EXAMINATION FOR THE DEGREES OF
M.A. AND B.Sc.

Mathematics 1

An electronic calculator may be used provided that it is allowed under the School of Mathematics and Statistics Calculator Policy. A copy of this policy has been distributed to the class prior to the exam and is also available via the invigilator.

Candidates must attempt the ALL *of Section A and* ALL *of Section B.*

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Section A Multiple Choice

Attempt ALL *questions from this section. Each question has only ONE correct answer. Enter your answers as well as your student number and name on the provided scanning sheet with a* BLACK BALL POINT PEN. *Return the sheet together with your script book.*

Q1. Suppose a connected planar graph G has 5 vertices and 3 faces. How many edges does G have? **3**

A) 5 edges

B) 6 edges

C) 7 edges

D) 8 edges

E) None of the above

Q2. Calculate the derivative of $g(x) = \sin^2 x \cos x$. **3**

A) $2 \sin x \cos x$

B) $2 \sin x \cos x - \sin^3 x$

C) $2 \sin x \cos x - \sin x$

D) $2 \sin x \cos^2 x - \sin^3 x$

E) None of the above.

Q3 Find all values of x such that $3|x-6| \leqslant |x-2|$.

A) $5 \leqslant x \leqslant 8$.

B) $x \leqslant 5$ and $x \geqslant 8$.

C) $x \geqslant 5$.

D) $x \leqslant 8$.

E) None of the above.

Q4. Let $f(x) = x^2 + 4x + 3$ with domain $[-2, \infty)$. What is the value of $(f^{-1})'(3)$? **3**

A) 1/3

B) 3

C) 4

D) 1/4

E) None of the above.

Q5. Which of the following statements is true of the equation $y^3 = x^5$? **3**

A) There are no integer solutions.

B) There is a unique integers solution.

C) There are exactly two integer solutions.

D) There are an infinite number of integer solutions.

E) None of the above.

Q6. What is the third nonzero term in the Taylor series for $f(x) = 2\ln x$ centered at $a = 1$? **3**

A) $4x^3$

B) $\frac{1}{3}x^3$

C) $\frac{2}{3}(x-1)^3$

D) $\frac{4}{3}(x-1)^3$

E) None of the above.

Q7. Let $p(x) = x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ be a polynomial with real coefficients $a_{n-1}, \ldots, a_1, a_0$ with $a_0 \neq 0$. Which one of the following statements is correct? **3**

A) The polynomial p has n distinct complex roots.

B) If n is odd, then p must have at least one real root.

C) The roots of p come in complex conjugate pairs, so there must be an even number of roots.

D) One of the roots of $p(x)$ is 0.

E) None of the above.

Q8. Find an equation of the tangent line to the curve defined by $\sinh(xy) = \cos(x+y)$ at the point $(x, y) = (0, \pi/2)$. **3**

A) $y = \frac{\pi}{2} - (1 + \frac{\pi}{2})x$

B) $y = \frac{\pi}{2} - x$

C) $y = \frac{\pi}{2} - \frac{x}{2}$

D) $y = \frac{\pi}{2} - \frac{\pi}{2}x$

E) None of the above.

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Q9. Identify the remainder r (between 0 and 12) that we get when we divide 6^{82} by 13. **3**

A) $r = 0$.

B) $r = 3$.

C) $r = 6$.

D) $r = 9$.

E) None of the above.

Q10. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be vectors. Which of the following expressions is **not** mathematically meaningful? **3**

A) $|\mathbf{a}|(\mathbf{b} \times \mathbf{c})$

B) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

C) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$

D) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

E) $|\mathbf{a}|(\mathbf{b} \cdot \mathbf{c})$

Section B

Attempt ALL *questions from this section.*

B1. (i) State the Principle of Mathematical Induction. **2**

(ii) Prove the following formula holds for all $n \in \mathbb{N}$.

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$ **3**

(iii) Compute the rational form of $0.8\overline{594}$. **3**

B2. (i) State the Mean Value Theorem. **2**

(ii) Find the derivative of $f(x) = \frac{x}{x+4}$. **2**

(iii) Find all numbers, c, that satisfy the conclusion of the Mean Value Theorem for $f(x)$ defined on the interval $[-1, 8]$. **4**

B3. (i) State the relationship between the cross product of two vectors $\mathbf{u}$ and $\mathbf{v}$ and the angle, θ, between them. **2**

(ii) Let $\mathbf{a} = \langle 1, 2, 0 \rangle$ and $\mathbf{b} = \langle 0, 4, 1 \rangle$ and show that $\langle 2, -1, 4 \rangle$ is orthogonal to both vectors. **2**

(iii) Find the (scalar) equation of the plane that contains the vectors $\mathbf{a}$ and $\mathbf{b}$ and the point $(1, 1, 1)$. **3**

B4. (i) State Fermat's Little Theorem. **2**

(ii) Prove by contradiction or otherwise that there are infinitely many prime numbers. **5**

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