Formal Semantics III: Designing Rules (Part B)

CAS CS 320: Principles of Programming Languages

Thursday, April 4, 2024

REVIEWS FROM PRECEDING LECTURE

(March 28 & April 2)

Multi-Step Evaluation Rules

Review on Operational Semantics

We define a new relation $\mathrm{P} \rightarrow^{*} \mathrm{Q}$ to represent program P reduces to Q in 0 or more steps.

$$\frac{}{\mathrm{P} \rightarrow^{*} \mathrm{P}}\ \text{reflexivity}$$

$$\frac{\mathrm{P} \rightarrow \mathrm{Q} \qquad \mathrm{Q} \rightarrow^{*} \mathrm{R}}{\mathrm{P} \rightarrow^{*} \mathrm{R}}\ \text{transitive}$$

Review on Operational Semantics

We define a new relation $P \to^* Q$ to represent program P reduces to Q in 0 or more steps.

No premise, trivially fulfilled

$$\frac{\boxed{}}{P \to^* P}\ \text{reflexivity}$$

Any program P reduces to itself trivially in 0 steps

$$\frac{P \to Q \qquad Q \to^* R}{P \to^* R}\ \text{transitive}$$

Review on Operational Semantics

We define a new relation $P \to^* Q$ to represent program P reduces to Q in 0 or more steps.

Requires a proof of a single step reduction

No premise, trivially fulfilled

$$\frac{\boxed{}}{P \to^* P}\ \text{reflexivity} \qquad\qquad \frac{\boxed{P \to Q} \qquad Q \to^* R}{P \to^* R}\ \text{transitive}$$

Any program P reduces to itself trivially in 0 steps

Review on Operational Semantics

We define a new relation $P \to^* Q$ to represent program P reduces to Q in 0 or more steps.

No premise, trivially fulfilled

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Any program P reduces to itself trivially in 0 steps

Requires a proof of a single step reduction

Requires a proof of a multi-step reduction

$$\frac{\boxed{P \to Q} \qquad \boxed{Q \to^* R}}{P \to^* R}\ \text{transitive}$$

Review on Operational Semantics

We define a new relation $P \to^* Q$ to represent program P reduces to Q in 0 or more steps.

Requires a proof of a single step reduction

Requires a proof of a multi-step reduction

No premise, trivially fulfilled

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$$\frac{\boxed{P \to Q} \qquad \boxed{Q \to^* R}}{P \to^* R}\ \text{transitive}$$

Any program P reduces to itself trivially in 0 steps

Conclusion: P reduces to R in multiple steps.

Review on Operational Semantics

When deriving $\left(\text{add}\ 12\ (\text{add}\ 10\ 5)\right) \to^* 27$, we work our way “backwards”.

- We begin our derivation by proving that $(\text{add}\ 12\ 15) \to^* 27$.

$$\dfrac{\dfrac{12 \in \mathbb{Z} \qquad\qquad 15 \in \mathbb{Z}}{(\text{add}\ 12\ 15) \to 27}\ \text{add-ok} \qquad\qquad \dfrac{}{27 \to^* 27}\ \text{reflexivity}}{(\text{add}\ 12\ 15) \to^* 27}\ \text{transitive}$$

Review on Operational Semantics

When deriving $(\text{add } 12 \ (\text{add } 10 \ 5)) \to^* 27$, we work our way "backwards".

- We begin our derivation by proving that $(\text{add } 12 \ 15) \to^* 27$.

Ending configuration reduces to itself in 0 steps.

$$\dfrac{\dfrac{12 \in \mathbb{Z} \qquad 15 \in \mathbb{Z}}{(\text{add } 12 \ 15) \to 27}\ \text{add-ok} \qquad \boxed{\dfrac{}{27 \to^* 27}\ \text{reflexivity}}}{(\text{add } 12 \ 15) \to^* 27}\ \text{transitive}$$

Review on Operational Semantics

When deriving $(\text{add } 12\ (\text{add } 10\ 5)) \to^{*} 27$, we work our way "backwards".

- We begin our derivation by proving that $(\text{add } 12\ 15) \to^{*} 27$.
- Next, apply the transitive rule and compose on the proof of $(\text{add } 12\ (\text{add } 10\ 5)) \to (\text{add } 12\ 15)$

$$
\dfrac{
\dfrac{12 \in \mathbb{Z} \qquad \dfrac{10 \in \mathbb{Z} \qquad 5 \in \mathbb{Z}}{(\text{add } 10\ 5) \to 15}\,\text{add-ok}}{(\text{add } 12\ (\text{add } 10\ 5)) \to (\text{add } 12\ 15)}\,\text{add-right}
\qquad
\dfrac{\dfrac{12 \in \mathbb{Z} \qquad 15 \in \mathbb{Z}}{(\text{add } 12\ 15) \to 27}\,\text{add-ok} \qquad \dfrac{}{27 \to^{*} 27}\,\text{reflexivity}}{(\text{add } 12\ 15) \to^{*} 27}\,\text{transitive}
}{(\text{add } 12\ (\text{add } 10\ 5)) \to^{*} 27}\,\text{transitive}
$$

Review on Operational Semantics

When deriving $(\text{add } 12\ (\text{add } 10\ 5)) \to^* 27$, we work our way "backwards".

- We begin our derivation by proving that $(\text{add } 12\ 15) \to^* 27$.
- Next, apply the transitive rule and compose on the proof of $(\text{add } 12\ (\text{add } 10\ 5)) \to (\text{add } 12\ 15)$
- For longer proofs, we can label proven conclusions for use in the premise of other rules.

$$\dfrac{12 \in \mathbb{Z} \qquad \dfrac{10 \in \mathbb{Z} \qquad 5 \in \mathbb{Z}}{(\text{add } 10\ 5) \to 15}\ \text{add-ok}}{(1)\quad (\text{add } 12\ (\text{add } 10\ 5)) \to (\text{add } 12\ 15)}\ \text{add-right}$$

$$\dfrac{(1) \qquad (2)}{(\text{add } 12\ (\text{add } 10\ 5)) \to^* 27}\ \text{transitive}$$

$$\dfrac{\dfrac{12 \in \mathbb{Z} \qquad 15 \in \mathbb{Z}}{(\text{add } 12\ 15) \to 27}\ \text{add-ok} \qquad \dfrac{}{27 \to^* 27}\ \text{reflexivity}}{(2)\quad (\text{add } 12\ 15) \to^* 27}\ \text{transitive}$$

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