Thursday, 14th December, 2017 4.30 pm *to* 6.00 pm

University of Glasgow

EXAMINATION FOR THE DEGREES OF
M.A. AND B.Sc.

Mathematics 2A - Multivariable Calculus

An electronic calculator may be used provided that it does not have a facility for either textual storage or display, or for graphical display.

Candidates must attempt all questions.

1. Let $z = \ln(x^2 + y)$, where $x = se^t$ and $y = te^{-s}$. Let $Z(s,t) = z(x(s,t), y(s,t))$. **Use the chain rule for functions of two variables** to find Z_s and Z_t. Give your answers in terms of s and t only. **5**

2. Let $f(x, y)$ be a scalar function of x and y. By making the change of variables $u = xy^4, v = xy$, find the general solution of the partial differential equation

$$xf_x - yf_y = 9xy.$$

Give your answer in terms of x and y. **7**

3. (i) Let $f(x, y, z)$ be a scalar field. State the definition of the Laplacian of f, denoted by $\nabla^2 f$, in terms of derivatives of x, y and z and in terms of the gradient and divergence operators. **2**

(ii) Let $r = \sqrt{x^2 + y^2 + z^2}$. Show that for $(x, y, z) \neq (0, 0, 0)$,

$$\nabla^2(1/r) = 0.$$ **5**

(iii) Let ϕ be a scalar field in $\mathbb{R}^3$. Show that

$$\nabla \times \nabla\phi = 0,$$

stating clearly any conditions on ϕ and its derivatives that you assume. **3**

4. Sketch the domain of integration for the integral

$$I = \int_1^3 \int_1^{y^4} \frac{y^2}{x}\, dx\, dy.$$

By changing the order of integration, evaluate I. **7**

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5. Use polar coordinates to evaluate

$$\int_D xy\,dx\,dy,$$

where the domain of integration D, is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 4$, the y-axis and the line $y = x$. **6**

6. Use spherical polar coordinates to evaluate

$$\iiint_V \left\{2 + 5(x^2 + y^2 + z^2)\right\}\,dx\,dy\,dz,$$

where V is the solid below the hemisphere $x^2 + y^2 + z^2 = 3$, such that $z > 0$. **6**

7. In $\mathbb{R}^3$, let S_1 be the part of the plane $3x+2y+z = 25$ lying above the quadrilateral given by the inequalities

$$0 \leqslant x \leqslant 1, \quad 0 \leqslant y \leqslant x + 3.$$

Evaluate the surface integral

$$\iint_{S_1} y\,dS.$$ **6**

8. Find a parametric equation for the curve C along the circle $x^2 + y^2 = 1$ from $(0, -1)$ to $(0, +1)$ in the positive direction. Hence evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F} = (-y, x)^T$. **6**

9. State the key equation in *Gauss' Divergence Theorem*, and define the domains of integration. Use this theorem to evaluate the integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\,dS,$$

where $\mathbf{F} = (-x^3 + 2y, -y^2, 3z^2)^T$, S is the surface of the cuboid described by

$$\{(x, y, z) : 0 \leqslant x \leqslant 1, -1 \leqslant y \leqslant 1, -1 \leqslant z \leqslant 2\},$$

and $\mathbf{n}$ is the unit outward normal to S. **7**

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