2A: Tutorial 1

School of Mathematics and Statistics

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Semester 1 2020–21

Instructions

Access your tutorial lecture

TU01,TU02,TU03,TU04,TU05: September 28th, 09:00-10:00

TU06,TU07,TU08,TU09,TU10: September 28th, 15:00-16:00

using the Zoom plug-in in the Moodle page of the course.

Please mute yourself when you are in the main room of the meeting.

Be ready to work in groups using Microsoft Whiteboard as instructed by your tutor. Having a method of writing on screen, as mouse, tablet or similar is useful.

From the Moodle forum

Only real numbers!

This course deals with real-valued functions $f : D \subset \mathbb{R}^n \to \mathbb{R}^m$.

Whenever D is not specified we will assume it is the maximal domain for f: the largest subset of $\mathbb{R}^n$ such that the expression defining f is meaningful.

Multiple Choice Question 1

Ex Sheet 1, T1(b)

The set of points $(x, y, z) \in \mathbb{R}^3$ that satisfy $x^2 + y^2 = 1$ is best described as

(A) A circle, radius 1

(B) A plane, distance 1 from the origin

(C) A cylinder, radius 1

(D) A sphere, radius 1

Multiple Choice Question 1

Ex Sheet 1, T1(b)

The set of points $(x, y, z) \in \mathbb{R}^3$ that satisfy $x^2 + y^2 = 1$ is best described as

(A) A circle, radius 1

(B) A plane, distance 1 from the origin

(C) A cylinder, radius 1

(D) A sphere, radius 1

ANSWER: **(C)** The equation imposes no constraint on z. The surface is invariant under translations in z.

The $z = 0$ cross-section (the x-y plane), gives a circle, centred at the origin, with radius 1. Translate this circle along the z-axis to produce a cylinder, radius 1, whose axis is the z-axis.

Multiple Choice Question 2

Ex Sheet 1, T5(b)

The set of points $(x, y, z) \in \mathbb{R}^3$ that satisfy $x^2 + y^2 = z$ is best described as

(A) A sphere

(B) A cone

(C) An ellipsoid

(D) A paraboloid

Multiple Choice Question 2

Ex Sheet 1, T5(b)

The set of points $(x, y, z) \in \mathbb{R}^3$ that satisfy $x^2 + y^2 = z$ is best described as

(A)	A sphere	**(B)**	A cone
(C)	An ellipsoid	**(D)**	A paraboloid

ANSWER: **(D)** The $x = 0$ cross-section gives $z = y^2$ and the $y = 0$ cross-section gives $z = x^2$. Both equations describe *parabolas*. There are no level curves for $z < 0$, while for $z \geq 0$ the level curves are circles of radius $\sqrt{z}$. This is a paraboloid.

Tutorial question

Ex Sheet 1, T7(d)

Compute the partial derivatives of

$$h(x, y, z) = \frac{yz + xz + xy}{xyz}.$$

Ex Sheet 1, T8

Let $u(x, y) = x^2 - y^2$ and $v(x, y) = 2xy$. Show that

$$\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} = 4(x^2 + y^2).$$

Tutorial question

Ex Sheet 1, T7(d)

Compute the partial derivatives of

$$h(x, y, z) = \frac{yz + xz + xy}{xyz}\,.$$

Note that

$$h(x, y, z) = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\,.$$

Hence,

$$\frac{\partial h}{\partial x} = -\frac{1}{x^2}\,, \quad \frac{\partial h}{\partial y} = -\frac{1}{y^2}\,, \quad \frac{\partial h}{\partial z} = -\frac{1}{z^2}\,.$$

Tutorial question

Ex Sheet 1, T8

Let $u(x, y) = x^2 - y^2$ and $v(x, y) = 2xy$. Show that

$$\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} = 4(x^2 + y^2).$$

We have

$$\frac{\partial u}{\partial x} = 2x, \quad \frac{\partial u}{\partial y} = -2y, \quad \frac{\partial v}{\partial x} = 2y, \quad \frac{\partial v}{\partial y} = 2x.$$

Hence,

$$\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} = 2x \cdot 2x - 2y \cdot (-2y) = 4(x^2 + y^2).$$

Bonus tutorial question

Ex Sheet 2, T1

Compute the partial derivatives (always assume limits exist)

$$\frac{\partial \phi}{\partial x} = \lim_{h \to 0} \frac{\phi(x+h, y) - \phi(x, y)}{h}, \frac{\partial \phi}{\partial y} = \lim_{h \to 0} \frac{\phi(x, y+h) - \phi(x, y)}{h}$$

in the cases

(a) $\phi(x, y) = g(x + y)$, (b) $\phi(x, y) = f(x)g(y)$.

Bonus tutorial question

In the first case (assuming the limits exist)

$$\frac{\partial \phi}{\partial x} = \lim_{h \to 0} \frac{g(x+h+y) - g(x+y)}{h}$$

which is the derivative of g evaluated at $x+y$, so $\partial \phi / \partial x = g'(x+y)$. Similarly for $\partial \phi / \partial y$.

Bonus tutorial question

In the second case (assuming the limits exist)

$$\begin{aligned}\frac{\partial\phi}{\partial x} &= \lim_{h\to 0}\frac{f(x+h)g(y)-f(x)g(y)}{h}\\ &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}g(y)\\ &= f'(x)g(y)\end{aligned}$$

and similarly for the other derivative.