Let a and b be integers and let m be a positive integer. Show that $a \equiv b \pmod{m}$ if and only if $a \pmod{m} = b \pmod{m}$. Explain your steps.

"If direction": if $a \pmod{m} = b \pmod{m}$, then $a = k{\cdot}m + r$ and $b = l{\cdot}m + r$ for some integers k, l, r such that $0{\le}r{<}m$. Hence

$$\begin{aligned} a - b = &\ (k{\cdot}m + r) - (l{\cdot}m + r) && \\ = &\ k{\cdot}m + r - l{\cdot}m - r && \text{rearranging} \\ = &\ m{\cdot}(k - l) && \text{rearranging} \end{aligned}$$

and therefore, since k and l are integers, we have $m|(a - b)$, and hence by definition $a \equiv b \pmod{m}$ as required.

"Only if direction": if $a \equiv b \pmod{m}$, then by definition m divides $a{-}b$, and hence $a{-}b = k{\cdot}m$ for some integer k which rearranging yields $a = b + k{\cdot}m$. Now by the division algorithm, we have $b = q{\cdot}m + r$ for some integers q and r, where $0{\le}r{<}m$ and by definition $b \pmod{m} = r$. Combining these equations for a and b we have:

$$a = b + k{\cdot}m = q{\cdot}m + r + k{\cdot}m = (q + k){\cdot}m + r$$

Now since $0{\le}r{<}m$, by definition $a \pmod{m} = r$, and since $b \pmod{m} = r$, we have $a \pmod{m} = b \pmod{m} = r$ as required.

For each of these arguments, explain which rules of inference are used for each step.

If I do the tutorial questions and attend the labs, then I can understand the material. If I understand the material, I will be an excellent computer scientist. Therefore, if I do the tutorial questions and attend the labs, then I will be an excellent computer scientist.

We need the following propositions:

- p – I do the tutorial exercises;
- q – I attend the labs;
- r – I understand the material;
- s – I will be an excellent computer scientist.

Using these we have the following argument.

1. $(p \wedge q) \to r$	premise
2. $r \to s$	premise
3. $(p \wedge q) \to s$	hypothetical syllogism using 1 and 2

Ella, a student in the AF2 class, knows how to prove an algorithm is correct. Every student who knows how to prove an algorithm is correct can get a job with Google. Therefore, someone in the AF2 class can get a job with Google.

We need the following:

- *Ella* - the student Ella;

- S_{AF2} - the set of students in the AF2 class;
- S - the set of students;
- $PA(x)$ - knows how to prove an algorithm is correct;
- $Google(x)$ - can get a job with Google.

Using these we have the following argument.

1. $PA(Ella)$	premise
2. $\forall x \in S. (PA(x) \rightarrow Google(x))$	premise
3. $PA(Ella) \rightarrow Google(Ella))$	universal instantiation of 2
4. $Google(Ella)$	modus ponens of 1 and 3
5. $\exists x \in S_{AF2}. Google(x)$	existential generalisation of 4

Using the universe of all students, write out the following argument using quantifiers, connectives, and symbols to stand for propositions as necessary, explaining which rules of inference are used for each step.

> *All AF2 students are second years. There exists an AF2 student from Glasgow. Therefore, there is a second year student from Glasgow.*

We need the following:

- S - the set of students;
- $AF2(x)$ - an AF2 student;
- $Sec(x)$ - a second year student;
- $Gla(x)$ - from Glasgow;

Using these we have the following argument.

1. $\forall x \in S. (AF2(x) \rightarrow Sec(x))$	premise
2. $\exists x \in S. (AF2(x) \wedge Gla(x))$	premise
3. $AF2(s) \wedge Gla(s)$	for some $s \in S$ existential instantiation of 2.
4. $AF2(s)$	simplification of 3
5. $AF2(s) \rightarrow Sec(s)$	universal instantiation of 1
6. $Sec(s)$	modus ponens of 4 and 5
7. $Gla(s)$	simplification of 3
8. $Sec(s) \wedge Gla(s)$	conjunction of 6 and 7
9. $\exists x \in S. (Sec(x) \wedge Gla(x))$	existential generalisation of 8