INFORMATION THEORY AND CODING

(15ELD511)

January 2016 2 Hours

Answer 4 out of 5 questions.

Each question carries **25 marks**.

Any University approved calculator is permitted.

1. A 1/2 rate recursive systematic encoder is described by the following generator matrix:

$$G(D) = \left[1, \quad \frac{1+D^2}{1+D+D^2}\right]$$

a) Draw the encoder structure for the generator matrix $G(D)$. [4 marks]

b) Draw the state transition diagram for the above encoder. [7 marks]

c) Suppose a bit sequence of 11 01 00 01 11 is received (assume the pair of bits on the right is received first). Decode this received sequence using the Viterbi algorithm and determine the transmitted code word and the data word. You should draw the Trellis diagram and show the distance metrics at every node (state). [14 marks]

2. For a recursive systematic encoder with a generator matrix $G(D) = \left[1, \quad \frac{1}{1+D}\right]$, the following received signal was obtained.

$$\boldsymbol{r} = [(1,1), \quad (-0.5,0.5), \quad (0,1), \quad (-1,1), \quad (1,-1)]$$

a) Assuming the encoder started and ended at state 0 and the pair of samples on the left is received first, determine the transmitted data bits using the Soft-Output Viterbi Algorithm (SOVA). You should compute the soft output values. [18 marks]

b) Using block diagrams for the encoder and decoder, discuss the operation and principles of turbo coding and decoding. You should explain clearly the extrinsic information and the role of interleavers. [7 marks]

3. A seventh order polynomial $x^7 + 1$ is factorised as follows

$$x^7 + 1 = (x+1)(x^3 + x + 1)\, g(x)$$

a) A systematic (7,4) cyclic code is constructed using the above generator polynomial $g(x)$. Determine the polynomial $g(x)$. You should show all the steps involved in the polynomial long division. [5 marks]

b) Construct the systematic code word for the data word 0001. [5 marks]

c) Using the results in (b), determine the code words for the data words 1000, 0100 and 0010. [4 marks]

d) For the received code word 1000001, generate the corresponding syndrome and use the syndrome table provided below, to determine the transmitted data word. [5 marks]

e) Without explicitly performing polynomial long division, but using the result in (b) and the syndrome table provided below, determine the remainders of the following divisions:

i. $\dfrac{x^6 + x^3 + x^2 + 1}{g(x)}$.

ii. $\dfrac{x^2 + 1}{g(x)}$.

iii. $\dfrac{x^3 + x^2}{g(x)}$. [6 marks]

Syndrome **s**	Error **e**
001	0000001
010	0000010
011	0100000
100	0000100
101	0001000
110	1000000
111	0010000

4.

a) The following questions are on mutual information.

i. X and Y are random variables representing the input and the output of a binary input binary output channel

$P(Y=1/X=0)=P(Y=0/X=1)=p$ and
$P(Y=1/X=1)=P(Y=0/X=0)=1-p$.

Assuming the input symbol probabilities $P(X=0)=1/2$ and $P(X=1)=1/2$, determine the mutual information $I(X=0;Y=1)$, $I(X=0;Y=0)$, $I(X=1;Y=1)$ and $I(X=1;Y=0)$. [7 marks]

ii. Using the following equation for average mutual information, determine the capacity of the above channel as a function of p.

$$I(X;Y)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)I(x_i;y_j)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)\log\frac{p(x_i,y_j)}{p(x_i)p(y_j)}$$ [6 marks]

b) The distances between two base stations (BTS1 and BTS2) and two receiver terminals (MS1 and MS2) are shown in Figure 1 below. BTS1 is serving MS1 while BTS2 is serving MS2 in the same frequency band simultaneously. There is no thermal noise; however, the interference from one base station to the receiver served by other base station is treated as additive white Gaussian noise. Assuming free space path loss model, the received signal power P_R is written in terms of the transmitted signal power P_T as

$$P_R = \alpha P_T d^{-2}$$

where $\alpha = (4\pi/\lambda)^{-2}$ and λ is the wavelength of the RF signal.

i. If the transmitted signal powers of BTS1 and BTS2 are $P1$ and $P2$, compute the Signal to Noise Ratio (SNR) of the received signal at terminals MS1 and MS2. Your answer should be a function of $P1$ and $P2$. [8 marks]

ii. Suppose $P1 = 15W$ and $P2 = 16W$. Determine the capacity of the channel between BTS1 and MS1. (Assume bandwidth is 1Hz) [4 marks]

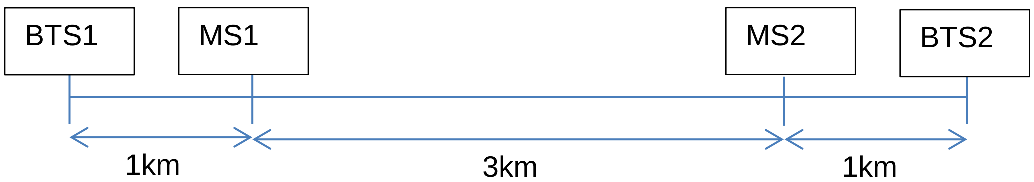

Figure 1: Distances between various base stations and receiver terminals.

5. A multiple input and multiple output (MIMO) wireless communication channel with two antennas at the transmitter (T1 and T2) and two antennas at the receiver (R1 and R2) is shown below in Figure 2. The channel gains between the transmitter and the receiver antennas are given by $h_{11} = 1$, $h_{12} = 0.5$, $h_{21} = 0.5$ and $h_{22} = 1$. The power of the Additive White Gaussian Noise (AWGN) present at the receiver antenna is $\sigma_n^2 = 0.1\ W$. The total available power at the transmitter is $1\ W$.

a) Determine the eigenvalues of the matrix $\mathbf{H} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$. Considering the eigenvectors of the matrix $\mathbf{H}$ as $\mathbf{v}_1$ and $\mathbf{v}_2$, show how this MIMO channel can be diagonalised using these eigenvectors. Your answer should include a block diagram depicting the precoding at the transmitter and processing at the receiver. You do not need to compute the eigenvectors explicitly. [5 marks]

b) Suppose $p\ W$ of power is allocated to the first diagonalized channel and $(1 - p)\ W$ of power is allocated to the second diagonalized channel, determine the total channel capacity of the MIMO channel in terms of p. You can assume $0 \leq p \leq 1$. [5 marks]

c) Determine the optimum p that maximises the total channel capacity. Compute the total capacity for this optimum power allocation. [9 marks]

d) Suppose the channel gains h_{11} and h_{22} are both zeros, determine the optimum capacity of the MIMO communication channel. [6 marks]

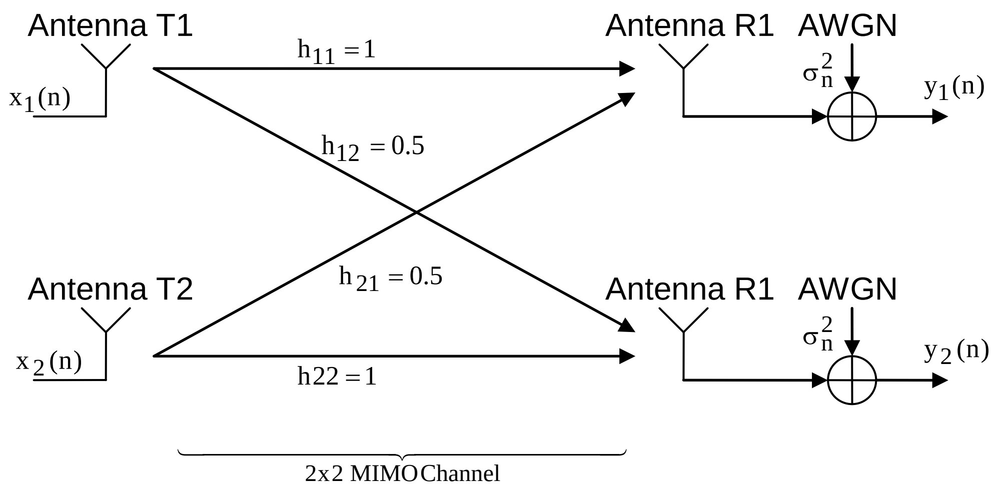

Figure 2: MIMO Channel

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