



# Supervisory Control of Discrete-Event Systems for Infinite-Step Opacity

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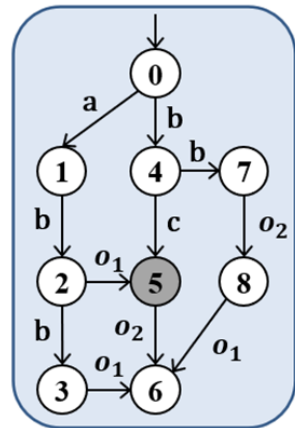
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## Motivation

- Security and privacy concerns in Cyber-Physical Systems
- Opacity: An information-flow property
- Application: Web services, Location-based services...
- Current-state opacity & **Infinite-step opacity**

# System Model and Intruder



Information-Flow  
 $P(\mathcal{L}(G)) \in \Sigma_o^*$



**The system has secrets**

- The system is modeled as a FSA  $G = (X, \Sigma, \delta, x_0)$
- The system has **secrets**, modeled a set of states  $X_s \subseteq X$
- $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo}$ ,  $P: \Sigma^* \rightarrow \Sigma_o^*$  is the **natural projection**
- The **intruder** is a passive observer seeing  $P(\mathcal{L}(G))$
- System  $G$  is **opaque** if the intruder cannot infer for sure that the system is in a secret state

# Delayed State Estimate



**Definition: (Delayed State Estimate).**

Let  $\alpha\beta \in P(\mathcal{L}(G))$  be an observable string. Then the delayed state estimate associated with  $(\alpha, \beta)$ , denoted by  $\hat{X}_G(\alpha \mid \alpha\beta)$ , is defined as the set of states the system could have been in  $|\beta|$ -steps earlier, after observing  $\alpha\beta$ .

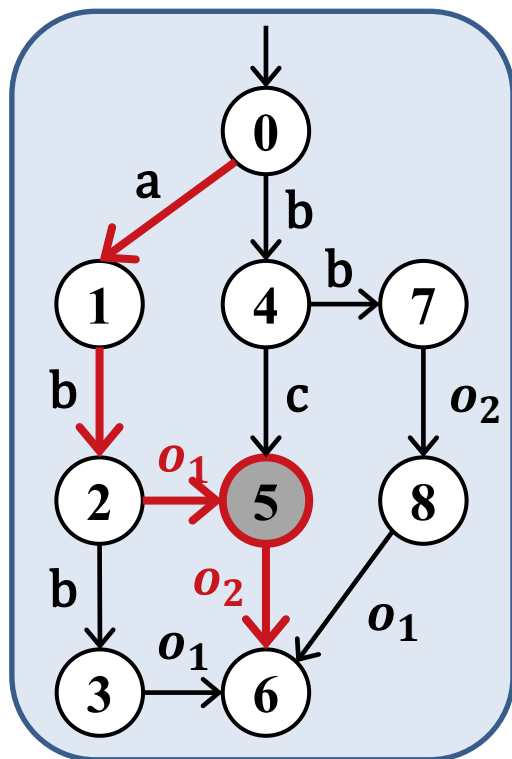
# Delayed State Estimate



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$$\hat{X}_G(o_1 \mid o_1 o_2) = \{5\}$$

- $\Sigma_o = \{o_1, o_2\}$
- Suppose string  $s = abo_1o_2$  with  $P(s) = o_1o_2$  is observed

**Definition: (Infinite-Step Opacity).**

System  $G$  is said to be infinite-step opaque w.r.t.  $G$  and  $X_s$  if

$$\forall \alpha\beta \in P(L(G)): \hat{X}_G(\alpha \mid \alpha\beta) \not\subseteq X_s$$

**The intruder can never know that the system was at a secret state**

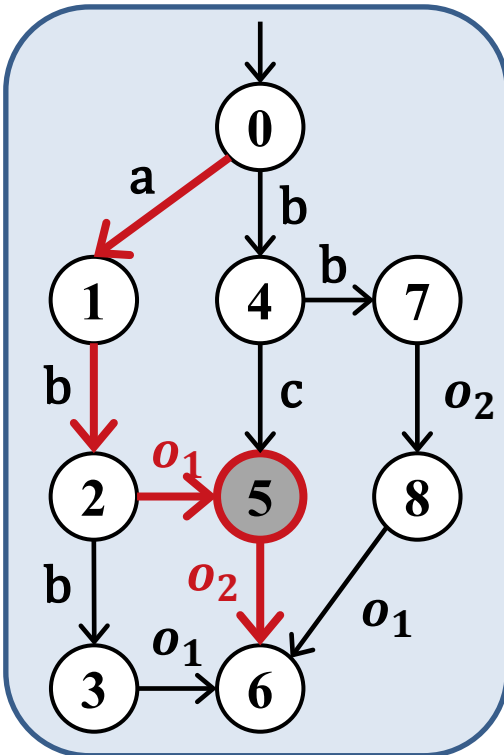
# Infinite-Step Opacity



**Definition: (Infinite-Step Opacity).**

System  $G$  is said to be infinite-step opaque w.r.t.  $\Omega$  and  $X_S$  if

$$\forall \alpha\beta \in P(L(G)): \hat{X}_G(\alpha \mid \alpha\beta) \not\subseteq X_S$$

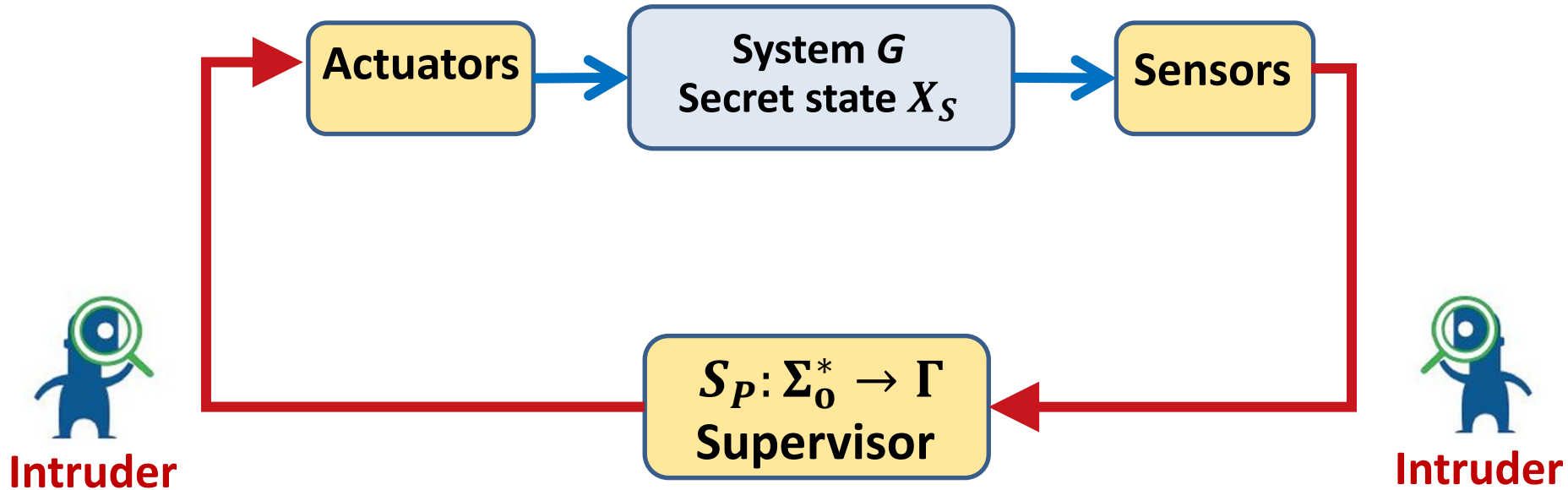


$$\hat{X}_G(o_1 \mid o_1 o_2) = \{5\} \subseteq X_S$$

**Not infinite-step opaque**

- $\Sigma_o = \{o_1, o_2\}$ ,  $X_S = \{5\}$
- Suppose string  $s = abo_1o_2$  with  $P(s) = o_1o_2$  is observed

# Supervisory Control Systems



- Sensors:  $\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo}$  and  $P: \Sigma^* \rightarrow \Sigma_o^*$
- Actuators:  $\Sigma = \Sigma_c \dot{\cup} \Sigma_{uc}$  and  $\Gamma := \{\gamma \in 2^\Sigma: \Sigma_{uc} \subseteq \gamma\}$
- Supervisor:  $S_P: P(\mathcal{L}(G)) \rightarrow \Gamma$  updates decisions dynamically
- Intruder: System model  $G$ , Observable events  $\Sigma_o^*$ , Control policy  $\Gamma$



**Definition: (Delayed State Estimate).**

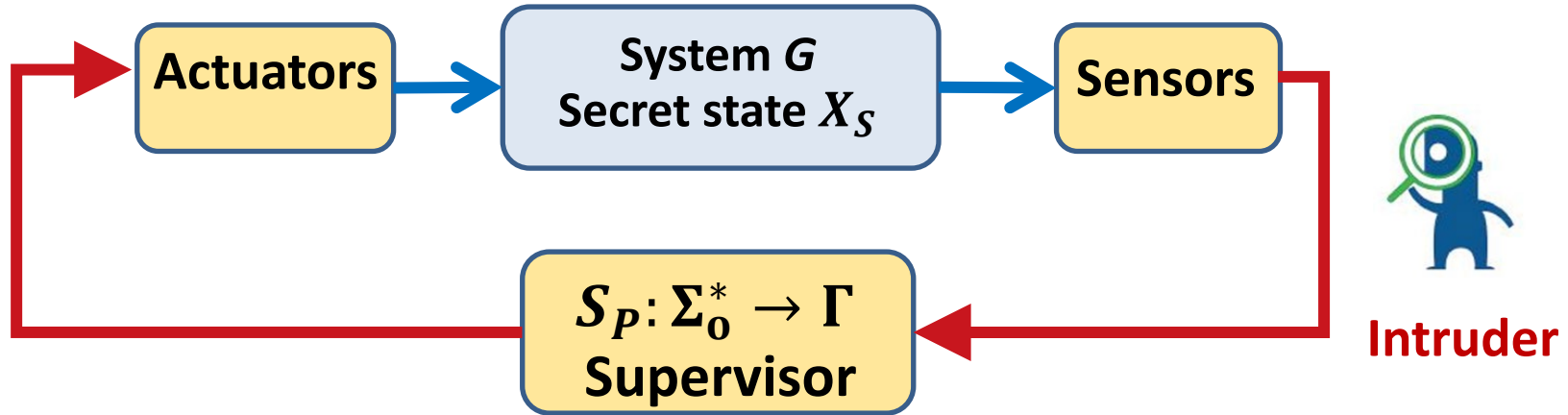
Let  $\alpha\beta \in P(\mathcal{L}(S_P/G))$  be an observable string. Then the delayed state estimate **in the closed-loop system** associated with  $(\alpha, \beta)$ , denoted by  $\hat{X}_{S_P/G}(\alpha \mid \alpha\beta)$ , is defined as the set of states the system could have been in  $|\beta|$ -steps earlier, after observing  $\alpha\beta$ .

**Definition: (Infinite-Step Opacity).**

Closed-loop system  $S_P/G$  is said to be infinite-step opaque w.r.t.  $\Sigma_o$  and  $X_S$  if

$$\forall \alpha\beta \in P(\mathcal{L}(S_P/G)): \hat{X}_{S_P/G}(\alpha \mid \alpha\beta) \not\subseteq X_S$$

# Synthesis Problem



- **Synthesis Problem:**

How to **design** a partial observation supervisor  $S_P: P(\mathcal{L}(G)) \rightarrow \Gamma$  ?

(1)  $S_P/G$  is infinite-step opaque

(2) For any supervisor  $S'_P$  satisfying (1), we have  $\mathcal{L}(S_P/G) \not\subseteq \mathcal{L}(S'_P/G)$

**The synthesized supervisor is maximal**

## • Main Idea for Enforcement

find a supervisor that **restricts the behavior** of the system dynamically such that the closed-loop system is opaque.

### Enforcement Algorithms for Current-state opacity

- J. Dubreil, P. Darondeau, and H. Marchand. Supervisory control for opacity. IEEE Trans. Automatic Control, 55(5):1089–1100, 2010.
- Y. Tong, Z. Li, C. Seatzu, and A. Giua. Current-state opacity enforcement in discrete event systems under incomparable observations. Discrete Event Dynamic Systems: Theory & Applications, 28(2):161–182, 2018.


$$\Sigma_c \subseteq \Sigma_o$$

Intruder doesn't know the control policy

**Difficulty:** need both current information and future information

**Challenge:**  $2^X$  is not sufficient due to the **delayed information**

**Question:** **what information** we need to synthesize for infinite-step opacity?

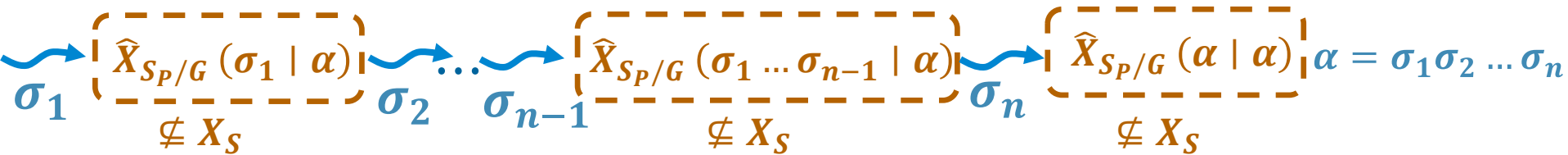
# Information State



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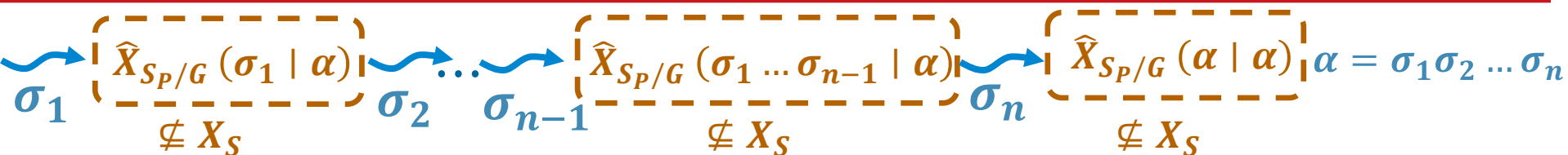


# Information State



**Challenge:**  $2^X$  is not sufficient due to the **delayed information**

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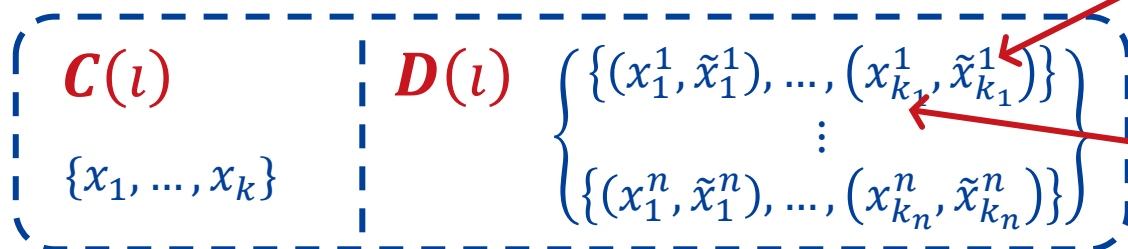
## Information State

We propose the following information-state space

$$I := 2^X \times 2^{2^{X \times X}}$$

**Current Estimate**      **Delayed Estimates for All Possible Previous Instant**

- $l = (C(l), D(l)) \in 2^X \times 2^{2^{X \times X}}$



**Current state**

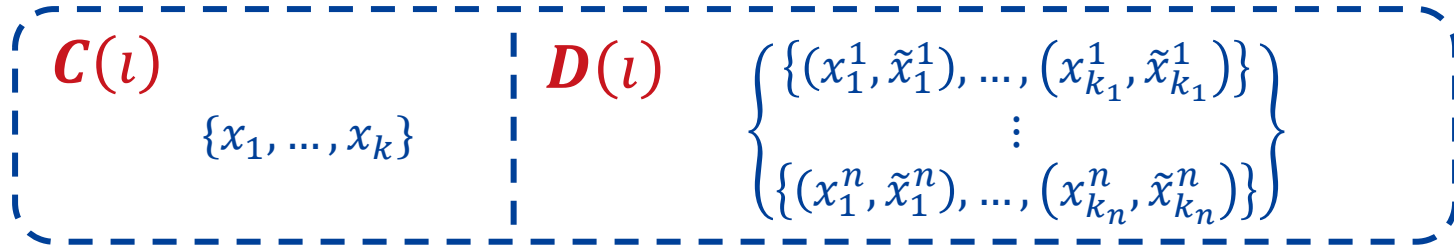
**State at some previous instant**

# Information State Update



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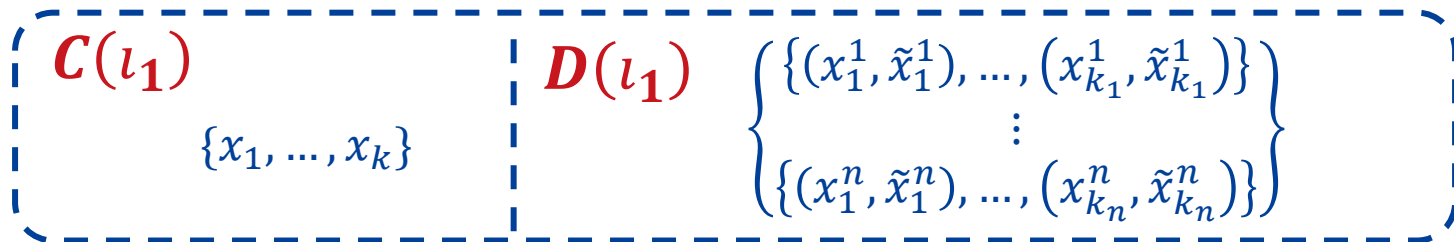
- **Update Rule:** new observation  $\sigma \in \Sigma_o$  & new control decision  $\gamma \in \Gamma$



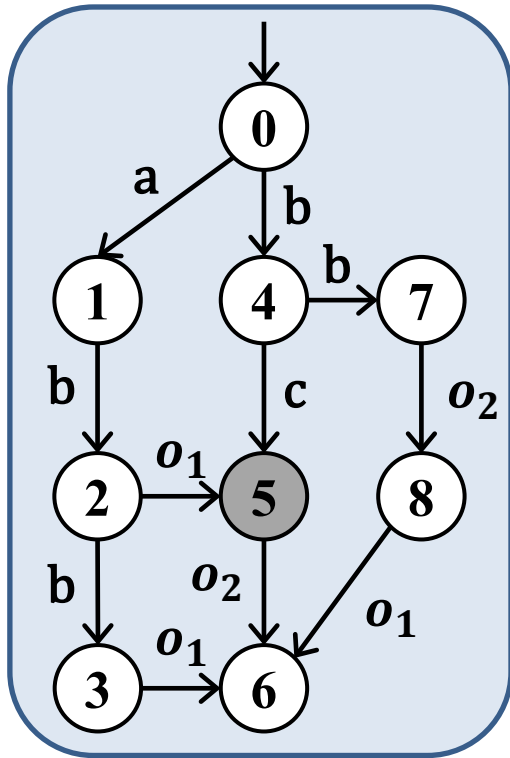
$$\mathbf{C}(l_1) = UR_{\gamma}(NX_{\sigma}(\mathbf{C}(l))) \quad \mathbf{D}(l_1) = \{\widetilde{UR}_{\gamma}(\widetilde{NX}_{\sigma}(\rho)) \in 2^{X \times X} : \rho \in \mathbf{D}(l)\} \cup \{\odot_{\gamma}(\mathbf{C}(l'))\}$$

- **Update the CSE**

- **Update all possible DSE**
- **Add CSE as DSE for the future**



# Example



$$\Sigma_o = \{o_1, o_2\}$$

$$\Sigma_c = \{a, b, c\}$$

$$S_P(\epsilon) = \{b, c\}$$



$$\{0, 4, 5, 7\}$$

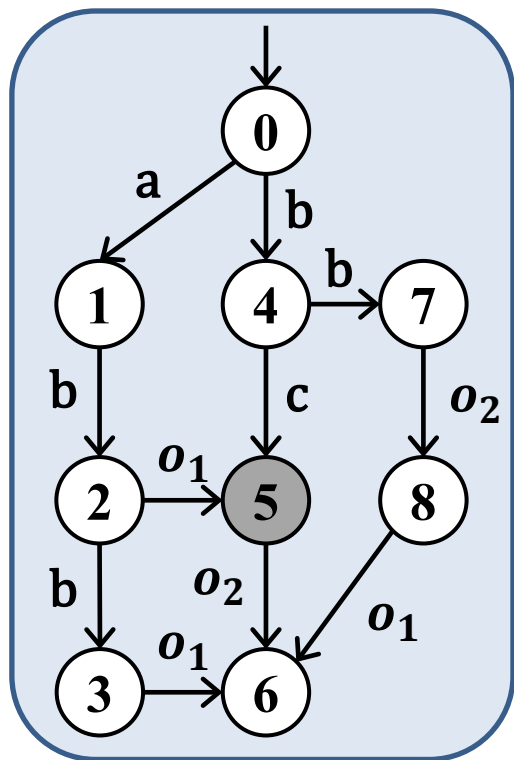
$$\left\{ \left\{ (0,0), (0,4), (0,5), (0,7) \right\} \right\}$$
$$\left\{ \left\{ (4,4), (4,5), (4,7), (5,5), (7,7) \right\} \right\}$$



# Example



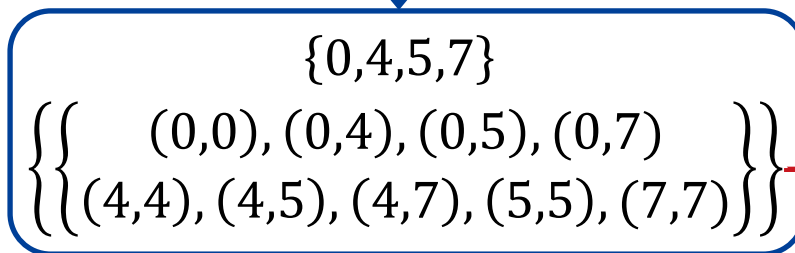
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$$\Sigma_o = \{o_1, o_2\}$$

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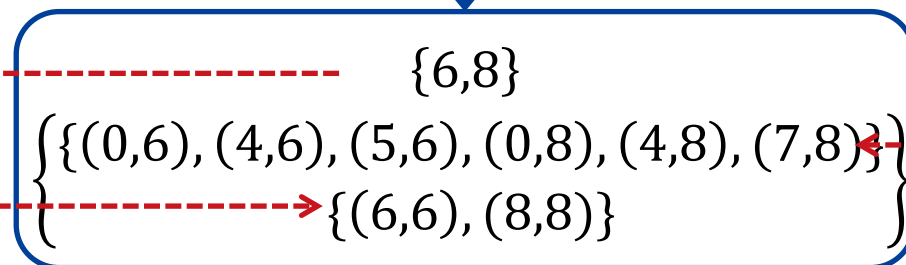


$o_2$

$$S_P(o_2) = \{a, b, c\}$$

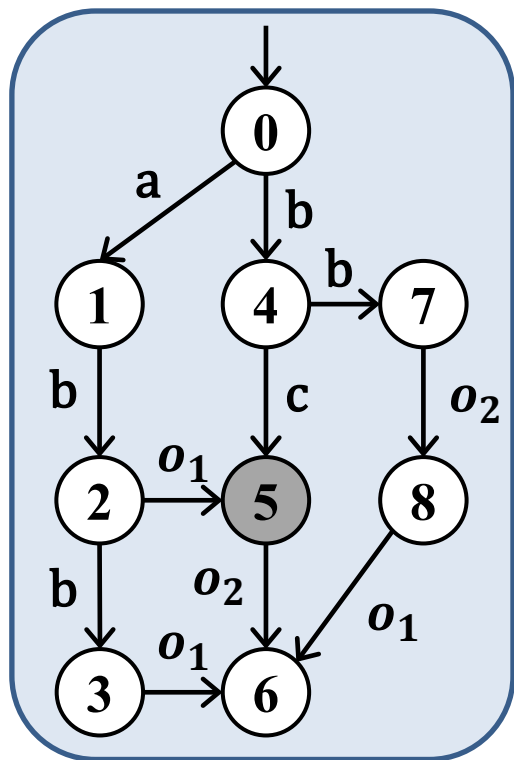
Update DSE

Add CSE



# Example

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$$\Sigma_o = \{o_1, o_2\}$$

$$\Sigma_c = \{a, b, c\}$$

$$D_1(l) = \{\{0, 4, 7\}, \{8\}, \{6\}\}$$

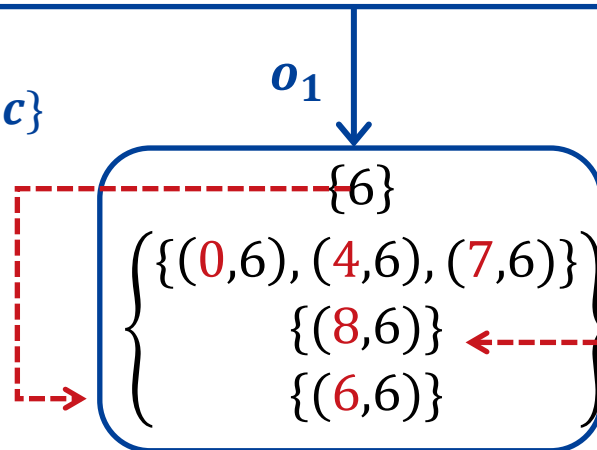
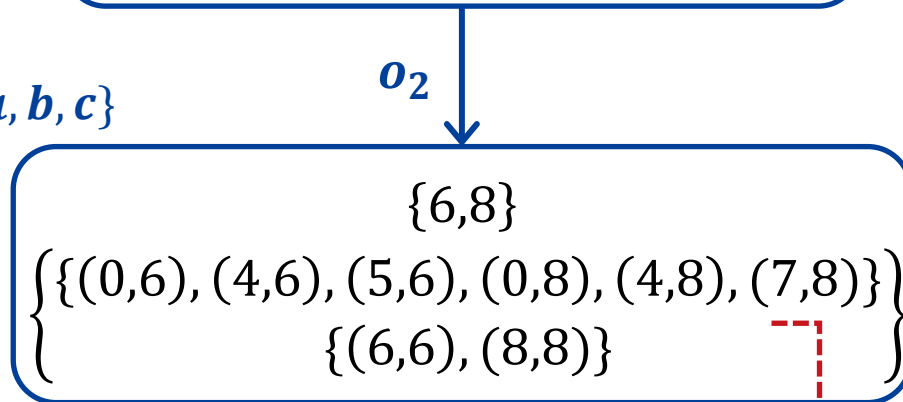
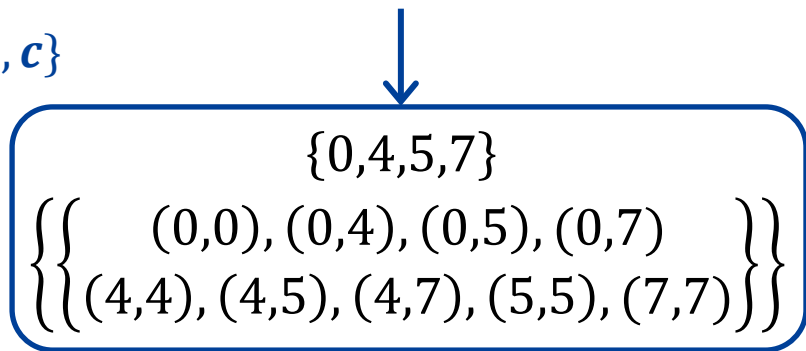
$$S_P(\epsilon) = \{b, c\}$$

$$S_P(o_2) = \{a, b, c\}$$

$$S_P(o_2 o_1) = \{a, b, c\}$$

Add CSE

Update DSE



- **Theorem**

Let  $I(\alpha)$  be the information state reached by  $\alpha \in P(\mathcal{L}(S_P/G))$ . Then

$$D_1(I(\alpha)) = \{\hat{X}_{S_P/G}(\beta|\alpha) \in 2^X : \beta \in \overline{\{\alpha\}}\}$$

- **Synthesis for infinite-step opacity**

- Construct the largest G-BTS
- Avoid states  $Q_{unsafe} = \{l \in I : \exists q \in D_1(l) \text{ s. t. } q \subseteq X_S\}$
- Delete all inconsistent states
- Maximal decision at each instant

A generalized bipartite transition system (G-BTS)  $T$  w.r.t.  $G$  is a 7-tuple

$$T = (Q_Y^T, Q_Z^T, h_{YZ}^T, h_{ZY}^T, \Sigma_o, \Gamma, y_0^T)$$

Unobservable reach

Observable reach

game structure between the controller and the environment

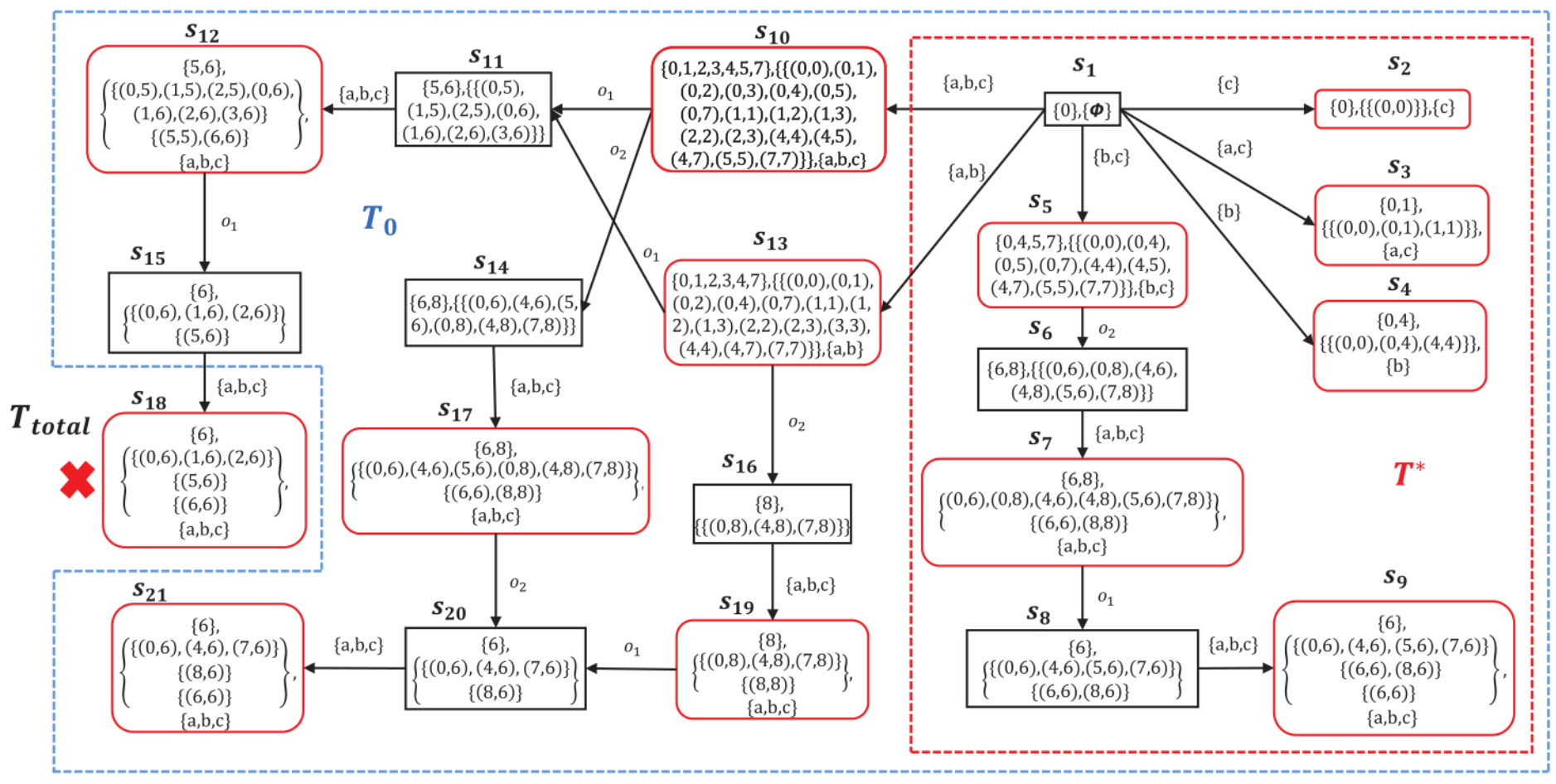
## Inconsistent states

- A Y-state is consistent if at least one control decision is defined.
- A Z-state is consistent if all feasible events are defined.

# Example



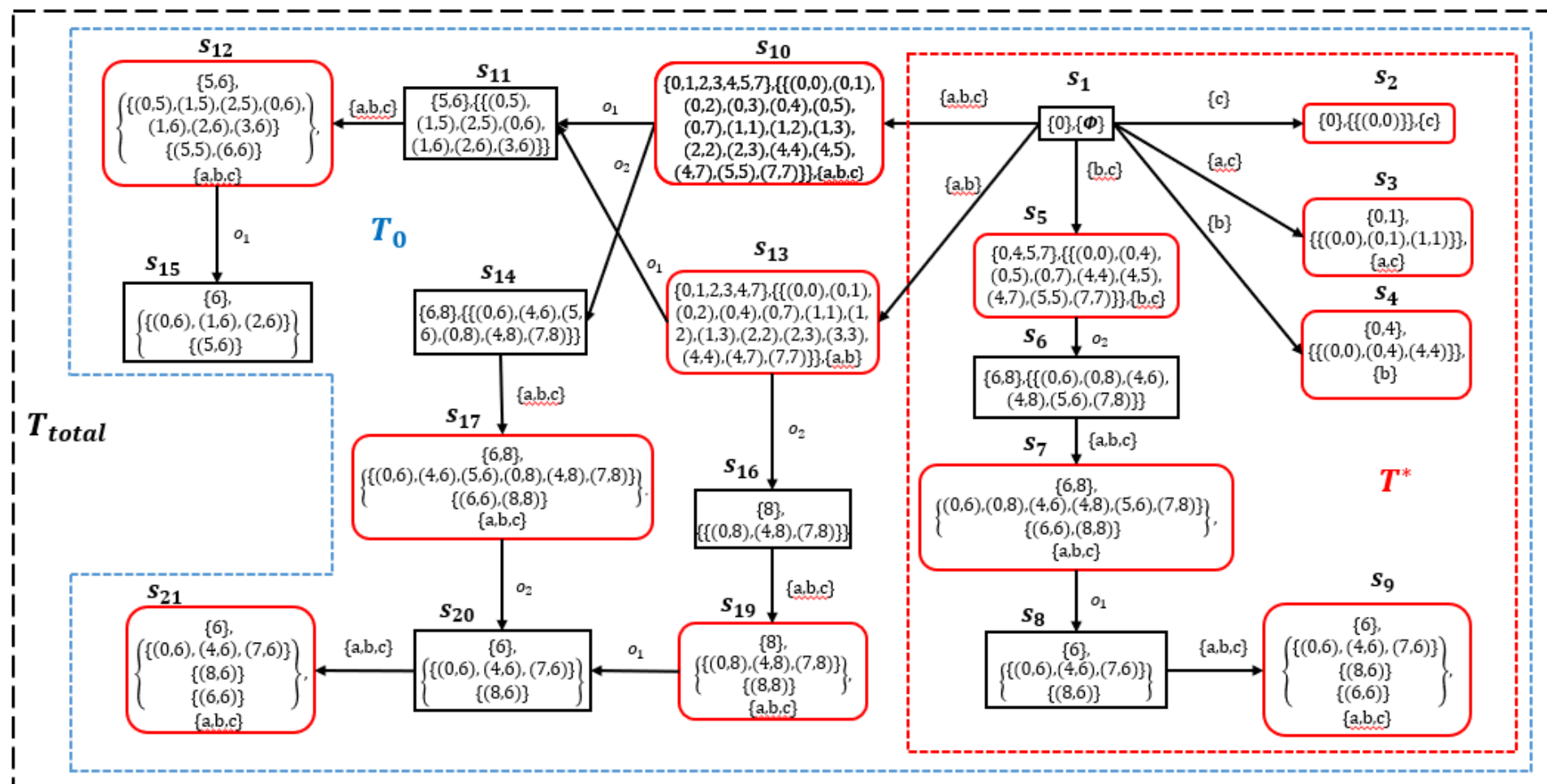
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# Example



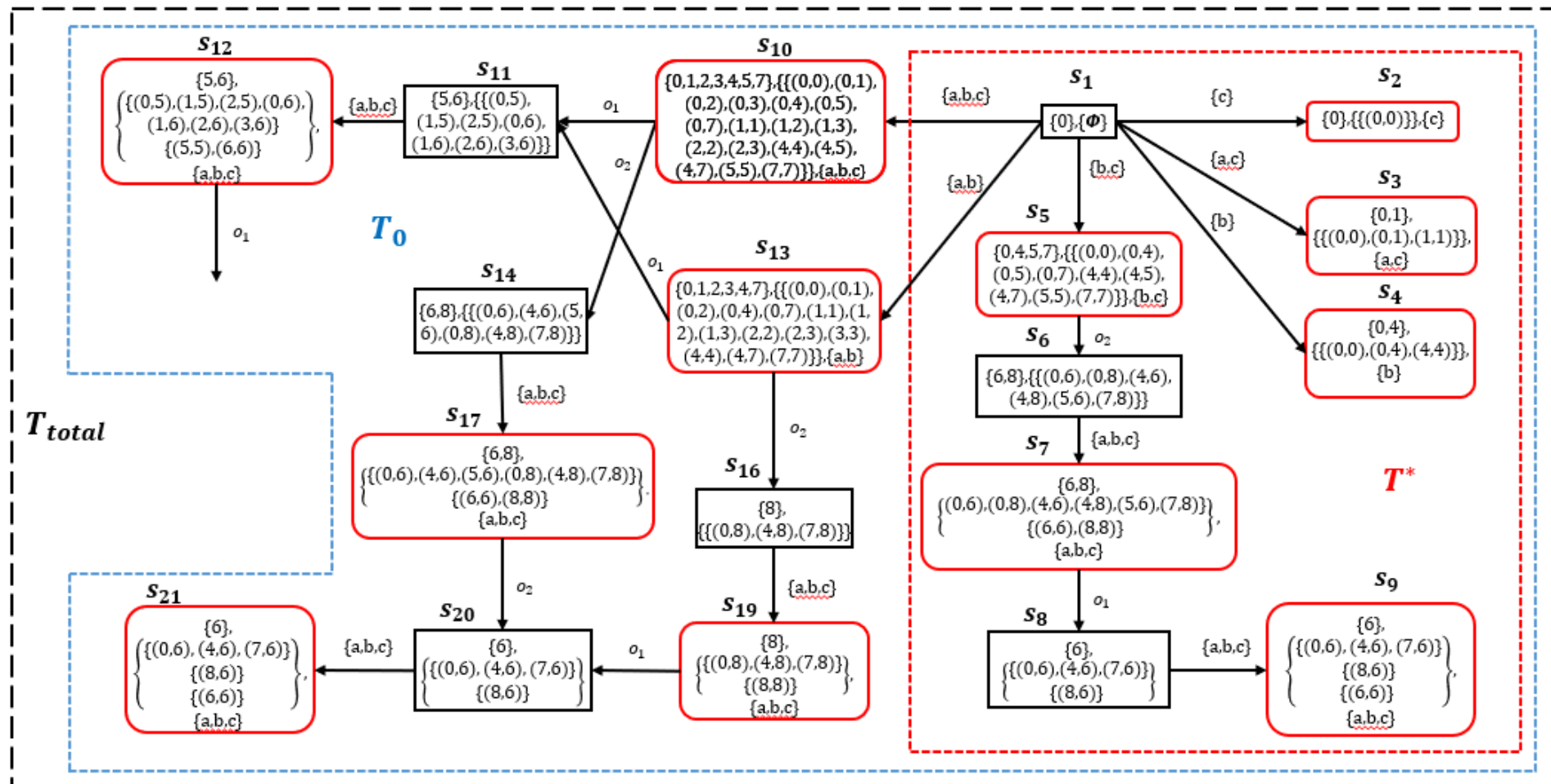
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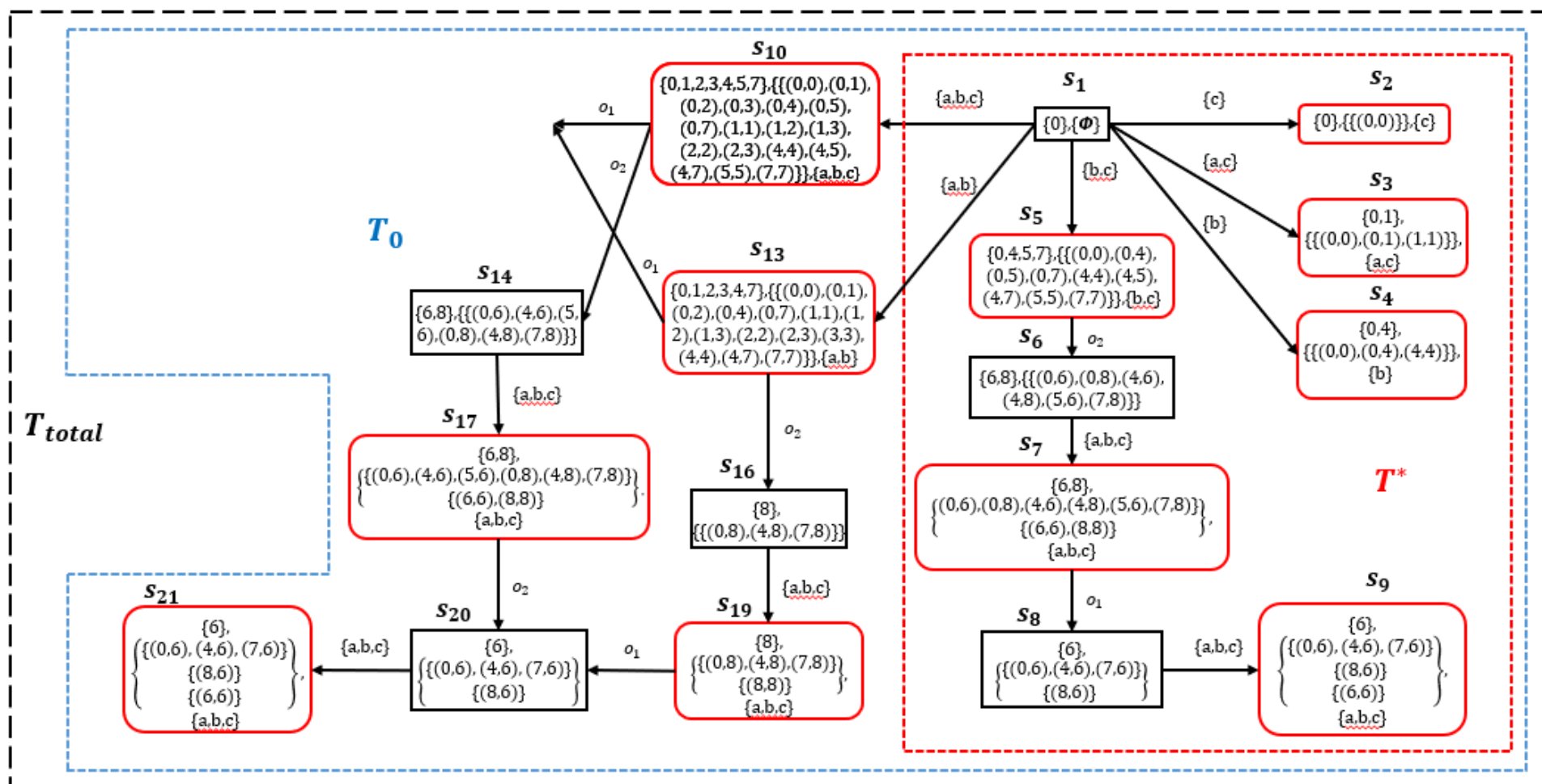
# Example



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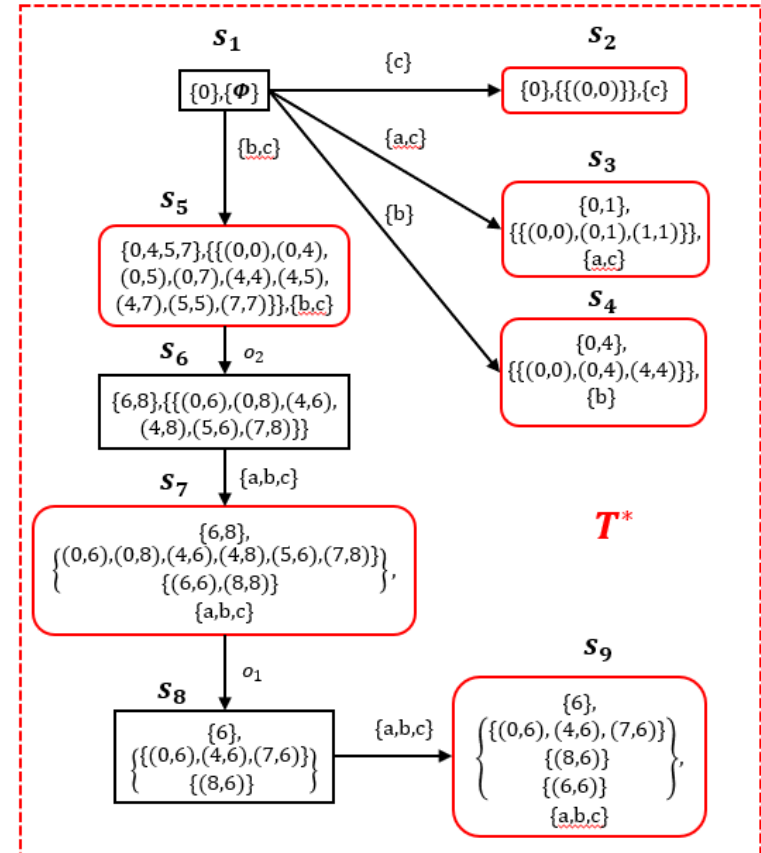


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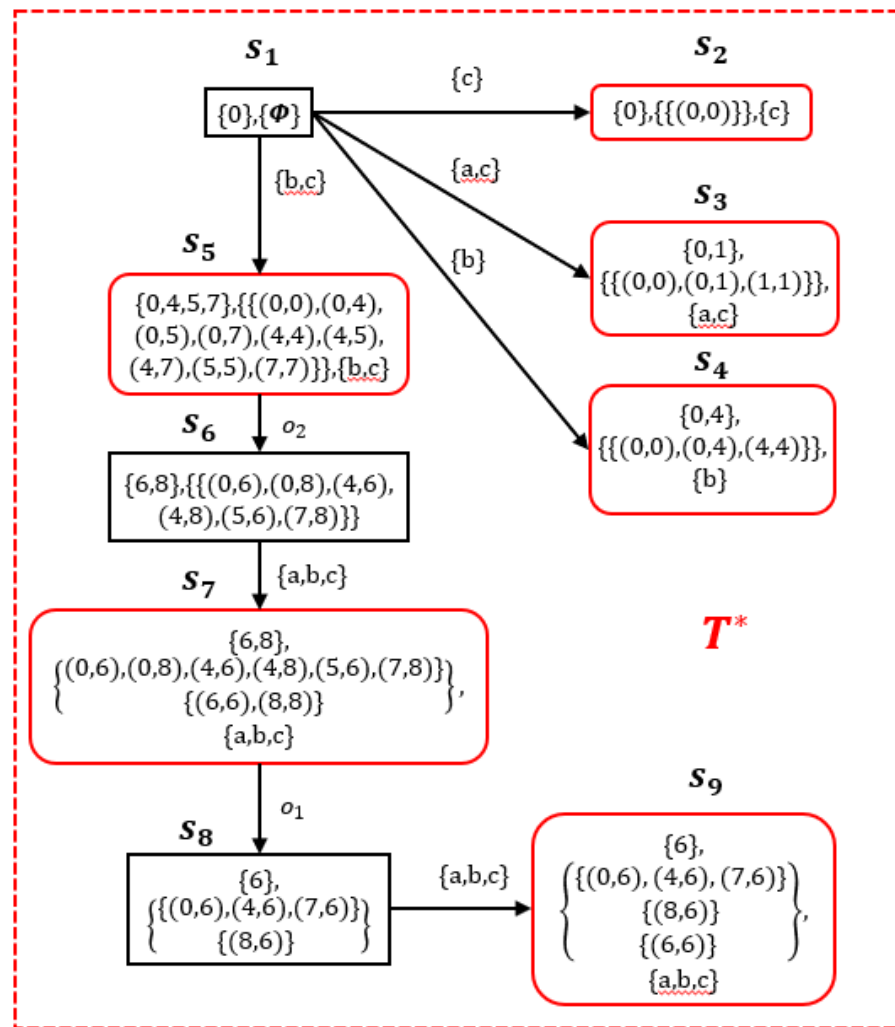
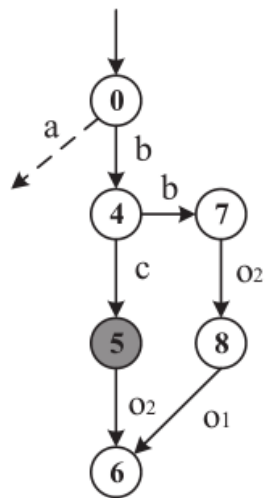
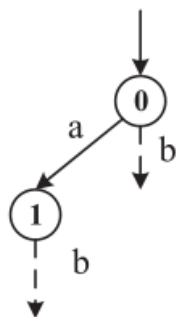
# Example



# Example



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- **Synthesis of supervisor for infinite-step opacity**
- **New type of information state for delayed information**
- **Effective synthesis procedure based on the proposed new IS**

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**Thank You!**