Proposal for revised level 2 mathematics

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Abstract

This is a **draft** proposal for changes to be made to the level 2 mathematics courses, motivated by the changes made already to level 1 mathematics.

Warning - this is a draft, and subject to change based on the feedback we receive from stakeholders!

1 Introduction

Recently, it was agreed that major changes would happen to level 1 mathematics in Glasgow; not only in the content of what is being taught, but also in the method of delivery and the way the students learn the material. This new level 1 will run for the first time during the 2019-2020 academic year. It will have an immediate knock-on effect on level 2 mathematics. Therefore, changes need to be made to level 2 mathematics BEFORE September 2020.

This document is a draft proposal of the changes that need to be made. It has been written by the Level 2 implementation group, which has meet numerous times over the course of the summer of 2019.

Level 2 mathematics is an important period in the development of mathematics undergraduates in Glasgow, acting as a bridge between the more basic material of level 1 and the advanced material taught in honors years. At the same time, it is the place where most other STEM students at the university will learn the mathematical underpinning of their degrees. Both these aspects need to be considered when planning the changes to be made. Crucially, we view level 2 mathematics as consisting of three distinct strands:

- Core courses - mathematical methods; consisting of 2A, 2B and 2D.
- Core courses - foundational mathematics; consisting of 2C, 2E and 2F.
- Optional courses; consisting of 2P and 2T.

1.1 Core courses - mathematical methods

Mathematics today is flourishing for one key reason - it is applicable! More than ever, mathematics is being applied to solve difficult problems in every aspect of life. Therefore, at the heart of level 2 mathematics are the courses 2A, 2B and 2D. These courses aim to teach students how to compute with, and apply, the theories of calculus, linear algebra, differential equations and Fourier analysis, in order to solve explicit numerical problems. We view these courses as a natural continuation of the new Mathematics 1 course.

Not only are these courses designed to teach mathematics students how to apply the powerful techniques contained within these theories, they also contain the material that forms the mathematical bedrock for any student on a numerically intensive course such as physics, stats or computer science. The course 2A, 2B and 2D will be compulsory for students wishing to take mathematics, statistics or physics (with 2A, 2B compulsory for physical chemistry) courses at honors. We expect that they will also be appealing to students taking computer science at honors too.

1.2 Core courses - foundational mathematics

Much of the progress that has been made in mathematics over the past two centuries stems from the rigorous, and systematic, development of the foundations of mathematics. Today, anyone wishing to learn the more advanced topics encountered in a honors mathematics programme must first learn the proper foundations of the subject - this is as true in applied mathematics as it is in pure mathematics. In addition to 2A, 2B and 2D, our core courses for students wishing to enter honors mathematics are 2C, 2E and 2F. These courses cover real analysis, classical mechanics, group theory and abstract linear algebra. More importantly, they are designed to teach the students how to argue rigorously, prove precise mathematics statements, and write formal mathematics correctly. As such, they will be delivered in a style that is much more similar to that of 3rd year honors courses. This will teach students how to *properly learn* mathematics; a skill that will be essential in honors mathematics.

The courses 2C, 2E and 2F will be compulsory for students wishing to enter honors mathematics.

1.3 Optional courses

The mathematical sciences is a vast field, ranging from applied number theory (such as cryptography or coding theory) to theoretical mathematical physics (such as general relativity or quantum field theory). It is hard for a student to grasp this breadth from our core courses. The optional courses 2P and 2T are designed to give students a flavour of this breadth by covering subjects that students will not have previously seen, but are not necessarily essential for studying honors level mathematics. Crucially, students can experience these courses having only taken 1G at level 1.

2 Syllabus

We explain in more detail the content of each of the new level 2 courses.

2.1 2A - Multivariable calculus

Short description

This course on multivariate calculus gives a practical introduction to differentiating and integrating in multiple dimensions, and to fundamental concepts found in diverse fields such as geometry and physics. It is an essential course for intending honours students. The emphasis is on methods and applications.

Course aims

This course covers fundamental topics in multivariable calculus. The aim of the first part of the course is teach students how effectively compute partial derivatives of multivariate functions. This is used to solve simple partial differential equations using change of variables. The aim of the second part of the course is to understand the concepts div, grad and curl and learn to use the identities they satisfy to solve concrete problems. The third part of the course aims to teach students how to compute double and triple integrals. In particular, we aim to understand how to perform line and surface integrals by using appropriate change of variables. The final aim of the course is to understand Green's theorem and the divergence theorem and learn how to apply these theorems in concrete situations.

Intended learning outcomes

By the end of the course, students should have attained the following learning objectives:

- Be able to draw three dimensional surfaces using cross-sections and contours; use the chain rule for partial derivatives;
- Solve simple PDEs by using a given change of variables.
- Give parametrisations of curves in two and three dimensions; recall the definition of div, grad and curl; use identities involving these derivatives in concrete problems.
- Compute volume under a surface and double integrals; obtaining limits for a given domain; perform change of order of integration; perform general change of variables and use the Jacobian in integration.
- Compute line integrals; prove path-independence of line integrals for conservative vector fields; give a parametric description of a surface and compute surface integrals.
- Apply Green's theorem to evaluating closed line integrals; apply the divergence theorem to evaluate closed surface integrals.
- Be able to learn and apply formulae used in this course.

Detailed syllabus

1. Partial differentiation
    - Functions of several variables
    - Implicit function theorem
    - The chain rule

- Partial differential equations

2. Double and triple integration
   - Double integration
   - Change of variables
   - Triple integration
3. Differentiation of scalar and vector fields
   - Vector and scalar fields
   - Gradient, divergence and curl
   - Nabla identities
4. Line and surface integrals
   - Line integrals
   - Green's Theorem
   - Surface integrals
   - Surface integrals of vector fields

Assessment

As currently, assessment is 80% exam and 20% continuous assessment. The continuous assessment is split 10% webassign and 10% for group work in tutorials. There are ten webassign and six tutorials.

2.2 2B - Linear algebra

Short description

This course covers the fundamentals of linear algebra that are applicable throughout science and engineering, and in particular in the physical, chemical and biological sciences, statistics and other parts of mathematics. It is an essential course for intending honours students. The emphasis is on methods and applications.

Course aims

This course covers the fundamentals of linear algebra that are applicable throughout science and engineering, and in particular in the physical, chemical and biological sciences, statistics and other parts of mathematics. The aim of the first part of the course is to introduce the idea of a finite dimensional vector space, including the concepts of linear independence, basis, dimension and linear map. The relation between linear maps and matrices will be explained, and this will motivate further study of matrices in the second part of the course, in which the determinant, eigenvalues and eigenvectors of a matrix will be studied. Throughout, all new ideas will be illustrated by examples drawn from applications in low dimensions.

Intended learning outcomes

By the end of the course, students should have attained the following learning objectives:

- Handle fluently problems involving matrices and their entries.
- Recognise vector spaces and subspaces over R and C.
- Test sets of vectors for linear independence and spanning properties, and understand methods for obtaining bases for a specified subspace of a vector space.
- Decide whether or not a map between spaces is linear and describe a linear map in matrix form.
- Evaluate determinants recursively and using elementary row and column operations, factorize algebraic determinants and apply results about determinants in theoretical problems.
- Find the characteristic polynomial of a square matrix and use it to determine the eigenvalues and eigenvectors of the matrix, and deal with theoretical problems involving eigenvalues.
- Write down the symmetric matrix associated to a quadratic forms. Determine the rank and signature of a quadratic form; diagonalise quadratic forms.

- Construct inner products on a real vector space. Use the Gram-Schmidt algorithm to construct an orthonormal basis of a Euclidean vector space.
- Be familiar with all definitions and results covered in lectures and understand the proofs of the results.

Detailed syllabus

1. Vectors and linear systems of equations
    - spanning sets
    - linear independent sets
    - (In)homogenous systems of equations
2. Matrix algebra
    - Addition and multiplication of matrices
    - Elementary matrices
3. Subspaces
    - Basis
    - Dimension of a subspace
    - The rank theorem
4. Linear transformations
    - Determinant and trace of a matrix
    - Similar matrices
    - Change of basis matrices
5. Diagonalization of a matrix
    - Symmetric matrices
    - Quadratic forms
    - Signatures
6. Hermitian and orthogonal matrices

- Hermitian matrices
- Orthogonal matrices
- The Gram-Schmidt algorithm

Assessment

As currently, assessment is 80% exam and 20% continuous assessment. The continuous assessment is split 10% webassign and 10% for group work in tutorials. There are ten webassign and six tutorials.

2.3 2C - Introduction to Real Analysis

Short description

The common thread running through this is the notion of limit. This course will give a precise definition of this notion for both sequences and series. It is an essential course for intending honours students. The emphasis is on developing and applying standard techniques of proof to give rigorous arguments from basic definitions.

Course aims

The common thread running through this is the notion of limit. The aim of the course is to give a precise definition of this notion for both sequences and series. The notion of continuity for functions will be discussed and related to convergence of sequences. Some important consequences of continuity to be studied are the intermediate value theorem and its applications, and the existence of extrema. The emphasis is on developing and applying standard techniques of proof to give rigorous arguments from basic definitions.

Intended learning outcomes

By the end of the course, students should have attained the following learning objectives:

- Manipulate implications and equivalences; interpret the negation of a statement involving quantifiers; recognise various methods of proof (direct, contrapositive, counterexample, contradiction, induction); show that a function is bounded/unbounded.

- Show, directly from the definition, that a given number is the limit of a given sequence; evaluate sequence limits using arithmetic and order properties;
- Show that a given sequence is monotonic; investigate sequences defined recursively; use subsequences to establish non-convergence; test series for convergence/divergence; test series for absolute/conditional convergence;
- Determine, directly from the definition, whether a function is continuous; use the sequential characterisation to establish discontinuity; solve problems using the intermediate value and extreme value theorems.
- Students should understand and be able to recall the definitions and proofs and be able to apply the results to the types of problem covered in lectures and tutorials.

Detailed syllabus

1. Logic and inequalities
    - qualifiers
    - negation of a statement
2. The real numbers
    - Axioms of an ordered field
    - Bounds
    - Completeness
3. Sequences
4. Series
5. Continuity

Assessment

As currently, assessment is 80% exam and 20% continuous assessment. The continuous assessment is split 10% webassign and 10% feedback exercises. There are ten webassign and four feedback exercises.

2.4 2D - Mathematical Methods and Modelling

Short description

This course aims to introduce theory and methods used in mathematical modelling. Topics covered in the course include: population modelling, local minima and maxima of functions, Fourier series and integral transforms. It is an essential course for intending honours students. The emphasis is on methods and applications.

Course aims

This course aims to introduce theory and methods used in mathematical modelling. The first part of the course aims to provide an introduction to the modelling of populations using difference and differential equations. Students will learn to construct the solution of the equations arising in such models and qualitatively analyse these solutions. The second part of the course aims to teach students important methods from calculus that are used in mathematical modelling. These include: analysis of local minima and maxima of functions, Fourier series and integral transforms.

Intended learning outcomes

By the end of the course, students should have attained the following learning objectives:

- Derive and analyse single-species dynamical systems, including the exponential and logistic models with predation.
- Analyse the stability of equilibrium points in first order autonomous differential equations and to construct simple bifurcation diagrams for single-species dynamics systems.
- Use diagonalization of a matrix to solve systems of ordinary differential equations and difference equations.
- Find stationary points for functions of several variables; classify them using first principles and the Hessian criterion; use the method of Lagrange multipliers to solve practical extreme value problems.
- Find Fourier series for given functions defined on finite intervals.

- Compute the Fourier transform and Laplace transform of a given function; use the properties of Beta and Gamma functions to evaluate certain integrals.
- Be able to learn and apply formulae used in this course.

Detailed syllabus

1. Population Modelling
   - Single species models.
   - Matrix diagonalization and systems of differential equations.
   - Analysis of stability of solutions
   - Multiple species models.
2. Local minima and maxima - Hessians and Lagrange multipliers.
3. Fourier series
4. Integral transforms
   - Laplace transform
   - Fourier transform
   - Gamma and Beta functions

Assessment

As currently, assessment is 80% exam and 20% continuous assessment. The continuous assessment is split 10% webassign and 10% for group work in tutorials. There are ten webassign and six tutorials.

2.5 2E: Mechanics

Short description

This course provides an introduction to the mathematical modelling of mechanical phenomena, for example, the motion of a golf ball moving under the influence of gravity. The main mathematical tools used in this course is vector algebra and the analysis of solutions of differential equations. It is an essential course for intending honours students.

Course aims

The aim of this course is to provide an introduction to the kinematics and dynamics of a single point mass particle. Equations of motion are constructed via the application of Newton's laws. The Lagrangian formalism of mechanics is introduced.

Intended learning outcomes

By the end of the course, students should have attained the following learning objectives:

- Analyse the dimensions of quantities and make predictions of parameter dependence based on dimensional analysis.
- Calculate the tangent vector and arc-length of a parametric curve and to sketch such curves; use plane, cylindrical and spherical polar coordinates to describe particle motion; explain the terms velocity, speed, acceleration, displacement and distance travelled in connection with the motion of a point particle and to connect these terms to properties of parametric curves.
- Formulate problems in particle mechanics in mathematical terms using vectors and/or differential equations as appropriate, and solve the resulting equations to determine the motion of the particle.
- Define the terms linear momentum, impulse and force and use them in calculations; construct and solve problems using kinematics;
- Apply the concepts of energy, work and power in physical situations; compute projectile motion and motion governed by Hooke's Law; construct and solve problems in elementary mechanics using plane polar coordinates; formulate problems in particle mechanics in mathematical terms using vectors and/or differential equations as appropriate, and solve the resulting equations to determine the motion of the particle.
- Determine appropriate Lagrangians, and derive and solve the equations of motions for simple mechanical systems subject to the principle of least action.
- Learn and apply formulas which are taught in the course.

Detailed syllabus

1. Physical Preliminaries
   - Dimensional analysis
   - Introduction to Mechanics; Degrees of freedom
2. Motion (or Kinematics)
3. Newtonian Dynamics
4. Conservation Laws
5. Introduction to Lagrangian Dynamics

Assessment

As currently, assessment is 80% exam and 20% continuous assessment. The continuous assessment is split 10% feedback exercises and 10% for group work in tutorials. There are six feedback exercises and six tutorials.

2.6 2F - Groups, Transformations and Symmetries

Short description

This course covers fundamental concepts in pure mathematics. Building on the definition of group given in level 1, deeper properties of these objects will be studied. Student's intuition for groups will developed through examples. The abstract concept of vector spaces, and linear transformations between these spaces will be introduced. By considering basis for vector spaces, the concept of linear transformation will be related back to matrices. In the final part the course, groups and linear transformations come together. This is done by considering the symmetries of spaces and shapes.

Course aims

The course covers fundamental concepts in pure mathematics. The aim of the first part of the course is to develop the student's understanding of the concept of group. Throughout, a strong emphasis is put on examples in order to develop students intuition for these objects. The aim of the second part of the course is to introduce abstract vector space, and understand their basic properties. Linear transformations between abstract vector spaces will be defined and the role of basis in relating linear transformations to matrices will be explained. The final aim of the course is to tie together the concept of group and linear transformation by studying the action of groups on geometric objects. This is done by considering the symmetries of spaces and shapes.

Intended learning outcomes

By the end of the course, students should have attained the following learning objectives:

- Decide whether a given function is injective, surjective or bijective.
- Give the definition of a group and a subgroup; produce examples of abelian and non-abelian groups; state, prove and apply Lagrange's theorem; describe all subgroups of a given group.
- Give examples of fields and decide if a given set is a vector space; compute the matrix form of a linear transformation with respect to a fixed bases and relate two basis via a change of basis matrix.
- Prove that the set of cosets of a normal subgroup has the structure of a group.
- Produce examples of groups acting on vector spaces; construct the group of all symmetries of the n-gon.
- Be familiar with all definitions and results covered in lectures and understand the proofs of the results.

Detailed syllabus

1. Sets and functions (1 week)
    - Intersection and union of sets.
    - Definition of a function.
    - Injective, surjective and bijective functions.
    - Images and preimages of sets under functions.

- Intersections and unions of sets.

2. Groups (4 weeks)
    - Binary operations
    - Definition of a group, and a subgroup. Order of an element.
    - Examples! Based around permutation groups and groups of matrices.
    - Abelian groups. More examples based on $\mathbb{Z}, \mathbb{Z}/n\mathbb{Z}$ etc.
    - Lagrange's theorem. Including revision of equivalence and partitions of a set.
    - Group homomorphisms, normal subgroups and quotients.

3. Fields, vector spaces and transformations (4 weeks)
    - Definition of field. Basic examples.
    - Vector spaces and subspaces.
    - Linear transformations.
    - Change of basis.

4. Symmetries in geometry (2 weeks)
    - Group action, motivated through the example of $GL(2, \mathbb{R})$ acting on $\mathbb{R}^2$.
    - The orthogonal group and rotations. Classification of conic sections using the orthogonal group.
    - Symmetries of an n-gon. Realization of the dihedral group as (a) abstract group of symmetries (b) permutation group (c) group of linear transformations.
    - Symmetries of platonic solids. Possibly just cover tetrahedron and octahedron.

Assessment

Assessment is 80% exam and 20% continuous assessment. The continuous assessment is split 10% feedback exercises and 10% for group work in tutorials. There are six feedback exercises and six tutorials.

3 The structure of level 2 mathematics

Now that we have described what material we propose to cover in the new level 2, we turn to the practicalities of how these courses are to be delivered.

3.1 Delivery, assessment and feedback

In general, delivery, assessment and feedback will be uniform over 2A, 2B and 2D; similarly for 2C, 2E and 2F. However, the guiding principals are different. In these aspects, 2A, 2B and 2D should feel to the student like a continuation of Mathematics 1. But 2C, 2E and 2F should be viewed as an introduction to the style of working found in honors mathematics.

3.2 Core courses - Mathematical methods

Lecturers are encourage to make extensive use of Class Response in lectures to develop a sense of interaction with these very large classes. This system also allows for quick feedback on student's comprehension of the material. For students it will feel like a continuation of Mathematics 1.

In 2A, 2B and 2D students will continue to complete weekly webassign assignments. There will no longer be feedback exercises for 2B.

A much more important role will be given to tutorials in these courses. In the fortnightly tutorials, students will be told to work in groups of around 5. Each tutorial they will be given a worksheet that has a clearly stated goal/task, broken up into a number of easy steps. The group's task will be to to work together to complete each of these steps, and write up the solutions (individually). Tutors will then quickly grade the solution at the end of the tutorial. This will be based on the current system in Mathematics 1, which appears to be very successful. The stated goal should either centre around an involved example to be understood, or the introduction of a new (examinable, but basic) concept that they should explore. For example, in 2B one could create a tutorial based on the concept of trace of a matrix, with ultimate goal to prove that $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$. Then trace would be examinable but not covered in lectures.

In 2A, 2B and 2D, continuous assessment will count for 20% of a student's grade. This is 10% derived from the tutorials and 10% from webassign.

Though students will no longer have the opportunity to go over exercises questions in the tutorials, students will still be given weekly exercise sheets via moodle. It will be made clear that (a) these exercise questions will help in the task in the forthcoming tutorial (b) they are important for learning the material and for exam revision. Students will continue to have access to extensive office hours where they can ask questions about the exercises on the exercise sheets.

3.3 Core courses - Foundations of mathematics

Lectures for 2C, 2E and 2F should be delivered in the same style as honors mathematics. We expect more use of the blackboard and more direct interaction with the students (though we do not wish to discourage the use of Class Response, should the lecturer wish).

In 2C, 2E and 2F students will complete a series of feedback exercises, which are designed to develop their mathematical writing and ability to argue rationally. We expect 2E to continue to use webassign too.

As in 2A, 2B and 2D, a much more important role will be given to tutorials. In the fortnightly tutorials, students will be told to work in groups of around 5. Each tutorial they will be given a worksheet that has a clearly stated goal/task, broken up into a number of easy steps. The group's task will be to to work together to complete each of these steps, and write up the solutions (individually). Tutors will then quick grade the solution at the end of the tutorial. This will be based on the current system in Mathematics 1, which appears to be very successful. The stated goal should either centre around an involved example to be understood, or the introduction of a new (examinable, but basic) concept that they should explore.

In 2C, 2E and 2F, continuous assessment will count for 20% of a student's grade. This is 10% derived from the feedback exercises and 10% from tutorials (with a slight variation for 2E).

Though students will no longer have the opportunity to go over exercises questions in the tutorials, students will still be given weekly exercise sheets via moodle. It will be made clear that (a) these exercise questions will help in the task in the coming tutorial (b) they are important for learning the material and for exam revision. Students will continue to have access to extensive office hours where they can ask questions about the exercises on the exercise sheets.

3.4 Group work in tutorials

As in the current level 1, the group work in tutorials that we propose involves a large degree of interaction with other students. This may be difficult, or impossible, for students with certain disabilities. These students will be identified from the disability reports for the courses. If they do not wish to participate in the group activity then they will be placed in a separate tutorial, where they can work individually on the same worksheet. This provision will mirror that in place for Mathematics 1.

3.5 Schedule

Currently the timetable is highly constrained, and we see no benefit in changing it. Therefore, we recommend the schedule remain the same. Specifically, 2A,2B and 2C will run in semester 1, occupying the lecturing slots taken up by the current 2A, 2B and 2F respectively. Similarly, 2D,2E and 2F will run in semester 2, occupying the lecturing slots taken up by the current 2D, 2E and 2C respectively.

We propose that 2A and 2B will have separate fortnightly tutorials. We will also require that 2C, 2E and 2F have all fortnightly tutorials on the same day i.e. we cannot have tutorials on alternate weeks for 2C etc.

3.6 Prerequisites and progression to honors

We propose to keep the progression requirements to honors mathematics the same. We would recommend that the progression requirements (in mathematics) for progression to honors in statistics, physics and physical chemistry also remain unchanged.

Currently, Mathematics 1 is the only required prerequisites for the core courses 2A–2F. We propose to leave this unchanged. Students taking only 1C/1G or semester 1 of Mathematics 1 and 1G will *not* be allowed to take these courses. The prerequisites for 2P and 2T will remain one of:

- Mathematics 1
- 1G

A Changes to level 2

In this appendix we explain what changes would need to be done to the current level 2 mathematics in order to implement the new level 2 proposed above.

A.1 2A

The content of 2A will not change. We note that the content of this course is covered by Stuart's Calculus. The method of delivery, assessment and feedback will not change, except that tutorials will be replaced by group work.

A.2 2B

The following content changes are based on the current set of lecture notes for 2B Linear Algebra. We propose to:

- Keep all material up to part 6 of the lecture notes.
- Keep the material on diagonalization of matrices forming the first half of part 7.
- Remove the material on abstract definition and axioms of a vector space that form the second half of part 7.
- Remover part 8, subspaces of a vector space.
- Remove part 9, linear maps between vector spaces, but keep change of basis matrix (for $\mathbb{R}^n$).
- Remove part 10, kernel range etc of a linear map.
- Add material from current 2D on diagonalization (including of symmetric matrices).
- Add material from current 2D on quadratic forms.
- Add material from current 2D on Hermitian matrices, orthogonal matrices and Gram-Schmidt.

The new material from 2D to be added is computational in nature and should fit in well with the material in parts 1-7. We note that the relation between linear transformations and matrices forming part 5 is important for stats and for applied mathematics. Similarly, quadratic forms are important in some 3rd year applied courses, and also may appear in 3rd year physics (check this!). The content of this course will be covered by Poole (however, the section on unitary matrices in Poole is inadequate).

A.3 2C

The content, method of delivery, and forms of assessment for 2C will be identical to that of the old 2E.

A.4 2D

The new 2D course consists of the applied content of the old 2D course, together with some material from the old 2C and other new material on Fourier transforms.

1. Population Modelling
    - Single species models. (Ch 2 of old 2C)
    - Matrix diagonalization and systems of differential equations. (Ch 1 of old 2D)
    - Multiple species models. (Ch 3 of old 2C)
2. Local minima and maxima - Hessians and Lagrange multipliers. (Ch 5 of old 2D)
3. Fourier series (Ch 6 of old 2D)
4. Integral transforms (partly new)
    - Laplace transform (new)
    - Fourier transform (new)
    - Gamma and Beta functions (Ch 7 of old 2D)

A.5 2E

This is a new course, but the content is very similar to the old 2C.

1. Physical Preliminaries (Ch 1 and 4 of old 2C)
   - Dimensional analysis (Ch 1 of old 2C)
   - Introduction to Mechanics; Degrees of freedom (Ch 4 of old 2C)
2. Motion (or Kinematics) (Ch 6 of old 2C)
3. Newtonian Dynamics (Ch 7 of old 2C)
4. Conservation Laws (Ch 8 of old 2C)
5. Introduction to Lagrangian Dynamics (New)

A.6 2F

The course 2F will be developed from scratch. Comments on the content; the numbering below corresponds to the numbering in section 2.6.

1. This topic is in theory already covered in Maths 1. It is viewed as essential for pure courses and poorly understood by students. So this first week should be a revision of material seen before. The use of quantifiers should be emphasized throughout, continuing the message that they are given in first year about the importance of these things.
2. Only the last point on group homomorphisms, normal subgroups and quotients is actually new here. In theory they have seen everything else before in Maths 1. So emphasis should be on being formally precise and also on the examples. In Liebeck they will see lots of permutations, but I think we should still do lots of permutations again. They will not have seen other examples (except in passing remarks) such as matrix groups and $\mathbb{Z}^n, \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$. Though the proof of Lagrange's theorem is in Liebeck, it is quite concise and will be the very last thing they do. I suggest it is done in its entirety again. I have included "Group homomorphisms, normal subgroups and quotients"; this is the only new theory they see. We expect this material will be recapped in 3H Algebra, but giving students a first taste here allows one to spend less time on it in 3H.

3. Essentially this should cover exactly the same as is covered in the last 3 weeks of 2B at the moment (down to coping the lecture notes across). The only difference is that currently vector spaces are defined over "scalars" which means either $\mathbb{R}$ or $\mathbb{C}$ (this is also Poole's approach). I would be inclined to at least give the definition of field and say examples are $\mathbb{Q}, \mathbb{R}$ and $\mathbb{C}$; you'll see more advanced examples in 3H algebra... Rather, I would then use some of the extra time to go more slowly through change of basis. I think 4 weeks is just a more sensible amount of time to cover the same material (also if you look at the notes there are not many proofs in these last 3 sections, so we could cover same results but a bit more proof). Could also add the fact that determinant and trace of a linear transformation "make sense".