Formula sheet

Theorem 1.1 Algebraic properties of addition and scalar multiplication
(a) $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ (commutativity of vector addition)
(b) $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$ (associativity of vector addition)
(c) $\mathbf{u}+\mathbf{0}=\mathbf{u}$ ($\mathbf{0}$ is additive identity)
(d) $\mathbf{u}+(-\mathbf{u})=\mathbf{0}$ (Every element has an inverse under addtion)

(e) $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$ (Multiplication distributes across addition)

(f) $(c-d)\mathbf{u}=c\mathbf{u}-d\mathbf{u}$ (Note: these two $+$'s are different!)

(g) $c(d\mathbf{u})=d(c\mathbf{u})$

(h) $1\mathbf{u}=\mathbf{u}$

Theorem 2.4 A system of linear equations with augmented matrix $[A|\mathbf{b}]$ is consistent if and only if $\mathbf{b}$ is a linear combination of the columns of A.

$$\begin{gathered}
a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=b_1\\
a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n=b_2\\
\vdots\\
a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n=b_m
\end{gathered}$$

$$x_1\begin{bmatrix}a_{11}\\a_{21}\\\vdots\\a_{m1}\end{bmatrix}+x_2\begin{bmatrix}a_{12}\\a_{22}\\\vdots\\a_{m2}\end{bmatrix}+\cdots+x_n\begin{bmatrix}a_{1n}\\a_{2n}\\\vdots\\a_{mn}\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\\vdots\\b_m\end{bmatrix}$$

Theorem 2.5 Vectors $\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_k$ in $\mathbb{R}^n$ are linearly dependent if and only if at least one of them can be expressed as a linear combination of the others.

Theorem 2.6 Let $\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_m$ be (column) vectors in $\mathbb{R}^n$ and let

$$A=[\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_m]$$

be the $n \times m$ matrix with columns $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m$. Then $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m$ are linearly dependent if and only if the homogenenous linear system with augmented matrix $[A|\mathbf{b}]$ has non-trivial solution.

Theorem 2.8 If $m > n$ then any set of m vectors in $\mathbb{R}^n$ is linearly dependent.

Theorem 3.2 Let A, B and C be $m \times n$ matrices and c and d be scalars. Then

1. $A + B = B + A$,
2. $(A + B) + C = A + (B + C)$,
3. $A + 0 = A$,
4. $A + (-A) = 0$,
5. $c(A + B) = cA + cB$,
6. $(c + d)A = cA + dA$,
7. $c(dA) = (cd)A$
8. $1A = A$.

Theorem 3.3 Let A, B and C be matrices and k be a scalar. The following identities hold whenever the operations involved can be performed.

1. $A(BC) = (AB)C$, (associativity of matrix multiplication)
2. $A(B + C) = AB + AC$, (left multiplication disctributes across addition)
3. $(A + B)C = AC + BC$, (right multiplication distributes across addition)
4. $k(AB) = (kA)B = A(kB)$, (scalar multiplication commutes with matrix multiplication)
5. $\mathbb{I}_m A = A = A\mathbb{I}_n$ if A is $m \times n$ (left/right multiplicative identities).

Theorem 3.4 Let A and B be matrices. The following identities hold whenever the operation involved can be performed.

1. $(A^T)^T = A$,
2. $(A+B)^T = A^T + B^T$,
3. $(kA)^T = k(A^T)$,
4. $(AB)^T = B^T A^T$,
5. $(A^m)^T = (A^T)^m$ for all integer $m \geq 0$.

Theorem 3.5

(a) If A is a square matrix then $A + A^T$ is a symmetric matrix,

(b) For any matrix A, AA^T and $A^T A$ are symmetric matrices.

Theorem 3.6 If an $n \times n$ matrix is invertible then its inverse is unique.

Theorem 3.7 If A is an invertible $n \times n$ matrix then the system of linear equations given by $A\mathbf{x} = \mathbf{b}$ has the unique solution given by $\mathbf{x} = A^{-1}\mathbf{b}$.

Theorem 3.8 If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then A is invertible if $ad - bc \neq 0$, in which case

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Theorem 3.9

1. If A is an invertible matrix, then A^{-1} is invertible and $(A^{-1})^{-1} = A$.
2. If A is an invertible matrix and $c \neq 0$ is a scalar then cA is invertible and $(cA)^{-1} = \frac{1}{c}A^{-1}$.
3. If A and B are invertible matrices of the same size, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.
4. If A is an invertible matrix, then A^T is invertible and $(A^T)^{-1} = (A^{-1})^T$.

5. If A is invertible matrix then A^n is invertible for all integers $n \geq 0$ and $(A^n)^{-1} = (A^{-1})^n$.

Theorem 3.10 Let E be the elementary matrix obtained by performing an ERO on $\mathbb{I}_n$. If the same ERO is performed on an $n \times r$ matrix A, then the result is the matrix EA.

Theorem 3.11 Each elementary matrix is invertible, and its inverse is an elementary matrix of the same type.

Theorem 3.12 The Fundamental theorem of invertible matrices

Let A be an $n \times n$ matrix. The following statements are equivalent:

1. A is invertible
2. $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$ in $\mathbb{R}^n$
3. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution
4. The reduced echelon form of A is $\mathbb{I}_n$
5. A is a product of elementary matrices

Theorem 3.13 A one-sided inverse is a two-sided inverse Let A be a square matrix. If B is a square matrix such that either $AB = \mathbb{I}_n$ or $BA = \mathbb{I}_n$, then A is invertible and $A^{-1} = B$.

Theorem 3.14 Let A be a square matrix. If a sequence of elementary row operations reduces A to $\mathbb{I}$, then the same sequence reduces $\mathbb{I}$ to A^{-1}.

Theorem 3.19 A span is a subspace Let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be vectors in $\mathbb{R}^n$. The $\text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a subspace of $\mathbb{R}^n$.

Theorem 3.20 Let B be any matrix which is row equivalent to A. Then $row(B) = row(A)$.

Theorem 3.21 Let A be an $m \times n$ matrix. Let N be the set of solutions to the homogeneous linear sustem $A\mathbf{x} = \mathbf{0}$. Then N is a subspace of $\mathbb{R}^n$.

Theorem 3.22 Let A be an $m \times n$ real matrix. Then for any system $A\mathbf{x} = \mathbf{b}$ of linear equations exactly one of the following is true:

1. There is no solution;
2. There is a unique solution;
3. There are infinitely many solutions.

Theorem 3.23 The basis theorem Let S be a subspace of $\mathbb{R}^n$. Then any two bases for S have the same number of vectors.

Theorem 3.24 The row and column spaces of a matrix A have the same dimension.

Theorem 3.25 For any matrix A, $\operatorname{rank}(A) = \operatorname{rank}(A^T)$.

Theorem 3.26 The rank theorem If A is an $m \times n$ matrix then

$$\operatorname{rank}(A) + \operatorname{nullity}(A) = n,$$

where n is the number of columns of A.

Theorem 3.29 Let S be a subspace of $\mathbb{R}^n$ and let $B : \mathbf{v}_1 \ldots, \mathbf{v}_k$ be an ordered basis for S. Then for every $\mathbf{v} \in S$, there is exactly one way to write $\mathbf{v}$ as an ordered linear combination of the basis vector in B.

Theorem 6.12 Let $\mathcal{B} : \mathbf{u}_1, \ldots, \mathbf{u}_n$ and $\mathcal{C} : \mathbf{v}_1, \ldots, \mathbf{v}_n$ be ordered bases for $\mathbb{R}^n$ and let $P_{\mathcal{C}\leftarrow\mathcal{B}}$ be the change of basis matrix from $\mathcal{B}$ to $\mathcal{C}$. Then

1. For all $\mathbf{x} \in \mathcal{C}$, $P_{\mathcal{C}\leftarrow\mathcal{B}}[\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{C}}$;

2. $P_{\mathcal{C}\leftarrow\mathcal{B}}$ is the unique $n\times n$ matrix P such that $P[\mathbf{x}]_{\mathcal{B}}=[\mathbf{x}]_{\mathcal{C}}$;
3. $P_{\mathcal{C}\leftarrow\mathcal{B}}$ is invertible and $P^{-1}_{\mathcal{C}\leftarrow\mathcal{B}}=P_{\mathcal{B}\leftarrow\mathcal{C}}$

Theorem 6.13 Gauss Jordan method for computing a change of basis matrix Let $B:\mathbf{u}_1,\ldots,\mathbf{u}_n$ and $C:\mathbf{v}_1,\ldots,\mathbf{v}_n$ be ordered bases for a vector space V. For any basis ε for V (such as the standard basis if $V=\mathbb{R}^n$), let B be the matrix with columns $[\mathbf{u}_1]_\varepsilon,\ldots,[\mathbf{u}_n]_\varepsilon$ and C the matrix with columns $[\mathbf{v}_1]_\varepsilon,\ldots,[\mathbf{v}_n]_\varepsilon$. Then applying row reduction to the $n\times 2n$ augmented matrix produces

$$[C|B]\rightarrow[\mathbb{I}_n|P_{C\leftarrow B}]$$

Theorem 6.14 Let $T:\mathbb{R}^n\rightarrow\mathbb{R}^m$ be a linear transformation. Then

1. $T(\mathbf{0}_{\mathbb{R}^n})=\mathbf{0}_{\mathbb{R}^m}$;
2. $T(-\mathbf{v})=-T(\mathbf{v})$ for $\mathbf{v}\in\mathbb{R}^n$;
3. $T(\mathbf{u}-\mathbf{v})=T(\mathbf{u})-T(\mathbf{v})$ for $\mathbf{u},\mathbf{v}\in\mathbb{R}^n$.

Theorem 6.15 Let $T:\mathbb{R}^n\rightarrow\mathbb{R}^m$ be a linear transformation and let $B:\mathbf{v}_1,\ldots,\mathbf{v}_n$ br a basis for $\mathbb{R}^n$. Then T is completely determined by its effect on B. More precisely, if $\mathbf{v}\in\mathbb{R}^n$ has

$$\mathbf{v}=c_1\mathbf{v}_1+\cdots+c_n\mathbf{v}_n$$

for scalars $c_1,\ldots,c_n$, then

$$T(\mathbf{v})=c_1T(\mathbf{v}_1)+\cdots+c_nT(\mathbf{v}_n).$$

Theorem 6.16 If $Y:\mathbb{R}^n\rightarrow\mathbb{R}^m$ and $S:\mathbb{R}^m\rightarrow\mathbb{R}^p$ are linear transformations, then $S\circ T:\mathbb{R}^n\rightarrow\mathbb{R}^p$ is also linear.

Theorem 6.17 If $T:\mathbb{R}^n\rightarrow\mathbb{R}^n$ is an invetible linear transformation then its inverse is unique.

Theorem 3.33 Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be an invertible linear transformation. Then the matrices od T and T^{-1} with respect to any basis of $\mathbb{R}^n$ are also in inverse:

$$[T^{-1}] = [T]^{-1}.$$

Theorem 4.1 Let A be an $n \times n$ matrix. Then for any i we can expand along the ith row:

$\det(A) = (-1)^{i+1} a_{i1} \det(A_{i1}) + (-1)^{i+2} a_{i2} \det(A_{i2}) + \cdots +$
$(-1)^{i+n} a_{in} \det(A_{in})$
$= \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij})$

and for any j we can expand along the jth column:

$\det(A) = (-1)^{1+j} a_{1j} \det(A_{1j}) + (-1)^{2+j} a_{2j} \det(A_{2j}) + \cdots +$
$(-1)^{n+j} a_{nj} \det(A_{nj})$
$= \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij}).$

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$= \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij})$

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$(-1)^{n+j} a_{nj} \det(A_{nj})$
$= \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det(A_{ij}).$

Theorem 4.3 Let $A = (a_{ij})$ be a square matrix.

a) If B is obtained from A by swapping any two adjacent rows then

$\det(B) = -\det(A).$

b) If B is obtained from A by multiplying a row by k, then $\det(B) = k \det(A)$.

c) If B is obtained from A by adding a multiple of one row of A to another row of A, then $\det(B) = \det(A)$.

Theorem 4.4 Let E be an $n \times n$ elementary matrix

a) If E results from swapping two rows of $\mathbb{I}_n$ then $\det(E) = -1$.

b) If E results from multiplying one row of $\mathbb{I}_n$ by $k \neq 0$ then $\det(E) = k$.

c) If E results from adding a multiple of one row of $\mathbb{I}_n$ to another row then $\det(E) = 1$.

Lemma 4.5 Let B be an $n \times n$ matrix and E be an $n \times n$ elementary matrix. Then

$$\det(EB) = \det(E)\det(B).$$

Theorem 4.6 A square matrix A is invertible if and only if $\det(A) \neq 0$.

Theorem 4.7-4.10 Let A and B be $n \times n$ matrices.

1 $\det(kA) = k^n \det(A)$ for all scalars k.

2 $\det(AB) = \det(A)\det(B)$.

3 If A is invertible then

$$\det(A^{-1}) = \frac{1}{\det(A)}.$$

4 $\det(A) = \det(A^T)$.

Theorem 4.15 The eigenvalues of an upper- or lower-triangular matrix A are the entries on its main diagonal.

Theorem 4.16 A square matrix A is invertible if and only if zero is **not** an eigenvalue of A.

Theorem 4.20 Let A be an $n \times n$ matrix and let $\lambda_1, \lambda_2, \cdots, \lambda_m$ be **distinct** eigenvalues of A with corresponding eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m$. Then

$\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m\}$ is linearly independent.

Theorem 4.21 Let A, B and C be $n \times n$ matrices. Then

1. $A \sim A$
2. If $A \sim B$, then $B \sim A$
3. If $A \sim B$ and $B \sim C$, then $A \sim C$

Theorem 4.22 Let A and B be $n \times n$ matrices with $A \sim B$. The:

1. $\det(A) = \det(B)$
2. A is invertible if and only if B is invertible
3. A and B have the same rank
4. A and B have the same characteristic polynomial
5. A and B have the same eigenvalues

Theorem 4.23 Let A be an $n \times n$ matrix. Then A is diagonalisable if and only if it has n linearly independent eigenvectors.

Theorem 4.25 Let A be an $n \times n$ matrix with n distinct eigenvalues. Then A is diagonalisable.

Theorem 4.27 (The diagonalisation theorem) Let A be an $n \times n$ matrix whose distinct eigenvalues are $\lambda_1, \ldots, \lambda_k$. The following are equivalent:

a) A is diagonalisable

b) The union of a basis for each of the eaigenspaces of A contains n vectors

c) The algebraic multiplicity of each eigenvalue is equal to its geometric multiplicity and the sum of these multiplicites across all eigenvalues

Theorem 1.2 Properties of inner products Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w} \in \mathbb{R}^n$ and $c \in \mathbb{R}$. Then

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ (commutativity of inner product)

2. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$ (distributivity of inner product)
3. $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$
4. $\mathbf{u} \cdot \mathbf{u} \geq 0$ and $\mathbf{u} \cdot \mathbf{u} = 0$ if and only if $\mathbf{u} = \mathbf{0}$.

Theorem 1.4 Cauchy-Schwartz Inequality Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Then

$$|\mathbf{u} \cdot \mathbf{v}| \leq ||\mathbf{u}|| \, ||\mathbf{v}||$$

Moreover, we have equality if and only if $\mathbf{u}$ and $\mathbf{v}$ are linearly dependent.

Theorem 1.5 The Triangle Inequality Let $\mathbf{u}, \ \mathbf{v} \in \mathbb{R}^n$. Then

$$||\mathbf{u} + \mathbf{v}|| \leq ||\mathbf{u}|| + ||\mathbf{v}||$$

Theorem 5.1 Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ be an orthogonal set of non-zero vectors in $\mathbb{R}^n$. Then S is linearly independent set.

Theorem 5.2 Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ be an orthogonal basis for subspace V of $\mathbb{R}^n$. For any $\mathbf{v} \in V$ then there are $c_!, c_2, \ldots, c_k \in \mathbb{R}$ sych that

$$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k,$$

where $c_i = \frac{\mathbf{v} \cdot \mathbf{v}_i}{\mathbf{v}_i \cdot \mathbf{v}_i}$, for $i = 1, \ldots, k$.

Theorem 5.15 The Gram-Schmidt Process Let $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n\}$ be a basis for subspace W of $\mathbb{R}^n$. Define:

$$\begin{aligned}
\mathbf{v}_1 &= \mathbf{w}_1 \\
\mathbf{v}_2 &= \mathbf{w}_2 - \left(\frac{\mathbf{w}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\right)\mathbf{v}_1 \\
\mathbf{v}_3 &= \mathbf{w}_3 - \left(\frac{\mathbf{w}_3 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\right)\mathbf{v}_1 - \left(\frac{\mathbf{w}_3 \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2}\right)\mathbf{v}_2 \\
&\vdots \\
\mathbf{v}_n &= \mathbf{w}_n - \sum_{i=1}^{n-1}\left(\frac{\mathbf{w}_n \cdot \mathbf{v}_i}{\mathbf{v}_i \cdot \mathbf{v}_i}\right)\mathbf{v}_i
\end{aligned}$$

The vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are an orthogonal basis for W. If we set $\mathbf{u}_i = \frac{\mathbf{v}_i}{||\mathbf{v}_i||}$ then $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n$ are an orthonormal basis for W.

Theorem 5.5 A square matrix Q is orthogonal if and only if $Q^{-1} = Q^T$.

Theorem 5.6 Let Q be an $n \times n$ matrix. The following are equivalent,

a) Q is orthogonal

b) $||Q\mathbf{x}|| = ||\mathbf{x}||$ for all $\mathbf{x} \in \mathbb{R}^n$

c) $Q\mathbf{x} \cdot Q\mathbf{y} = \mathbf{x} \cdot \mathbf{y}$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$

This says every orthogonal matrix is an isometry, that is the matrix transformation preserves length.

Theorem 5.8 Let Q be an $n \times n$ orthogonal matrix. Then,

(a) Q^{-1} is orthogonal

(b) $\det Q = \pm 1$

(c) If λ is an eigenvalue of Q, then $|\lambda| = 1$

(d) If Q_1 and Q_2 are orthogonal $n \times n$ matrices then so is $Q_1 Q_2$

Theorem 5.17 If A is orthogonally diagonalisable, then A is symmetric.

Theorem 5.18 and 5.19 If A is a real symmetric matrix then

a) The eigenvalues of A are all real.

b) Eigenvector from different eigenspaces are orthogonal.

Theorem 5.18 and 5.19 If A is a real symmetric matrix then

a) The eigenvalues of A are all real.

b) Eigenvector from different eigenspaces are orthogonal.

Theorem 5.21 Principal Axes Theorem Every quadratic form can be diagonalised. If A is an $n \times n$ symmetric matrix such that there exists a quadratic form $q(\mathbf{x}) = \mathbf{x}^T A\mathbf{x}$, $\mathbf{x} \in \mathbb{R}^n$, and if Q is an orthogonal matrix such that $Q^T AQ = D$ is diagonal matrix, then the change of variables $\mathbf{x} = Q\mathbf{y}$ transforms the quadratic form q into $\mathbf{y}^T D\mathbf{y}$, which has no cross product terms. If the eigenvalues of A are $\lambda_1, \ldots, \lambda_n$ and $\mathbf{y} = [y_1, \ldots, y_n]^T$ then

$$\mathbf{x}^T A\mathbf{x} = \mathbf{y}^T D\mathbf{y} = \lambda_1 y_1^2 | \cdots | y_n^2$$

Theorem 5.22 Let A be an $n \times n$ symmetric matrix. The quadratic form $f(\mathbf{x}) = \mathbf{x}^T A\mathbf{x}$ is
a. positive definite if and only if all of the eigenvalues of A are positive. (signature is n)

b. positive semidefinite if and only if all of the eigenvalues of A are nonnegative. (signature = rank)

c. negative definite if and only if all of the eigenvalues of A are negative. (signature is $-n$)

d. negative semidefinite if and only if all of the eigenvalues of A are non positive. (signature = - rank)

e. indefinite if and only if A has both positive and negative eigenvalues. (-rank < signature < rank)

Note: The following don't have a number even in Poole:

Theorem If $A, B \in M_{n\times n}(\mathbb{C})$ be matrices and $c \in \mathbb{C}$. Then

a) $(A^*)^* = A$

b) $(A+B)^* = A^* + B^*$

c) $(CA)^* = \overline{c}A^*$

d) $(AB)^* = B^*A^*$

Theorem If $A, B \in M_{n\times n}(\mathbb{C})$ be matrices and $c \in \mathbb{C}$. Then

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Theorem Every Hermitian matrix A is *unitarily diagonalisable*.

Theorem A square complex matrix A is *unitarily diagonalisable* if and only if

$$A^*A = AA^*.$$

Theorem Every Hermitian matrix, every unitary matrix, and every *skew Hermitian matrix* $(A^* = -A)$ is normal.