



University
of Glasgow

Monday, 29 April 2019
9.30 am – 11.00 am
(1 hour 30 minutes)

DEGREES OF MSci, MEng, BEng, BSc, MA and MA (Social Sciences)

ALGORITHMS AND DATA STRUCTURES 2: COMPSCI2007

Answer all questions

This examination paper is worth a total of 60 marks.

The use of calculators is not permitted in this examination.

INSTRUCTIONS TO INVIGILATORS: Please collect all exam question papers and exam answer scripts and retain for school to collect. Candidates must not remove exam question papers.

1. Consider the algorithm described by the following pseudocode:

```

1: F( $A, x, p, r$ )
2:   if  $p > r$ 
3:     return  $-1$ 
4:    $q := (p + r) / 2$ 
5:   if  $A[q] = x$ 
6:     return  $q$ 
7:   else
8:     if  $A[q] > x$ 
9:       return F( $A, x, p, q - 1$ )
10:    else
11:      return F( $A, x, q + 1, r$ )

```

The inputs of algorithm **F** are: a sorted array of integers A , an integer x , and two indices for A p, r .

- (a) Briefly explain what algorithm **F** implements. [5]
- (b) What is the output of **F**($A, 13, 0, 7$), where $A = [0, 4, 4, 5, 7, 8, 13, 14]$? [2]
- (c) Draw the recursion trace for **F**($A, 13, 0, 7$). [3]
- (d) Is **F** tail recursive? Justify your answer. [2]
- (e) Is **F** linear recursive? Justify your answer. [2]
- (f) Write the recurrence equation for the running time $T(n)$ of algorithm **F** (with $n = r - p + 1$). [2]
- (g) Solve the recurrence equation in 1(f) to compute the complexity of algorithm **F** using big-Oh notation. [4]

2. Rank the following functions by order of growth; that is, find an arrangement f_1, f_2, \dots, f_8 of the functions satisfying: $f_{i-1} = O(f_i)$.

$\log \log n$ 1 n^3 2^n n \sqrt{n} $n \log n$ $\log n$

[5]

3. (a) Describe using pseudocode or otherwise the merge sort algorithm for sorting a list of integers contained in an array. Illustrate your description by sorting the list: 6, 3, 1, 7, 4, 5, 2. Note that you do not need to describe the merge procedure in detail. [8]
- (b) What is the time complexity of merge sort? [1]
- (c) Assume the quicksort algorithm is implemented using a partitioning scheme that selects the middle element (i.e. element $A[q]$ in subarray $A[p..r]$ where $q = \lfloor (p+r)/2 \rfloor$) as the pivot. Show an input of length 8 exhibiting worst case complexity $O(n^2)$. Explain your answer. [4]
4. (a) Briefly describe the two primary operations of the queue ADT. [2]
- (b) Briefly describe the linked list implementation of the queue ADT. What is the complexity of the primary operations? [4]
5. (a) What is a binary search tree? Does the order in which a set of values is inserted into a binary search tree matter? Justify your answer. [6]
- (b) Give a recursive definition of the preorder, inorder and postorder traversals of a binary tree. [6]
- (c) Define using pseudocode or otherwise the insert operation on binary search trees. [4]