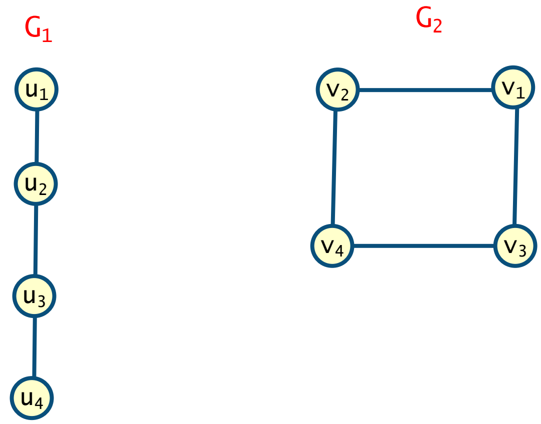
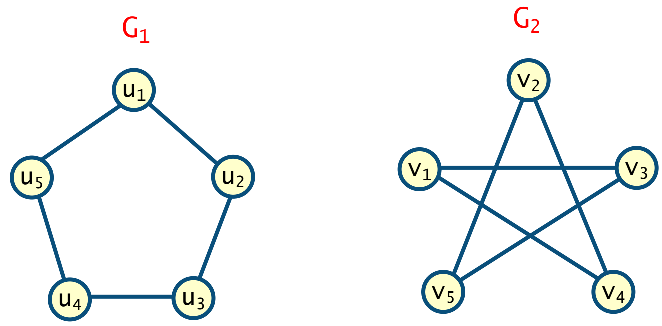
Are the following two graphs are isomorphic.

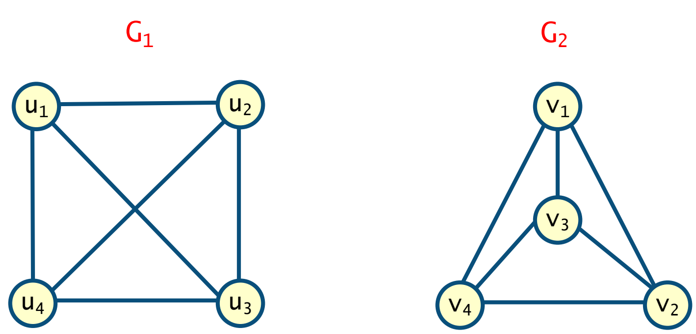


These graphs are not isomorphic since G_1 has 3 edges while G_2 has 4 edges (also the first has two vertices with degree 1 and all the vertices in G_2 have degree 2.



These graphs are isomorphic, first the have the same number of vertices and edges and the degrees of the vertices are all equal. One such isomorphism is given by the bijection:

$$u_1 \mapsto v_1,\ u_2 \mapsto v_3,\ u_3 \mapsto v_5,\ u_4 \mapsto v_2,\ u_5 \mapsto v_1,$$



These graphs are isomorphic, first the have the same number of vertices and edges and the degrees of the vertices are all equal. In this case since in both graphs all vertices are adjacent to all others any bijection between the vertices will work. One such isomorphism is given by the bijection:

$$u_1 \mapsto v_1,\ u_2 \mapsto v_2,\ u_3 \mapsto v_3,\ u_4 \mapsto v_4,$$

Which of these relations on the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

(a) $\{(f, g) \mid f(1) = g(1)\}$

- **Reflexive.** It is reflexive since $f(1) = f(1)$ for any function f.
- **Symmetric.** It is symmetric since if $f(1) = g(1)$, then clearly $g(1) = f(1)$ for any functions f, g.
- **Transitive.** It is transitive since if $f(1) = g(1)$ and $g(1) = h(1)$, then $f(1) = h(1)$ for any functions f, g, h.

(b) $\{(f, g) \mid f(0) = g(0) \vee f(1) = g(1)\}$

- **Reflexive.** It is reflexive since $f(0) = f(0)$ and $f(1) = f(1)$ for any function f.
- **Symmetric.** It is symmetric since if $f(0) = g(0)$ or $f(1) = g(1)$, then clearly $g(0) = f(0)$ or $g(1) = f(1)$ for any functions f, g.
- **Transitive.** It is not transitive for example if we consider any functions f, gv and h where $f(0) = g(0) = 3$ and $h(0) = 4$ and $f(1) = 4$ and $g(1) = h(1) = 3$, (f, g) and (g, h) are in the relation but (f, h) is not in the relation.

(c) $\{(f, g) \mid f(x) - g(x) = 1 \text{ for all } x \in \mathbb{Z}\}$

- **Reflexive.** It is not reflexive since $f(x) - f(x) = 0 \neq 1$ for any function f.
- **Symmetric.** It is not symmetric since if $f(x) - g(x) = 1$, then $g(1) - f(1) = -(f(x) - g(x)) = -1 \neq 1$ for any functions f, g.
- **Transitive.** It is not transitive since if $f(x) - g(x) = 1$ and $g(x) = h(x) = 1$, then:

$$\begin{aligned}
f(x) - h(x) &= f(x) + (g(x) - g(x)) - h(x) && \text{since } g(x) - g(x) = 0 \\
&= (f(x) - g(x)) + (g(x) - h(x)) && \text{rearranging} \\
&= 1 + 1 && \text{since } f(x) - g(x) = 1 \text{ and } g(x) = h(x) = 1 \\
&= 2 \neq 1
\end{aligned}$$

(d) $\{(f, g) \mid \exists C \in \mathbb{Z}. \forall x \in \mathbb{Z}. (f(x) - g(x) = C)\}$

- **Reflexive.** It is reflexive since $f(x) - f(x) = 0$ for any function f.
- **Symmetric.** It is symmetric since if $f(x) - g(x) = C$, then clearly $g(x) - f(x) = -(f(x) - g(x)) = -C$ for any functions f, g.
- **Transitive.** It is transitive since if $f(x) - g(x) = C_1$ and $g(x) - h(x) = C_2$, then:

$$\begin{aligned}
f(x) - h(x) &= f(x) + (g(x) - g(x)) - h(x) && \text{since } g(x) - g(x) = 0 \\
&= (f(x) - g(x)) + (g(x) - h(x)) && \text{rearranging} \\
&= C_1 + C_2 && \text{since } f(x) - g(x) = C_1 \text{ and } g(x) = h(x) = C_2
\end{aligned}$$

Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

- **Reflexive.** It is reflexive since $a + b = b + a$ for any (a, b).
- **Symmetric.** It is symmetric since if $a + d = b + c$, then clearly $c + b = d + a$ for any functions (a, b) and (b, a).
- **Transitive.** It is transitive since if $a + d = b + c$, then clearly $c + e = d + f$, then adding both sides of these equations we preserve equality and therefore:

$$(a + d) + (c + e) = (b + c) + (d + f)$$

Regarraning we have

$$(a + e) + (c + d) = (b + f) + (c + d)$$

which simplifying yields $(a + e) = (b + f)$ as required.

Find the lexicographic ordering of the bit strings `0`, `01`, `11`, `001`, `010`, `011`, `0001`, and `0101` based on the ordering $\texttt{0} \sqsubseteq \texttt{1}$.

First `11` is the only element with a `1` in the first position, and therefore this is the "largest element". From the remaining the elements:

$$\texttt{01},\ \texttt{010},\ \texttt{011},\ \texttt{0101}$$

have a `1` in the second position. From these only `011` has a `1` in the third so this comes next. The remaining are prefixes of each other so we have `01`*sqsubset*`010` $\sqsubset$ `0101`. What is left are `0`, `001` and `0001`. The string `001` comes next as it has a `1` in the third position and finally $\texttt{0} \sqsubset \texttt{0001}$ since the first is a prefix of the second. This yields the final ordering:

$$\texttt{0} \sqsubset \texttt{0001} \sqsubset \texttt{001} \sqsubset \texttt{01} \sqsubset \texttt{010} \sqsubset \texttt{0101} \sqsubset \texttt{011} \sqsubset \texttt{11}$$