Exam solution corrections 2020

Handout

Some solutions to exam papers from previous years contain a few typos and minor errors. I have listed below those I am aware of - if you happen to spot further mistakes please do get in touch.

2018/19 Degree Exam

Question 5 (ii). The last sentence should read "The series $\sum(-1)^n b_n$ then converges by the Leibniz Test."

2017/18 Degree Exam

Question 5 (ii). There is a typo in the line after the formula: it should read "But by the standard limit $a^{\frac{1}{n}} \to 1, n \to \infty$ for $a > 0$..."

2016/17 Degree Exam

Question 6 (ii). There is an arithmetic mistake in the calculation. A correct estimate is

$$\begin{aligned} a_{n+1} - a_n &= \frac{n+1}{n^2+3n+3} - \frac{n}{n^2+n+1} \\ &= \frac{(n+1)(n^2+n+1) - n(n^2+3n+3)}{(n^2+3n+3)(n^2+n+1)} \\ &= \frac{1-n-n^2}{(n^2+3n+3)(n^2+n+1)} \\ &\leq \frac{-n^2}{(n^2+3n+3)(n^2+n+1)} \end{aligned}$$

for $n \in \mathbb{N}$, which implies $a_{n+1} - a_n < 0$ for $n > 1$.

2015/16 Degree Exam

Question 4 (ii). The last line of the computation should read

$$|f(x) - f(1)| = \frac{|x+1|}{|x+2|}|x-1| < \frac{3}{2}|x-1| < \varepsilon.$$

Question 8 (ii). The question asks for an example of a function f and sequence $(x_n)_{n=1}^{\infty}$ such that $(f(x_n))_{n=1}^{\infty}$ does not converge, but the function f given in the solution provides no such example: the constant sequence $f(x_n) = 1$ is convergent (albeit not to $f(0)$). A valid example of a function as requested is $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} \frac{1}{x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

together with the sequence $x_n = \frac{1}{n}$.