18.4

(1) $F(x)$: x是有理数, $G(x)$: x能表示成分数.

$\neg\exists x(F(x)\wedge\neg G(x))$.

(2). $F(x)$: x去八达岭长城游玩. $G(x)$ x是外地人.

$\neg\forall x(F(x)\rightarrow G(x))$

(3). $F(x)$: x是乌鸦, $G(x)$: x是黑色的.

$\forall x(F(x)\rightarrow G(x))$.

(4). $F(x)$: x是人, $G(x)$: x天天锻炼身体.

$\exists x(F(x)\wedge G(x))$.

18.5.

(1). $F(x)$: x是火车, $G(y)$: y是轮船. $H(x,y)$. x比y快.

(2). $F(x)$: x是火车 $G(y)$ y是汽车. $H(x,y)$: x比y快.

$\exists x\exists y(F(x)\wedge(G(y)\wedge H(x,y)))$

(3). $F(x)$: x是火车, $G(y)$: y是汽车. $H(x,y)$. x比y快.

$\neg\exists x(G(x)\wedge\forall y(F(y)\rightarrow H(x,y)))$.

(4). $F(x)$ x是火车 $G(y)$ y是汽车. $H(x,y)$: x比y慢.

$\neg\forall x(G(x)\rightarrow\forall y(F(y)\rightarrow H(x,y)))$.

18.9. (1). $\forall x((x<-1)\rightarrow\exists(x=y))$. 真值为1.

(2). $\forall y((1-y=0)\rightarrow \cancel{\forall x}\ \forall x(x<y))$. 真值为0.

(3). $\exists x((x<-1)\rightarrow\forall y(1-y=0))$. 真值为0.

(4). $\forall y((1-y<0)\rightarrow\exists x(x=-1))$, 真值为1.

18.11. (1). $F(x,y)\rightarrow(G(x,y)\rightarrow F(x,y))$.

为永真式. 由真值表可得.

(2). $\forall x(F(x)\rightarrow F(x))\rightarrow\exists y(G(y)\wedge\neg G(y))$. 为矛盾式.

(3). $\forall x \exists y F(x,y) \to \exists x \forall y F(x,y)$.

I_1: 个体域为自然数集 N, $F(x,y): x \le y$.

$\Rightarrow$ 该公式为真

I_2: 个体域为 N. $F(x): x=y$.

$\forall x \exists y F(x,y)$ 为真. $\exists x \forall y F(x,y)$ 为假.

为可满足式.

(4). $\exists x \forall y F(x,y) \to \forall y \exists x F(x,y)$

永真式.

(5). $\forall x \forall y (F(x,y) \to F(y,x))$.

$F_1(x,y): x=y$.

$F_2(x,y): x<y$.

可满足式.

(6). $\neg(\forall x F(x) \to \exists y G(y)) \land \exists y G(y)$.

$\Rightarrow$ $\neg(A \to B) \land B$

$\Leftrightarrow$ $A \land \neg B \land B$

$\Leftrightarrow 0$.

为矛盾式.

19. 2. $D=\{a, b, c\}$.

(1). $\forall x \exists y (F(x) \land G(y))$.

$(F(a) \land F(b) \land F(c)) \land (G(a) \lor G(b) \lor (G(c))$.

(2). $\forall x \forall y (F(x) \lor G(y))$.

$(F(a) \land F(b) \land F(c)) \lor (G(a) \land G(b) \land G(c))$

(3). $\forall x F(x) \to \forall y G(y)$.

$(F(a) \land F(b) \land F(c)) \to (G(a) \land G(b) \land G(c))$

(4). $\forall x (F(x,y) \to \exists y G(y))$.

$(F(a,y) \lor F(b,y) \lor F(c,y)) \to (G(a) \lor G(b) \lor G(c))$.

19.12.

(1). $\forall x F(x) \to \forall y G(x,y)$.

$\exists z \forall y (F(x) \to G(x,y))$

(2). $\forall x (F(x,y) \to \exists y G(x,y,z))$

$\forall x \exists t (F(x,y) \to G(x,t,z))$.

(3). $\forall x F(x,y) \leftrightarrow \exists x G(x,y)$.

$\exists x_1 \exists x_2 \forall x_3 \forall x_4 ((F(x_1,y) \to G(x_2,y)) \wedge (G(x_3,y) \to F(x_4,y)))$.

(4). $\forall x_1 (F(x_1) \to G(x_1,x_2)) \to (\exists x_1 H(x_2) \to \exists x_3 L(x_2,x_3))$.

$\exists y_1 \forall y_2 \exists y_3 ((F(y_1) \to G(y_1,x_2)) \to (H(y_2) \to L(x_2,y_3)))$.

(5). $\exists x_1 F(x_1,x_2) \to (F(x_1) \to \neg \exists x_2 G(x_1,x_2))$.

$\forall y_1 \forall y_2 (F(y_1,x_2) \to (F(x_1) \to \neg G(x_1,y_2)))$.

19.13.

(1). ~~$\exists x \exists y (F(x)$~~

$F(x)$: x是汽车, $G(y)$: y是火车, $H(x,y)$: x比y跑得快.

$\exists x \exists y (F(x) \wedge G(y) \wedge H(x,y))$

(2). $F(x)$: x是火车. $G(y)$: y是汽车. $H(x,y)$: x比y跑得快.

$\exists x \forall y (F(x) \wedge (G(y) \to \neg H(x,y)))$.

(3). ~~$\exists x \exists y ($~~ $F(x)$: x是火车. $G(y)$: y是汽车. $H(x,y)$: x比y跑得快.

$\exists x \exists y (F(x) \wedge G(x) \wedge \neg H(x,y))$.

(4). $F(x)$: x是飞机, $G(y)$: y是汽车, $H(x,y)$: x比y慢.

$\forall x \forall y (F(x) \wedge (G(y) \to \neg H(x,y)))$.

19、25.

$F(x)$：x是科学工作者，$G(x)$：x是刻苦钻研的，$H(x)$：x是聪明的.

$I(x)$：x在事业中获得成功，a：王大海.

前提：$\forall x(F(x)\to G(x))$，$\forall x((G(x)\wedge H(x))\to I(x))$，$F(a)$，$H(a)$.

结论：$I(a)$.

证明：① $F(a)$ 前提

② $\forall x(F(x)\to G(x))$ 前提.

③. $F(a)\to G(a)$. ②$\forall-$

④ $G(a)$. ①③

⑤. $H(a)$. 前提.

⑥ $\forall x(G(x)\wedge H(x))\to I(x))$ 前提.

⑦. $(G(a)\wedge H(a))\to I(a)$. ~~前提~~⑥

⑧. $G(a)\wedge H(a)$ ④⑤

⑨ $I(a)$. ⑦⑧