Mathematics 1
Week 3: Introduction to Integration

Peter Stewart

peter.stewart@glasgow.ac.uk

2019-20

LOGIN: `https://classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

21. MULTIPLE RESPONSE QUESTION

Consider the area enclosed by the curves $y = 4 - x^2$, $y = 0$ and $x = 0$. Select the appropriate width of strip to divide the x-domain into n rectangles:

(A) $\Delta x = 1/n$;	(B) $\Delta x = 2/n$;
(C) $\Delta x = 4/n$;	(D) None of the above.

21. Multiple response question

Consider the area enclosed by the curves $y = 4 - x^2$, $y = 0$ and $x = 0$. Select the appropriate width of strip to divide the x-domain into n rectangles:

(A) $\Delta x = 1/n$; (B) $\Delta x = 2/n$;

(C) $\Delta x = 4/n$; (D) None of the above.

Solution: (B). The region enclosed by the integral along the x-axis is between $x = 0$ and $x = 2$, so we use strips of width $\Delta x = (2 - 0)/n$.

Dividing a domain into intervals

For a general domain $[a, b]$ we divide the domain into n equally sized strips of width $\Delta x = (b - a)/n$ with boundaries at $x_0 (= a)$, $x_1, \cdots, x_n (= b)$ such that $x_i = a + i\Delta x$ $(i = 0, 1, 2, \cdots, n)$.

RIEMANN SUMS

AREA

We approximate the area A under the graph of the continuous function f by the sum of the areas of the approximating rectangles
Form the right Riemann sum R_n

$$\begin{aligned} A \approx R_n &= [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x] \\ &= \sum_{i=1}^{n} f(x_i)\Delta x \end{aligned}$$

by choosing the sample point as the value of f at the right hand side of the ith subinterval.

[DIAGRAMS OF AREA IN EACH CASE]

RIEMANN SUMS

AREA

We approximate the area A under the graph of the continuous function f by the sum of the areas of the approximating rectangles
Form the left Riemann sum L_n

$$\begin{aligned} A \approx L_n &= [f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x] \\ &= \sum_{i=0}^{n-1} f(x_i)\Delta x \end{aligned}$$

by choosing the sample point as the value of f at the left hand side of the ith subinterval.

[DIAGRAMS OF AREA IN EACH CASE]

AREA

We approximate the area A under the graph of the continuous function f by the sum of the areas of the approximating rectangles
Form the lower Riemann sum l_n

$$A \approx l_n = [f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x] = \sum_{i=1}^{n} f(x_i^*)\Delta x$$

by choosing the sample points x_i^* so that $f(x_i^*)$ is the minimum value of f on the ith subinterval.

[DIAGRAMS OF AREA IN EACH CASE]

AREA

We approximate the area A under the graph of the continuous function f by the sum of the areas of the approximating rectangles

Form the upper Riemann sum u_n

$$
\begin{aligned}
A \approx u_n &= [f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x] \\
&= \sum_{i=1}^{n} f(x_i^*)\Delta x
\end{aligned}
$$

by choosing the sample points x_i^* so that $f(x_i^*)$ is the maximum value of f on the ith subinterval.

[DIAGRAMS OF AREA IN EACH CASE]

Class response

Login: https : //classresponse.gla.ac.uk

Session number: 570 (10am) 571 (11am) 572 (11am OF)

22. Multiple Choice question

Consider the area below the function $f(x) = \sin x$ between $x = 0$ and $x = \pi$. Using two grid intervals, which of the following is the upper Riemann sum for the area under the graph of f:

(A) 0; (B) $\frac{1}{2}\pi$; (C) π; (D) None of the above.

22. MULTIPLE CHOICE QUESTION

Consider the area below the function $f(x) = \sin x$ between $x = 0$ and $x = \pi$. Using two grid intervals, which of the following is the upper Riemann sum for the area under the graph of f:

(A) 0; (B) $\frac{1}{2}\pi$; (C) π; (D) None of the above.

Solution: (C). With two grid intervals we have $\Delta x = \pi/2$ and so

$$u_2 = \sum_{i=1}^{2} f(x_i^*)\Delta x = \tfrac{1}{2}\pi(f(\tfrac{1}{2}\pi) + f(\tfrac{1}{2}\pi)) = \tfrac{1}{2}\pi(1 + 1) = \pi.$$

Class response

Login: `https://classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

23. Multiple Choice question

Consider the area below the function $f(x) = \sin x$ between $x = 0$ and $x = \pi$. Using two grid intervals, which of the following is the lower Riemann sum for the area under the graph of f:

(A) 0; (B) $\frac{1}{2}\pi$; (C) π; (D) None of the above.

23. MULTIPLE CHOICE QUESTION

Consider the area below the function $f(x) = \sin x$ between $x = 0$ and $x = \pi$. Using two grid intervals, which of the following is the lower Riemann sum for the area under the graph of f:

(A) 0; (B) $\frac{1}{2}\pi$; (C) π; (D) None of the above.

Solution: (A). With two grid intervals we have $\Delta x = \frac{1}{2}\pi$ and so

$$I_2 = \sum_{i=1}^{2} f(x_i^*)\Delta x = \tfrac{1}{2}\pi(f(0) + f(\pi)) = \tfrac{1}{2}\pi(0 + 0) = 0.$$

Class response

Login: `https : //classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

24. Multiple select question

Consider the area below the function $f(x) = x$ between $x = 0$ and $x = 1$ using n grid intervals, with corresponding left (denoted L_n), right (R_n), lower (l_n) and upper (u_n) Riemann sums. Which of the following statements is true:

(A) : $l_n = L_n$; (B) : $u_n = R_n$; (C) : $u_n = L_n$;

(D) : $l_n = R_n$; (E) : $L_n > R_n$; (F) : None of the above.

Class response

24. Multiple select question

Consider the area below the function $f(x) = x$ between $x = 0$ and $x = 1$ using n grid intervals, with corresponding left (denoted L_n), right (R_n), lower (l_n) and upper (u_n) Riemann sums. Which of the following statements is true:

(A) : $l_n = L_n$; (B) : $u_n = R_n$; (C) : $u_n = L_n$;
(D) : $l_n = R_n$; (E) : $L_n > R_n$; (F) : None of the above.

Solutions: (A), (B). As the function is monotonically increasing in the given range, the minimum value of the function across each interval is always at the left hand boundary, and the maximim value of the function across each interval is always at the right hand boundary.

AREA

The area A under the graph of the continuous function f is the limit of the sum of the areas of the approximating rectangles (as the width of each rectangle goes to zero)
Take the limit of the right Riemann sum R_n

$$\begin{aligned} A &= \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x] \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \end{aligned}$$

by choosing the sample point as the value of f at the right hand side of the ith subinterval.

AREA

The area A under the graph of the continuous function f is the limit of the sum of the areas of the approximating rectangles (as the width of each rectangle goes to zero)
Take the limit of the left Riemann sum L_n

$$A = \lim_{n\to\infty} L_n = \lim_{n\to\infty} [f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x]$$
$$= \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i)\Delta x$$

by choosing the sample point as the value of f at the left hand side of the ith subinterval.

Areas

Area

The area A under the graph of the continuous function f is the limit of the sum of the areas of the approximating rectangles (as the width of each rectangle goes to zero)
Take the limit of the lower and upper Riemann sums

$$\begin{aligned} A &= \lim_{n\to\infty} [f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x] \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \end{aligned}$$

by choosing the sample points x_i^* so that $f(x_i^*)$ is the minimum (maximum) value of f on the ith subinterval.

DEFINITE INTEGRAL

If f is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0\,(= a)$, x_1, x_2,...,$x_n\,(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \cdots, x_n^*$ be any sample points in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the definite integral of f from a to b is

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that this limit exists and gives the same value for all possible choices of sample points. If it does exist, we say f is integrable on $[a, b]$.

EXAMPLE 3.1

Compute an expression for the definite integral

$$\int_1^2 x^3 \, dx$$

as a limit of a right Riemann sum and evaluate this limit.

[EXAMPLE 3.1]

Theorems of integration

Theorem

If f is continuous on $[a, b]$, or if f has only a finite number of jump discontinuities, then f is integrable on $[a, b]$; that is, the definite integral

$$\int_a^b f(x)\,\mathrm{d}x$$

exists.

Theorem

If f is integrable on $[a, b]$, then

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$, $(i = 0, 1, 2, \cdots, n)$.

Properties of the definite integral

The definite integral has the following properties

1. $$\int_a^b c\,\mathrm{d}x = c(b-a)$$

   where c is any constant;

2. $$\int_a^b [f(x)+g(x)]\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_a^b g(x)\,\mathrm{d}x;$$

3. $$\int_a^b cf(x)\,\mathrm{d}x = c\int_a^b f(x)\,\mathrm{d}x$$

   where c is any constant;

4. $$\int_a^b [f(x)-g(x)]\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x - \int_a^b g(x)\,\mathrm{d}x;$$

Properties of the definite integral

The definite integral has the following properties

5
$$\int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x;$$

6 If $f(x) \geq 0$ for $a \leq x \leq b$ then
$$\int_a^b f(x)\,\mathrm{d}x \geq 0;$$

7 If $f(x) \geq g(x)$ for $a \leq x \leq b$ then
$$\int_a^b f(x)\,\mathrm{d}x \geq \int_a^b g(x)\,\mathrm{d}x;$$

8 If $m \leq f(x) \leq M$ for $a \leq x \leq b$ then
$$m(b-a) \leq \int_a^b f(x)\,\mathrm{d}x \leq M(b-a).$$

Example

Example 3.2

Consider the definite integral

$$I = \int_{-\pi}^{\pi} \cos^2\left(\tfrac{1}{2}x\right)\, \mathrm{d}x.$$

Show that $0 \leq I \leq 2\pi$. Evaluate I given that

$$\int_{-\pi}^{\pi} \cos(x)\, \mathrm{d}x = 0.$$

[EXAMPLE 3.2]

LOGIN: `https : //classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

25. MULTI-SELECT QUESTION

Use left and right Riemann sums with two grid intervals (denoted L_2 and R_2, respectively) to approximate the integral

$$I = \int_0^{\pi} \sin x \, \mathrm{d}x.$$

Select all that apply:

$(A): \ L_2 = \frac{1}{2}\pi;$ $(B): \ R_2 = \pi;$ $(C): \ L_2 = 0;$

$(D): \ R_2 = \frac{1}{2}\pi;$ $(E): \ R_2 > L_2;$ $(F): \ R_2 = 0.$

25. MULTI-SELECT QUESTION

Use left and right Riemann sums with two grid intervals (denoted L_2 and R_2, respectively) to approximate the integral

$$I = \int_0^{\pi} \sin x \, \mathrm{d}x.$$

Select all that apply:

$(A): \; L_2 = \frac{1}{2}\pi;$ $(B): \; R_2 = \pi;$ $(C): \; L_2 = 0;$

$(D): \; R_2 = \frac{1}{2}\pi;$ $(E): \; R_2 > L_2;$ $(F): \; R_2 = 0.$

Solutions: (A), (D). We compute $L_2 = R_2 = \frac{1}{2}\pi$. We will see next week that the exact value of this integral is $I = 2$.

RECALL: DEFINITE INTEGRAL

If f is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0\,(= a)$, x_1, x_2,...,$x_n\,(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \cdots, x_n^*$ be any sample points in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the definite integral of f from a to b is

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that this limit exists and gives the same value for all possible choices of sample points. If it does exist, we say f is integrable on $[a, b]$.

The midpoint rule

In numerical integration we approximate the integral using a finite number of intervals n similar to Riemann sums.

Midpoint rule

We define the midpoint rule by choosing the sample point $x_i^* = \overline{x}_i$, the midpoint of interval i

$$\int_a^b f(x)\,\mathrm{d}x \approx M_n = \Delta x[f(\overline{x}_1) + \cdots + f(\overline{x}_n)],$$

where $\Delta x = (b-a)/n$ and $\overline{x}_i = \frac{1}{2}(x_i + x_{i-1})$, $(i = 1, \cdots, n)$.

[DIAGRAM OF MIDPOINT RULE]

Class response

Login: `https : //classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

26. Mutliple choice question

Select the value of the integral

$$I = \int_0^{\pi} \sin(x)\,\mathrm{d}x$$

approximated using the midpoint rule with two intervals.

$(A)\ M_2 = 2$; $(B)\ M_2 = \pi/\sqrt{2}$; $(C)\ M_2 = \pi/2$; $(D)\ M_2 = 2\pi/3$.

26. Mutliple choice question

Select the value of the integral

$$I = \int_0^{\pi} \sin(x)\, dx$$

approximated using the midpoint rule with two intervals.

$(A)\; M_2 = 2$; $(B)\; M_2 = \pi/\sqrt{2}$; $(C)\; M_2 = \pi/2$; $(D)\; M_2 = 2\pi/3$.

Solution (B): We choose two intervals $[0, \pi/2]$ and $[\pi/2, \pi]$ so $\Delta x = \pi/2$. The midpoints are $\overline{x}_1 = \pi/4$ and $\overline{x}_2 = 3\pi/4$, while $\sin(\overline{x}_1) = 1/\sqrt{2}$ and $\sin(\overline{x}_2) = 1/\sqrt{2}$. Hence

$$\begin{aligned}\int_0^{\pi} \sin(x)\, dx \approx M_2 &= \Delta x[f(\overline{x}_1) + f(\overline{x}_2)] \\ &= (\pi/2) \times (2 \times 1/\sqrt{2}) = \pi/\sqrt{2} \approx 2.22144.\end{aligned}$$

[EXAMPLE 3.3]

THE TRAPEZIUM RULE

RECALL: LEFT AND RIGHT RIEMANN SUMS

The left and right Riemann sums take the form

$$\int_a^b f(x)\,\mathrm{d}x \approx L_n = \sum_{i=1}^{n} f(x_{i-1}), \qquad \int_a^b f(x)\,\mathrm{d}x \approx R_n = \sum_{i=1}^{n} f(x_i).$$

THE TRAPEZIUM RULE

The trapezium rule is the average of the left and right Riemann sums in the form

$$\begin{aligned}\int_a^b f(x)\,\mathrm{d}x \approx T_n &= \tfrac{1}{2}(L_n + R_n)\\ &= \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)].\end{aligned}$$

[DIAGRAM OF TRAPEZIUM RULE]

Class response

Login: `https : //classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

27. Mutliple choice question

Select the value of the integral

$$I = \int_0^\pi \sin(x)\,\mathrm{d}x$$

approximated using the trapezium rule with two intervals.

$(A)\ T_2 = 2$; $(B)\ T_2 = \pi/\sqrt{2}$; $(C)\ T_2 = \pi/2$; $(D)\ T_2 = 2\pi/3$.

27. MUTLIPLE CHOICE QUESTION

Select the value of the integral

$$I = \int_0^{\pi} \sin(x)\,\mathrm{d}x$$

approximated using the trapezium rule with two intervals.

$(A)\ T_2 = 2;\quad (B)\ T_2 = \pi/\sqrt{2};\quad (C)\ T_2 = \pi/2;\quad (D)\ T_2 = 2\pi/3.$

Solution (C): We choose two intervals $[0, \pi/2]$ and $[\pi/2, \pi]$ so $\Delta x = \pi/2$ and so the points are $x_0 = 0$, $x_1 = \pi/2$ and $x_2 = \pi$ where $\sin(x_0) = 0$, $\sin(x_1) = 1$ and $\sin(x_2) = 0$. Hence

$$\int_0^{\pi} \sin(x)\,\mathrm{d}x \approx T_2 = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + f(x_2)]$$

$$= (\pi/4) \times (0 + 2 + 0) = \pi/2 \approx 1.5708.$$

[EXAMPLE 3.4]

Simpson's rule

Simpson's rule

Use parabolae instead of straight lines to approximate the curve

Divide $[a, b]$ into n subintervals of equal length $\Delta x = (b - a)/n$ (in this case n must be even)

In Simpson's rule we consider consecutive pair of intervals and approximate the intergral over each pair (i odd)

$$I_i = \int_{x_i - \Delta x}^{x_i + \Delta x} f(x)\,\mathrm{d}x = \int_{x_{i-1}}^{x_{i+1}} f(x)\,\mathrm{d}x, \qquad (i = 1, 3, \cdots, n-1)$$

and so the total integral takes the form

$$I = \int_a^b f(x)\,\mathrm{d}x \approx \sum_{j=0}^{n/2-1} I_{2j+1}.$$

[DIAGRAM OF SIMPSON'S RULE]

Approximating the parabola

For i odd we approximate the curve that passes through three consecutive points

$$(x_{i-1}, y_{i-1}) = (x_{i-1}, f(x_{i-1})), \quad (x_i, y_i) = (x_i, f(x_i)),$$

$$(x_{i+1}, y_{i+1}) = (x_{i+1}, f(x_{i+1})),$$

as the parabola $y = A_i x^2 + B_i x + C_i$

The integral of this parabola (next week!) takes the form

$$\int_{x_i - \Delta x}^{x_i + \Delta x} (A_i x^2 + B_i x + C_i)\, \mathrm{d}x = \frac{\Delta x}{3}(2A_i \Delta x^2 + 6C_i)$$

Approximating the parabola

We obtain a linear system for A_i, B_i and C_i

$$\begin{aligned} y_{i-1} &= A_i x_{i-1}^2 + B_i x_{i-1} + C_i, \\ y_i &= A_i x_i^2 + B_i x_i + C_i, \\ y_{i+1} &= A_i x_{i+1}^2 + B_i x_{i+1} + C_i, \end{aligned}$$

with solution in the form

$$A_i = \frac{-x_{i+1}y_i + x_{i-1}y_i + x_i y_{i+1} - x_{i-1}y_{i+1} - x_i y_{i-1} + x_{i+1}y_{i-1}}{\Delta x^3},$$

$$B_i = \frac{-x_{i+1}^2 y_i + x_{i-1}^2 y_i + x_i^2 y_{i+1} - x_{i-1}^2 y_{i+1} - x_i^2 y_{i-1} + x_{i+1}^2 y_{i-1}}{\Delta x^3},$$

$$C_i = \frac{-x_{i+1}^2 x_{i-1} y_i + x_i x_{i-1}^2 y_i + x_i^2 x_{i-1} y_{i+1} - x_i x_{i-1}^2 y_{i+1} - x_i^2 x_{i+1} y_{i-1} + x_i x_{i+1}^2 y_{i-1}}{\Delta x^3},$$

and so

$$\int_{x_i-\Delta x}^{x_i+\Delta x} y(x)\,\mathrm{d}x = \int_{x_{i-1}}^{x_{i+1}} (A_i x^2 + B_i x + C_i)\,\mathrm{d}x = \frac{\Delta x}{3}(y_{i-1} + 4y_i + y_{i+1}).$$

Login: `https://classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

28. Multiple choice question

Select the parabola passing through the points (0,0), ($\frac{1}{2}\pi$,1) and (π,0):

$(A): \; y = \frac{1}{3}(x^2 + \pi x + \pi^2);$ $(B): \; y = -x^2 + 2x + 2;$

$(C): \; y = \dfrac{4}{\pi^2}(-x^2 + \pi x);$ $(D):$ None of the above.

Class response

28. Multiple choice question

Select the parabola passing through the points (0,0), ($\frac{1}{2}\pi$,1) and (π,0):

$(A):\ y = \frac{1}{3}(x^2 + \pi x + \pi^2);$ $(B):\ y = -x^2 + 2x + 2;$

$(C):\ y = \dfrac{4}{\pi^2}(-x^2 + \pi x);$ $(D):$ None of the above.

Solution: (C). Verify by substituing into the formulae for A_i, B_i and C_i by setting $(x_{i-1}, y_{i-1}) = (0,0)$, $(x_i, y_i) = (\frac{1}{2}\pi, 1)$ and $(x_{i+1}, y_{i+1}) = (\pi, 0)$.

Simpson's rule

Simpson's rule

Summing all the contributions together we obtain

$$\int_a^b f(x)\,\mathrm{d}x \approx S_n = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots$$
$$\cdots + 2y_{n-2} + 4y_{n-1} + y_n)$$

CLASS RESPONSE

LOGIN: `https : //classresponse.gla.ac.uk`

Session number: 570 (10am) 571 (11am) 572 (11am OF)

29. MUTLIPLE CHOICE QUESTION

Select the value of the integral

$$I = \int_0^\pi \sin(x)\,\mathrm{d}x$$

approximated using Simpson's rule with two intervals.

$(A)\ S_2 = 2$; $(B)\ S_2 = \pi/\sqrt{2}$; $(C)\ S_2 = \pi/2$; $(D)\ S_2 = 2\pi/3$.

29. MUTLIPLE CHOICE QUESTION

Select the value of the integral

$$I = \int_0^{\pi} \sin(x)\,\mathrm{d}x$$

approximated using Simpson's rule with two intervals.

$(A)\ S_2 = 2$; $(B)\ S_2 = \pi/\sqrt{2}$; $(C)\ S_2 = \pi/2$; $(D)\ S_2 = 2\pi/3$.

Solution (D): We choose two intervals $[0, \pi/2]$ and $[\pi/2, \pi]$ so $\Delta x = \pi/2$ and $x_0 = 0$, $x_1 = \pi/2$ and $x_2 = \pi$ where $y_0 = \sin(x_0) = 0$, $y_1 = \sin(x_1) = 1$ and $y_2 = \sin(x_2) = 0$. Hence

$$\int_0^{\pi} \sin(x)\,\mathrm{d}x \approx S_2 = \frac{\Delta x}{3}(y_0 + 4y_1 + y_2)$$
$$= (\pi/6) \times (0 + 4 + 0) = 2\pi/3 \approx 2.0944.$$

[EXAMPLE 3.5]

29. Mutliple choice question

Select the value of the integral

$$I = \int_0^{\pi} \sin(x)\,\mathrm{d}x$$

approximated using Simpson's rule with two intervals.

$(A)\ S_2 = 2;\quad (B)\ S_2 = \pi/\sqrt{2};\quad (C)\ S_2 = \pi/2;\quad (D)\ S_2 = 2\pi/3.$

Note

Note that the integral S_2 is equal to the area under the parabola constructed in Class Response question 28

$$S_2 = \int_0^{\pi} \frac{4}{\pi^2}(-x^2 + \pi x)\,\mathrm{d}x = \frac{2\pi}{3}.$$