Mathematics 1 2019-29

Feedback **9**

Feedback Exercise 1

These questions relate to the material in Weeks 18 and 19. These feedback exercises are due by **4pm on Tuesday March 10th, handed in electronically through the course Moodle page.** Late submissions will not be accepted. You must submit your solutions to the feedback exercises through Moodle. Your work must be legible and be in a single PDF file. A PDF scan of your work can be created by scanning your work on any photocopier in the university, for upload to Moodle. **We will not accept or grade photographed work.**

One of the aims of this course is to develop mathematical writing skills. These will be assessed explicitly in the feedback exercises using assessment criteria outlined in **SOLVE: communicating mathematics in writing** (see the course information sheets on Moodle). **These feedback exercises will be graded out of 30 marks, with 15 marks awarded for writing and 15 for mathematics.**

We encourage you to talk to each other about mathematics, and work in groups to understand the material in this course and the exercises. However, your final submitted feedback exercise must be your own work. This means that it is suitable to talk about the problems and get the ideas together, but you should do the final write up alone. You also should not show another student your final submission.

FB1 Suppose that $f\colon A \to B$ is a surjective function. Define the following relation on A:

$$a_1 \sim a_2 \text{ if and only if } f(a_1) = f(a_2).$$

Show that this is an equivalence relation. Denote by $A/{\sim}$ the set of equivalence classes of $\sim$. Prove that

$$|A/{\sim}| = |B|.$$

FB2 Suppose that G is a group with identity element e. Let $\alpha, \beta, \gamma \in G$ be arbitrary. Prove the following statements.

(i) $\alpha\beta\gamma = e$ implies $\beta\gamma\alpha = e$.

(ii) $\beta\alpha\gamma = \alpha^{-1}$ implies $\gamma\alpha\beta = \alpha^{-1}$.

FB3 A parametric curve is described by the following equations

$$\frac{dx}{dt} = x, \quad y = \cos t, \quad z = \sin t,$$

and passes through $\langle 1, 1, 0\rangle$ when $t = 0$. By solving the ODE for $x(t)$, or otherwise, find an expression for x in terms of t and use this to write the space curve as a vector function. Hence, find the unit tangent to the curve $\mathbf{T}(t)$ at the point $\langle 1, 1, 0\rangle$.