Loughborough University

# INFORMATION THEORY AND CODING
## (13ELP011)

January 2014 2 Hours

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Answer **4** out of **5** questions.

Each question carries **25 marks**.

Any University approved calculator is permitted.

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**1.**

a)

i. A discrete memoryless source emits symbols $x_1, x_2, x_3, x_4$ and $x_5$ with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{16}$ respectively. Determine Huffman codes, and compare the average number of bits required per source symbol to the entropy of the source. [9 marks]

ii. Provide an upper and lower bound on the average number of bits required per source symbol in terms of entropy and discuss how the efficiency can be improved when *J* symbols are jointly encoded.

[3 marks]

b) *X* and *Y* are random variables representing the input and the output of a binary input binary output channel. $P(Y=1/X=0)=P(Y=0/X=1)=q$ and $P(Y=1/X=1)=P(Y=0/X=0)=1-q$.

i. Assuming the input symbol probabilities $P(X=0)=1/2$ and $P(X=1)=1/2$, determine the mutual information *I(X=0;Y=1)*, *I(X=0;Y=0)*, *I(X=1;Y=0)* and *I(X=1;Y=1)*. [4 marks]

ii. Using the following equation for average mutual information, determine the capacity of the above channel as a function of $q$.

$$I(X;Y)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)I(x_i;y_j)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)\log\frac{p(x_i,y_j)}{p(x_i)p(y_j)}$$

[5 marks]

iii. Determine the value of *p* that maximises the channel capacity and comment upon this result. [4 marks]

2. The input-output relationship of a multiple-input multiple-output (MIMO) channel is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

where $\mathbf{x}$ is the input signal vector, $\mathbf{y}$ is the output signal vector and $\mathbf{w}$ is the noise vector whose elements are assumed to be zero mean additive white Gaussian noise (AWGN) variables with variance 0.1. The MIMO channel matrix is given by

$$\mathbf{H} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

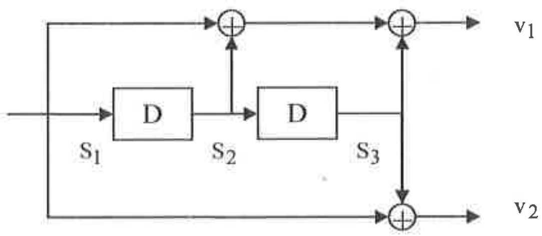
a) Determine the eigenvalues of the above symmetric MIMO channel matrix **H**. [5 marks]

b) Using the principle of eigenvalue decomposition of a symmetric matrix (or generally using singular value decomposition), show how the above channel matrix can be diagonalised. You will need to show the results using a block diagram and there is no need to determine the eigenvectors (or singular vectors) explicitly. [4 marks]

c) Assume that the total available transmission power is 1W. Determine the optimum power allocation for each of the diagonalised MIMO channels. [10 marks]

d) Determine the total capacity when the transmission power is distributed optimally. [6 marks]

3. A ½ rate convolutional code is generated using the generator polynomials $1+D^2$ and $1+D+D^2$ as shown below




a) Draw the state transition diagram for the above encoder. [6 marks]

b) Assuming zero initialization, determine the coded bit sequence (output) for a data input sequence 0011101 (assume the bit on the right enters first). The outputs should be pairs of bits in the form of $v_1v_2 \quad v_1v_2 \quad v_1v_2 \quad \cdots$ [6 marks]

c) Suppose a bit sequence of 00 11 10 01 01 00 11 is received (assume the pair of bits on the right is received first), decode this received sequence using the Viterbi algorithm. You should show the Trellis diagram together with distance metric at each node (state). [13 marks]

4. A seventh order polynomial $x^7 + 1$ is factorised as follows

$$x^7 + 1 = (x+1)(x^3 + x + 1)\,g(x)$$

a) A systematic (7,4) cyclic code is constructed using the generator polynomial $g(x)$. Determine the generator polynomial $g(x)$. [5 marks]

b) Construct systematic code words for the data words 1001 and 0110. [8 marks]

c) The codewords for the data words 0100 and 0011 are obtained as 0100011 and 0011010. Obtain codewords for the data words 1111, 1011 and 1000. [4 marks]

d) For the received code words 1101110 and 1101100, generate the corresponding syndromes and use the syndrome table provided below, to determine transmitted datawords. [8 marks]

| Syndrome **s** | Error **e** |
|---|---|
| 001 | 0000001 |
| 010 | 0000010 |
| 011 | 0100000 |
| 100 | 0000100 |
| 101 | 0001000 |
| 110 | 1000000 |
| 111 | 0010000 |

5. Assume a speech signal $s[n]$ is generated using a first order autoregressive (AR) process described by the difference equation

$$s[n] = x[n] - a\,s[n-1]$$

where $a$ is the AR parameter of the process and $x[n]$ is a zero mean uncorrelated white noise of variance $\sigma_x^2$.

a) Draw the diagram for a first order linear predictor to compute the parameter $a$. [4 marks]

b) Plot the error performance curve for this problem, showing the minimum point of the curve in terms of the autocorrelation sequence of $s[n]$. [8 marks]

c) Compute the autocorrelation sequence of $s[n]$ in terms of the parameter a and the autocorrelation sequence of $x[n]$, and determine the minimum point of error performance curve in terms of $a$. [9 marks]

d) Discuss various types of speech coders and the basic principle of linear predictive coding (LPC) of speech signal. [4 marks]

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