5 \rightarrow 4$
$v$	–	–
$w$	–	–
$x$	$z$	6
$y$	–	–
$z$	–	–



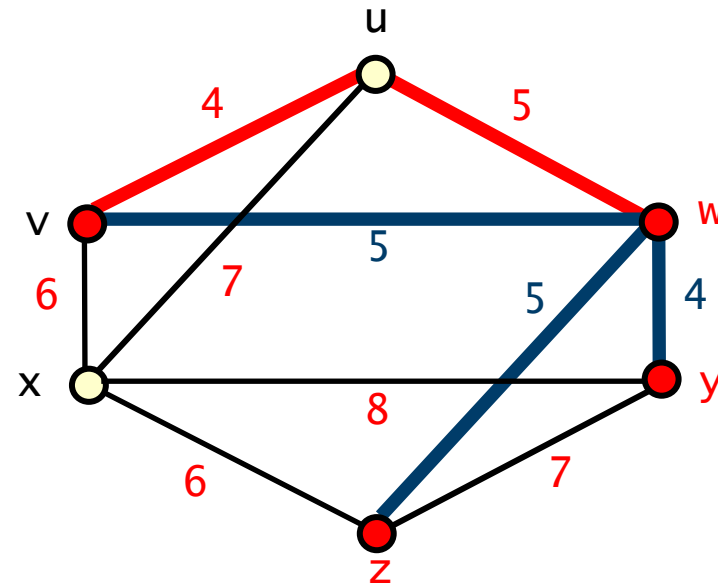
● tree vertices:  $v, w, y, z$   
○ non-tree vertices:  $u, x$

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
<b>u</b>	<b>w</b> → <b>v</b>	<b>5</b> → <b>4</b>
v	–	–
w	–	–
x	z	6
y	–	–
z	–	–



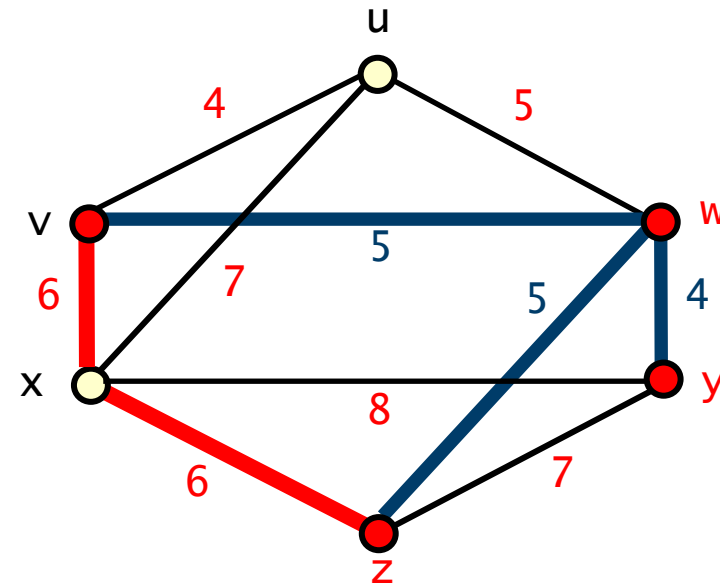
● tree vertices: v, w, y, z  
○ non-tree vertices: u, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
$u$	$w \rightarrow v$	$5 \rightarrow 4$
$v$	–	–
$w$	–	–
<b><math>x</math></b>	<b><math>z</math></b>	<b><math>6</math></b>
$y$	–	–
$z$	–	–



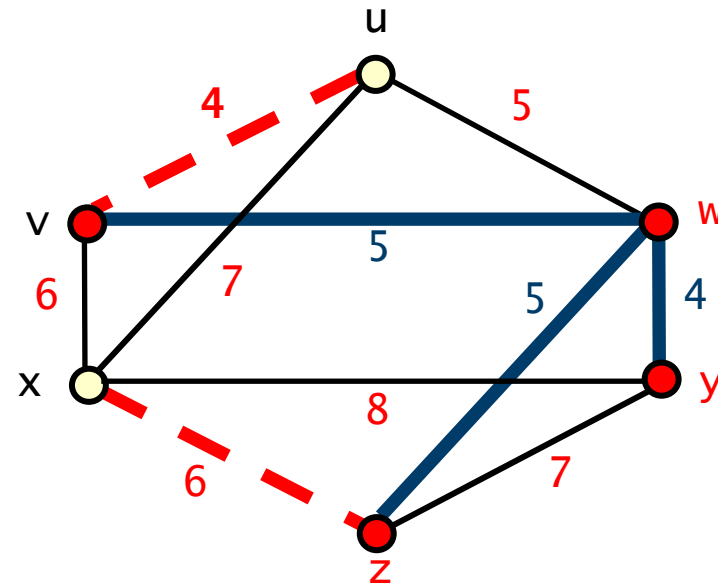
● tree vertices:  $v, w, y, z$   
○ non-tree vertices:  $u, x$

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
$u$	$v$	4
$v$	–	–
$w$	–	–
$x$	$z$	6
$y$	–	–
$z$	–	–



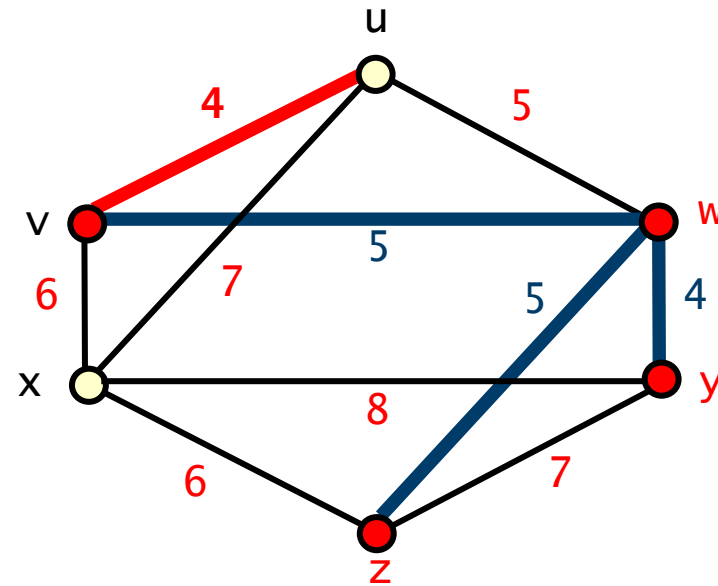
● tree vertices:  $v, w, y, z$   
○ non-tree vertices:  $u, x$

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $wt(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$wt(\{q.\text{bestTV}, q\})$
<b>u</b>	<b>v</b>	<b>4</b>
v	–	–
w	–	–
x	z	6
y	–	–
z	–	–



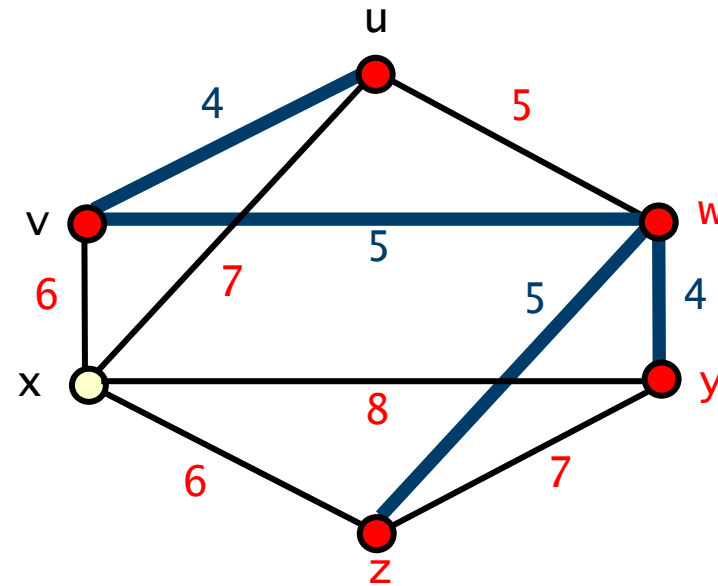
● tree vertices: v, w, y, z  
○ non-tree vertices: u, x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	v	4
v	–	–
w	–	–
x	z	6
y	–	–
z	–	–



● tree vertices: u, v, w, y, z

○ non-tree vertices: x

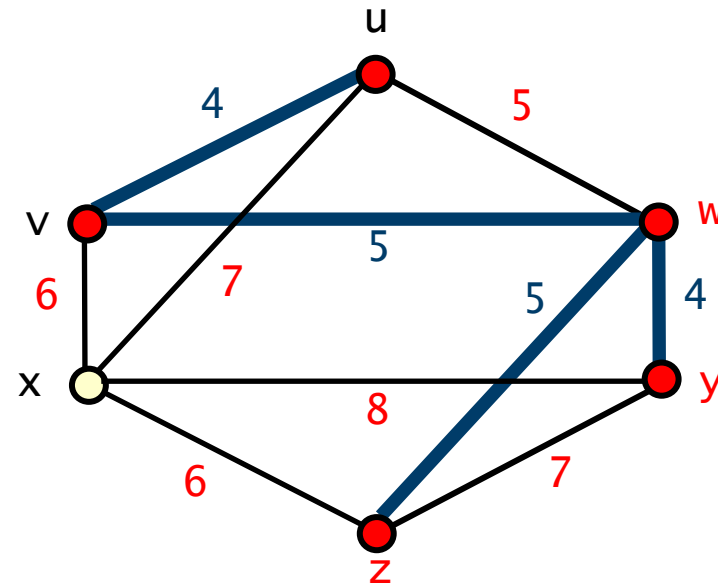


# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	–	–
v	–	–
w	–	–
x	z	6
y	–	–
z	–	–



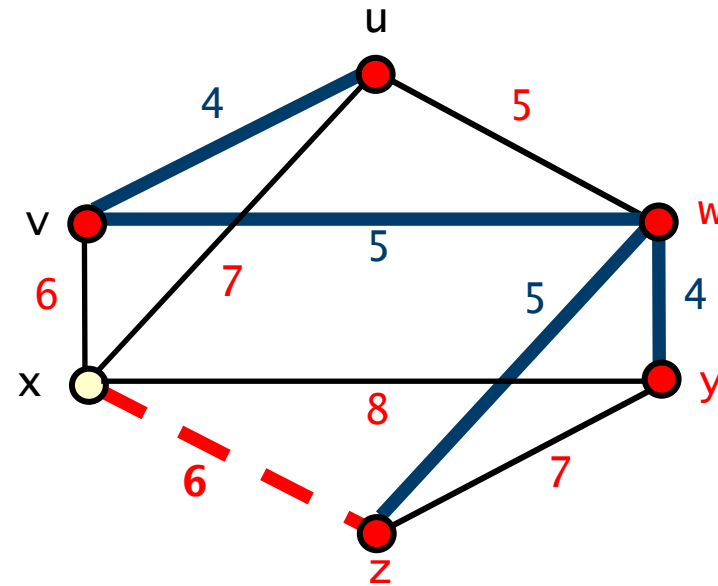
● tree vertices: u, v, w, y, z  
○ non-tree vertices: x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	–	–
v	–	–
w	–	–
x	z	6
y	–	–
z	–	–



● tree vertices: u, v, w, y, z

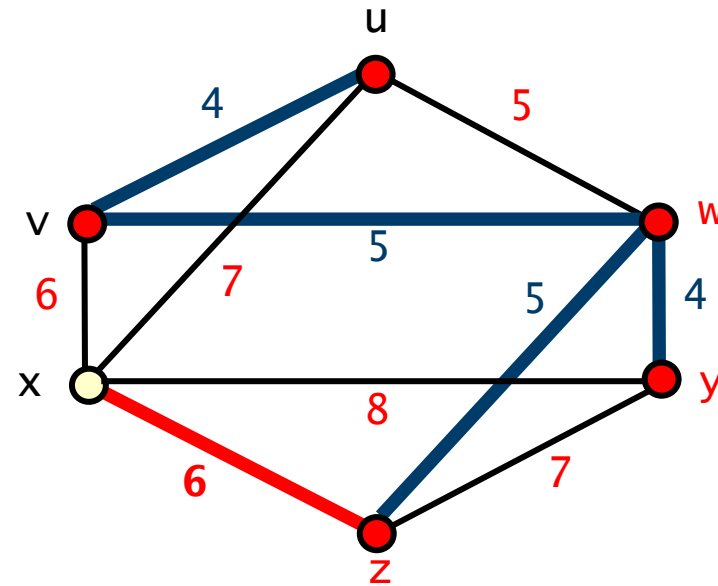
○ non-tree vertices: x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	–	–
v	–	–
w	–	–
x	z	6
y	–	–
z	–	–



● tree vertices: u, v, w, y, z

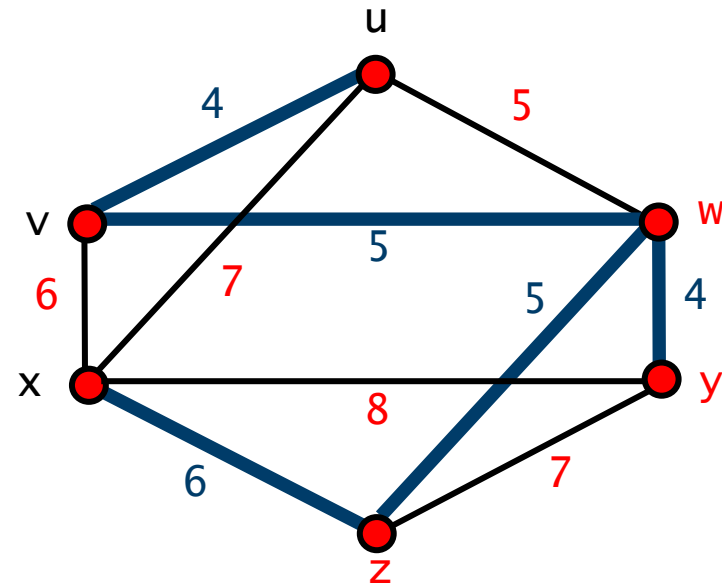
○ non-tree vertices: x

# Dijkstra's refinement – Example

## Weighted graph $G$

- choose **ntv**  $q$  for which  $\text{wt}(\{q, q.\text{bestTV}\})$  is minimal and make  $q$  a **tv**
- update  $s.\text{bestTV}$  for all **ntv**  $s$

$q$	$q.\text{bestTV}$	$\text{wt}(\{q.\text{bestTV}, q\})$
u	–	–
v	–	–
w	–	–
x	–	–
y	–	–
z	–	–



- **tree vertices:** u, v, w, x, y, z
- **non-tree vertices:**

# Dijkstra's refinement – Example

Weighted graph **G**

Minimum spanning tree for **G**

– weight **24**

q	q.bestTV	wt({q.bestTV, q})
u	–	–
v	–	–
w	–	–
x	–	–
y	–	–
z	–	–

