

Formal Semantics III: Designing Rules (Part B)

CAS CS 320: Principles of Programming Languages

Thursday, April 4, 2024

REVIEWS FROM PRECEDING LECTURE

(March 28 & April 2)

Multi-Step Evaluation Rules

Review on Operational Semantics

We define a new relation $P \rightarrow^* Q$ to represent program P reduces to Q in 0 or more steps.

$$\frac{}{P \rightarrow^* P} \text{ reflexivity}$$

$$\frac{P \rightarrow Q \quad Q \rightarrow^* R}{P \rightarrow^* R} \text{ transitive}$$

Review on Operational Semantics

We define a new relation $P \rightarrow^* Q$ to represent program P reduces to Q in 0 or more steps.

No premise, trivially fulfilled

$$\frac{\boxed{}}{P \rightarrow^* P} \text{ reflexivity}$$



Any program P reduces to itself trivially in 0 steps

$$\frac{P \rightarrow Q \quad Q \rightarrow^* R}{P \rightarrow^* R} \text{ transitive}$$

Review on Operational Semantics

We define a new relation $P \rightarrow^* Q$ to represent program P reduces to Q in 0 or more steps.

Requires a proof of a single step reduction

No premise, trivially fulfilled

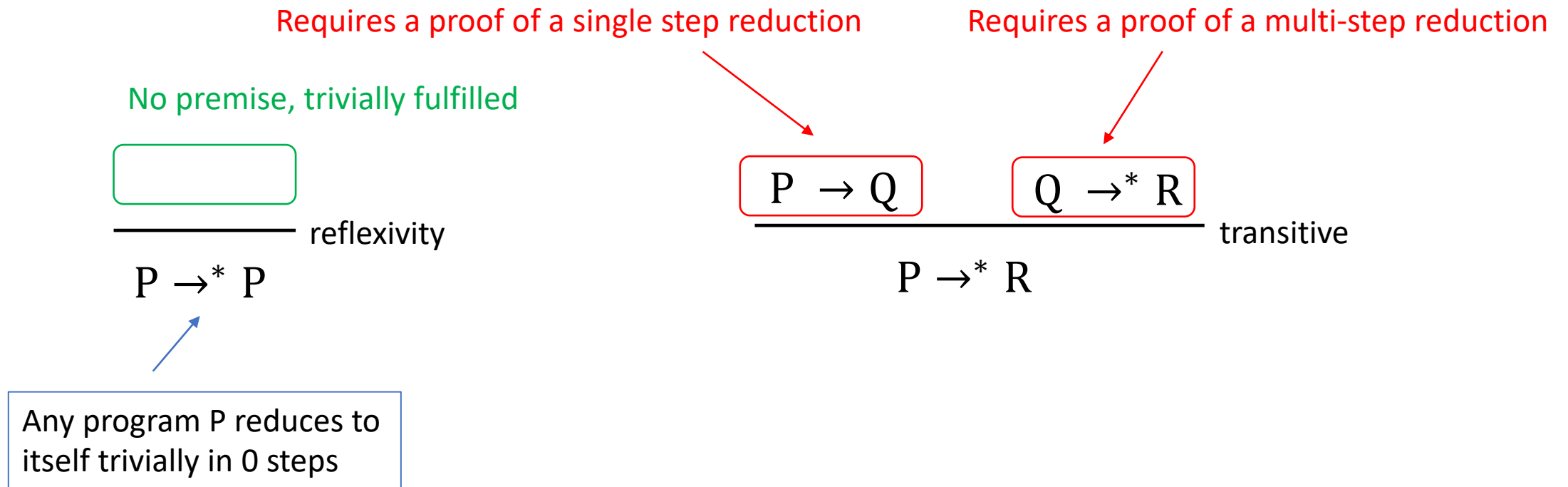
$$\frac{\boxed{}}{P \rightarrow^* P} \text{ reflexivity}$$

Any program P reduces to itself trivially in 0 steps

$$\frac{\boxed{P \rightarrow Q} \quad Q \rightarrow^* R}{P \rightarrow^* R} \text{ transitive}$$

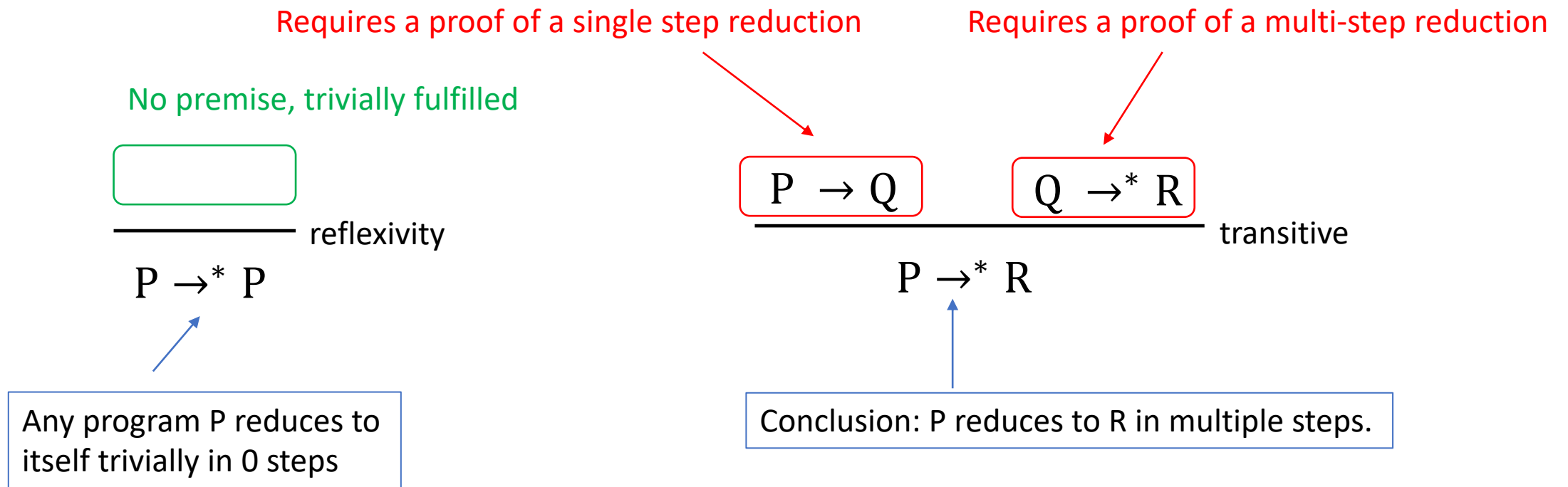
Review on Operational Semantics

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Review on Operational Semantics

When deriving $(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow^* 27$, we work our way “backwards”.

- We begin our derivation by proving that $(\text{add } 12 \ 15) \rightarrow^* 27$.

$$\frac{\frac{12 \in \mathbb{Z} \quad 15 \in \mathbb{Z}}{(\text{add } 12 \ 15) \rightarrow 27} \text{ add-ok} \quad \frac{}{27 \rightarrow^* 27} \text{ reflexivity}}{(\text{add } 12 \ 15) \rightarrow^* 27} \text{ transitive}$$

Review on Operational Semantics

When deriving $(\text{add } 12 (\text{add } 10 5)) \rightarrow^* 27$, we work our way “backwards”.

- We begin our derivation by proving that $(\text{add } 12 15) \rightarrow^* 27$.

$$\frac{\frac{12 \in \mathbb{Z} \quad 15 \in \mathbb{Z}}{(\text{add } 12 15) \rightarrow 27} \text{ add-ok} \quad \boxed{\frac{}{27 \rightarrow^* 27} \text{ reflexivity}}}{(\text{add } 12 15) \rightarrow^* 27} \text{ transitive}$$

Ending configuration reduces to itself in 0 steps.

Review on Operational Semantics

When deriving $(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow^* 27$, we work our way “backwards”.

- We begin our derivation by proving that $(\text{add } 12 \ 15) \rightarrow^* 27$.
- Next, apply the transitive rule and compose on the proof of $(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow (\text{add } 12 \ 15)$

$$\begin{array}{c}
 \begin{array}{c}
 \frac{12 \in \mathbb{Z} \quad \frac{\frac{10 \in \mathbb{Z} \quad 5 \in \mathbb{Z}}{(\text{add } 10 \ 5) \rightarrow 15} \text{ add-ok}}{(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow (\text{add } 12 \ 15)} \text{ add-right}
 \end{array}
 \quad
 \begin{array}{c}
 \frac{12 \in \mathbb{Z} \quad 15 \in \mathbb{Z}}{(\text{add } 12 \ 15) \rightarrow 27} \text{ add-ok}
 \end{array}
 \quad
 \begin{array}{c}
 \frac{}{27 \rightarrow^* 27} \text{ reflexivity}
 \end{array}
 \\
 \hline
 \frac{(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow (\text{add } 12 \ 15) \quad (\text{add } 12 \ 15) \rightarrow^* 27}{(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow^* 27} \text{ transitive}
 \end{array}$$

Review on Operational Semantics

When deriving $(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow^* 27$, we work our way “backwards”.

- We begin our derivation by proving that $(\text{add } 12 \ 15) \rightarrow^* 27$.
- Next, apply the transitive rule and compose on the proof of $(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow (\text{add } 12 \ 15)$
- For longer proofs, we can label proven conclusions for use in the premise of other rules.

$$\begin{array}{c}
 12 \in \mathbb{Z} \qquad \frac{10 \in \mathbb{Z} \qquad 5 \in \mathbb{Z}}{(\text{add } 10 \ 5) \rightarrow 15} \text{ add-ok} \\
 \hline
 (\text{add } 12 (\text{add } 10 \ 5)) \rightarrow (\text{add } 12 \ 15) \text{ add-right}
 \end{array}$$

(1) $(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow (\text{add } 12 \ 15)$

$$\frac{\text{(1)} \qquad \text{(2)}}{(\text{add } 12 (\text{add } 10 \ 5)) \rightarrow^* 27} \text{ transitive}$$

$$\begin{array}{c}
 12 \in \mathbb{Z} \qquad 15 \in \mathbb{Z} \\
 \hline
 (\text{add } 12 \ 15) \rightarrow 27 \text{ add-ok}
 \end{array}
 \qquad
 \frac{}{27 \rightarrow^* 27} \text{ reflexivity}$$

$$\frac{(\text{add } 12 \ 15) \rightarrow 27 \qquad 27 \rightarrow^* 27}{(\text{add } 12 \ 15) \rightarrow^* 27} \text{ transitive}$$

(2) $(\text{add } 12 \ 15) \rightarrow^* 27$

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