# 2A Multivariable Calculus 2020

Ex Sheet 0

## Differentiation

**Q1** Find the derivatives of the following functions

a) $y = (2x+3)^6$

b) $y = \frac{3}{\sin(2x)}$

c) $y = \sqrt{x+7}$

d) $y = x^3 \tan(x)$

e) $y = \frac{\ln x}{x}$

f) $y = \cos(3x+2)$

g) $y = \frac{1+\sqrt{x}}{1-\sqrt{x}}$

h) $y = e^{x^2}$

i) $y = \sec(\sqrt{x}) + 3$

j) $y = \sin^{-1} x$

**Solution**

a) $12(2x+3)^5$ b) $-6\mathrm{cosec}(2x)\cot(2x)$ c) $\frac{1}{2\sqrt{x+7}}$ d) $3x^2\tan(x) + \frac{x^3}{\cos^2 x}$ e) $\frac{1-\ln x}{x^2}$ f) $-3\sin(3x+2)$
g) $\frac{1}{\sqrt{x}(1-\sqrt{x})^2}$ h) $2xe^{x^2}$ i) $\frac{\sec\sqrt{x}\tan\sqrt{x}}{2\sqrt{x}}$ j) $\frac{1}{\sqrt{1-x^2}}$

## Integration

**Q2** Compute the following integrals

a) $\int xe^x \, dx$

b) $\int \frac{x}{2x^2+5x+2}$

c) $\int e^{-x}\sin(2x)\, dx$

d) $\int \frac{3}{4x^2-1}\, dx$

e) $\int \frac{\cos x}{4+\sin^2 x}\, dx$

f) $\int xe^{-4x^2}\, dx$

g) $\int \sin x \cos x \, dx$

h) $\int_0^8 \frac{\cos(\sqrt{x+1})}{\sqrt{x+1}}\, dx$

i) $\int \ln x\, dx$

j) $\int \frac{2+3x+x^2}{x(x^2+1)}\, dx$

**Solution**

a) $xe^x - e^x + C$ b) $\frac{2}{3}\ln|x+2| - \frac{1}{6}\ln|2x+1| + C$ c) $-e^{-x}\left(\frac{2}{5}\cos 2x + \frac{1}{5}\sin 2x\right) + C$
d) $\frac{3}{4}\ln\left|\frac{2x-1}{2x+1}\right| + C)$ e) $\frac{1}{2}\tan^{-1}\left(\frac{\sin x}{2}\right) + C$ f) $\frac{-1}{8}e^{-4x^2} + C$ g) $\frac{-1}{4}\cos(2x) + C$ h) $2\sin 3 - 2\sin 1$
i) $x\ln x - x + C$ j) $2\ln x + 3\tan^{-1} x - \frac{1}{2}\ln(x^2+1) + C$.

## Vector algebra

**Q3** Let $\mathbf{a} = (1,5,3)$ , $\mathbf{b} = (2,4,7)$, $\mathbf{c} = (2,0,-1)$. Find

$$\mathbf{a}\cdot\mathbf{b}, \quad \mathbf{a}\times\mathbf{b}, \quad \mathbf{b}\times\mathbf{c}, \quad [\mathbf{a},\mathbf{b},\mathbf{c}].$$

**Solution**

$\mathbf{a}\cdot\mathbf{b} = 43, \quad \mathbf{a}\times\mathbf{b} = (23,-1,-6), \quad \mathbf{b}\times\mathbf{c} = (-4,16,-8), \quad [\mathbf{a},\mathbf{b},\mathbf{c}] = 52.$

**Q4** Suppose that $\mathbf{u}$ and $\mathbf{v}$ are unit vectors and the angle between them is $\pi/4$. Let $\mathbf{a} = \mathbf{u} + 3\sqrt{2}\mathbf{v}$. By considering $\mathbf{a}\cdot\mathbf{a}$, find $|\mathbf{a}|$.

**Solution**

$|\mathbf{a}| = \sqrt{\mathbf{a}\cdot\mathbf{a}} = \sqrt{25} = 5.$

**Q5** Let $\mathbf{u}$ and $\mathbf{v}$ be non-zero *parallel* vectors. Find $\mathbf{u}\times\mathbf{v}$.

**Solution**

$\mathbf{u}\times\mathbf{v} = 0.$

**Q6** Let $\mathbf{a}$ and $\mathbf{b}$ be non-zero vectors, simplify $(\mathbf{a}+2\mathbf{b})\times(3\mathbf{a}-4\mathbf{b})$.

**Solution**

$(\mathbf{a} + 2\mathbf{b}) \times (3\mathbf{a} - 4\mathbf{b}) = -10\mathbf{a} \times \mathbf{b}$.

**Q7** Let $\mathbf{a} = (-5, 4, 2)$ and $\mathbf{b} = (-2, 1, 2)$. Calculate $\mathbf{a} \times \mathbf{b}$ and hence find the two unit vectors perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

**Solution**

$\mathbf{a} \times \mathbf{b} = (6, 6, 3)$. The two unit vectors are $\pm\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$.

**Q8** Let $\mathbf{a} = (1, 2, 3)$, $\mathbf{b} = (2, -1, 1)$ and $\mathbf{c} = (1, 4, -1)$.

a) Obtain the value of $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ by working out the value of a $3 \times 3$ determinant.

b) Without performing separate determinant expansions, write down the values of $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$ and $\mathbf{c} \cdot (\mathbf{b} \times \mathbf{a})$.

**Solution**

a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 30$  b) $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = 30$, $\mathbf{c} \cdot (\mathbf{b} \times \mathbf{a}) = -30$.