Tutorial group	
Name	
Student Id	

University of Glasgow | School of Computing Science

Assessed Coursework

Course Name	**Algorithmic Foundations 2**			
Coursework Number	**2**			
Deadline	**Time:**	**16:30**	**Date:**	**23/11/2020**
% Contribution to final course mark	**10%**			
Solo or Group ✓	**Solo**	✓	**Group**	
Anticipated Hours	**10**			
Submission Instructions	**Use the latex template and submit the generated pdf through moodle (do not submit the source latex file). Failure to follow the submission instructions will lead to a penalty of 2 bands.**			
Please Note: This Coursework cannot be Re-Assessed				

Code of Assessment Rules for Coursework Submission

Deadlines for the submission of coursework which is to be formally assessed will be published in course documentation, and work which is submitted later than the deadline will be subject to penalty as set out below.

The primary grade and secondary band awarded for coursework which is submitted after the published deadline will be calculated as follows:

(i) in respect of work submitted not more than five working days after the deadline
   a. the work will be assessed in the usual way;
   b. the primary grade and secondary band so determined will then be reduced by two secondary bands for each working day (or part of a working day) the work was submitted late.

(ii) work submitted more than five working days after the deadline will be awarded Grade H.

Penalties for late submission of coursework will not be imposed if good cause is established for the late submission. You should submit documents supporting good cause via MyCampus.

Algorithmic Foundations 2

Assessed Exercise 2

Notes for guidance

1. There are two assessed exercises. Each is worth 10% of your final grade for this module. Your answers must be the result of your own individual efforts.

2. Please use the latex template and submit your the generated pdf via moodle (do not submit the latex source file).

3. Please ensure you have filled out your tutorial group, name and student id.

4. **Failure to follow the submission instructions will lead to a penalty for non-adherence to submission instructions of 2 bands**.

5. As stated on the cover sheet deadline for completing this assessed exercise is **16:30 Monday, November 23, 2020**.

6. The exercise is marked out of 30 using the included marking scheme. Credit will be given for partial answers.

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1. (a) Show that if n divides m where n and m are positive integers greater than 1, then $a \equiv b \pmod{m}$ implies $a \equiv b \pmod{n}$ for any positive integers a and b. [4]

   (b) Show that $a{\cdot}c \equiv b{\cdot}c \pmod{m}$ with a, b, c and m integers with $m{\geq}2$ does not imply $a \equiv b \pmod{m}$. [3]

   (c) Using the Euclidean Algorithm, find $\texttt{gcd}(3084, 1424)$. Show your working. [3]

2. A company has a contract to cover the four walls, ceiling, and floor of a factory building with fire-retardant material. The building is rectangular where of width 280m, length 336m and height 168m. Square panels can be manufactured in any size of whole metres. For safety reasons, the building must be covered in complete panels (i.e. panels cannot be cut). What is the minimum number of equally sized square panels that are required to line the interior of the building? Explain your answer. [5]

3. Prove that least significant digit of the square of an even integer is either 0, 4, or 6. [5]

   Hint: considering splitting into cases where integers are of the form $a{\cdot}k{+}b$ or $-(a{\cdot}k{+}b)$ for $k \in \mathbb{N}$ where a and b are fixed for a given case, b varies over the cases and the least significant digit of the integer depends on only b.

   Note: the *least significant digit* of an integer is the digit farthest to the right in a integer. For example, the least significant digits of 1007 and 26 are 7 and 6 respectively.

4. Use mathematical induction to show that for any $n \in \mathbb{N}$, if $n \geq 2$, then [5]

$$\prod_{i=2}^{n} \left(1 - \frac{1}{i^2}\right) = \frac{n{+}1}{2{\cdot}n}\,.$$

5. Use mathematical induction to show that 2 divides $n^2 - n$ for all $n \in \mathbb{N}$. [5]