13.15. $\forall x \in G,\ x^2 = e$.

则 $\forall x \in G$ 有 $x^{-1} = x$.

现 $\forall x, y \in G$, 有 $xy = (xy)^{-1} = y^{-1}x^{-1} = yx$

$\therefore G$ 是交换群.

13.17. 设 $|abc| = r$, $|bca| = s$, $|cab| = t$.

由 $(abc)^{s+1} = a(bca)^s bc = abc$.

$\Rightarrow (abc)^s = e$, $\Rightarrow r \mid s$.

同理得 $s \mid t$, $t \mid r$,

由 $s \mid t$ 和 $t \mid r$ 得, $s \mid r$.

$\Rightarrow r = s$.

13.21. $\because a \in N(a)$, 故 $N(a) \neq \emptyset$.

任取 $x, y \in N(a)$, 有 $xa = ax$, $ya = ay$.

$(xy^{-1})a = (xy^{-1})(yay^{-1}) = xy^{-1}yay^{-1}$

$= xay^{-1} = axy^{-1} = a(xy^{-1})$.

13.22. $e = xex^{-1} \in xHx^{-1}$,

任取 $xh_1x^{-1}, xh_2x^{-1} \in xHx^{-1}$, 有 $h_1h_2^{-1} \in H$, 得

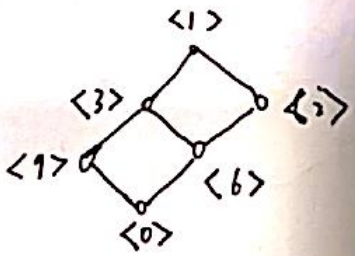
$(xh_1x^{-1})(xh_2x^{-1})^{-1} = xh_1x^{-1}xh_2^{-1}x^{-1} = x(h_1h_2^{-1})x^{-1} \in xHx^{-1}$.

13.23. $\mathbb{Z}_{18}$ 有 6 个子群.

$\langle 1 \rangle = \mathbb{Z}_{18}$ $\langle 2 \rangle = \{0, 2, 4, 6, 8, 10, 12, 14, 16\}$.

$\langle 3 \rangle = \{0, 3, 6, 9, 12, 15\}$. $\langle 6 \rangle = \{0, 6, 12\}$.

$\langle 9 \rangle = \{0, 9\}$ $\langle 0 \rangle = \{0\}$.



习. 28. (1) G 的生成元有 $a^1, a^2, a^4, a^7, a^8, a^{11}, a^{13}, a^{14}$.

(2). G 的子群有 $\langle e\rangle = \{e\}$, $\langle a\rangle = G$, $\langle a^3\rangle = \{e, a^3, a^6, a^9, a^{12}\}$, $\langle a^5\rangle = \{e, a^5, a^{10}\}$.