Communicating Mathematics in Writing – Marking Scheme

We believe that mathematics is a vehicle for improving reasoning and clarity in communications; indeed, the language of mathematics is created precisely for this purpose.

This document outlines five attributes of well written, effectively communicated mathematics, and details the marking of your feedback exercises. The sections below indicate these attributes (which give the acronym *SOLVE*) together with an indication of the marking scheme for your feedback exercises where each attribute attracts a mark of 0–3.

Marks of 0 or 1 for any individual attribute indicate you have not demonstrated satisfactorily this attribute and that this is an area for improvement in your subsequent submission.

For attributes where you have scored 0–2, your marker will indicate to you some instances where you have not demonstrated these attributes with circled letters (or the full attribute name). You should ensure you can identify all of these instances, or seek support to do so.

Sentence (S)

Mathematics is written in sentences.

Any graphs or figures are suitably labelled (including axes, functions, any relevant points of interest).

The litmus test to apply is: pick a point in the writing, go forwards to find a full stop; go backwards to find a capital letter. Starting a sentence with a symbol which cannot be capitalised could be considered as poor style or not — it's certainly not a popular typographical style choice.

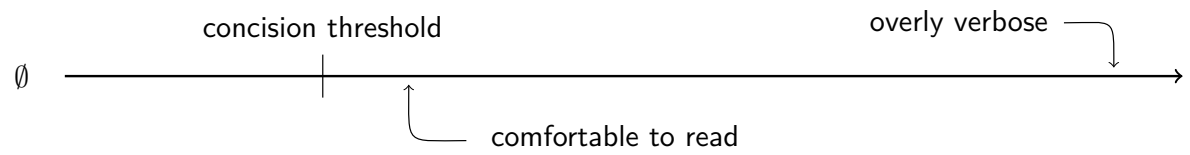
mark	description of submitted work
0	*no attempt to write in sentences for one or more question responses* and/or *graphs and figures unlabelled*
1	*at least one sentence in one question response* and/or *graphs and figures contain at least one label*
2	*most question responses written in sentences* and/or *graphs and figures mostly suitably labeled in all question responses*
3	*all question responses written in sentences (occasional omission of full stops, capital letters permissable)* and/or *all graphs and figures suitably labeled*

cOncision (O)

Mathematics is written concisely.

Steps of the proof or solution are clearly communicated with an appropriate level of verbosity. This means thinking about the intended audience (who you can normally reasonably assume is yourself before having completed the proof or solution) and unambiguously communicating enough detail to convince them without loosing the point in deatils. You do not need to include every step that you took to initially solve the problem, or any "dead-ends" you explored. You can reasonably expect that the reader will need to perform simple calculations to comprehend your writing. 1

There is a concision threshold that lies between writing nothing and an overly verbose submission where everything that is required to be stated is and nothing more (see Figure 1). Human mathematicians normally prefer to read immediately above this threshold, so that for example, signposting the type of proof (*by contradiction, by induction, etc.*) is included and mathematics written to teach concepts usually lies further above this concision threshold again. In automated theorem proving, computers usually work at the concision threshold. Without any "framing" of the arguement, it is challenging for to read and process mathematics presented in this way. Writing below the concision threshold will loose you marks for your mathematics independently of the communication mark.



mark	description of submitted work
0	*overly verbose submission* (e.g. many pages of solutions where one would suffice)
1	*at least one part of the submission has an appropriate level of verbosity*
2	*most question responses are at or somewhat above the concision threshold*
3	*all question responses are at or immediately above the concision threshold*

Logic (L)

Mathematics has a clear and logical structure.

Common errors include making statements like "If A $\implies$ B" when "A $\implies$ B" or "If A then B" is intended; the use of arrows to lead the eye around the page; equations with no linking text (hence, that is, thus, such that, etc.) stacked on the page one above the other.

As a sidenote, it is advisable not to use symbols that you are unsure how to "read out loud"! Reading mathematics invovles connecting the symbols with the underlying ideas

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1	*at least one part of the submission has a clear and/or logical structure*
2	*most question responses have a clear and/or logical structure*
3	*all question responses have a clear and/or logical structure*

Variable (V)

Symbols that identify mathematical objects are introduced and used appropriately.

For example, symbols used to identify variables are introduced before or as they are first used; symbols identifying vectors are underlined when handwritten and boldface when typeset; constants of integration are introduced as such.

mark	description of submitted work
0	*symbols are not identified and/or correctly used throughout submission*
1	*at least one symbol in the submission is identified and correctly used*
2	*most symbols in the submission are identified and correctly used*
3	*symbols in the submission are identified and correctly used throughout*

Equality & comparison (E)

Compatible mathematical objects are compared accurately and precisely.

Examples of comparing incompatible objects are, claiming a matrix is an element of $\mathbb{R}$, confusing a set with an element of a set, or using "$=$" where the appropriate word is "is".

Examples of inaccurate and/or imprecise comparisons include writing

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

when it is intended to claim they are row equivalent and '$\sim$' should have been used; or claiming something similar to

$$\sqrt{2} = 1.41421356237,$$

when we all know that $\sqrt{2}$ is irrational! Likely it was one of the following that was intended to be communicated:

$$\sqrt{2} = 1.41421356237 \text{ (11 d.p.), or } \sqrt{2} \approx 1.41421356237.$$

An illuminating examples is to ask yourself if the following statement is true or false:

$$\frac{1}{100} = \frac{1}{1000}.$$

Undoubtedly this question is ridiculous. However, it is equally ridiculous to state that an irrational number is rational.
Of course the above statement is false as written, nonetheless the statements

$$\frac{1}{100} = 0.0 \text{ (1 d.p.)}, \quad \frac{1}{1000} = 0.0 \text{ (1 d.p.)} \quad \text{and} \quad \frac{1}{100} \approx \frac{1}{1000}.$$

are true.

mark	description of submitted work
0	*comparison of mathematical objects is inaccurate and/or imprecise throughout submission* (e.g. use of "=" to compare row equivalent matrices, claiming irrational numbers are rational or infinite power series are polynomials, etc.)
1	*at least one type of comparison is accurate and/or precise in the submission*
2	*most comparisons in the submission are accurate and/or precise*
3	*comparisons in the submission are accurate and/or precise throughout*

SOLVE Attributes — Summary

Sentence (S)

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Logic (L)

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2	*most question responses have a clear and/or logical structure*
3	*all question responses have a clear and/or logical structure*

Variable (V)

mark	description of submitted work
0	*symbols are not identified and/or correctly used throughout submission*
1	*at least one symbol in the submission is identified and correctly used*
2	*most symbols in the submission are identified and correctly used*
3	*symbols in the submission are identified and correctly used throughout*

Equality & comparison (E)

mark	description of submitted work
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1	*at least one type of comparison is accurate and/or precise in the submission*
2	*most comparisons in the submission are accurate and/or precise*
3	*comparisons in the submission are accurate and/or precise throughout*