INFORMATION THEORY AND CODING

(16ELD511, 16ELP011)

January 2017 2 Hours

Answer **<u>ONLY</u> FOUR** questions out of **FIVE**.

All questions carry equal marks.

Any approved University calculator is permitted.

1.

a) A memoryless source emits symbols s_1, s_2 and s_3. The probabilities of emitting symbols s_1 and s_2 are p and q, respectively. Determining the probability of emitting the symbol s_3, compute the entropy of the source in terms p and q. [4 marks]

b) Compute the values of p and q that maximise the entropy (you need to show all of the steps involved). [5 marks]

c) X and Y are random variables that represent the input and the output of a binary input binary output channel. The transient probabilities are $P(Y=1/X=0)=P(Y=0/X=1)=p$ and $P(Y=1/X=1)=P(Y=0/X=0)=1-p$. Assume that the input symbol probabilities are $P(X=0)=1/2$ and $P(X=1)=1/2$.

Computing $P(Y=0)$ and $P(Y=1)$, determine the mutual information $I(X=0;Y=1)$, $I(X=0;Y=1)$, $I(X=0;Y=1)$ and $I(X=0;Y=1)$. [6 marks]

d) Using the following equation, determine the average mutual information as a function of p. [6 marks]

$$I(X;Y)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)I(x_i;y_j)=\sum_{i=1}^{2}\sum_{j=1}^{2}p(x_i,y_j)\log\frac{p(x_i,y_j)}{p(x_i)p(y_j)}.$$

e) Evaluate the value of p that maximises the average mutual information and sketch the average mutual information against p. [4 marks]

2. A seventh order polynomial $x^7 + 1$ is factorised as follows

$$x^7 + 1 = (x + 1)(x^3 + x + 1)\,g(x)$$

a) A systematic (7,4) cyclic code is constructed using the above generator polynomial $g(x)$. Determine the polynomial $g(x)$. You should show all the steps involved in the polynomial long division. [3 marks]

b) Construct the systematic code word for the data word 1101. [4 marks]

c) Using the result in (b) only (i.e., without performing long polynomial division), determine the code words for the data words 1000, 0100, 0001 and 1001. [4 marks]

d) For the received code word 0111101, generate the corresponding syndrome and use the syndrome table provided below, to determine transmitted data word. [5 marks]

e) Without explicitly performing polynomial long division, but using the result in (b) and the syndrome table provided below, determine the remainders of the following divisions:

i) $\dfrac{x^6 + x^5 + x^3 + x}{g(x)}$.

ii) $\dfrac{x^6 + x^5 + x^3 + 1}{g(x)}$.

iii) $\dfrac{x^6 + x^5}{g(x)}$. [9 marks]

Syndrome **s**	Error **e**
001	0000001
010	0000010
011	0100000
100	0000100
101	0001000
110	1000000
111	0010000

3. The input-output relationship of a multiple-input multiple-output (MIMO) channel is given by

$$\boldsymbol{y} = \boldsymbol{H}\,\boldsymbol{x} + \boldsymbol{w}$$

where $\boldsymbol{x}$ is the input signal vector, $\boldsymbol{y}$ is the output signal vector and $\boldsymbol{w}$ is the noise vector whose elements are assumed to be zero mean additive white Gaussian noise (AWGN) variables. The MIMO channel matrix is given by

$$\boldsymbol{H} = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

a) One of the eigenvalues of the above symmetric matrix is 7. Determine the remaining two eigenvalues. [8 marks]

b) Show how the MIMO channel $\boldsymbol{H}$ can be diagonalised using the eigenvalue decomposition (EVD). Your answer should include a block diagram depicting the precoding at the transmitter and processing at the receiver. You do not need to compute the EVD. [4 marks]

c) Suppose total transmission power of 1W is available and the noise variance is 0.25. The total tramision power is split between the sub-channels as p, q and $1 - p - \ q$. Assuming allocated power for each channel turns out to be a positive value, determine the optimum power allocation for each sub-channel that maximises the MIMO channel capacity. (Note that the power gain of each sub-channel is the square of the corresponding eigenvalue). [13 marks]

4.

a) For a recursive systematic encoder with the generator matrix $G(D) = \left[1, \ \ \frac{1}{1+D}\right]$, the following received sequence was obtained.

$$\boldsymbol{r} = [(1,1), \quad (1,-1), \quad (1, 0.5), \quad (-1,1), \quad (1,-1)]$$

Assume the encoder started and ended at state 0 and the pair of samples on the left is received first. Using Soft-Output Viterbi Algorithm (SOVA), compute the soft outputs of the decoder and determine the transmitted data word. [18 marks]

b) Using block diagrams for the encoder and decoder, discuss the operation and principles of turbo coding and decoding. You should explain clearly the extrinsic information and the role of interleavers. [7 marks]

5. A 1/3 rate convolutional code is generated using the generator polynomials $1 + D + D^2$, $1 + D^2$ and $D + D^2$

a) Draw the state transition diagram for the above encoder. [7 marks]

b) Suppose a bit sequence of 111 010 001 001 010 011 111 is received (assume the group of three bits on the right is received first), decode this received sequence using Viterbi algorithm and determine the transmitted codeword and the dataword. You should draw the Trellis diagram and show distance metric at each node (state). [18 marks]

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