Opacity Enforcing Supervisory Control using Non-deterministic Supervisors

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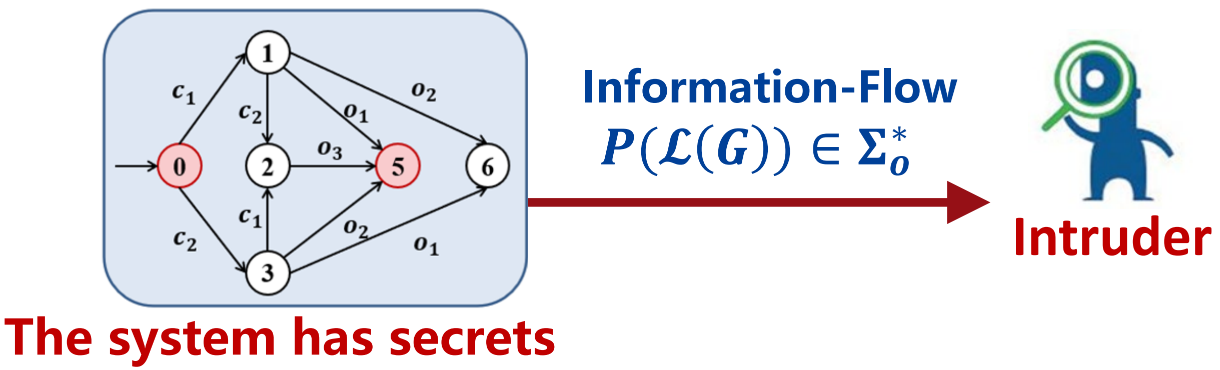
SHANGHAI JIAO TONG UNIVERSITY

Introduction

Motivation

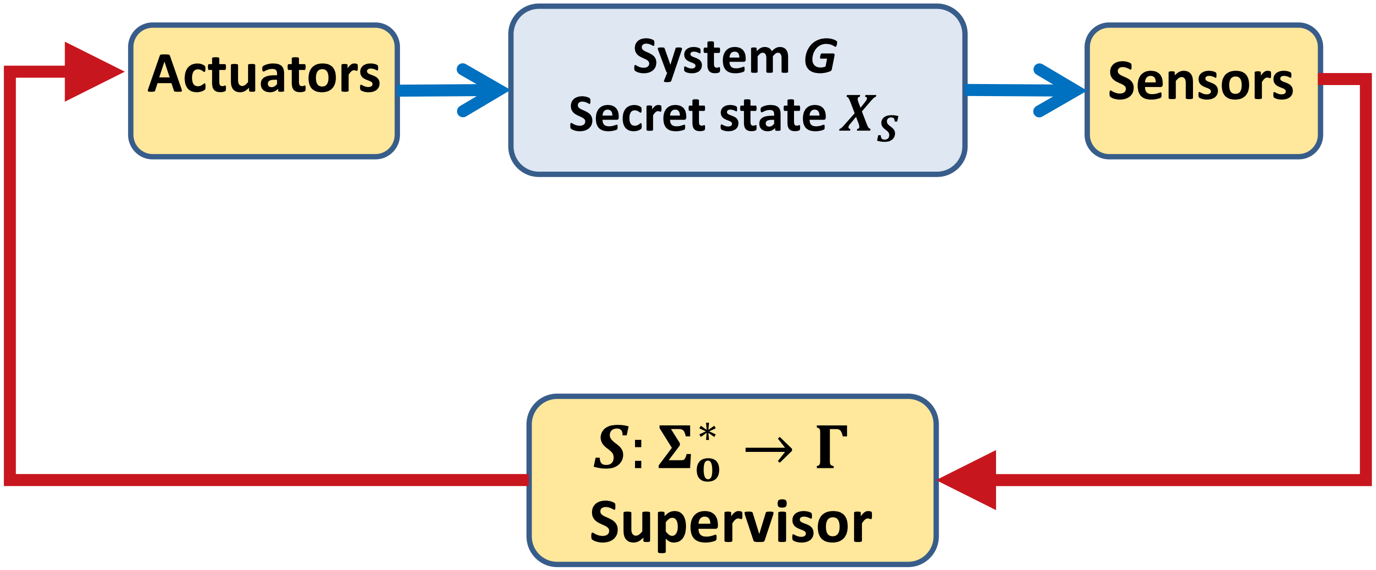
- **Security and privacy concerns in Cyber-Physical Systems**
- **Opacity: An information-flow property**
- **Application: Web services, Location-based services…**

System Model and the Intruder



- The system is modeled as a FSA $G = (X, \Sigma, \delta, x_0)$
- The system has secrets, modeled a set of states $X_s \subseteq X$
- $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo}$, $P: \Sigma^* \to \Sigma_o^*$ is the natural projection
- The intruder is a passive observer seeing $P(\mathcal{L}(G))$

Supervisory Control Systems



- **Sensors:** $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo}$ **and** $P: \Sigma^* \to \Sigma_o^*$
- **Actuators:** $\Sigma = \Sigma_c \dot{\cup} \Sigma_{uc}$ **and** $\Gamma := \{\gamma \in 2^{\Sigma}: \Sigma_{uc} \subseteq \gamma\}$
- **Supervisor:** $S: P(L(G)) \to \Gamma$ **updates decisions dynamically**

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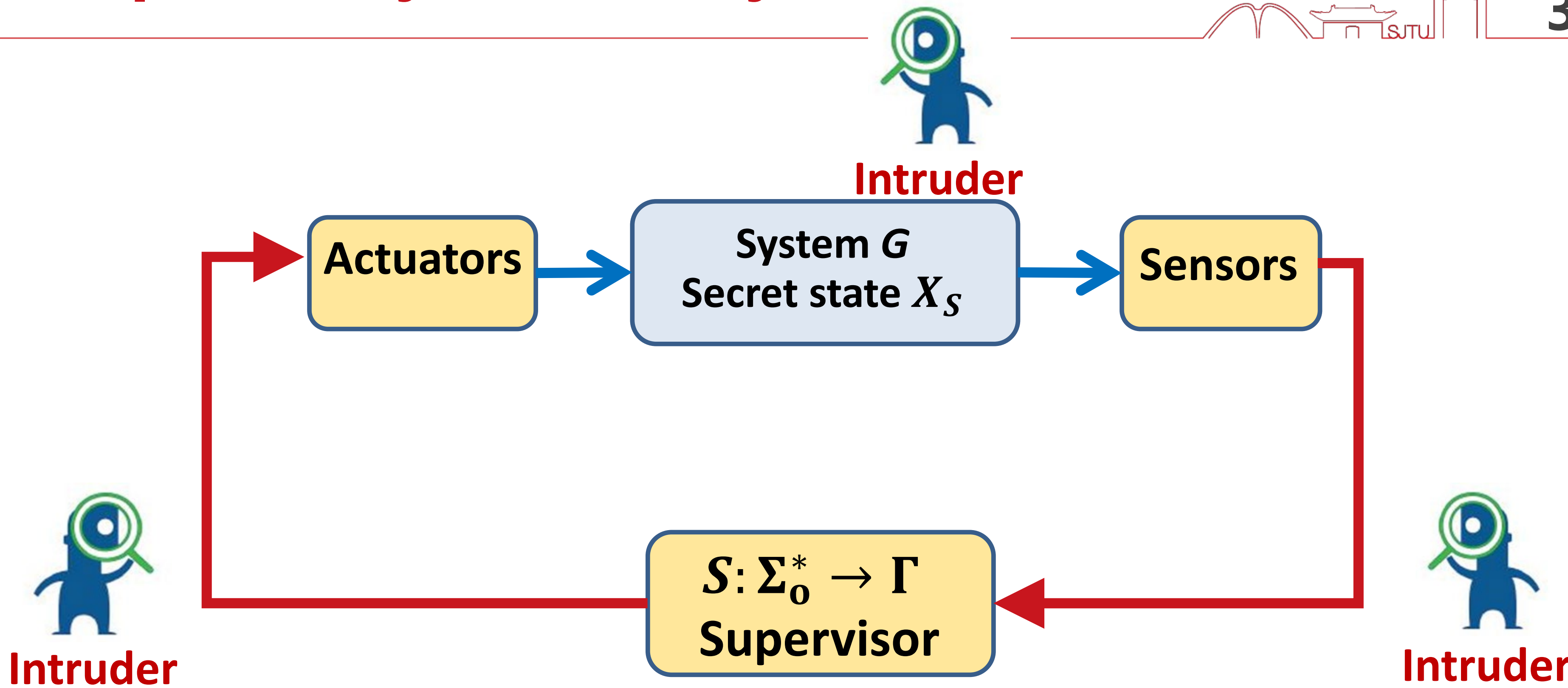

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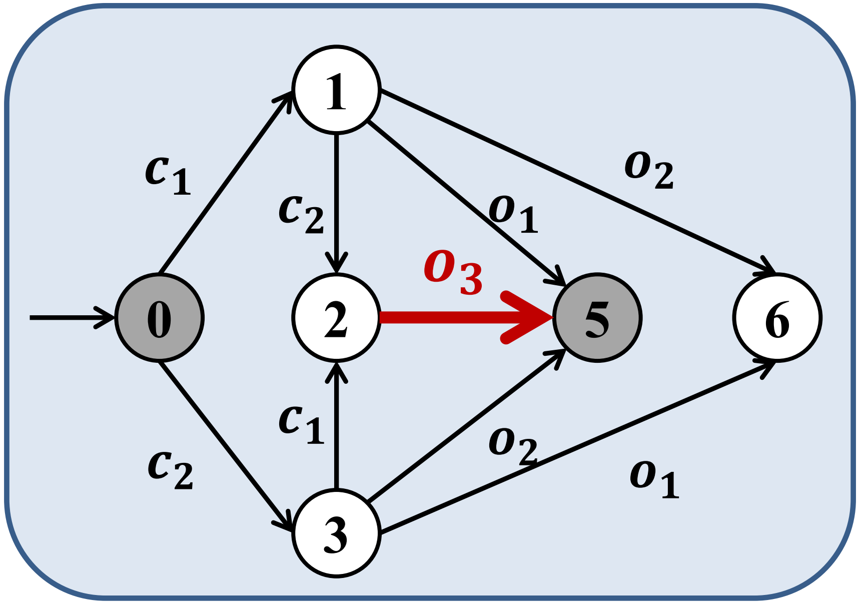
- **Supervisor:** $S: P(L(G)) \rightarrow \Gamma$ updates decisions dynamically
- **Intruder:** System model G, Observable events Σ_o^* , Control policy Γ

Motivating Example

$$\Sigma_o = \{o_1, o_2, o_3\}$$
$$\Sigma_c = \{c_1, c_2\}$$

$$\Gamma = \left\{\begin{matrix} \emptyset \\ \{c_1\} \\ \{c_2\} \\ \{c_1, c_2\} \end{matrix}\right\}$$



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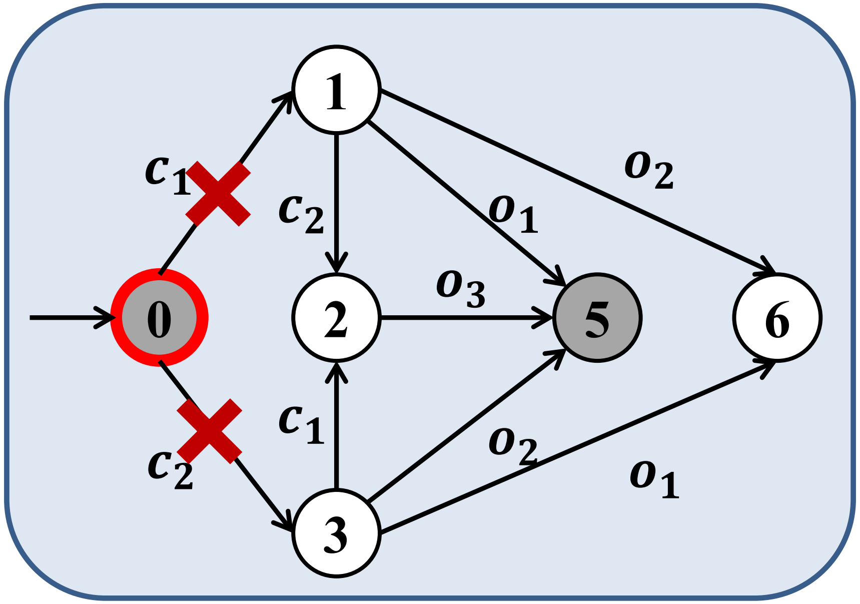

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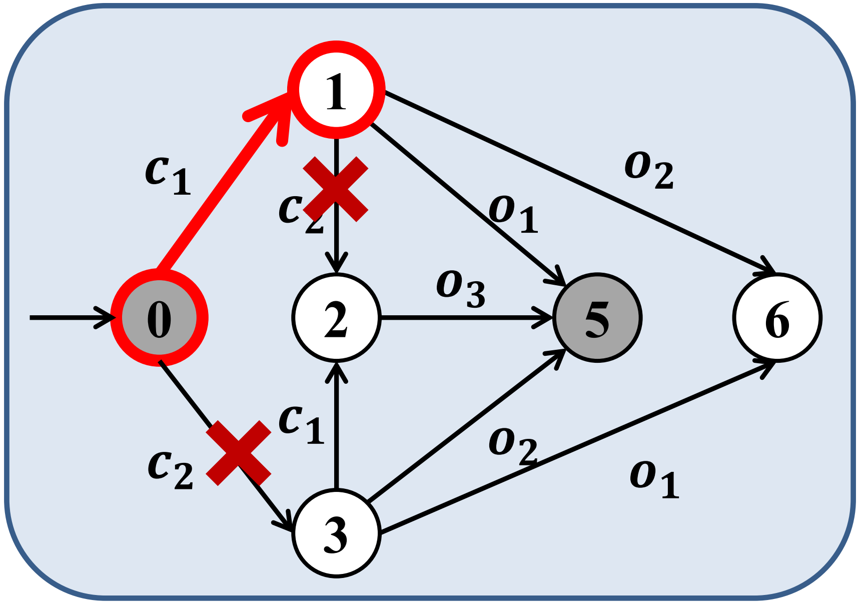

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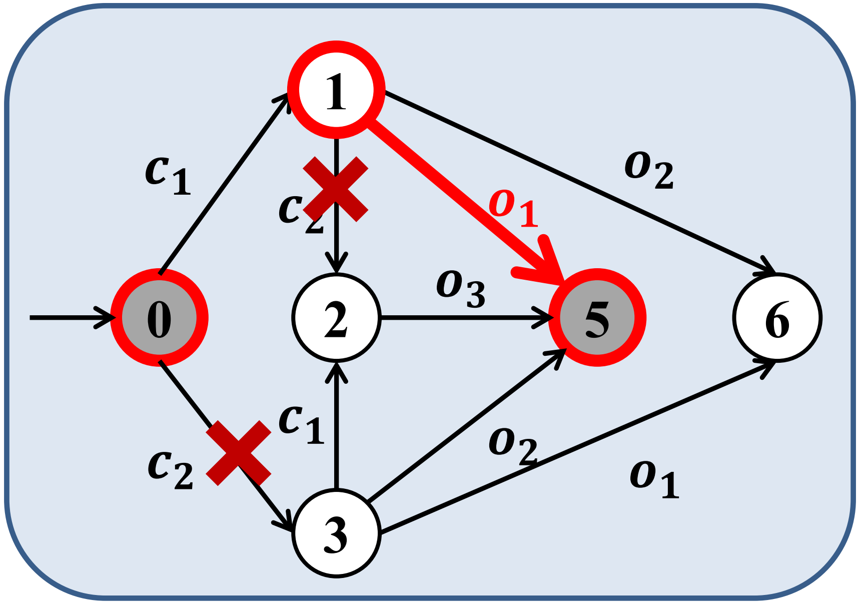

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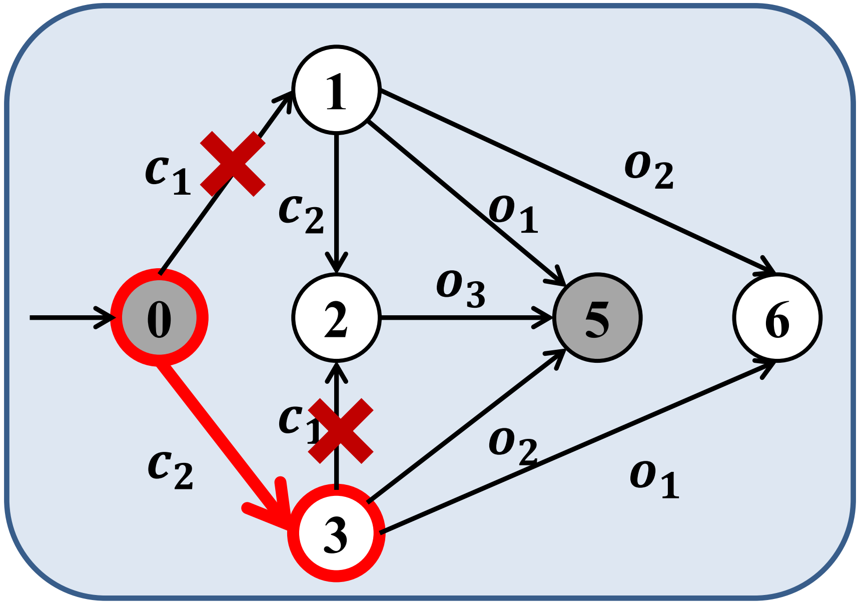

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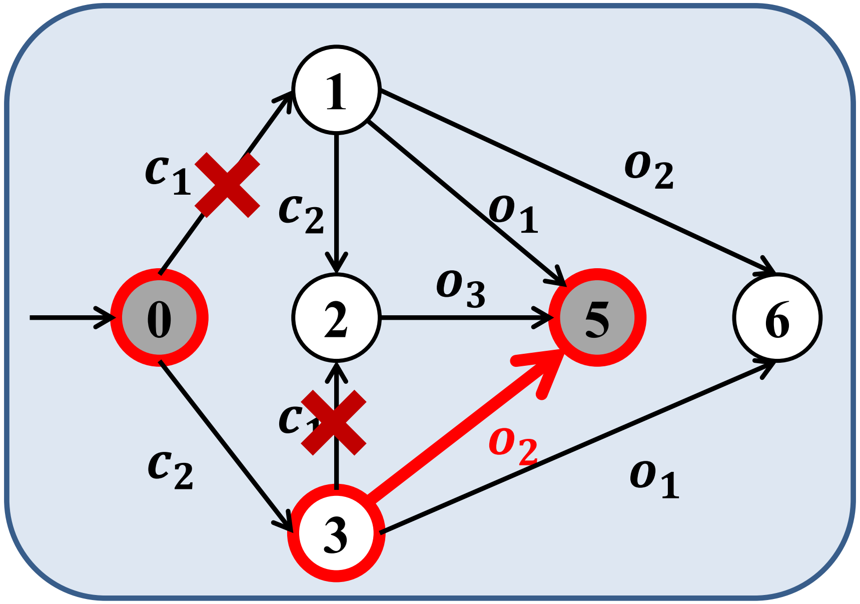

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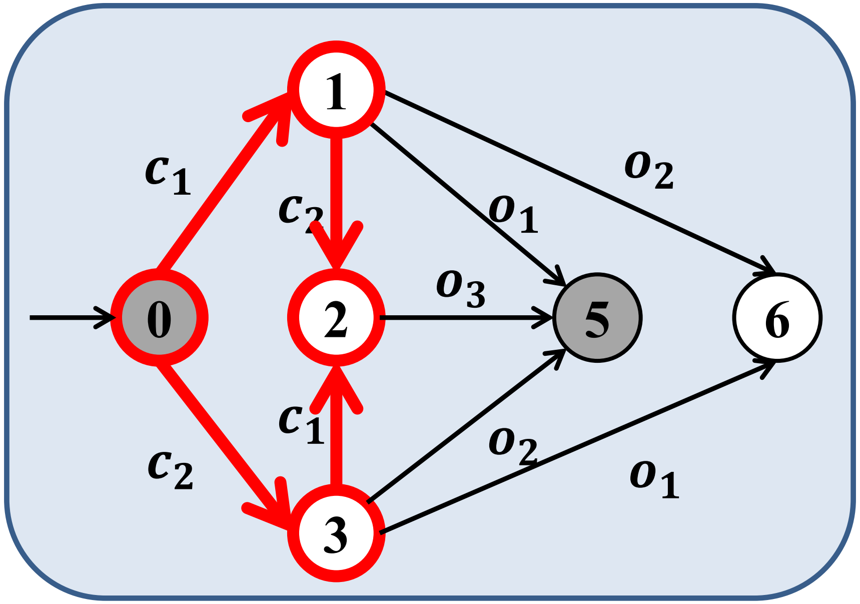

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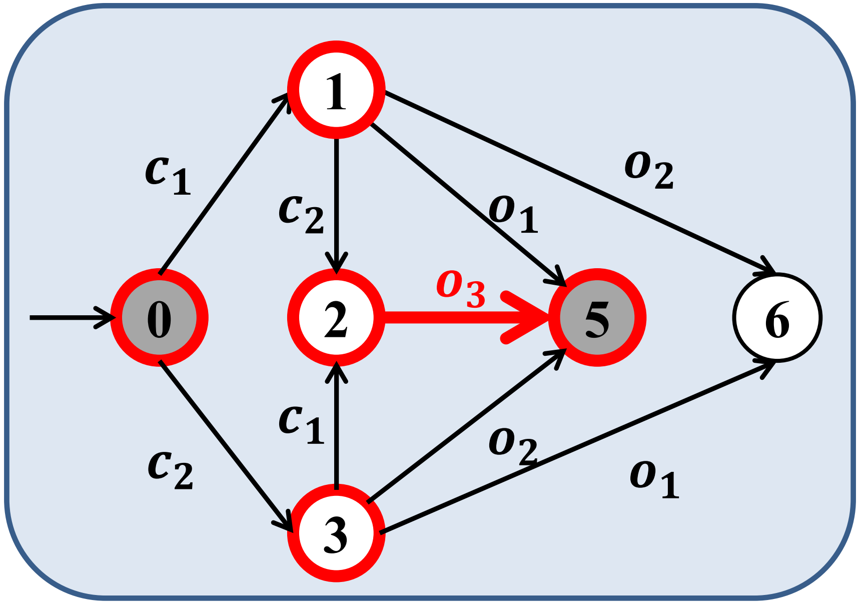

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Non-deterministic Supervisor

A set of possible control decisions $\Longrightarrow$ A specific control decision

The non-deterministic supervisor is defined as a function

$$S_N\colon\ (\Gamma\Sigma_o)^* \to 2^{\Gamma}$$

that maps a decision history to a set of possible control decision.

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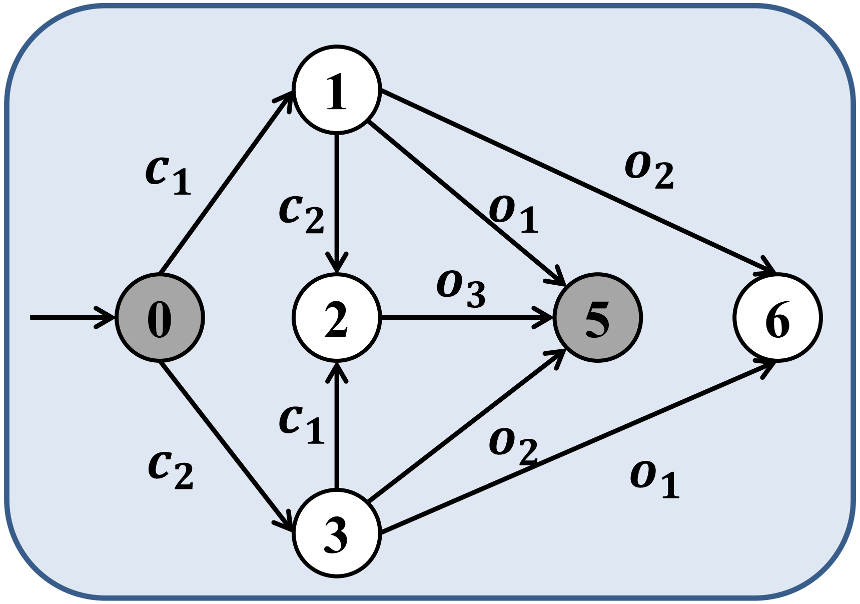

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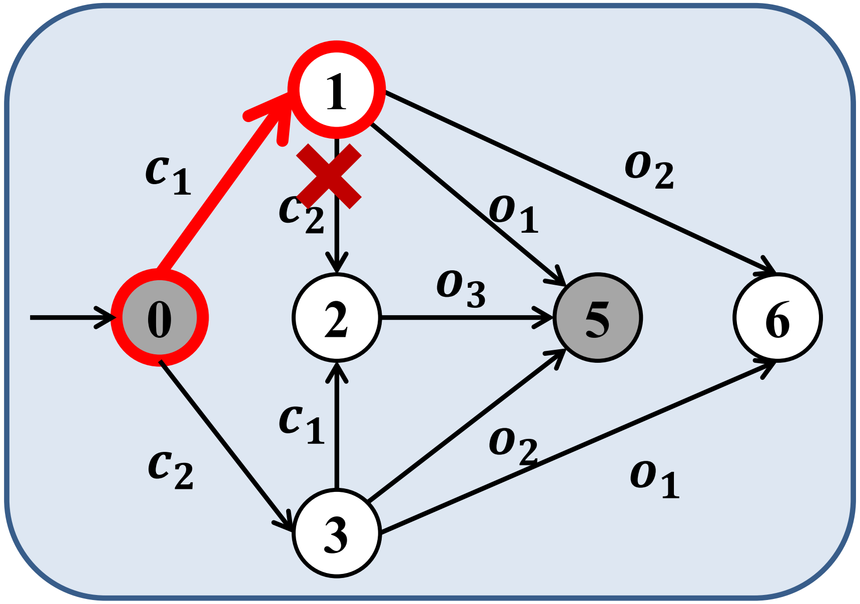

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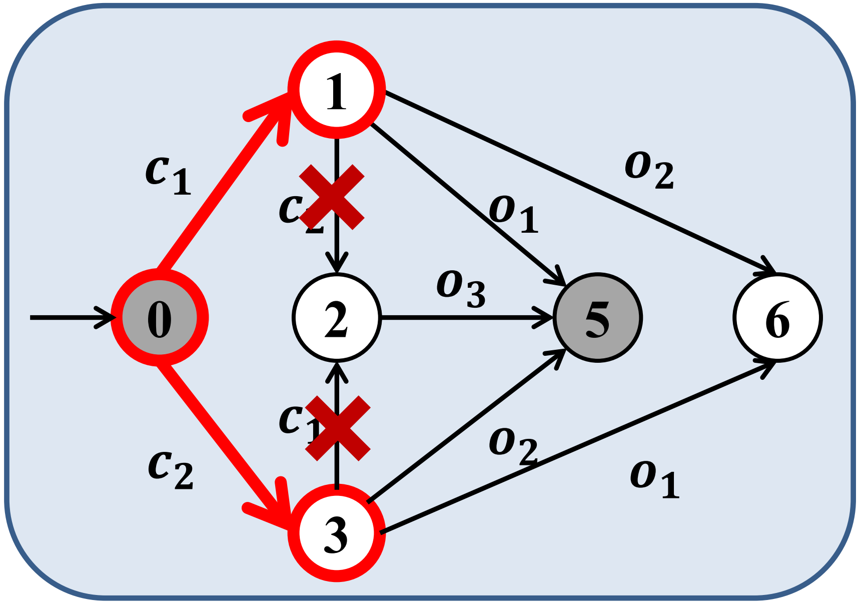

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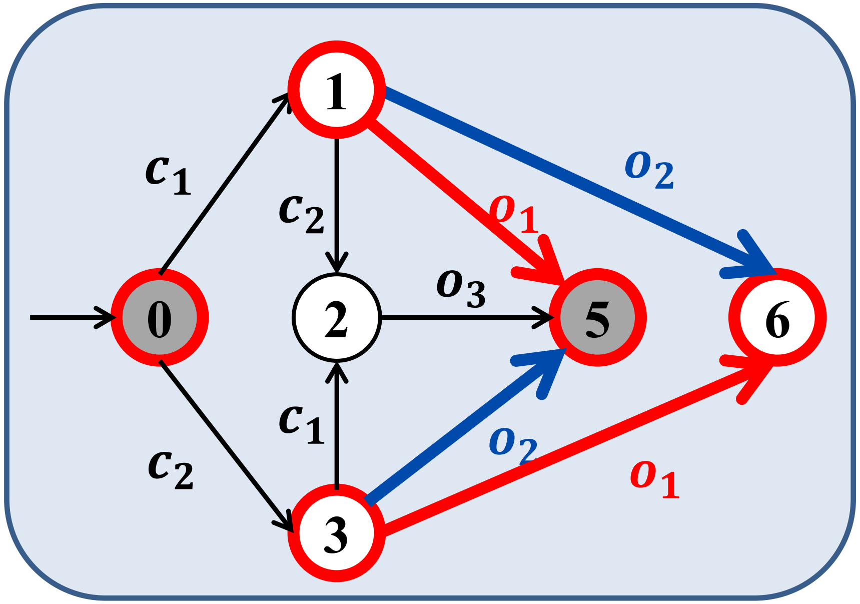

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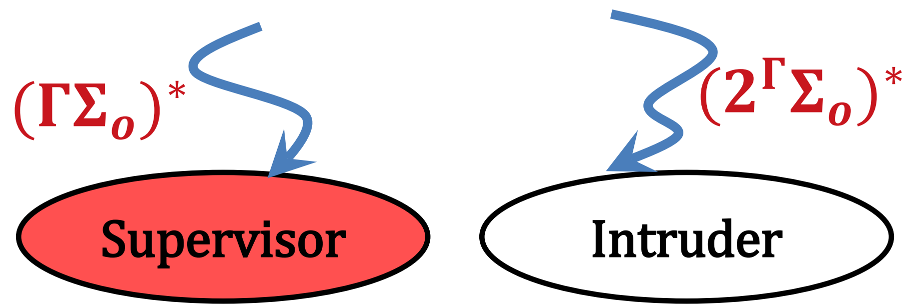
Supervisor: specific control decision

Intruder: the set of all possible control decisions



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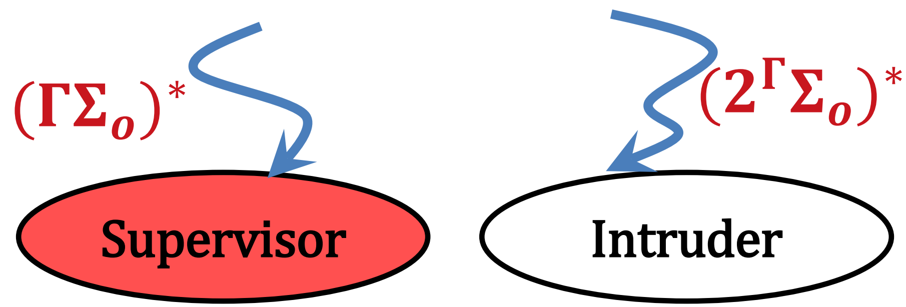

Opacity under Non-deterministic Supervisor

Let $S_N \colon (\Gamma\Sigma_o)^* \rightarrow 2^{\Gamma}$ be a non-deterministic supervisor. We say the closed-loop system S_N/G is opaque (w.r.t. Σ_o and X_S) if $\forall s \in P(\mathcal{L}(S_N/G)) \colon X_I(s) \nsubseteq X_S$.

Information State and its Flow

Information State

We propose the following information-state space

$$I := 2^X \times 2^{2^X}$$

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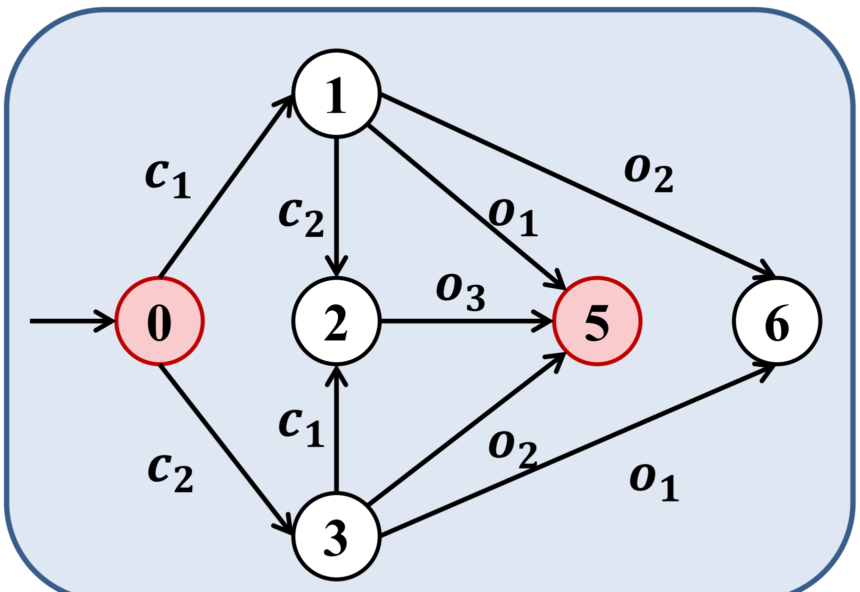

$$0 \xrightarrow{\{\{c_1\},\{c_2\}\}} \xrightarrow{\{c_1\}}$$

- **From supervisor's point of view**

$$\{0, 1\} \Rightarrow 2^X$$

- **From the intruder's point of view**

$$\begin{Bmatrix} \{0, 1\} \\ \{0, 3\} \end{Bmatrix} \Rightarrow 2^{2^X}$$

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Information State Non-deterministic Supervisor

$$S_N : I \to 2^\Gamma$$

which makes control decision based on the proposed information state.

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- Micro-state: $m \in 2^X$
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- Macro-control-decision:

$$d = \{(m_1, \Gamma_1), (m_2, \Gamma_2), \cdots, (m_n, \Gamma_n)\} \subseteq 2^X \times \Gamma$$

d is compatible with **m** if *d* essentially assigns each micro-state a non-deterministic control decision.

Information State and its Flow

Information State Flow

Suppose that the intruder observes $\sigma_1 \cdots \sigma_n \in P(\mathcal{L}(S_N/G))$ and by knowing the fact that S_N is an IS-based supervisor

$$\mathbf{m}_0 \xrightarrow{d_0} \mathbf{m}_0^+ \xrightarrow{\sigma_1} \mathbf{m}_1 \xrightarrow{d_1} \cdots \xrightarrow{\sigma_n} \mathbf{m}_n \xrightarrow{d_n} \mathbf{m}_n^+$$

where $\mathbf{m}_0 = \{\{x_0\}\}$, $d_i = d_{S_N}(\mathbf{m}_i)$, $\mathbf{m}_i^+ = \odot(d_i)$ and $m_{i+1} = \widehat{NX}_{\sigma_{i+1}}(\mathbf{m}_i^+)$

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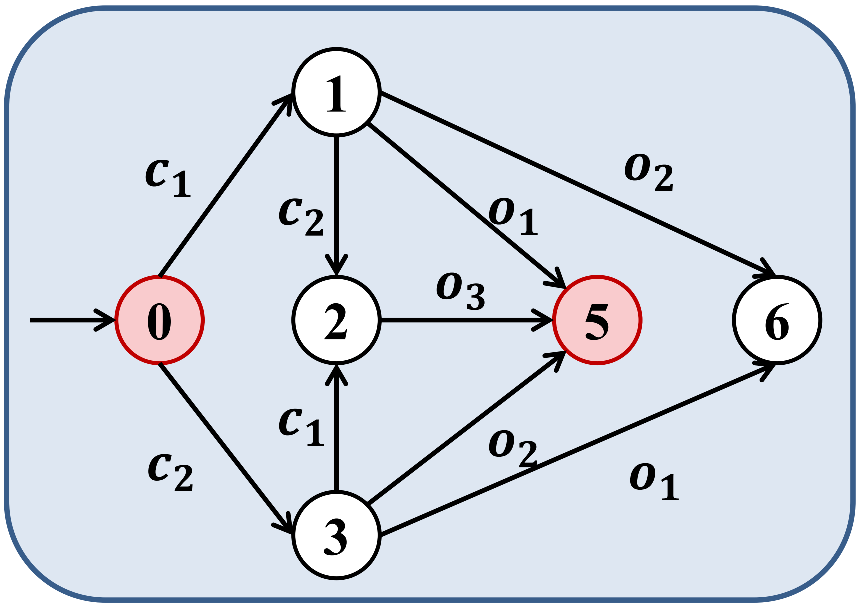

$$\gamma_1 = \{o_1, o_2, o_3\}$$
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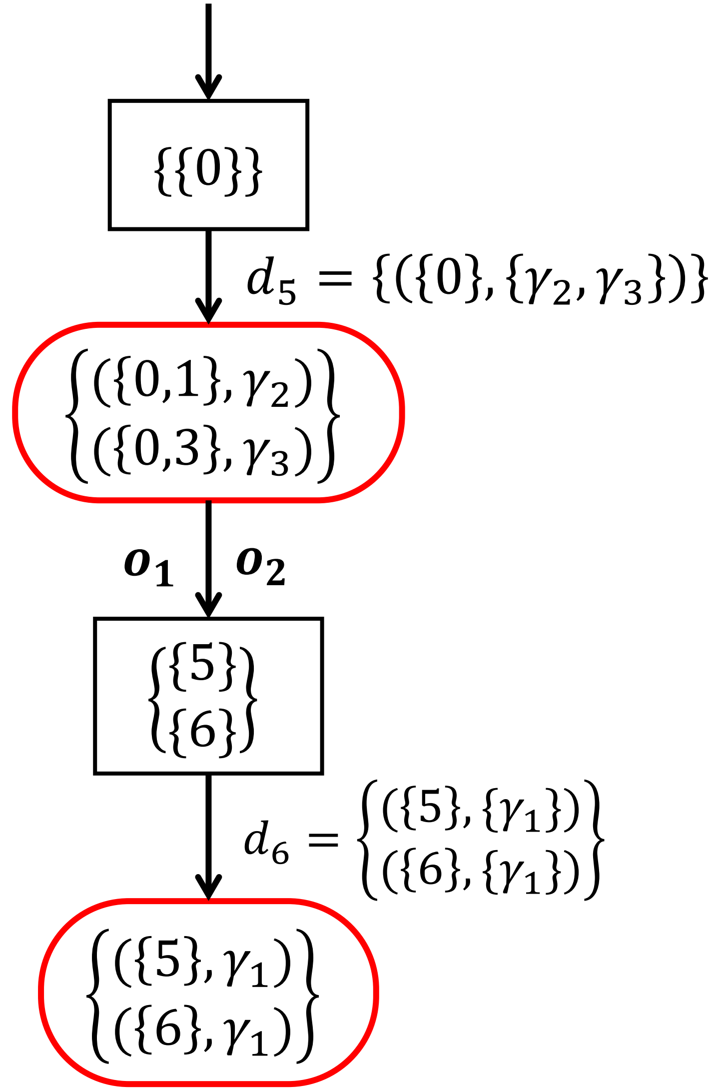

Supervisor Synthesis Procedure

1. Enumerates all feasible transitions

new macro-control-decision & new observation

A generalized bipartite transition system (G-BTS) *T* w.r.t. G is a 7-tuple

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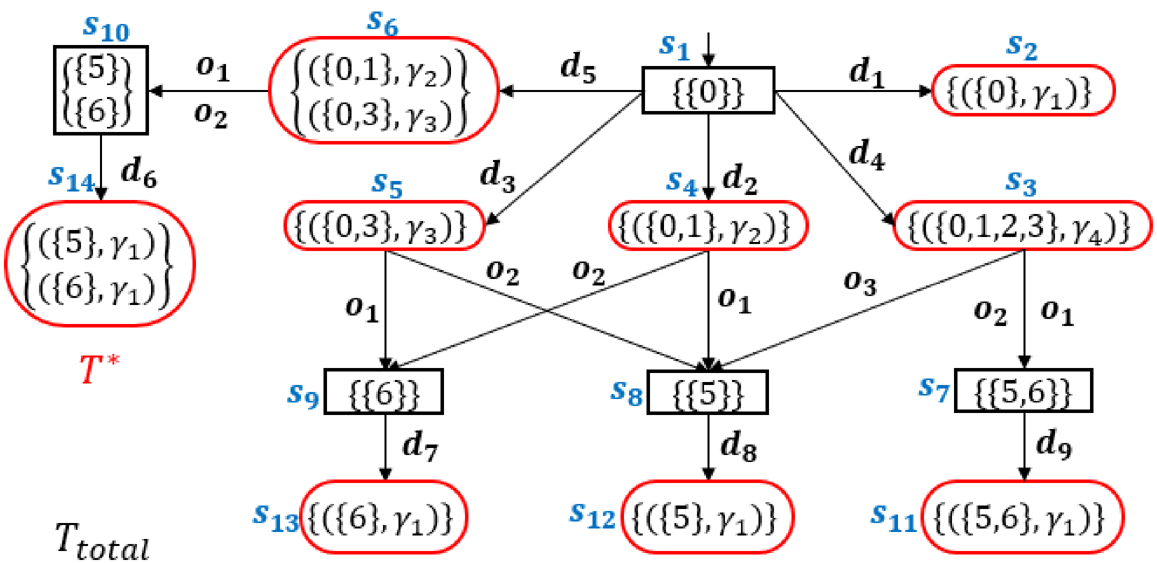

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2. Delete all secret-revealing states and inconsistent states

Secret-revealing Z-states

$$Q_{reveal} = \{z \in \mathbb{M}^+ : \cup\, \Xi(z) \subseteq X_s\} \qquad \cup\, \Xi(z) = X_I(s)$$

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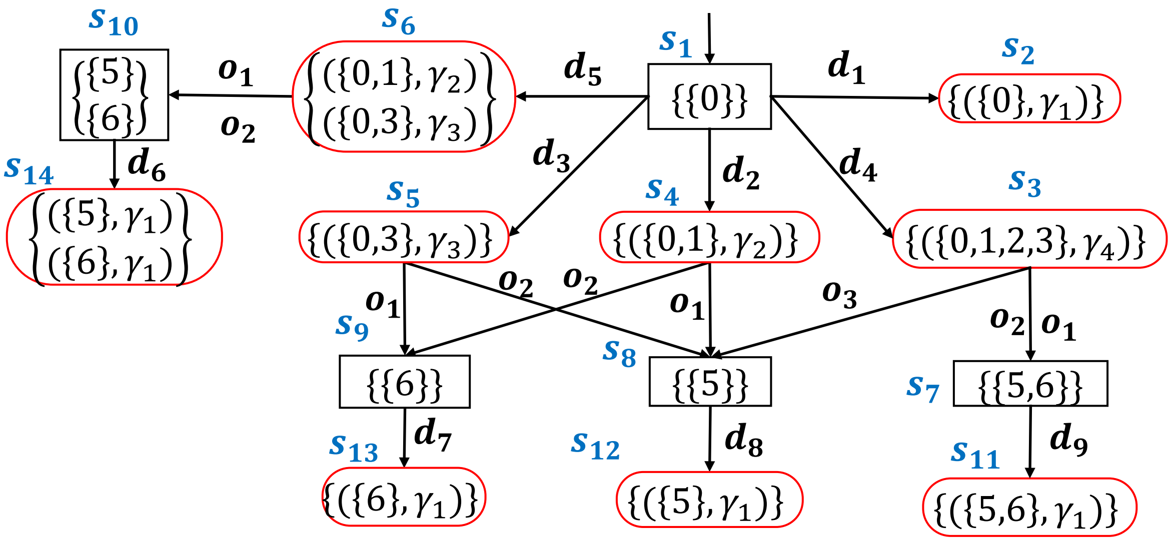

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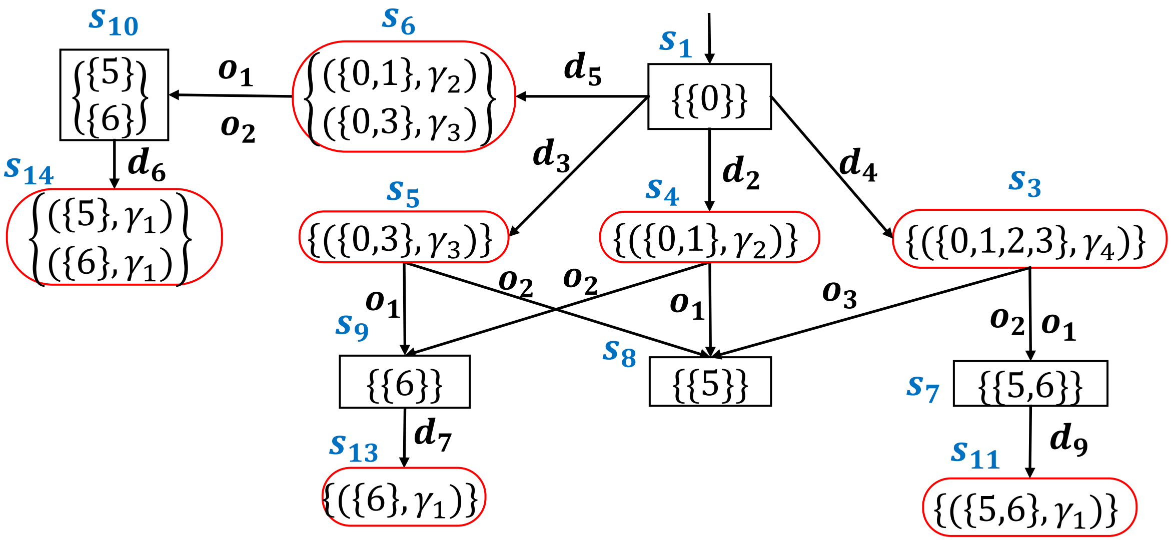

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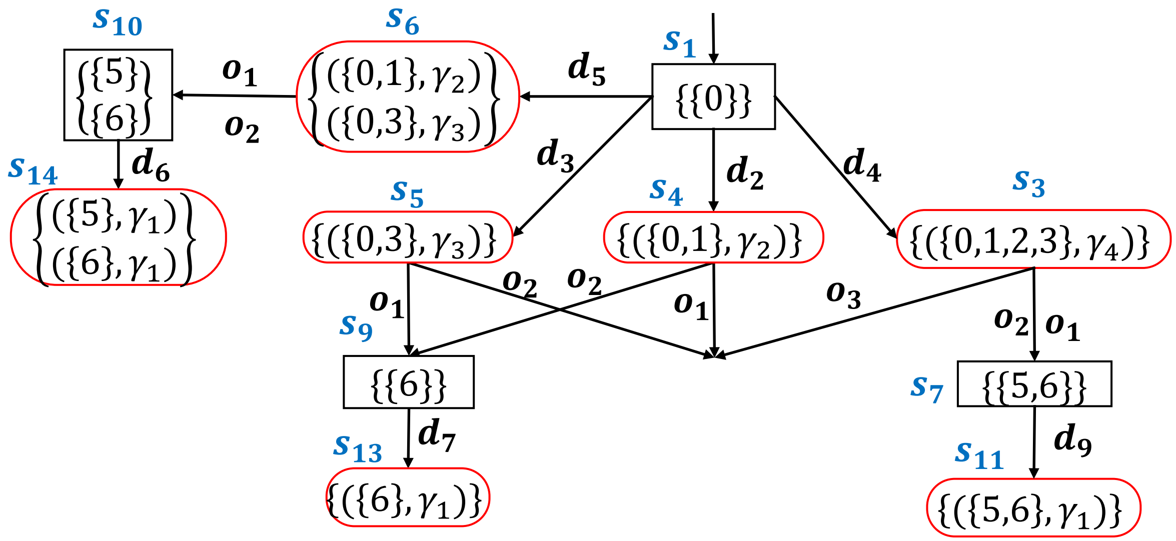

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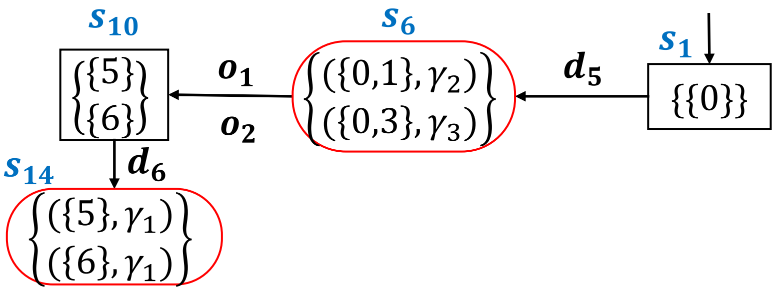
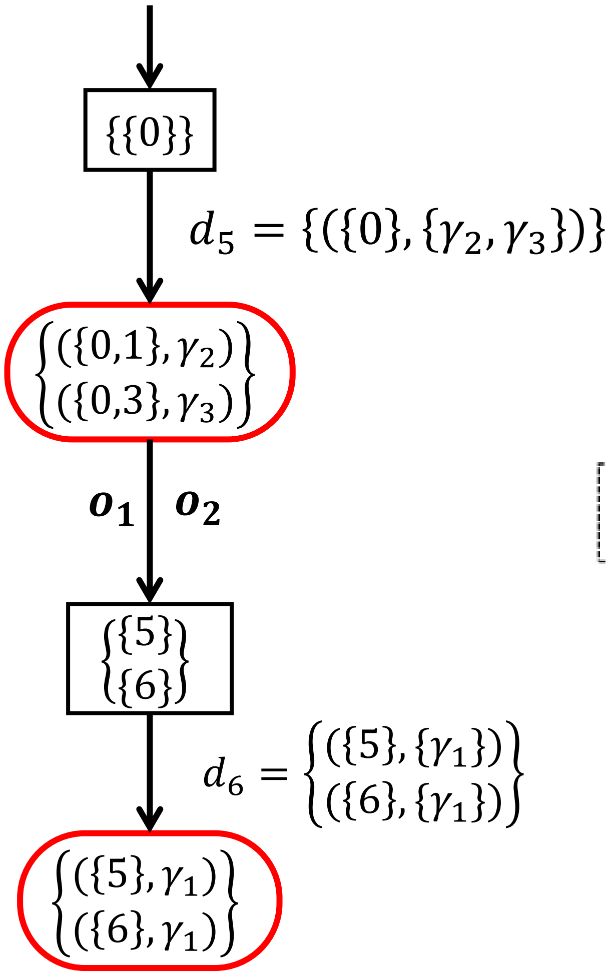
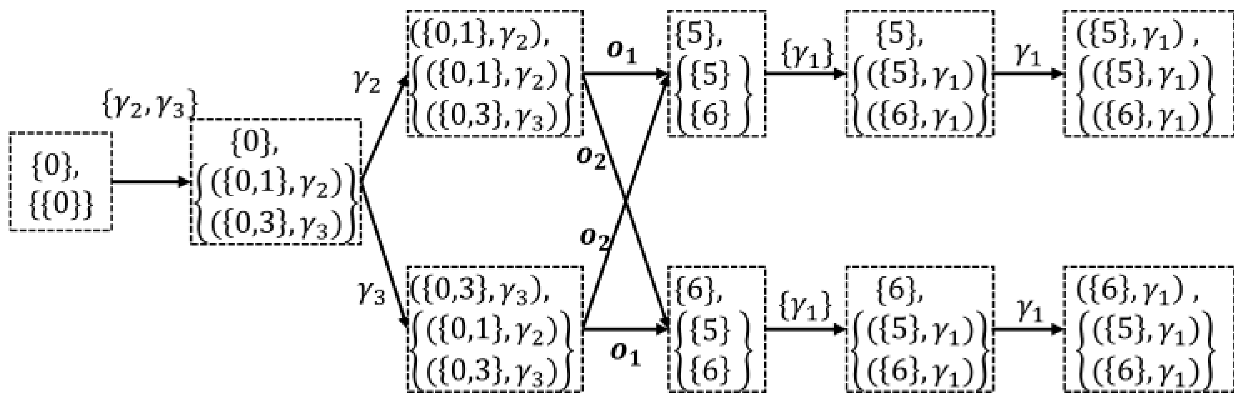


Supervisor Synthesis Procedure

3. Arbitrarily pick a macro-control decision for each Y-state





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Thank You!