INFORMATION THEORY & CODING
(14ELP011)

January 2015 2 Hours

Answer 4 out of 5 questions.

Each question carries 25 marks.

Any University approved calculator is permitted.

1.

a) A discrete memoryless source emits symbols x_1 and x_2 with probabilities ⅓ and ⅔ respectively. Determine Huffman codes, and compare the average number of bits required per source symbol to the entropy of the source. [7 marks]

b) Determine the Huffman codes for the symbols emitted from the source in (a), but based on jointly encoding two symbols. Compare the average number of bits required per symbol to the entropy of the source and comment upon the result. [8 marks]

c) Suppose a discrete memoryless source emits 32 symbols $s_1, s_2, \cdots, s_{31}$ and s_{32} with equal probabilities. Comment upon the required Huffman codes, entropy and the number of bits required per symbol. [6 marks]

d) State the Shannon theorem for source coding. [4 marks]

2.

a) The input-output relationship of a multiple-input multiple-output (MIMO) channel is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

where $\mathbf{x}$ is the input signal vector, $\mathbf{y}$ is the output signal vector and $\mathbf{z}$ is the noise vector whose elements are assumed to be zero mean additive white Gaussian noise (AWGN) variables with variance 0.5. The MIMO channel matrix is given by

$$\mathbf{H} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine the eigenvalues of the above symmetric MIMO channel matrix $\mathbf{H}$. [6 marks]

b) Using the principle of eigenvalue decomposition of a symmetric matrix (or generally using singular value decomposition), show how the above channel matrix can be diagonalised. You will need to show the results using a block diagram and there is no need to determine the eigenvectors (or singular vectors) explicitly. [4 marks]

c) Assume that the total available transmission power is 1W and the bandwidth is 1Hz. Also assume that equal power is allocated to channels with identical gains. Determine the optimum power allocation for each of the diagonalised MIMO channels. [10 marks]

d) Determine the total capacity when the transmission power is distributed optimally. [5 marks]

3.

a) The following questions are on mutual information.

i. X and Y are random variables representing the input and the output of a binary input binary output channel.
$P(Y=1/X=0)=P(Y=0/X=1)=p$
and $P(Y=1/X=1)=P(Y=0/X=0)=1-p$.

Assuming the input symbol probabilities $P(X=0)=1/2$ and $P(X=1)=1/2$, determine the mutual information $I(X=0;Y=1)$, $I(X=0;Y=0)$, $I(X=1;Y=1)$ and $I(X=1;Y=0)$ [7 marks]

i) Using the following equation for average mutual information, determine the capacity of the above channel as a function of p.

$$I(X;Y)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)I(x_i;y_j)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)\log\frac{p(x_i,y_j)}{p(x_i)p(y_j)}$$

[6 marks]

b) The following questions are on channel capacity.

i. For a band limited additive white Gaussian noise (AWGN) channel with signal power P_{av} and noise power σ_n^2, determine the Shannon capacity. [3 marks]

ii. Determine the average signal power P_{av} in terms of capacity C and bit energy E_b and modify the Shannon capacity formula to write E_b / N_0 in terms of C / W. Determine E_b / N_0 for the limiting cases of $C / W \rightarrow 0$ and $C / W \rightarrow \infty$, and plot C / W versus E_b / N_0. Comment upon your results. [9 marks]

4. A ½ rate convolutional code is generated using the generator polynomials $1+\mathrm{D}^2$ and $1+\mathrm{D}+\mathrm{D}^2$ as shown below

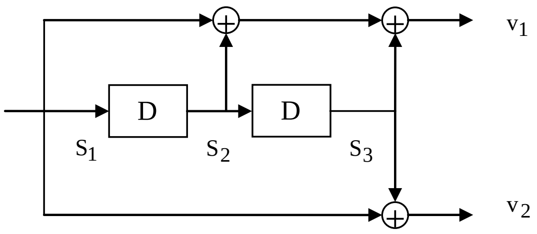

Figure 1: Convolutional Encoder

a) Draw the state transition diagram for the above encoder. [6 marks]

b) Assuming zero initialization, determine the coded bit sequence (output) for a data input sequence 0011101 (assume the bit on the right enters encoder first). The outputs should be pairs of bits of the form $\mathrm{v}_1\mathrm{v}_2 \quad \mathrm{v}_1\mathrm{v}_2 \quad \mathrm{v}_1\mathrm{v}_2 \quad \cdots$ [6 marks]

c) Suppose a bit sequence of 00 11 10 01 01 00 11 is received (assume the pair of bits on the right is received first), decode this received sequence using the Viterbi algorithm. You should show the Trellis diagram together with distance metric at each node (state). [13 marks]

5.

a) Using a block diagram, explain the steps involved in transform coding. [5 marks]

b) For a 4x4 image block shown on the next page, determine the missing discrete cosine transform coefficient $\boldsymbol{F}_{10}$. The formula for the two dimensional DCT is shown at the end of the question 5. [8 marks]

2	2	2	3
2	2	3	3
2	2	2	2
1	1	2	2

Image Samples

8.25	-1.25	0.25	0.25
F_{10}	-0.07	0.33	-0.33
-0.75	-0.33	0.25	-0.14
-0.25	-0.68	0.14	-0.43

DCT Coefficients

c) Round off the DCT coefficients to the nearest integer and reorder these quantized DCT coefficients. Apply zigzag scanning and determine the first five run-level pairs. [7 marks]

d) With the help of a diagram, explain and state the differences between video image coding and video sequence coding. [5 marks]

$$F_{xy} = \frac{C(x)C(y)}{N/2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f_{ij} \cos\left(\frac{(2i+1)\pi x}{2N}\right) \cos\left(\frac{(2j+1)\pi y}{2N}\right)$$

where $C(n) = \begin{cases} 1\sqrt{2} & for \quad n = 0 \\ 1 & for \quad n \neq 0 \end{cases}$

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